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Vibration mechanism of a mistuned bladed-disk

Y.J. Yan[a,*], P.L. Cui[a], H.N. Hao[b]

[a]School of Mechanics, Civil and Architecture, Northwestern Polytechnical University, Xi'an 710072, PR China
[b]School of Science, Xi'an Shi You University, Xi'an 710065, PR China

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Abstract

For coupling vibration of the mistuned bladed-disk in aeroengine, the vibration differential equation of the system is established using component-mode synthesis methods. Experimental mode analysis and mode correction are used to calculate some low-order modes of tuned blade and the disk with cone-flange. The models of actual mistuned bladed-disk is constructed by exerting some small perturbation on the mode stiffness of blades. The forced vibration experimentation of an actual mistuned bladed-disk in non-rotating status is carried out to verify the vibration model and calculation formulae. The results show that excessive vibration response of a single blade is caused by blade mistuning, and the random mistuning distribution of a small frequency difference is more profitable than other arrangements for depressing the maximal vibration level of the hapless blade. In this study, the better natural frequency difference amplitude of blades and the right working frequency range of aeroengine are also suggested according to the result analysis of several numerical calculations.

1. Introduction

For long time, vibration of the rotating blade in an aeroengine was taken as blade-root not to vibrate in many existed publications [1–4], i.e., the disk effect on blade vibration is ignored. For an initial jet aeroengine, since the disk is thicker and its vibration is very small, the hypothesis on the fixed blade root is probably close to real status between the blade and the disk. With developments in advanced performance and the high thrust-weight rate of a jet aeroengine, the rotating disk with installed multiple blades is gradually thinned and many inscrutable problems of blade vibration often emerge. For example, the same multiple blades installed on a disk behave very differently in their response features of forced vibration, i.e., only one or a few blades undergo severe vibration so as to anteriorly cause fatigue damage of a single blade. In fact, the above-mentioned issue has previously been discussed. In the 1960s, the concept of ''Rogue Blade'' was put forward [5,6]. For the origin caused to the Rouge Blade, the common comprehension is the influence of a disk on blade vibration, e.g., coupling vibration between a disk and blades. Griffin and Hoosac [7] showed that the maximum response amplitude of blades on a mistuned bladed-disk may be several times greater than that of

*Corresponding author. Tel.: +86 29 88492895.
E-mail address: yjyan_2895@nwpu.edu.cn (Y.J. Yan).

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the tuned one. Kaneko et al. [8] indicated that weakly coupled systems with low damping are very sensitive to mistuning. Mignolet and Hu [9] presented an approach to determine reliable estimates of the moments of the steady-state resonant response of a randomly mistuned bladed-disk and described the use of these moments to predict the corresponding distribution of the amplitude of blade vibration. Because of the large numbers of blades and the complex shapes of the blade and the disk, it is very difficult to establish an accurate model of structural dynamics for a mistuned blade-disk. Previously, multifarious simplified models, such as the model expressed by spring mass [10,11], the model of elastic bar and lumped mass [7], etc., were often adopted because of the limitations of computer capability. Due to differences of the built dynamics model and the adopted analysis methods, the results of coupling vibration analysis for disks and blades do not agree with one another, especially, which blade on the same disk will undergo the severest vibration level.

In the 21st century, a more advanced aeroengine with a higher thrust-weight rate needs to be developed. On the other hand, there is always some difference in the natural vibration frequencies of all blades installed on the same disk, which is called ‘‘blades mistuning’’, because of some unavoidable reasons, such as manufacturing tolerance, materials non-uniformity, installation differentia, etc. Obviously, blade mistuning is unavoidable. Therefore, the coupling vibration among disk and mistuned blades cannot be avoided and still needs to be explored. To date, the vibration fatigue damage of several blades on the rotating bladed-disk of an aeroengine sometimes occur, and it is very necessary to further study the coupling vibration of a mistuned bladed-disk. In 2001, Huang and Kuang [12] presented an investigation of the effects of centrifugal and Coriolis forces, on the mode localization of a mistuned blade–disk which is presented in this paper. A disk comprising of periodically shrouded blades is used to simulate the weakly coupled periodic structure. The Galerkin method is employed to derive the mode localization equations of the mistuned system with the consideration of Coriolis force. The blades are approximated as cantilever beams, and five axial and lateral modes of each blade are used to present the dynamic behavior of the system. Ten modal coordinates have been considered for each blade. The effects of Coriolis force and the magnitude of disorder on the localization phenomenon of a rotating blade–disk system were investigated numerically. Numerical results obtained herein indicate that the Coriolis force may enhance the localization phenomenon. In 2002, Feiner and Griffin [13] proposed a new reduced-order model of mistuned bladed-disk vibration. This new approach is shown to accurately represent the response of real turbine geometries when only a single family of modes is excited. Yet its mathematical form is even simpler than that of a mass–spring model. Because it requires only minimal input data, this model is much easier to use than previous reduced-order methods. Furthermore, its simplicity allows the fundamental parameters that control mistuning to be readily identified. In addition, Bladh et al. [14] also presented a study on the effects of random blade mistuning on the dynamics of an advanced industrial compressor rotor using a component-mode-based reduced-order model formulation for tuned and mistuned bladed-disks, and found mistuned forced response amplitudes and stresses vary considerably with mistuning strength and the degree of structural coupling between the blades. In 2003, Rivas-Guerra and Mignolet [15] presented a method to determine the maximum amplification of the steady-state forced response of bladed-disks due to mistuning, and proposed an optimization strategy in which partially mistuned bladed-disks are considered as physical approximations of the worst-case disk and the mistuned properties are sought to maximize the response of a specific blade. In 2005, Hou and Cross [16] proposed a study on minimizing the maximum dynamic response in a mistuned bladed-disk through design optimization. The problem was formulated as a constrained, nonlinear optimization process. It was found that the dynamic amplification factor of the maximum responding blade can be reduced to a range between 20% and 40% less than the tuned system for several combinations of engine excitation orders and coupling ratios. Recently, Castanier and Pierre [17] presented a review on reduced-order modeling, simulation and analysis of the vibration of bladed-disks found in gas-turbine engines, and showed that key developments in the last decade have enabled a better prediction and understanding of the forced response of mistuned bladed-disks, especially with respect to assessing and mitigating the harmful impact of mistuning on blade vibration, stress increases and attendant high-cycle fatigue.

