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Optimal sensor placement for spatial lattice structure based on genetic algorithms

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Abstract

Optimal sensor placement technique plays a key role in structural health monitoring of spatial lattice structures. This paper considers the problem of locating sensors on a spatial lattice structure with the aim of maximizing the data information so that structural dynamic behavior can be fully characterized. Based on the criterion of optimal sensor placement for modal test, an improved genetic algorithm is introduced to find the optimal placement of sensors. The modal strain energy (MSE) and the modal assurance criterion (MAC) have been taken as the fitness function, respectively, so that three placement designs were produced. The decimal two-dimension array coding method instead of binary coding method is proposed to code the solution. Forced mutation operator is introduced when the identical genes appear via the crossover procedure. A computational simulation of a 12-bay plain truss model has been implemented to demonstrate the feasibility of the three optimal algorithms above. The obtained optimal sensor placements using the improved genetic algorithm are compared with those gained by exiting genetic algorithm using the binary coding method. Further the comparison criterion based on the mean square error between the finite element method (FEM) mode shapes and the Guyan expansion mode shapes identified by data-driven stochastic subspace identification (SSI-DATA) method are employed to demonstrate the advantage of the different fitness function. The results showed that some innovations in genetic algorithm proposed in this paper can enlarge the genes storage and improve the convergence of the algorithm. More importantly, the three optimal sensor placement methods can all provide the reliable results and identify the vibration characteristics of the 12-bay plain truss model accurately.

1. Introduction

Structural modal parameter identification using measured dynamic data has received much attention over the years because of its importance in structural model updating, structural health monitoring and structural control. In particular, the quality of a modal parameter identification process strongly depends on the quality of the measured response data, which further depends substantially on the numbers and locations of sensors in the structure [1]. So determining the optimal numbers and locations of sensors is a critical issue encountered in the construction and implementation of an effective structural health monitoring system. Its basic idea is to

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select the optimal numbers and locations of the sensors such that the resulting measured data are most informative, the identified modal parameters are quite accurate and the structural health monitoring system are quite robust.

Many authors [2–10] have researched the optimal sensor placement problem for structural modal parameter identification and structural health monitoring in the past few years. Kammer [2,3] presented and developed the effective independence (EI) method, which maximizes a combination of target mode signal strength and linear independence. The method starts with the large candidate sensor set, ranks all the sensors based on their contributions to the determinant of a Fisher information matrix (FIM), and then eliminates the lowest ranked sensor. The new candidate sensor set is then re-ranked and the lowest ranked sensor is again discarded. In an iterative fashion, the initial candidate set is reduced to the desired number of locations. Lim [4] employed the generalized Hankel matrix, a function of the system controllability and observability, to develop an approach which can determine sensor locations based on a given rank for the system observability matrix while satisfying modal test constraints. Papadimitriou et al. [1,5] introduced the information entropy norm as the measure that best corresponds to the objective of structural testing which is to minimize the uncertainty in the model parameter estimates. The optimal sensor configuration is selected as the one that minimizes the information entropy measure since it gives a direct measure of this uncertainty. An important advantage of the information entropy measure is that it allows us to make comparisons among sensor configurations involving a different number of sensors in each configuration.

Genetic algorithms (GA) have also been proposed as an effective alternative [6–8] to the previous heuristic algorithm, which is not guaranteed to give the optimal solution. Yao et al. [6] had taken GA as an alternative to the EI method and the determinant of the FIM is chosen as the objective function. Worden and Burrows [7] reviewed the recent work on sensor placement and applied the GA and the simulated annealing to determine the optimal sensor placement in structural dynamic test. Then it described an approach to fault detection and classification using neural networks and combinatorial optimization. Gao et al. [8] developed a new framework of sensor placement optimization for structural health monitoring. The optimization problem is to minimize the damage misdetection rate as well as to minimize the number of sensors by searching the optimized patterns of sensor placement topology on the feasible region of the monitored structure. The program was applied to a sample sensor placement problem of an aging aircraft wing. Optimized sensor placement designs are obtained.

Some comparison work can be seen in Refs. [9,10]. Larson et al. [9] made a comparison between some actuator and sensor placement techniques including the EI method, the kinetic energy (KE) method, average kinetic energy (AKE) and eigenvector component product (EVP). All methods proceed by sequentially deleting the worst candidate points until the correct number of sensors is obtained. Meo and Zumpano [10] investigated six different optimal sensor placement techniques on a bridge structure with the aim of maximizing the data information. Three of them are based on the maximization of the FIM, one is based on the properties of the covariance matrix coefficients, and last two are based on energetic approaches. The results showed that the effective independence driving-point residue (EFI-DPR) method can provide an effective method for optimal sensor placement to identify the vibration characteristics of the bridge.

The research presented in this paper is aimed to develop some optimal sensor placement techniques for damage detection and structural health monitoring on the spatial lattice structure. Based on the criterion of optimal sensor placement for modal test, an improved GA is introduced to find the optimal placement of sensors. The layout of the paper is as follows: Section 2 gives the basic theory of the improved GA including the selection of the fitness function, the presented coding system and genetic operator. Section 3 describes the computational simulation using a 12-bay plain truss model and the presented optimization strategies are demonstrated and compared using the modal parameters identified by the data-driven stochastic subspace identification (SSI-DATA) method. Section 4 discussed the concerning work and the conclusions.

2. Basic theory

2.1. GA

For the sake of completeness, a brief discussion of GA will be given here. For more details, readers could refer to the standard introduction in Ref. [11]. GA is optimization algorithm, which evolves in an analogous manner as the Darwinian principle of natural selection. To obtain the optimal solution for design problems, the GA has been implemented so that it progresses in a similar way as the natural evolution of a species. It means that the fundamental concepts of reproduction, chromosomal crossover, occasional mutation of genes and natural selection are reflected in the different stages of the GA process. The process is initiated by selecting a number of candidate design variables either randomly or heuristically in order to create an initial population. Then the initial population is encouraged to evolve over generations to produce new designs, which are better or fitter. The quality or fitness of the designs is evaluated according to an objective function, i.e. the fitness function, which must be formulated in relation to the specific optimization problem. By definition the optimal design corresponds to the maximum of this objective function. To implement the GA, it is necessary to devise a general coding system for the representation of the design variables first. Most commonly the design variables are coded by the binary representation. Since the search for the optimal solution proceeds with the population of design alternatives, the GA has a distinct advantage over traditional optimization techniques, which start from a single point in the design space [7,12].

