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Damage prediction of rotating blades using displacement residuals

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Abstract

This paper presents an optimization methodology for prediction of damage in rotating blades. Blade damage is often characterized by relatively high amplitudes of response due to a local reduction in stiffness factors. An explicit relation between the amplitudes of response and stiffness reduction coefficients can therefore be conveniently developed. Based on the available response amplitudes along various points in a harmonically excited blade with modal data, a displacement residue is defined in terms of unknown stiffness reduction coefficients. These coefficients are predicted by minimizing the norm of the residual vector using genetic algorithms. The approach is illustrated with a finite element model of a viscously damped rotating blade of aerofoil cross-section.

1. Introduction

A primary failure mode of blade is nucleation of a crack caused through unexpected impacts. As the crack grows, the structure suffers catastrophic failure. Hence, the prediction of a crack in blades is an important issue. Often blades undergo critical responses at low engine orders due to aerodynamic forces, flow non-uniformities and blade-passing frequency excitations. A limitation of the Campbell diagram is that no distinction could be made between resonant response levels that are likely to compromise the life of a component and those that can be tolerated. The industrial requirement is to identify and quantify engine order harmonics arising from the unsteady forces. This knowledge permits calculation of the fatigue life of a component.

Damage prediction in rotating blades is now at a mature level. Common approaches of damage detection are: (i) identification of candidate damage elements using artificial intelligence techniques [1] or (ii) use of residue minimization schemes [2]. Several authors [3–8] reported analytical approaches for predicting the damage in blades using vibration characteristics. More commonly, finite element models are employed with the damage represented in terms of element stiffness reduction coefficients. These coefficients are predicted by

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minimizing a residual difference defined in terms of known dynamic characteristics such as natural frequencies, forced and transient responses. In the present paper, a displacement residue vector is directly defined using available modal data. The approach is illustrated with a finite element model of a standard rotating blade of aerofoil cross-section. In the first stage, modal analysis is conducted on some selected damage cases to obtain forced responses. These are termed as simulated experimental responses in the present context. In the second stage, stiffness reduction coefficients are obtained back by solving it as a residue minimization problem using genetic algorithms. Results are shown for a two-element damage case.

2. Mathematical background

Usually blades have complex asymmetrical aerofoil sections involving coupled bending-torsional modes. They are modeled as rotating cantilever beams of uniform cross-section A and length L mounted over a rigid disc of radius r_2 at a stagger angle ϕ as shown in Fig. 1.

In a finite element formulation of rotating blades, the stiffness and mass matrices of an element are obtained from energy considerations. Strain energy V is the sum of the energy due to flapwise bending and torsional modes (V_s) and additional energy due to rotation ω of the blade (V_f).

Mathematically

$$V_s = \int_0^L \left[\frac{EI_{xx}}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)^2 + \frac{C}{2}\left(\frac{\partial \theta}{\partial z}\right)^2\right] \mathrm{d}z \tag{1}$$

$$V_f = \frac{\omega^2 AL\rho}{2}\int_0^L \left[(r_2 + z)\int_0^z \left(\frac{\partial v_1}{\partial z}\right)^2 \mathrm{d}z - v_1^2 \sin^2\phi\right] \mathrm{d}z \tag{2}$$

where $v_1 = v + \theta d_x$ is the displacement of centroid in the y-direction, v is the displacement of the center of flexure, θ is torsional displacement and d_x refers to the distance between the center of flexure and centroid (asymmetry) as shown in Fig. 1(b). Also the terms E, ρ and C are modulus of the elasticity, density and torsional rigidity of the system, respectively.