In this study, by classifying and analyzing various bladed-disk structures in first stage of compressor in aeroengine, vibration model of a real bladed-disk is built. First, experimental mode analysis and mode correction are used to calculate the actual vibration modes of a blade and a disk, respectively. Compared with similar studies, the main contributions of this study are as follows: (1) owing to utilizing real solutions of bending–torsion coupling vibration of mistuned blades and theoretic solutions of disk and its flange vibration in establishing a dynamics model of a mistuned disk-blade using the modal synthetical method, the constructed dynamics model of the mistuned bladed-disk is more in accordance with a practical bladed-disk compared with the existing one; (2) the experimental validations for vibration modes of blades, disk and real bladed-disk are carried out for establishing the dynamics model; (3) real distribution laws of mistuned blades are experimentally measured, and the results show that the proposed optimal collocation of blade location can effectively restrain the greatest vibration level of blades.

Because the bladed-disk of an aeroengine always rotates at high speed when it operates, the rotor system consisting of a multiple-stage bladed-disk undergoes strong centrifugal force. Owing to the action of centrifugal force on the disk and blades, the "dynamic stiffening" in a bladed-disk will be very strong so that the natural frequencies of disk and blades will increase with rotating speed. Based on two causes, the effect of rotational speed is not considered in the dynamics model in this study. First, it is difficult and expensive experimentally to measure the vibration of a bladed-disk at high rotating speed with tremendous aerodynamic load. Since the experimental data with rotational speed are unavailable, the numerical analytical results for coupling vibration of a bladed-disk cannot be compared with the experimental result. Secondly, coupling between disk and blades should not be affected by centrifugal force field, and in most of the publications on vibration analysis for a bladed-disk, centrifugal force caused by rotational speed is almost ignored.

2. Vibration equation of a bladed-disk

In an aeroengine compressor, to assemble many working blades onto a disk, a cone-flange with a disk is necessary. A typical structural model of the first stage bladed-disk of an aeroengine compressor can be generalized as a model as shown in Fig. 1(a) and (b). Generally, a bladed-disk can be divided into three parts as sub-structures: (1) a variable-thickness disk, the center of which gets a fixed stay, (2) a cone-flange connected to the disk rim, whose thickness is variable, (3) multiple blades, which are twisty along the blade center axes. While establishing the mechanical model of a bladed-disk, two reasonable assumptions are given.

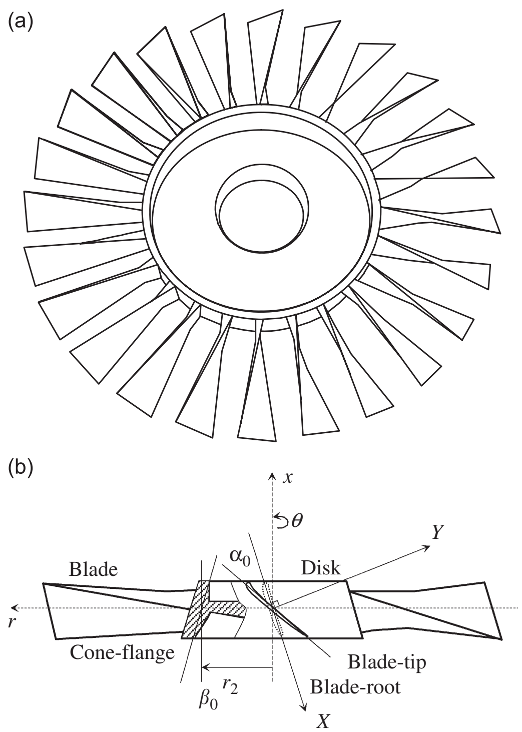

Fig. 1. Mechanical model of a typical bladed-disk: (a) framework figure of an experimental bladed-disk and (b) model of a bladed-disk.

First, one can assume that all blades are fixed on the cone-flange connected to the external rim of the disk. Secondly, one can be hypothetical that the longitudinal flexing of the cone-flange can be ignored because of the reinforced effect of multiple blade roots.

In order to acquire the equation of motion of a bladed-disk using component-mode synthesis methods, the experimental mode analysis and mode correction separately to the blades and the cone-flange with the disk are carried out. The obtained vibration modes of the blades and the disk can be described as follows:

The mode of the disk can be expressed as $w_{ij}(r)\cos j\theta$ $i, j = 0, 1, 2$, where i is the number of the node cycle, j the number of node diameter of the disk mode and $w_{ij}(r)$ is the displacement in normal direction of the disk middle plane.