2.2. *Fitness function*

The fitness functions presented in this paper are the modal strain energy (MSE) and the modal assurance criterion (MAC), respectively. The objective of MSE is to find a reduced configuration of sensor placements, which maximizes the measure of the MSE of the structure. The reason is that the signal-to-noise ratio of the measured response data is larger on the degree of freedoms (dofs) which have the larger MSE and it makes for parameter identification when the sensor are placed on these locations. At the same time, the MAC matrix is used to construct other two objective functions. The first is the average value of all the off-diagonal elements in MAC matrix. The second is the biggest value in all the off-diagonal elements in MAC matrix. The reason for the selection of these fitness functions is that the MAC matrix will be diagonal for an optimal sensor placement strategy so the size of the off-diagonal elements can be taken as an indication of the fitness.

Assume the mode shape matrix is $\mathbf{\Phi} = [\varphi_1, \varphi_2, \ldots, \varphi_p]$ (subscript p is the number of mode shape vectors) and the number of the measured points is q, the MSE fitness function can be given as

$$f = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{r \in q} \sum_{s \in q} \left| \varphi_{ri} k_{rs} \varphi_{sj} \right| \tag{1}$$

where k_{rs} represents the stiffness coefficient between the rth dof and sth dof, is just the element corresponding to the rth row and the sth column in the stiffness matrix. φ_{ri} represents the deformation of rth element in ith mode and φ_{sj} represents the deformation of sth element in jth mode. $r \in q$ and $s \in q$ represents that r and s are all included in the total measured point set.

The MAC can be defined as Eq. (2), which measures the correlation between mode shapes:

$$\mathrm{MAC}_{ij} = \frac{\left| \varphi_i^{\mathrm{T}} \varphi_j \right|^2}{(\varphi_i^{\mathrm{T}} \varphi_i)(\varphi_j^{\mathrm{T}} \varphi_j)} \tag{2}$$

where φ_i and φ_j represent the ith mode shape vector and the jth mode shape vector, respectively, and the superscript T represents the transpose of the vector.

Then the MAC1 fitness function is given as Eq. (3) and the MAC2 fitness function is given as Eq. (4):

$$f = 1 - \mathrm{average}(\mathrm{abs}(\mathrm{MAC}_{ij})), \quad i \neq j \tag{3}$$

$$f = 1 - \max(\mathrm{abs}(\mathrm{MAC}_{ij})), \quad i \neq j \tag{4}$$

where abs $(\cdot)$ represents the absolute value, average $(\cdot)$ represents the average value and max $(\cdot)$ represents the maximal value.

2.3. Coding system

Considering the characteristics of the optimal sensor placement problem, the decimal coding system is adopted for the representation of the design variables in this paper. Because the number of the dofs is enormous in large-scale spatial lattice structures, the requirement for the large storage space is increased to save the optimal solutions. So the decimal two-dimension array coding method instead of binary coding method is presented to code the solutions. If there are s sensors to place in the total n degrees of freedom, the coding length of a string is s. Every value of the string is the dof on which the sensor is located. For example, 2 6 12 14 22 29 30 35 38 43 is a string, it denotes that sensors are located on the second, sixth, 12th, 14th, etc. 10 dofs. If the size of the initial population of individuals is m, then the decimal two-dimension array coding method is formed as Table 1 in which the number of sensors s is 10 and the total degrees of freedom n is 44 in this paper. (Referred to the illustrative example presented subsequently in Section 3.) To demonstrate the advantage of the decimal two-dimension array coding method directly, two binary coding methods are introduced here briefly as Tables 2 and 3. Table 2 shows one kind of binary coding method in which the coding length of a string is the total degrees of freedom n. If the value of the ith bit position of the string is 1, it denotes that a sensor is located on the ith dof. In contrast, if the value of the ith bit position is 0, it denotes no sensor is located on the ith dof [12]. Table 3 shows another kind of binary coding method in which one sensor location is represented by a binary string then all the strings are connected in series as a total string [13]. The length of the binary string in one sensor location is l which should ensure that 2 to the power of l is the nearest integer to the total degrees of freedom n and larger than it. By comparison it is obvious that the dissipative storage space of the decimal two-dimension array coding method is minimal among them. This is very important in the optimal sensor placement problem for spatial lattice structure because the number of the dof is enormous in the large-scale spatial lattice structure. The convergence of the proposed coding system can be demonstrated in the next section.

2.4. Genetic operation

To implement the GA for the determination of the optimal sensor location, a number of candidate design variables have been selected randomly as an initial population (such as Table 1). Then the reproduction operation also called the natural selection is carried out, in which the fitness of the different individual of the population had been evaluated based on the above presented fitness functions and ranked by the ratio of individual fitness to the total fitness of the current population. Some new design variables that will become parent designs in the next circulation are selected directly according to their individual fitness ranking. Further is the crossover process. Some sections of the bit-string representations of the two parent designs (arbitrary two rows in Table 1) are swapped directly to create the two offspring design solutions. This process ensures that design information is transferred from one generation to the next. Following crossover, the mutation operation is introduced via the occasional switching of the bit value at a randomly selected location of the generated strings. This action is important since it guards against the premature convergence of the design towards a non-optimal solution [14].