The kinetic energy of the system T is given by

$$T = \frac{\rho}{2}\int_0^L \left[I_{cg}\left(\frac{\partial \theta}{\partial t}\right)^2 + A\left(\frac{\partial v_1}{\partial t}\right)^2\right] \mathrm{d}z \tag{3}$$

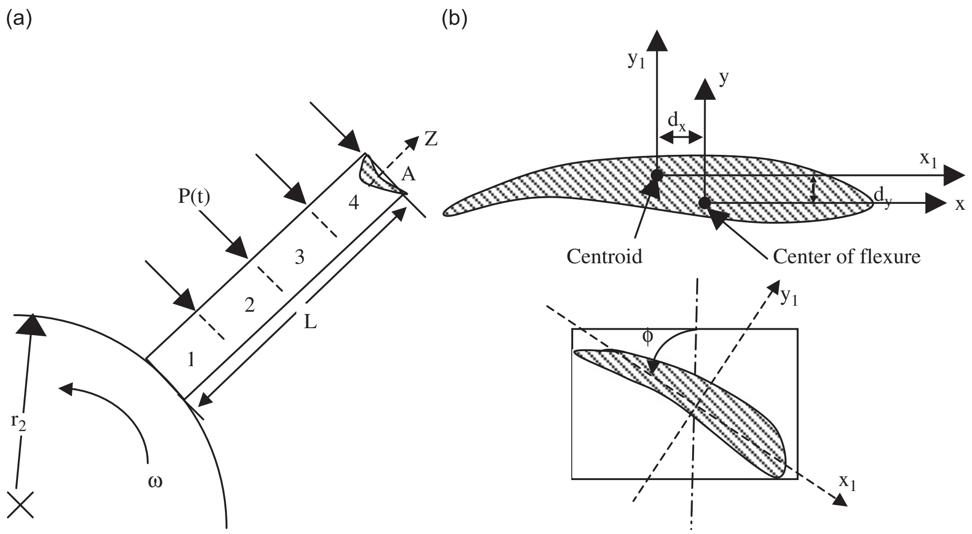

Fig. 1. Modeling of a rotating blade. (a) Blade on the disk and (b) cross-section and mounting of blade.

where

$$\left(\frac{\partial v_1}{\partial t}\right) = \left(\frac{\partial v}{\partial t}\right) + d_x\left(\frac{\partial \theta}{\partial t}\right) \tag{4}$$

Here I_{xx} and I_{yy} are moments of inertia about two bending planes; then $I_{cg}(=I_{xx}+I_{yy})$ is known as the polar moment of inertia.

By considering cubic polynomial expansion for bending motion v and linear polynomial for angular motion θ, the energy expressions become the summation over all the elements as follows:

$$V_s = \sum\{q\}^{\mathrm{T}}[k_b]\{q\},\ V_f = \sum\{q\}^{\mathrm{T}}[k_f]\{q\} \ \text{ and } \ T = \sum\{\dot{q}\}T[m_b]\{\dot{q}\} \tag{5}$$

where $q = \{v_1,\ \phi_1,\ \theta_1,\ v_2,\ \phi_2,\ \theta_2\}^{\mathrm{T}}$ is the vector of nodal degrees of freedom and $[k_b]$, $[k_f]$ and $[m_b]$ are the element bending stiffness, rotational stiffness and mass matrices, respectively.

The dynamic response of the blade $\{X\}$ as a conservative system is obtained from

$$[M]\{\ddot{X}\} + [B]\{\dot{X}\} + [K]\{X\} = \{P(t)\} \tag{6}$$

Here $[K] = [K_b]+[K_f]$ is the overall stiffness matrix, $[B]$ is the viscous damping matrix and $[M]$ is the overall mass matrix. Right-hand-side term $\{P(t)\}$ is a vector of either a harmonic force caused by non-uniform gas flow or a thermal excitation due to a difference in gas temperatures on either side of the blade or an axial component of aerodynamic or hydrodynamic forces acting on the blades. In representing local damage in elements, non-dimensional stiffness reduction coefficients $\beta_i\,(i = 1,2,\ \ldots)$ are used to define the overall stiffness matrix of the blade. That is the stiffness matrix $[K]$ in Eq. (6) should be replaced with damaged stiffness $[K_d]$ defined by

$$[K_d] = [K_f] + \sum_{i=1}^{n} \beta_i [k_b]_i \tag{7}$$

where $[k_b]_i$ is the bending stiffness matrix of the element-i and n is the number of elements considered.