The three components of the bend–torsion combination vibration mode for the blade can be, respectively, denoted as $u_k(r)$, $v_k(r)$, $\phi_k(r)$, $k = 1, 2$, where k is the number of the mode order. The $u_k(r)$ and $v_k(r)$ are transverse displacement components of a blade bending center in r location to a disk center, and $\phi_k(r)$ is the torsion component of the vibration mode of the blade.

Selecting multiple vibration modes of the blade and the disk, and using the mode synthesis techniques, the vibration displacement of every sub-structure can be expressed as:

For disk:

$$w(r, \theta, t) = \sum_{i=0}^{m} \sum_{i=0}^{n} w_{ij}(r) \cos j\theta q_{ij}(t) \tag{1}$$

where $q_{ij}(t)$ is the generalized coordinates, ij is an integer variable, m is an integer which denotes the maximum number of the node cycle and n is the maximum number of the node radius in the selected disk mode. Then, the integer variable ij can be expressed as $ij = (j+1) + i(n+1) = 1, 2, \ldots, (m+1)(n+1)$, where $i = 0$ and $j = 0$ denote the vibration mode with 0 node cycle and 0 node radius, respectively.

For the pth mistuned blade:

$$\left.\begin{aligned} u_p(r,t) &= -[w(r_2,\theta_p,t) + w'_r(r_2,\theta_p,t)r]\cos(\alpha_o) + \sum_{k=0}^{s} u_{Nkp}(t) \\ v_p(r,t) &= [w(r_2,\theta_p,t) + w'_r(r_2,\theta_p,t)r]\sin(\alpha_o) + \sum_{k=0}^{s} v_{Nkp}(t) \\ \phi_p(r,t) &= \sum_{k=1}^{s} \phi_{pk}(r) q_{Nkp} \end{aligned}\right\} \tag{2}$$

where $u_p(r, t)$, $v_p(r, t)$ and $\phi_p(r, t)$ are the vibration displacement of the pth blade fixed on the disk, and $u_{pk}(t)$, $v_{pk}(t)$ and $\phi_{pk}(t)$ are the kth order mode of the pth blade. α_o is the installed angle of blade root, and s is the number of selected blade vibration mode, $w'_r = \partial w(r, \theta, t)/\partial r$. q_{Nkp} is the generalized coordinate and Nkp is an integer variable which denotes the sequence of the selected vibration mode of the mistuned blades. Thus,

$$Nkp = (m+1)(n+1) + k + (p-1)s, \quad k = 1, 2, \ldots, s, \quad p = 1, 2, \ldots, N$$

and N is the total number of all blades installed on the disk.

For the cone-flange:

Set circular cylindrical coordinates $xh\theta$ in the cone-flange as shown in Fig. 1, and suppose that the axial (x), radial (h) and circumferential (θ) vibration displacements are $w_c(x, h, \theta, t)$, $v_c(x, h, \theta, t)$ and $u_c(x, h, \theta, t)$, respectively.

According to the displacement coincident relationship of the cone-flange with the disk rim and the distortion assumption, the vibration displacements of the cone-flange can be expressed as

$$\left.\begin{aligned} w_c(x,h,\theta,t) &= w(r_2,\theta,t) + hw'_r(r_2,\theta,t) \\ v_c(x,h,\theta,t) &= [h/(r_2 - xtg\beta_o)]w''_r(r_2,\theta,t) \\ u_c(x,h,\theta,t) &= -w'_r(r_2,\theta,t)x \end{aligned}\right\} \tag{3}$$

where $x_1 \leqslant x \leqslant x_2$, $h_1 \leqslant x \leqslant h_2$, $h_1 = -h_0(x)/2$, $h_2 = h_0(x)/2$. β_0 is the half cone angle and $h_0(x)$ is the average thickness of the cone-flange.

According to the vibration displacement of every sub-structure, their vibration speed can be expressed as

$$\begin{aligned} &\dot{w} = \partial w/\partial t \\ &\dot{u}_p = \partial u_p(r,t)/\partial t, \quad \dot{w}_c = \partial w_c(x,h,\theta,t)/\partial t \\ &\dot{v}_p = \partial v_p(r,t)/\partial t, \quad \dot{u}_c = \partial u_c(x,h,\theta,t)/\partial t \\ &\dot{\phi}_p = \partial \phi_p(r,t)/\partial t, \quad \dot{v}_c = \partial v_c(x,h,\theta,t)/\partial t \end{aligned}$$

Thus, the vibration kinetic energy and potential energy of all sub-structures can be calculated.

2.1. Vibration energy of every sub-structure

The kinetic energy of the disk is

$$T_1 = \frac{1}{2}\iint_D \rho_1 h(r)\dot{w}^2 r\,\mathrm{d}r\,\mathrm{d}\theta \tag{4}$$

where D is the integration field of the disk, ρ_1 is the material density of the disk and $h(r)$ is the thickness of the disk.

The kinetic energy of all N mistuned blades is

$$T_2 = \sum_{p=1}^{N}\frac{1}{2}\left\{\int_{r_2}^{r_2+L} \rho_2 A(r)[\dot{u}_p^2 + \dot{v}_p^2 + (J_{rp}/A(r))\dot{\phi}_p^2 + 2(x_r\dot{v}_p - y_r\dot{u}_p)\dot{\phi}_p]\,\mathrm{d}r\right\} \tag{5}$$

where L is the length of the blade, $A(r)$ is the sectional area of the blade, ρ_2 is the material density of the disk, $J_{rp}(r)$ is the rotary inertia about the bending center of the blade section, and x_r and y_r are the coordinates of the bending center, respectively.