In executing the optimal sensor placement searching via GA, the same location may be placed with two sensors synchronously in the crossover process. It is impractical and must be avoided. In this paper, we introduced the forced mutation operator to change the repeated sensor location in the generated strings. The detailed operation process is represented and shown in Table 4. The first two lines in Table 4 are the two

Table 1
The decimal two-dimension array coding method ($m \times s$, $s = 10$)

No.	1	2	3	4	5	6	7	8	9	10
Genes 1	2	3	6	12	20	28	33	36	38	44
Genes 2	2	5	8	18	24	30	35	38	40	43
…	…	…	…	…	…	…	…	…	…	…
Genes m	4	6	9	16	22	32	36	38	43	44

Table 2
Binary coding method 1 ($m \times s$, $s = 44$)

No.	1	2	3	4	5	6	7	8	9	10	…	35	36	37	38	39	40	41	42	43	44
Genes 1	0	1	1	0	0	1	0	0	0	0	…	0	1	0	1	0	0	0	0	0	1
Genes 2	0	1	0	0	1	0	0	1	0	0	…	1	0	0	1	0	1	0	0	1	0
…	…	…	…	…	…	…	…	...	…	…	…	…	…	…	…	…	…	…	…	…	…
Genes m	0	0	0	1	0	1	0	0	1	0	…	0	1	0	1	0	0	0	0	1	1

Table 3
Binary coding method 2 ($m \times s$, $s = 10 \times 6$)

No.	1	2	3	4	5	6	7	8	9	10
Genes 1	000010	000011	000110	001100	010100	011100	100001	100100	100110	101010
Genes 2	000010	000101	001000	010010	011000	011110	100011	100110	101000	101001
…	…	…	…	…	…	…	…	…	…	…
Genes m	000100	000110	001001	010000	010110	100000	100100	100110	101001	101010

Table 4
Operation process of forced mutation in genetic algorithm

No.		1	2	3	…	*i*	…	*j*	…	s-2	s-1	s
Individual gene pair before crossover	Parent 1	2	3	6	…	17	…	22	…	36	40	44
	Parent 2	4	6	9	…	22	…	25	…	38	41	44
New individual gene pair after crossover	Offspring 1	2	3	6	…	17	…	25	…	38	41	44
	Offspring 2	4	6	9	…	*22*	…	*22*	…	36	40	44
New individual gene pair after forced mutation	Offspring 1	2	3	6	…	17	…	25	…	38	41	44
	Modified offspring 2	4	6	9	…	*22*	…	*28*	…	36	40	44

selected parent design solutions. If the *j*th bit in the parent design solutions is chosen randomly as the cutting position, the two new offspring design solutions will be generated by each taking the first part from one parent and the second from the other as shown in the second two lines in Table 4. Unfortunately, the *j*th bit in one new offspring design solution (offspring 2) is the same with the *i*th bit in it (the italic numbers). That means that one location has been placed with two sensors synchronously. So the forced mutation operator is introduced to change one value of the same two numbers to the other value, which is different from the other numbers in offspring 2. In this example, the second value 22 is changed to 28 (the bold italic underlined numbers in the last line in Table 4), which is not included in the offspring 2 before. For the reasonable compatibility of GA, we reduced the ratio of the natural mutation operation as a compromise. The research results showed that the forced mutation operator obtained the expected intention and did not influence the convergence of the GA.

In practice, a convergence criterion must be specified in executing the GA. In this paper, a relative large number N is selected to avoid redundant iteration. The genetic process will be stopped automatically if the best individual in the population does not change in continuous N iteration. To sum up, the whole flowchart of the genetic search to find the optimal sensor locations presented in this paper is shown in Fig. 1.

3. Numerical example

3.1. Analytical model

In this section, the three different optimal sensor placement techniques presented above were tested for modal identification on a 12-bay plain truss model as shown in Fig. 2. The total size of 12-bay plain truss

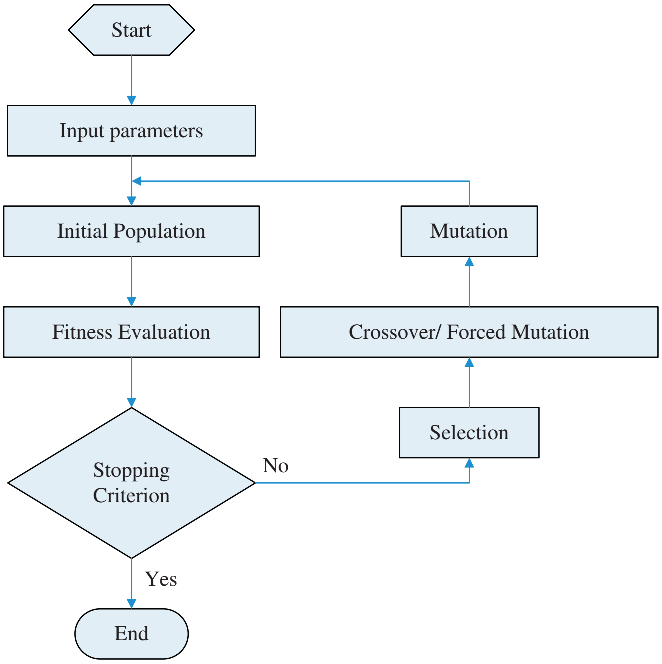

Fig. 1. Flowchart of the genetic algorithm.

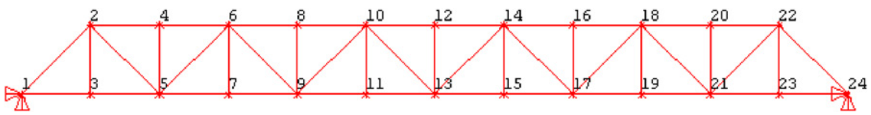

Fig. 2. A 12-bay plain truss model.

model is 4.8 m × 0.4 m, the number of girds is 12 × 1. All element sections are tubular and the dimensions are ∅ 16 mm × 2 mm. The material properties are taken from Q235 steel where the elastic modulus is 210 GPa and the density is 7850 kg/m^3. The deadweight of the members and the ball are treated as lumped mass concentrated at the nodes. The joints of the plain truss are hinged connection and the plain truss is hinged at fixed-point supports on the both sides. The analytical model has 24 nodes, 45 elements and 44 dofs. In order to provide the input data for the optimal sensor placement methods, the finite element model of the 12-bay plain truss model is developed using the universal finite element analysis package (ANSYS [15]). The vibration properties were calculated by performing modal analysis based on the subspace iteration method. The structural dynamic characteristics including the first six natural frequencies and mode shapes are obtained and shown in Table 11 and Fig. 3. It is obvious that mode shape 1 is the first vertical bending deflection, in sequence, mode shape 2 is the second vertical bending deflection, mode shape 3 is the third vertical bending deflection, mode shape 5 is the forth vertical bending deflection, mode shape 6 is the fifth vertical bending deflection, while mode shape 4 behaves as a coupled vibration mode shape between the third vertical bending deflection and the horizontal longitudinal deflection.