Eq. (6) is solved in modal coordinates by considering viscous damping in the system and the forced responses are obtained under different damage conditions. The corresponding amplitude vector at various points on the blade is termed as $\{X_0\}$. Displacement residue is then defined in terms of unknown coefficients β_i using available modal analysis data. Writing the vector $\{X\}$ in modal coordinates $\{Y\} = [y_1\, y_2 \ldots y_m]^{\mathrm{T}}$ using the relation: $\{X\} = [\phi]\{Y\}$, Eq. (6) becomes

$$\ddot{y}_j + 2\xi_j\omega_{nj}\dot{y}_j + \omega_{nj}^2 y_j = f_j(t), \quad j = 1,2,\ldots,m \tag{8}$$

where m is the number of modes considered and $[f_1\ f_2\ \ldots\ f_m]^{\mathrm{T}}$ is a vector of modal forces defined as $\{F(t)\} = [\phi]^{\mathrm{T}}\{P(t)\}$ with $[\phi]$ being the orthonormalized modal matrix.

Now the harmonic solution of Eq. (8) in each mode, expressed as amplitudes of modal coordinates is

$$Y_{0j} = \frac{F_{0j}}{2\xi_j\omega_{nj}^2}, \quad j = 1,2,\ldots,m \tag{9}$$

Here ω_{nj}, ξ_j, F_{0j} and Y_{0j} are, respectively the natural frequency, damping ratio, amplitudes of modal force and modal displacement in the jth mode.

With the knowledge of experimentally measured amplitudes $\{Z_0\}$ and the modal amplitudes $\{Y_0\}$ obtained from the finite element model, a residual displacement vector $\{R\}$ is defined as

$$\{R\} = [\phi]\{Y_0\} - \{Z_0\} \tag{10}$$

The unknown stiffness reduction coefficients (β_i) are obtained by minimizing the norm of vector $\{R\}$ using binary-coded genetic algorithms. The method starts from a population of initial points in the search space $0 \leqslant \beta_i \leqslant 1$. A new population set of β's is generated every time with selection, cross-over and mutation operators. By the end of presumed generations, all the solution sets in the population converge to a single point and the solution is reported as the state of damage for a given response.

3. Results and discussion

To generate simulated experimental data, the blade is discretized into four elements and assembled matrices are formulated with selected stiffness reduction coefficients in the first and the last element. A computer

Table 1
Properties of the blade considered [9]

Property	Value
Length of the blade	$L = 15.24\,\text{cm}$
Area of cross section	$A = 58.97\text{e}{-6}\,\text{m}^2$
Disc radius	$r_2 = L = 15.24\,\text{cm}$
Stagger angle	$\phi = 90^\circ$
Moments of area	$I_{xx} = 34.96\text{e}{-12}\,\text{m}^4$ $I_{yy} = 2.7928\text{e}{-9}\,\text{m}^4$
Modulus of elasticity	$E = 213.9\text{e}+9\,\text{N/m}^2$
Torsional stiffness	$C = 9.14\,\text{Nm}^2/\text{rad}$
Density	$\rho = 7859\,\text{Kg/m}^3$
Asymmetry in x and y directions	$d_x = 0.193\text{e}{-3}\,\text{m}$ $d_y = 0.193\text{e}{-3}\,\text{m}$

Table 2
Stiffness reduction Vs Natural frequencies of a non-rotating blade

β_1	β_4	Natural frequencies (Hz)							
		(1-bending)		(2-bending)		(1-torsion)		(3-bending)	
		Present	Ref. [9]	Present	Ref. [9]	Present	Ref. [9]	Present	Ref. [9]
1.0	1.0	96.738	96.78	605.44	606.55	1021.09	1079.12	1714.14	1699.21
0.7	0.3	84.737	–	548.72	–	885.78	–	1493.5	–
0.4	0.8	67.59	–	534.15	–	781.758	–	1493.69	–
0.5	0.5	74.19	–	541.68	–	830.27	–	1498.17	–