The kinetic energy of the cone-flange is

$$T_3 = \frac{1}{2}\iiint_c \rho_1(\dot{w}_C + \dot{u}_C^2 + \dot{v}_C^2)R\,\mathrm{d}x\,\mathrm{d}h\,\mathrm{d}\theta \tag{6}$$

where C is the integration field of the cone-flange and $R = (r_2 - xtg\beta_0 + h)$.

The elastc deformation energy of the disk is

$$\begin{aligned} U_1 = \frac{1}{2}\iint_D \frac{E_1 h^3(r)}{12(1-\mu_1^2)}&\left[\left(w''_{rr} + \frac{1}{r}w'_r + \frac{1}{r^2}w''_{\theta\theta}\right)^2 - 2(1-\mu_1)w''_{rr}\left(\frac{1}{r}w'_r + \frac{1}{r^2}w''_{\theta\theta}\right)\right. \\ &\left. +2(1-\mu_1)w''_{rr}\left(\frac{1}{r}w''_{r\theta} + \frac{1}{r^2}w'_\theta\right)\right] r\,\mathrm{d}r\,\mathrm{d}\theta \end{aligned} \tag{7}$$

where E_1 is the elastic modulus of the disk, μ_1 is Poisson's ratio, w'_r and w'_θ are the first-order partial derivatives of $w(r, \theta, t)$, w''_{rr} and $w''_{\theta\theta}$ are the second-order partial derivatives of $w(r, \theta, t)$ and $w''_{r\theta}$ is the mixed partial derivatives of $w(r, \theta, t)$.

The elastic deformation energy of the N mistuned blades is

$$\begin{aligned} U_2 = \sum_{P=1}^{N}\frac{1}{2}\int_{r_2}^{r_2+L}&\Big[E_2(J_{Gy}u''^2_p + J_{Gx}v''^2_p + 2J_{Gxy}u''_p v''_p + 2\alpha'(J_{Tx}v''_p \\ &+ J_{Ty}u''_p)\phi'_p + (G_2 I_T + E_2 J_2)\phi'^2_p\Big]\,\mathrm{d}r \end{aligned} \tag{8}$$

where E_2 is the elastic modulus of the blade materials, G_2 is the shear modulus, α' is the natural torsion ratio, I_r is the torsion-resistance coefficient, J_{Gx}, J_{Gy} and J_{Gxy} are the inertia moments and inertia product about the mass center of the blade sections, and J_{Tx}, J_{Ty} and J are the high-order moment of the blade section about its bending center.

According to elastic mechanics, the non-zero strain on the cone-flange can be written as

$$\varepsilon_\theta = (\partial v_c/\partial\theta + u_c)/R, \quad \tau_{x\theta} = R\partial(v_c/R)\partial x + (\partial w_c/\partial\theta)/R \tag{9}$$

So the *elastic deformation energy* of the *cone-flange* is

$$U_3 = \frac{1}{2}\iiint_C \left(\frac{E_1}{1-\mu_1^2}\varepsilon_\theta^2 + \frac{E_1}{2(1+\mu_1^2)}\tau_{x\theta}^2\right) R\,\mathrm{d}h\,\mathrm{d}x\,\mathrm{d}\theta \tag{10}$$

2.2. *The equation of a motion of mistuned bladed-disk*

Based on the known vibration modes, structural parameters, the expressions of vibration kinetic energy and elastic deformation energy can be written as a quadratic form function of generalized speed $\dot{q}(t)$ and generalized coordinate $q(t)$ follow as:

$$\left.\begin{aligned} T(\dot{q}_{ij}, \dot{q}_{Nkp}) &= T_1(\dot{q}_{ij}) + T_2(\dot{q}, \dot{q}_{Nkp}) + T_3(\dot{q}_{ij}) = (1/2)\{\dot{q}\}^{\mathrm{T}}[M]\{\dot{q}\} \\ U(q_{ij}, q_{Nkp}) &= U_1(q_{ij}) + U_2(q_{Nkp}) + U_3(q_{ij}) = (1/2)\{q\}^{\mathrm{T}}[K]\{q\} \end{aligned}\right\} \tag{11}$$

where $\{q\} = \{q_{ij}, q_{Nkp}\}^{\mathrm{T}}$ and $\{\dot{q}\} = \{\dot{q}_{ij}, \dot{q}_{Nkp}\}^{\mathrm{T}}$, $[M]$ and $[K]$ are the total mass matrix and total stiffness matrix of the system, and their element values can be calculated using numerical integration.

When the system vibrates freely, by taking T and U into Lagrange equation one can obtain

$$d(\partial T/\partial \dot{q}_t)\,\mathrm{d}t + \partial U/\partial q_t = 0, \quad l = 1, 2, \ldots, (m+1)(n+1) + Ns \tag{12}$$

So the equation of motion of the free vibration for a mistuned bladed-disk can be written as

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\} \tag{13}$$

where the $\{\ddot{q}\}$ is the generalized acceleration.