3.2. Optimization results

Based on the stiffness matrix and mode shape matrix calculated by finite element method (FEM), the above three approaches which differ only in the chosen of objective function based on GA are implemented to select the best sensor locations. The basic parameters of GA are listed as follows: population size is 300, probability of selection is 0.2, probability of crossover is 0.6, probability of mutation is 0.01 and the relative large number of generations selected for convergence is 100. All the best results for the 15, 10 and five sensor locations are

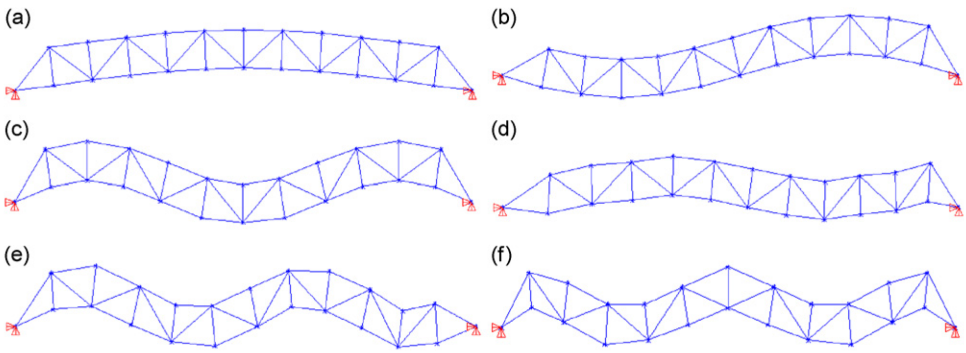

Fig. 3. Mode shapes calculated by finite element method (FEM): (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

Table 5
Comparison of the optimal sensor locations in 15 measured points' case

Fitness function	Optimal sensor locations														
MSE	2	5	6	8	12	18	21	22	23	35	36	37	38	42	43
MAC1	2	3	6	11	12	13	14	20	21	22	30	36	38	43	44
MAC2	1	2	4	9	16	18	19	21	22	24	34	35	38	39	42

Table 6
Comparison of the optimal sensor locations in ten measured points' case

Fitness function	Optimal sensor locations									
MSE	2	5	18	21	22	35	37	38	42	43
MAC1	2	6	12	14	22	29	30	35	38	43
MAC2	2	5	12	22	26	27	28	34	38	39

Table 7
Comparison of the optimal sensor locations in five measured points' case

Fitness function	Optimal sensor locations				
MSE	5	21	22	35	37
MAC1	4	14	30	35	44
MAC2	3	12	20	26	38

listed and compared in Tables 5–7. Each algorithm was used to select the best 15, 10 and five sensor locations placed on the 12-bay plain truss model to independently identify the modal parameters, respectively. The aim is to determine the optimal numbers and locations of sensors, which is enough to obtain the response data and the structural dynamic behavior of the 12-bay plain truss model thoroughly.

In order to evaluate the reliability of the above results, all the fitness convergence cures of different fitness function in different measured points' cases are shown as Figs. 4–6. It is obvious that all the maximum fitness values tend to a constant quickly and the average fitness value steadily tends to the maximum fitness value along with increasing number of generations. It shows a good characteristic of convergence. Further to demonstrate the effectiveness of the improvements in the GA, the existing GAs with binary coding method [12] are performed and compared with the one proposed in this paper. Each method was executed 10 times

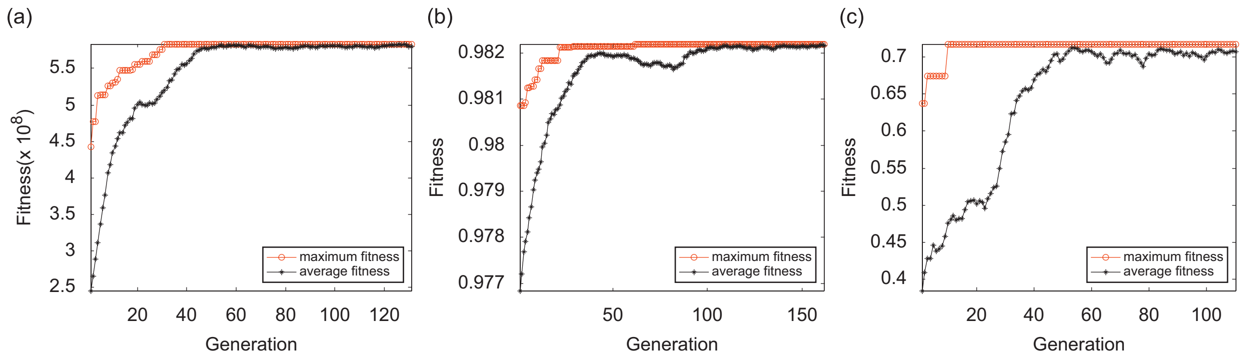

Fig. 4. Fitness curves of improved genetic algorithm with different fitness functions (15 sensors): (a) MSE; (b) MAC1 and (c) MAC2.

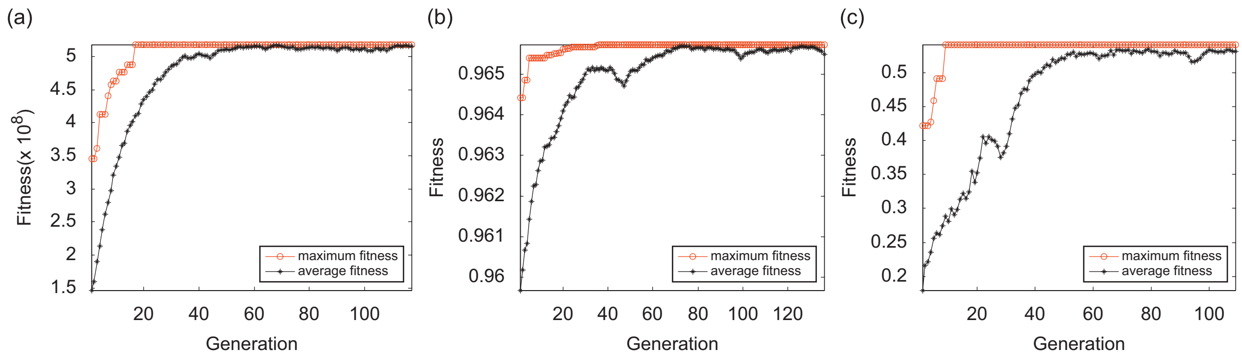

Fig. 5. Fitness curves of improved genetic algorithm with different fitness functions (10 sensors): (a) MSE; (b) MAC1 and (c) MAC2.