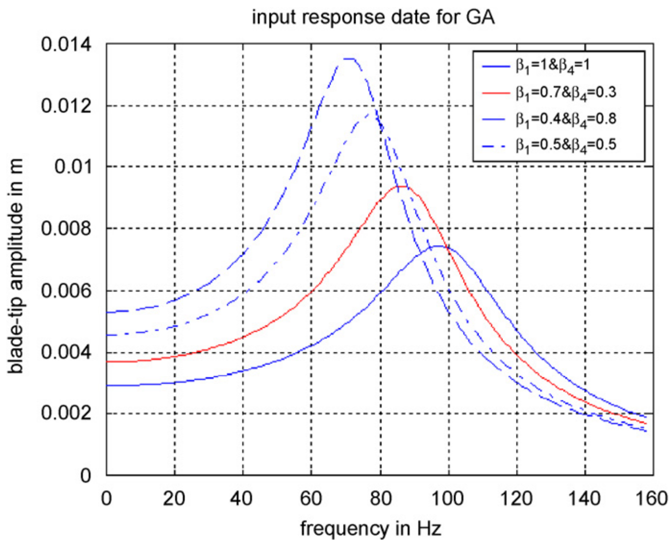

Fig. 2. Simulated forced response curves at the blade tip.

program is developed to obtain the modal data as well as the frequency response under different conditions of damage. To test the accuracy of the model and to obtain the minimum number of elements in the model, standard blade data from Ref. [9] as shown in Table 1 are employed.

Table 2 shows the variation of natural frequencies as a function of stiffness reduction factors (β) in two elements. The values of β and the specific elements considered are arbitrarily chosen.