For forced vibration of a bladed-disk, periodical excitation is taken into account. The periodical excitation on the operating cascade is created because of the wake of struts, guide vane or distortion of inlet flow field. The circumferential non-uniformity of the exciting force can be decomposed using Fourier series, and the radial non-uniformity can be decomposed by every mode of the blade. In a fixed coordinate system, any component force exerted on the blade can be expressed as $F_k \cos j\theta$, which corresponds to the component force of the kth order blade mode and the disk mode with the jth nodal diameter. In a coordinate system moving with the bladed-disk, the exciting force exerted on the pth blade can be written as

$$F_{Nkp} = F_k \cos j(\omega t + \theta_p) = F_k \cos j\omega t \cos j\theta_p - F_k \sin j\theta_p \sin j\omega t \tag{14}$$

where ω is the angular velocity of the bladed-disk, θ_p is the direction angle of the pth blade and F_k is amplitude of the exciting force. Apparently, $j\omega j\theta_p$ denotes the exciting frequency and phase. In numerical analysis, each part in the expanded form of F_{Nkp} can be calculated as one exciting force.

From Eq. (14), an exciting force vector $\{F(t)\}$ can be formed by changing $p = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, s$. In this study, only the vibration damp of blades is considered, and the mode damping ξ_{kp} is obtained by experimental modal analysis to every blade. Therefore, the component of the modal damping force exerted on the kth mode of the pth blade can be expressed as

$$D_{Nkp} = -\xi_{kp}\dot{q}_{Nkp} \tag{15}$$

Taking $\{F\}$ and $\{D\}$ into Eq. (13), the forced vibration equation of the mistuned bladed-disk can be written as

$$[M]\{\ddot{q}\} + [C]\{\ddot{q}\} + [K]\{q\} = \{F\} \tag{16}$$

2.3. The equation of motion of a tuned bladed-disk

When all blades installed on a disk are completely same, the bladed-disk is called the tuned bladed-disk. Obviously, this is only a kind of idealization case. Because the N blades installed on the disk only have s different modes, the corresponding generalized coordinate q_{Nkp} of the tuned bladed-disk is only related to the nodal diameter number j of the disk mode and the mode order k of a blade, while it is not relevant to the serial number of every blade. Thus, the number of the generalized coordinate q_{Nkp} can be reduced as the new generalized coordinate g_{skj} based on the nodal diameter number j of the disk mode. The relationship between g_{skj} and q_{Nkp} can be expressed as

$$q_{Nkp} = g_{skj}\cos(j\theta_p), \quad sk_j = (m+1)(n+1) = k = js \tag{17}$$

Irrespective of being mistuned or harmonious, the generalized coordinate of the disk and cone-flange does not change. When $j = 1, 2, \ldots, s$ and $p = 1, 2, \ldots, N$, a transformation relation can be acquired from Eq. (17) as follow

$$\{q\} = [\psi]\{g\} \tag{18}$$

where $[\psi]$ is a transformation matrix and $\{g\} = \{q_{ij}, g_{skj}\}^{\mathrm{T}}$ is the generalized coordinate vector of the tuned bladed-disk.

One can realign the displacement expressions of all sub-structures based on the generalized coordinate of the tuned bladed-disk $\{g\}$, so that the mass matrix $[M_g]$ and stiffness matrix $[K_g]$ of the tuned system are obtained using the foregoing procedures; Using the transformation matrix $[\psi]$, one can derive the exciting force of the tuned system as being $\{F\}_g = [\psi]\{F\}$, and the damping force$\{D\}_g = [\psi]\{D\}$. Therefore, the free vibration and forced vibration equations of the tuned bladed-disk can be written as follows:

$$[M_g]\{\ddot{g}\} + [K_g]\{g\} = \{0\} \tag{19}$$

$$[M_g]\{\ddot{g}\} + [C_g]\{\ddot{g}\} + [K_g]\{g\} = \{F_g\} \tag{20}$$

The solutions of the free vibration and forced vibration of the tuned and mistuned bladed-disk can be obtained by solving Eqs. (13), (16), (19) and (20).

3. Example of a mistuned bladed-disk

3.1. The model parameters and natural frequency experiments

The example model in this study is from the first-stage bladed-disk of the low-pressure compressor in some type of aeroengine. The model parameters are listed in Table 1. To obtain every sub-structure mode, their natural frequencies are experimentally measured. Because the disk and cone-flange intersects, they are taken as one. The natural frequency and corresponding nodal diameter and nodal cycle are listed in Table 2.

When a blade is taken as a sub-structure, it can be considered as the fixed root. The natural frequency experiment results in the status are listed in Table 3. If the interested frequency for the bladed-disk is confirmed as 1000 Hz, Tables 2 and 3, the first three order modes of the disk and blade should be taken into the mode synthesis. When the tuned bladed-disk model is build, the natural frequencies obtained by experiment for one blade can be reckoned as all blades on the disk.

As mentioned before, frequency mistuning of blades on a disk is not avoidable in a practice aeroengine, but the mistuning distributing can be changed artificially because this can be carried out by rearranging the position of all blades on the disk. First, a real blade mistuning distributing of the selected bladed-disk is obtained by measuring the first four natural frequencies of all 31 blades on the disk, and the first and second orders of measured natural frequencies are shown in Fig. 2. Fig. 2 shows that the mistuning distribution of actual blades is random. For the mistuned frequency difference, as the first order, the maximum is 161.25 Hz and the minimum is 127.75 Hz. The frequency difference is 23.7% relative to the average value. It may be a little large compared with the practical situation because the experimental measure is taken in the condition of the non-rotating bladed-disk. It is because the blade-roots are not fixed completely. When the bladed-disk is in

Table 1
Parameters of the bladed-disk

Structural parameters	Symbols	Values
Number of the blades on the disk	N	31
Blade length	L	0.184 m
Blade warping angle between the root and tip	θ	38°
Installed angle of blade root	α_0	3°
Disk radius	r_2	0.14 m
Half cone angle of the cone-flange	β	28°
Longitudinal length of the cone-flange	x_2-x_1	0.075 m