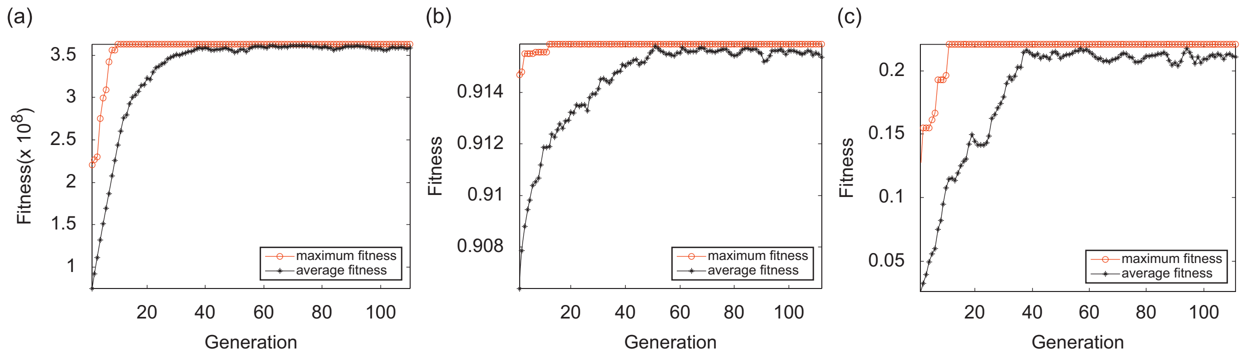

Fig. 6. Fitness curves of improved genetic algorithm with different fitness functions (five sensors): (a) MSE; (b) MAC1 and (c) MAC2.

with a different stochastic initial population. Numbers of the convergence generations are compared in Tables 8–10. The average number of convergence generations of different fitness function in different measured points' cases using decimal two-dimension array coding method is smaller than those using binary coding method. That means the convergence speed of the improved GA is far higher than that of binary coding method and 20–30% reduction in computational iterations is gained to reach a satisfactory solution. At the same time, the fitness function values are compared each other. They are almost identical and are omitted in this paper.

Table 8
Comparison of the convergence by different coding methods using different fitness functions (15 sensors)

No.	Number of convergence generations					
	Decimal two-dimension array coding method			Binary coding method		
	MSE	MAC1	MAC2	MSE	MAC1	MAC2
1	131	162	110	141	239	179
2	127	137	154	118	278	194
3	171	145	175	133	193	188
4	121	150	116	151	157	206
5	134	183	166	197	142	188
6	136	132	128	176	141	328
7	123	141	233	145	183	137
8	164	140	152	143	157	146
9	131	162	110	139	201	156
10	131	162	110	134	249	168
Average	136.9	151.4	145.4	147.7	194	189

Table 9
Comparison of the convergence by different coding methods using different fitness functions (ten sensors)

No.	Number of convergence generations					
	Decimal two-dimension array coding method			Binary coding method		
	MSE	MAC1	MAC2	MSE	MAC1	MAC2
1	134	139	129	127	172	157
2	150	141	180	131	223	135
3	131	133	137	157	158	129
4	117	136	109	128	200	173
5	126	157	180	126	202	136
6	145	146	109	121	119	219
7	129	164	123	130	160	185
8	132	158	129	141	191	147
9	134	139	129	165	190	187
10	134	139	129	132	167	180
Average	133.2	145.2	135.4	135.8	178.2	164.8

3.3. Comparison study

In order to demonstrate the capability of capturing the vibration behavior of the 12-bay plain truss model using the three optimal sensor placement techniques, the SSI-DATA method [16] is adopted to identify the modal parameters as the measured data set. To do this, the simulated excitation that is assumed as the independent band-limited white noises is applied to the y direction of nodes 12. Meantime, the outputs (15, 10 and five accelerations) are collected in the above-determined optimal sensor locations, respectively. To simulate the ambient vibration case, a 5% root mean square noise is added to the measured outputs and inputs are not collected. Comparison criteria based on the mean square error between the FEM mode shapes and the Guyan expansion [17] mode shapes measured at the selected sensor locations was employed to demonstrate the feasibility of the selected optimal sensor locations [10].

First, the modal parameters are obtained by SSI-DATA method in several different sensor placement cases. Considering the integrality of the paper, stability diagram obtained by SSI-DATA algorithm in 15 measured

Table 10
Comparison of the convergence by different coding methods using different fitness functions (five sensors)

No.	Number of convergence generations					
	Decimal two-dimension array coding method			Binary coding method		
	MSE	MAC1	MAC2	MSE	MAC1	MAC2
1	110	112	111	115	187	150
2	116	114	108	109	158	117
3	114	116	109	111	124	122
4	107	107	113	117	167	111
5	113	115	104	114	120	126
6	115	104	113	111	197	118
7	107	137	106	113	134	170
8	110	112	111	112	141	135
9	110	112	111	112	116	143
10	116	114	108	109	143	120
Average	111.8	114.3	109.4	112.3	148.7	131.2

points' case using MSE fitness function is given in Fig. 7. The background curve is the sum of all the auto-spectral and cross-spectral density functions. The stabilization criteria are 1% for frequencies, 5% for damping and 5% for mode vectors. From it the first six natural frequencies and mode shapes can identify distinctly. All the modal frequencies are listed in Table 11. The identified modal frequencies results are compared with those calculated by FEM in the first row. It is obvious that the identification results by SSI-DATA are quite accurate. This means that the five measured points' case can obtain the modal frequencies accurately. It is not surprising because one sensor is enough to know the frequency of the structure in theory. Then the mean square errors between the FEM mode shapes and the Guyan expansion mode shapes, which are identified by SSI-DATA method and then expanded by the Guyan expansion technique are calculated and summarized in Table 12. As expected, the mean square error of the three optimal sensor placement methods is all very small. This implies that the identified mode shapes under different sensor placement cases may be consistent with the FEM modes shapes.