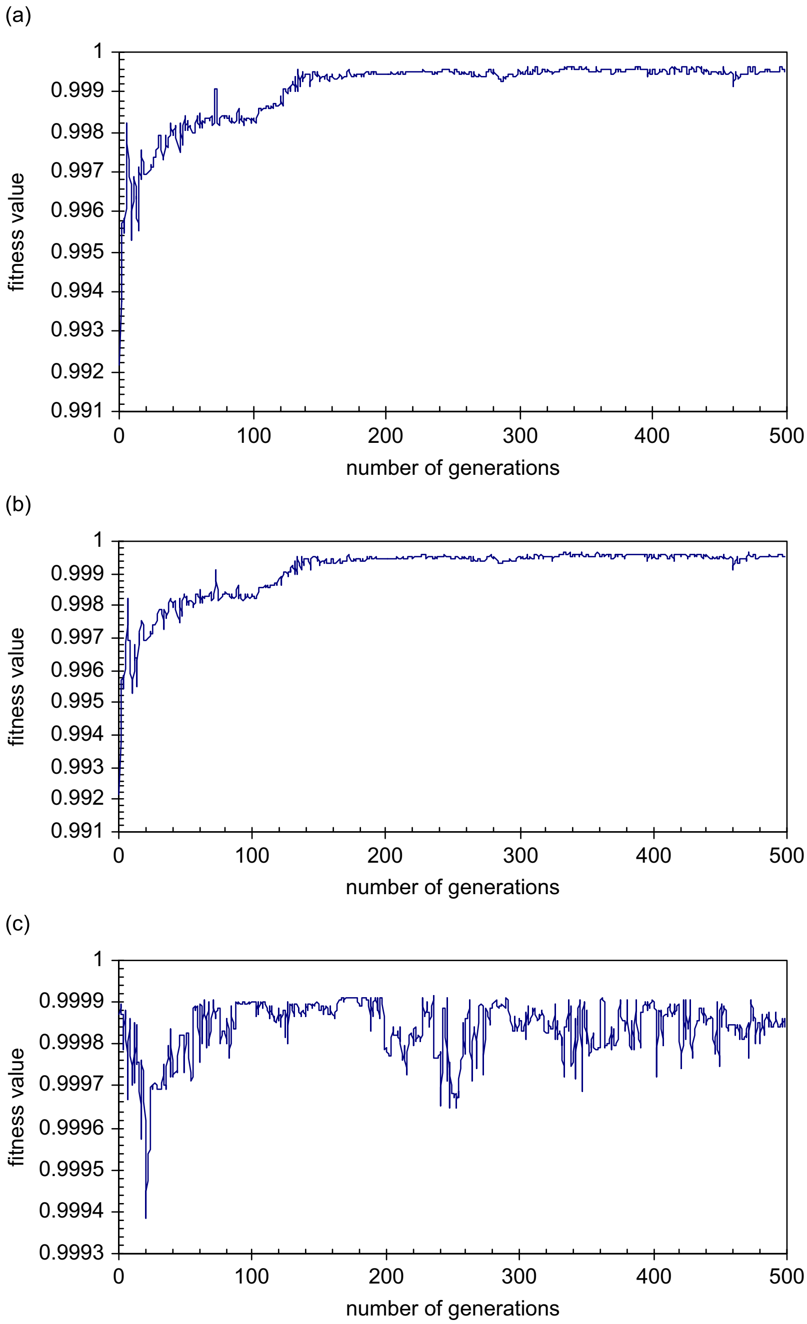

Fig. 3. Training of GA with known response values. (a) Case 1 ($\beta_1 = 0.7$ and $\beta_4 = 0.3$), (b) case 2 ($\beta_1 = 0.4$ and $\beta_4 = 0.8$) and (c) case 3 ($\beta_1 = 0.5$ and $\beta_4 = 0.5$).

Table 3
Outputs of the genetic algorithms at the end of 500 generations

Case ♯	β_1			β_4		
	Predicted	Actual	% Error	Predicted	Actual	% Error
1	0.7038	0.7000	0.54	0.3137	0.3000	4.56
2	0.4066	0.4000	1.65	0.8357	0.8000	4.46
3	0.5004	0.5000	0.08	0.5229	0.5000	4.58

It can be seen that a four-element model gives comparable accuracy in undamaged case, which shows that the accuracy of the present finite element model is not poor. However, it should be noted that the selection of the minimum number of elements depends on the dimensions and the required prediction accuracy.

Frequency responses are obtained by considering a modal-damping ratio of 0.2 in the system. A uniform amplitude of applied lateral harmonic load is taken as 10 N. The input blade-tip responses at a speed of 1000 rev/min during all the selected damage cases are depicted in Fig. 2.

Now the residue minimization scheme is employed with genetic algorithms to predict the values of β_i, under a constant excitation system. Different trails are conducted to select the following best set of variable in genetic algorithms (GA) using the tournament selection approach: crossover probability: 1.00, mutation probability: 0.05, population size: 40 and string length: 20. The program is executed independently for prediction of stiffness reduction factors in each damaged state. Fig. 3 shows the variation of fitness values as a function of number of generations in all three damaged cases. Here, as the first mode is predominant, response amplitudes in this mode are only accounted for in computing the residue.

The predicted damage coefficients along with the actual values used for simulation are presented in Table 3.

The predicted coefficients are very close to the actual values. The computational time is also small. Even in this paper, only two-element damage is considered to show the methodology; the approach can be extended for cases with a higher number of damaged elements. Based on these initial results, the variation in real systems cannot be completely explained only by stiffness reductions. In addition, realistic variations in density and damping may also be included. To implement the approach experimentally, the system damping and excitation are to be measured in addition to the response amplitudes along the blade length. Future work will explore the practical implementation of this routine.

4. Conclusions

A numerical damage prediction strategy using an analytical formulation has been presented in this paper. Damage in any element of the blade is modeled as local reduction in the stiffness matrix and element stiffness reduction coefficients were used to indicate the extent of damage. A displacement residue in terms of these unknown coefficients is defined from the original and computed values of the response amplitudes along the blade length. Residue minimization using genetic algorithms resulted in the optimum values of stiffness reduction coefficients. The method has been illustrated with response data obtained from a finite element model of a rotating blade of aerofoil cross-section. The results are quite encouraging and future work is required to include practical considerations in the model.

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