Table 2
Experimental results of disk and cone-flange natural frequency

Mode shape		Natural frequency (Hz)
Nodal diameter	Nodal cycle	
0	0	634.39
1	0	519.61
2	0	924.24
3	0	2854.60
0	1	5446.7

Table 3
Experimental results of a single blade

Mode order	Measured natural frequency (Hz)
1	141.49
2	406.94
3	935.56
4	1546.7

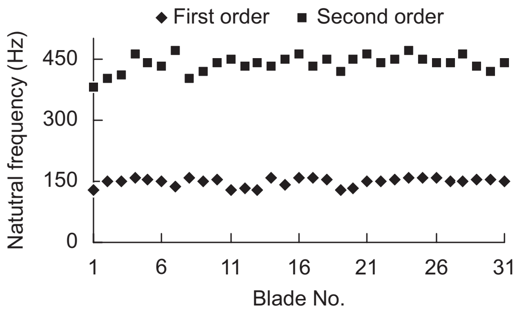

Fig. 2. Mistuning distributing of measured natural frequencies of blades on a disk.

a rotating status at high speed, the fixing of a blade-root can be reinforced because of centrifugal force. Hence the actual frequency difference is much smaller, but its distribution tendency should not change much.

In order to find the most appropriate blade mistuning distributing rule for reducing the vibration level of blades on a disk, different mistuning rules can be constructed using a small quantity change of the mode stiffness of blades in numerical simulations. The acquired experimental mode shape must be properly modified before they are taken into the mode synthesis.

3.2. Experimental validation

To validate the calculation formulae and model parameters, the forced vibration experiments of the selected bladed-disk are carried out. In the experiment, the central axis of the bladed-disk is fixed onto the vibration table, and the sine excitation is exerted on it. The vibration amplitude of all 31 blade-tip centers are measured, and they are compared with the numerical calculation results in the same condition. In numerical simulations, one supposes that the motion of the bladed-disk is

$$x(t) = A_O \sin \Omega t \tag{21}$$

where A_O is the exciting amplitude and Ω is the exciting frequency.

At the moment, the bladed-disk receives basic excitation, and the inertia exciting force created by the mode mass of the bladed-disk can be written as

$$\{F\} = -[M]\{\ddot{x}(t)\} = [M]\{A_0\Omega^2\} \sin \Omega t \tag{22}$$

where $[M]$ is the mass matrix of the mistuned bladed-disk system.

Because the measured data $c_p(t)$ is the amplitude of the mass center of the blade-tip, data u_p, v_p and ϕ_p obtained by numerical simulation should be transformed into $c_p(t)$ so that they can be contrasted. The relationship between them can be expressed as

$$c_p(t) = (u_p - y_r\phi_p) \cos \alpha_L + (v_p + x_r\phi_p) \sin \alpha_L + \sin(\alpha_0 + \alpha_L)A_0 \sin \Omega t \tag{23}$$

where p denotes the pth blade and α_L is the directional angle of the main inertia axis in the blade-tip section. u_p, v_p and ϕ_p are the same as in Eq. (2).

$c_p(t)$ in Eq. (23) can also be written in the compact form as

$$c_p(t) = c_{op} \sin(\Omega t + \beta_{op}) \tag{24}$$

In this experiment, the response amplitudes of all 31 blade tips under eight exciting frequencies are measured. The experimental and numerical results under 125 Hz exciting frequency are shown in Fig. 3. Results show that both are in accordance with each other. This indicates that the hypothesis about the model in this study is correct, and also demonstrates the credibility of the theoretical analysis method. As shown in Fig. 2, the first-order natural frequencies of the 1st, 7th, 11th and 19th blade of the mistuned bladed-disk are all smaller, but one of the 1st blade is smaller. In the distribution of forced response amplitude of blades in Fig. 3, the response amplitude of the 1st blade is the greatest; this is because the used excitation frequency 125 Hz is most closest to the first-order natural frequency of the 1st blade, which causes the resonance response of the 1st blade. In fact, this phenomenon can explain why some blades solely undergo the most severe vibration level.

3.3. Numerical results and analysis of bladed-disk vibration

In order to analyze influence of blade mistuning on the dynamic characteristic and the vibration response characteristic of a bladed-disk, six kinds of mistuning distribution models are factitiously designed, which are

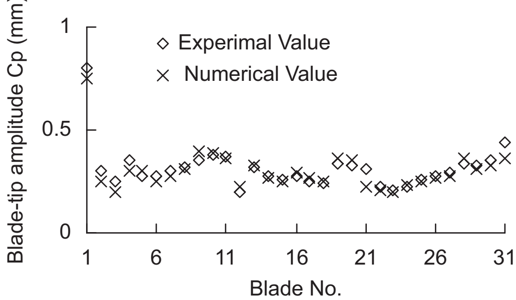

Fig. 3. Response amplitude distributing of blade tips under exiting frequency 125 Hz.

random distribution (RAM), crossed distribution (CRO), two kinds of cosine distribution ($\cos\theta$ and $\cos 2\theta$), and only single-order random mistuning (SIG1), and only second-order random mistuning (SIG2). As a comparable baseline, the tuned bladed-disk is marked TUN. The numerical simulations under the seven distributions, the exciting frequency range from 110 to 170 Hz, the mistuned frequency difference from 1% to 12%, and the four exciting forces ($F_1\cos\theta$, $F_2\cos\theta$, $F_1\cos 2\theta$ and $F_2\cos 2\theta$) are fulfilled.