To further demonstrate the feasibility of the selected optimal sensor locations, the identified six Guyan expansion mode shapes are shown in Figs. 8–13 for comparing with those calculated by FEM in Fig. 3. (The results in 15 measured points' case are omitted for the length of paper.) By comparison all the identified six mode shapes by SSI-DATA method with different optimal sensor methods in 15 measured points' case and 10 measured points' case are nearly consistent with those calculated by FEM. And in five measured point's case, the optimal sensor method with MSE fitness function can identify the first two mode shapes and the optimal sensor methods with MAC1 and MAC2 fitness function can identify the first five mode shapes. These results are also expected. Table 7 shows that the optimal five sensor points obtained by the MSE fitness function include one sensor in y direction and four sensors in x direction, and the optimal five sensor points obtained by the MAC1 and MAC2 fitness function include four sensors in y direction and one sensor in x direction, respectively. It is obvious that four sensors in y direction and one sensor in x direction can identify the first five mode shapes at most because mode shape 4 behaves as a coupled vibration mode shape between the third vertical bending deflection and the horizontal longitudinal deflection and mode shape 5 is the forth vertical bending deflection as described above. In conclusion, the two methods based on MAC fitness function in GA are better than the method based on MSE fitness function in the case of placing a few sensors. With the increasing of the number of sensors the three methods based on different fitness function in GAs can all obtain the reliable optimal sensor placement to identify the vibration characteristics of the 12-bay plain truss model accurately. To obtain the first six mode shapes of the 12-bay plain truss model under investigation, six sensors including five in y direction and one in x direction is required at least because the 12-bay plain truss model provides the vibration characteristic as the beam with both ends built-in.

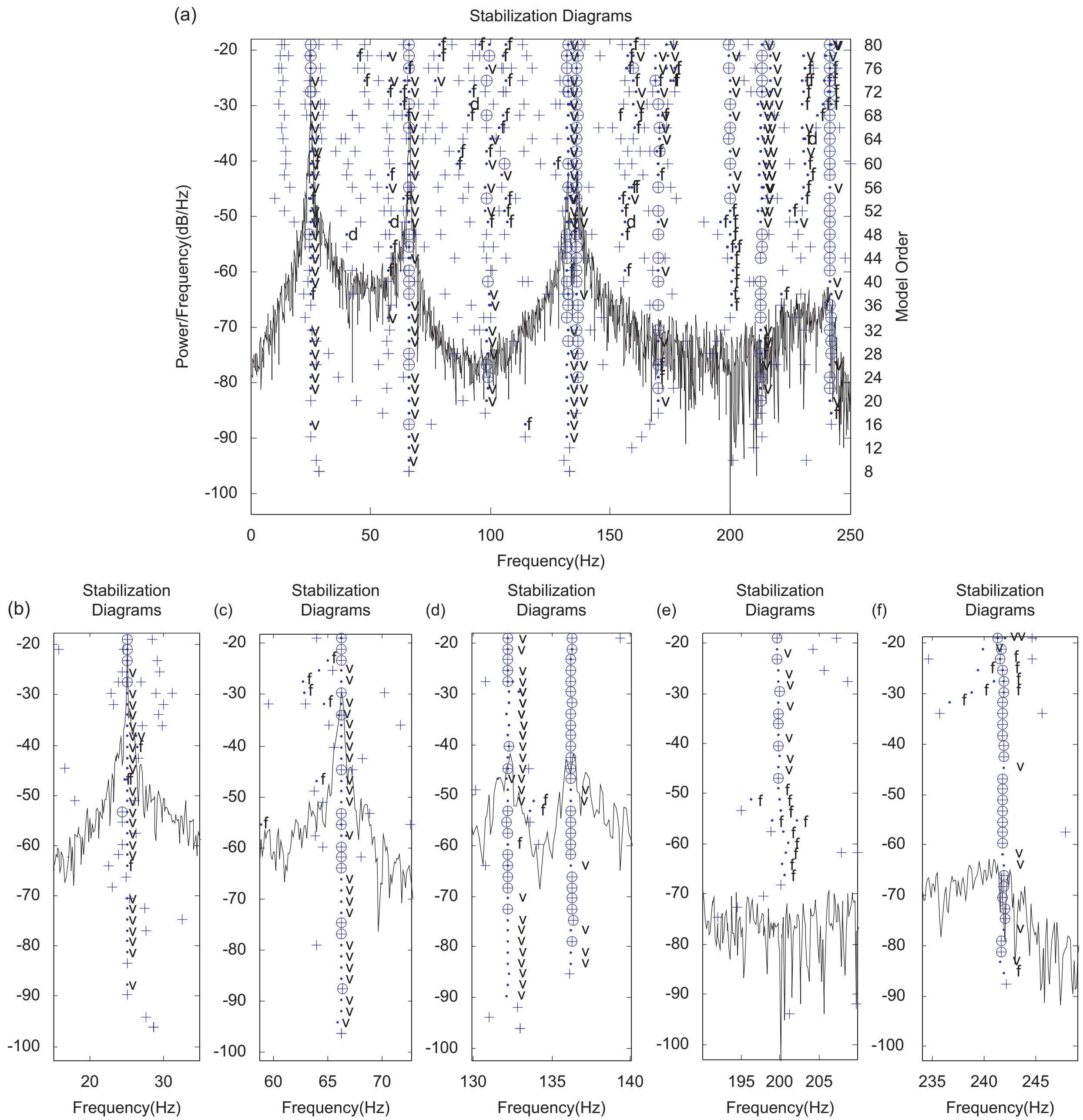

Fig. 7. Stability diagram obtained by SSI-DATA algorithm: (a) all the six modes; (b) partial enlarged result of the first mode; (c) partial enlarged result of the second mode; (d) partial enlarged result of the third mode and the forth mode; (e) partial enlarged result of the fifth mode and (f) partial enlarged result of the sixth mode. "⊕" for a stable pole; ".v" for a pole with stable frequency and vector; ".d" for a pole with stable frequency and damping; ".f" for a pole with stable frequency and "+" for a new pole.