3.3.1. Free vibration characteristic of a bladed-disk

First, modal analyses of the tuned and mistuned bladed-disk models are carried out using the given model parameters and the distribution data of the blade mistuning blades, which are experimentally measured. The natural frequency distributions of the two models are shown in Fig. 4. The mode shape of the TUN is shown in Fig. 5 which is plotted using amplitudes of the 31 blades on the disk. A typical mode shape of the mistuned is shown in Fig. 6.

The obtained natural frequencies and mode shapes of the bladed-disk show that the natural frequency of a mistuned bladed-disk is denser and has a wider scatter compared to that of the tuned harmonious one. This indicates that the more resonance may occur in the mistuned bladed-disk. On the other hand, the mode shape of the tuned bladed-disk is either regularly distributed in cosine wave or the constant, but the mode shape of the mistuned bladed-disk is very anomalous, i.e., the amplitudes of few mistuned blades are observably large. Since the forced vibration response is the sum of all modes at different proportions, the response amplitude of every blade on the tuned bladed-disk will be the same consequentially, but the total result in the mistuned bladed-disk may be that the response amplitude of the single blade is much larger than others. Obviously, it is very much possible that this kind of blade becomes the most hapless, that is the so-called "Rogde Blade". That is, the vibration mechanism of the mistuned bladed-disk can also give a logical explanation to the advanced damage of a single blade.

3.3.2. Resonance response characteristic of a bladed-disk

The research shows that the effects of blade mistuning are mainly in the resonance region of the system. When it is far from the resonance section, the effect of the mistuning is very small. The maxim amplitude response curves of the blade-tip on 6 mistuned and the tuned models with an exciting frequency range from 120 to 160 Hz are shown in Fig. 7.

Generally, in the resonance region, the response at the mistuned is larger than that at the tuned one, and it differs in different mistuning forms. Since the peak response at the resonance region depends mainly on system damping, the mistuning effect to the peak value is secondary. Perhaps, the worst problem is that the resonance range of the mistuned bladed-disk is much large than that of the tuned bladed-disk. It can be seen from Fig. 7 that the mistuned blade can produce resonance in a larger frequency range (i.e. rotating speed of the aeroengine). Thus, the dispersion degree of natural frequency for a mistuned blade has a direct effect on the width of the resonance bandwidth, and the detailed analysis is as follows.

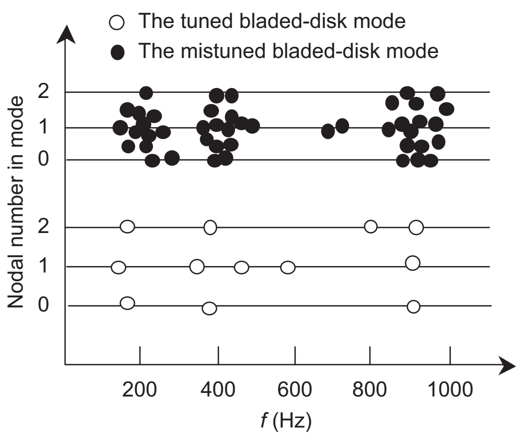

Fig. 4. Natural frequencies of the tuned and mistuned bladed-disks.

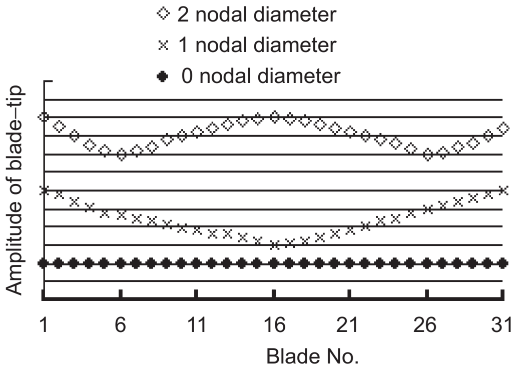

Fig. 5. Mode shapes of the tuned bladed-disk.

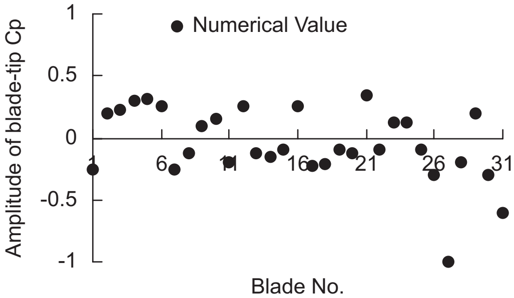

Fig. 6. Some order mode shape of a mistuned bladed-disk.

If the maximum resonance amplitude is $\overline{Q}_M$, one can obtain the two frequencies F_1 and F_2 corresponding to two points on the response curve at $\overline{Q}_M/2$ ($F_1 > F_2$). Let the dispersion degree of the mistuned blade frequency $\Delta F = (F_{\max} - F_{\min})/(F_{\max} + F_{\min})$, where $F_{\max}$ and $F_{\min}$ are the maximal and the minimal natural vibration frequency of all blades in the same order mode for a group of mistuned blades. Also suppose the non-dimensional resonance bandwidth coefficient $\eta = (F_2 - F_1)/(F_2 + F_1)$. The relationship between ΔF and η under random misting is showed in Fig. 8. It can be seen from Fig. 8 that the larger the ΔF, the larger the η. If ΔF is $>3\%$, then η enlarges obviously. If $F_n = n/60$ is the working frequency of an aeroengine where n is the rotating speed and F_c is the center frequency of the resonance amplitude, the maximal amplitude of the bladed-disk is always less than $\overline{Q}_M/2$ when $F_c(1+\Delta F) < F_n < F_c(1-\Delta F)$.