4. Conclusions

In this paper, the GA was studied and improved to find the optimal sensor placement based on the criterion of optimal sensor placement for modal test. Three optimal placement techniques were presented when the MSE and the MAC had been taken as the fitness function, respectively. Considering the characteristics of the optimal sensor placement techniques in the large-scale spatial lattice structure, some innovations in GA such

Table 11
Comparison of modal frequencies identified by SSI-DATA using different fitness functions under different sensor placement cases and the ones calculated by FEM

No.	1	2	3	4	5	6
Modal frequencies calculated by FEM (Hz)	25.12	66.27	132.24	136.26	199.73	241.67
Modal frequencies identified by SSI-DATA (Hz)						
Fifteen measured points						
MSE	25.11	66.29	132.23	136.27	198.38	241.71
MAC1	25.11	66.29	132.22	136.27	199.90	241.77
MAC2	25.11	66.29	132.27	136.25	200.12	241.83
Ten measured points						
MSE	25.11	66.30	132.23	136.20	199.57	241.78
MAC1	25.11	66.29	132.23	136.22	199.67	241.81
MAC2	25.11	66.28	132.23	136.26	199.97	241.86
Five measured points						
MSE	25.12	66.28	132.22	136.25	198.38	241.85
MAC1	25.12	66.28	132.21	136.27	199.90	241.69
MAC2	25.12	66.28	132.21	136.26	200.12	241.52

Table 12
Comparison of the mean square error between the FEM mode shape and the Guyan expansion mode shapes using different fitness functions under different sensor placement cases

	Mean square error						Total mean square error
	First mode	Second mode	Third mode	Fourth mode	Fifth mode	Sixth mode	
Fifteen measured points							
MSE	2.04e−005	2.86e−005	8.31e−005	2.84e−004	1.79e−003	8.37e−004	3.05e−003
MAC1	9.31e−007	6.10e−006	5.95e−005	1.11e−004	4.92e−004	3.95e−004	1.06e−003
MAC2	1.05e−005	2.04e−005	2.05e−004	1.17e−004	1.41e−003	6.18e−004	2.38e−003
Ten measured points							
MSE	3.47e−005	1.20e−004	8.72e−004	5.28e−004	5.44e−003	9.06e−003	1.61e−002
MAC1	1.36e−006	1.44e−005	1.39e−004	6.15e−004	1.25e−003	1.85e−003	3.87e−003
MAC2	2.76e−005	4.61e−005	1.34e−004	4.39e−004	3.70e−003	1.26e−003	5.61e−003
Five measured points							
MSE	7.21e−006	6.68e−004	1.01e−002	3.28e−003	1.39e−002	1.33e−002	4.12e−002
MAC1	5.99e−004	3.78e−005	3.08e−003	1.09e−003	3.99e−003	9.63e−003	1.84e−002
MAC2	1.90e−005	5.93e−005	6.55e−004	5.62e−003	2.89e−003	1.08e−002	2.00e−002

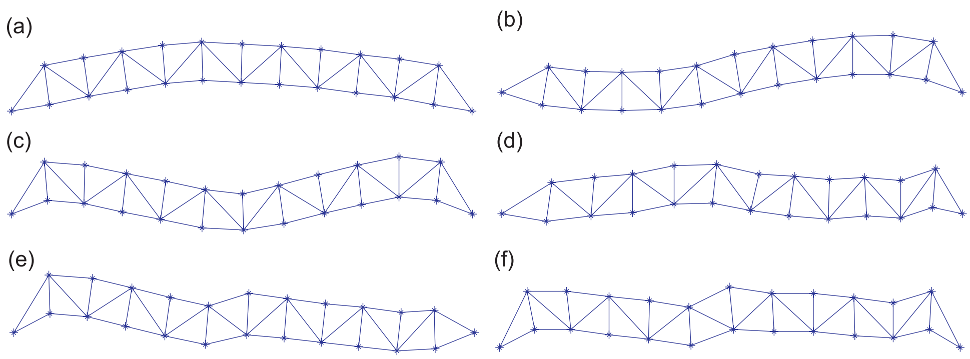

Fig. 8. Mode shapes identified in 10 measured points' case using MSE: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

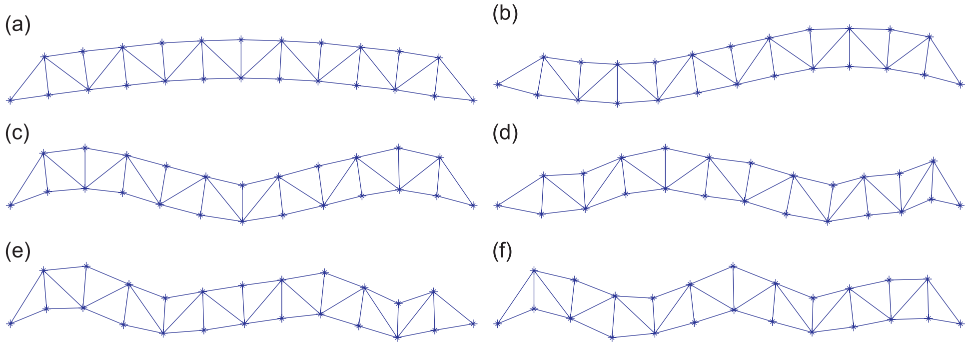

Fig. 9. Mode shapes identified in 10 measured points' case using MAC1: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

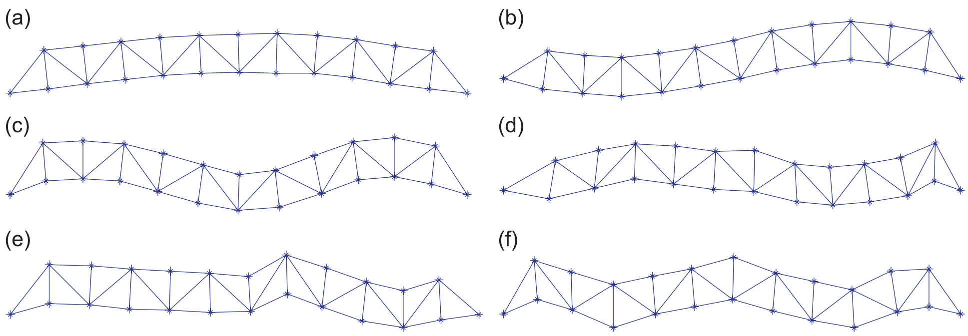

Fig. 10. Mode shapes identified in 10 measured points' case using MAC2: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

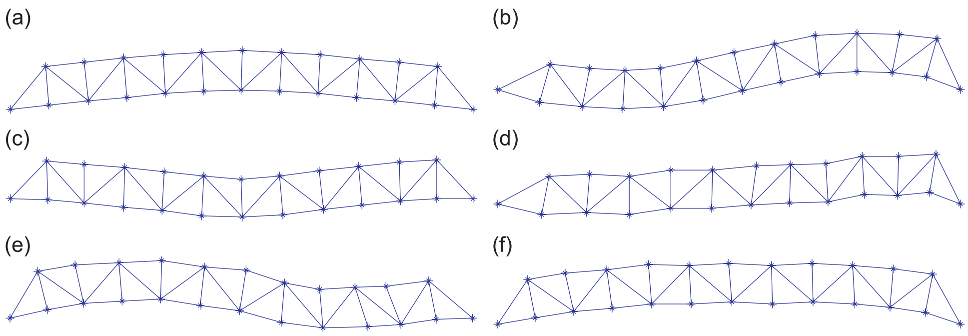

Fig. 11. Mode shapes identified in five measured points' case using MSE: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

as the decimal two-dimension array coding system and the forced mutation operator were proposed in this paper to enlarge the genes storage and improve the convergence of the algorithm. A 12-bay plain truss model was taken as the simulation example to demonstrate the feasibility of the three optimal sensor placement algorithms presented. The optimal sensor placement for 15, 10 and five sensors cases are obtained and studied in detail. Some conclusions and recommendations are summarized as follows:

(1) The dissipative storage space of the proposed decimal two-dimension array coding method is far less than the existing two kinds of binary coding methods. It is propitious for optimal sensor placement in spatial lattice structure because of the enormous dofs.

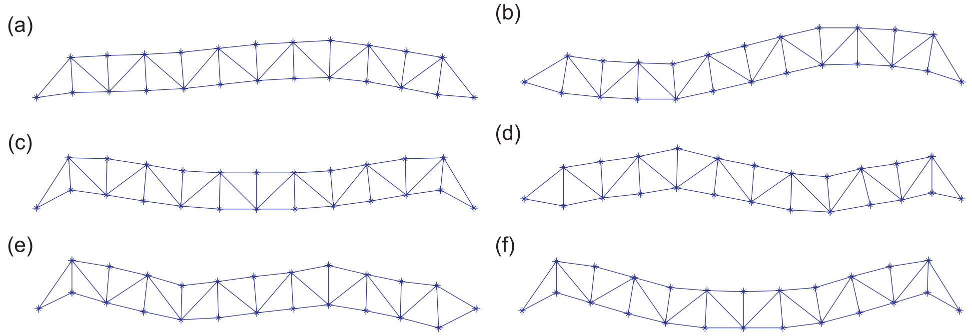

Fig. 12. Mode shapes identified in five measured points' case using MAC1: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

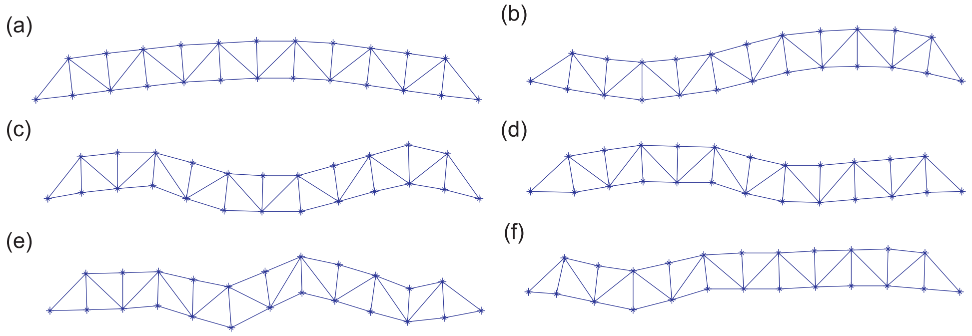

Fig. 13. Mode shapes identified in five measured points' case using MAC2: (a) the first mode shape; (b) the second mode shape; (c) the third mode shape; (d) the forth mode shape; (e) the fifth mode shape and (f) the sixth mode shape.

(2) The proposed forced mutation operator can avoid one location placed with two sensors synchronously in the crossover process. We can reduce the ratio of the natural mutation operation as a compromise for the reasonable compatibility of GA.
(3) The convergences of the improved GA using different fitness functions under different sensor placement cases are all better than those of the existing GA with binary coding method. In total, 20–30% reduction in computational iterations can be gained to reach the satisfactory solutions.
(4) The modal frequencies can be accurately identified even if only five sensors are optimally placed on the structure. And the mean square errors between the FEM mode shapes and the Guyan expansion mode shapes which are identified by SSI-DATA method and then expanded by the Guyan expansion technique are all very small.
(5) By comparing the identified Guyan expansion mode shapes with those calculated by FEM, the results obtained by the improved GA based on MAC fitness function is better than those obtained using MSE fitness function in five measured points' case. With the increasing of placed sensors, the three methods based on different fitness function can all provide the reliable optimal sensor placement to identify the vibration characteristics of the 12-bay plain truss model accurately.
(6) The GA is particularly effective in solving the combinatorial optimization problem such as optimal sensor placement problem when the performance tradeoffs are not unbearable and when the number of combinations is too large to preclude enumeration.

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