$\overline{Q}_M$ values with different mistuning distributing under the excitation of the $F_1 \cos\theta$ are listed in Table 4 where $F_{e\max}$ is the exciting frequency corresponding to $\overline{Q}_M$. The maximal response amplitudes of the blade-tip response with only a single-order natural frequency mistuning under different exciting forces are listed in Table 5.It can be seen from Table 4 that $\overline{Q}_M$ under $\cos\theta$ mistuning is the largest and $\overline{Q}_M$ under CRO is the second largest. This may be because that the exciting force is $\cos\theta$-type so that it is more possible for the $\cos\theta$ mistuned distribution to cause resonance. In the view of the requirement to decrease the vibration level, the CRO-type mistuned distribution is also not a good choice, and this distributing is not easy to realize in practice. The $\overline{Q}_M$ under the RAM-type and SIG1-type mistuning are almost the same because their first-order natural frequencies are all random mistuned. Since the frequencies distributing under the SIG2-type mistuning and the TUN are all tuned, their $\overline{Q}_M$ are quite similar and small. But both the SIG1-type and SIG2-type mistuning distributing need more than one tuned order, and these two types are not easy to be realized in practice. Summarizing the above, the random mistuned distribution with a small frequency difference should have a small vibration response and be easy to be realized.

According to data in Table 5, only the mode with the mistuning is excited, the vibration response level will be obviously increscent. For example, under the same exciting force $F_1 \cos\theta$, the maximal response of the SIG2-type mistuning is 17.4, but that of the SIG1-type is 37.2; this increases mostly to 114%. Obviously, if the

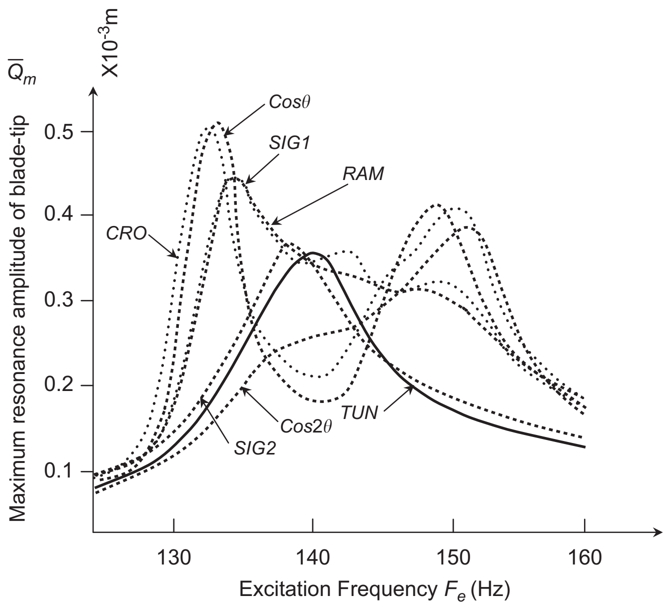

Fig. 7. Frequency responses of different mistuning distributing.

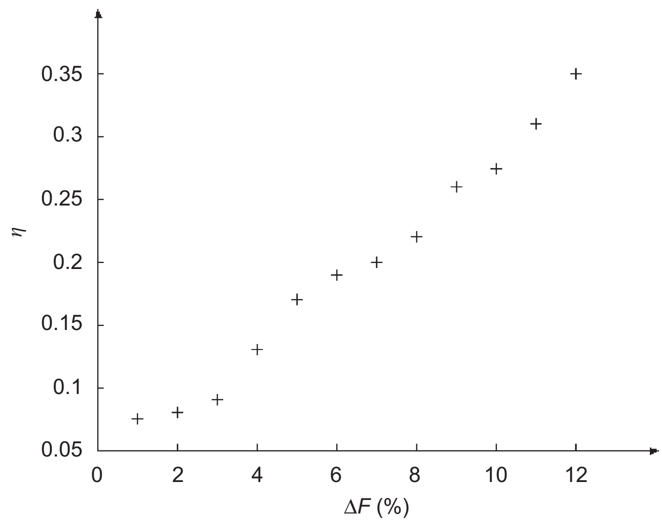

Fig. 8. Mistuning frequency difference η versus resonance bandwidth ΔF.

Table 4
$\overline{Q}_M$ of different mistuned distribution

Mistuned model	TUN	RAM	CRO	$\cos\theta$	$\cos 2\theta$	SIG1	SIG2
$\overline{Q}_M$	34.66	40.16	48.49	49.78	35.46	40.16	35.2
$F_{e\max}$ (Hz)	142	136	134	134	138	136	140

exciting frequency is close to some order natural frequency and we make this order frequency to be tuned as possible, this method can effectively reduce vibration response level. Especially, this method has a better effect on low-order vibration.

Table 5
Maximal response values under different exciting forces for single-order mistuning

Exciting frequency	135 Hz		415 Hz	
Exciting force	$F_1 \cos\theta$	$F_2 \cos\theta$	$F_1 \cos 2\theta$	$F_2 \cos 2\theta$
SIG1	37.2	0.013	0.36	2.09
SIG2	17.4	0.015	0.45	1.91

The coupling degree between the blade and the disk has a greater influence on the distribution of the natural vibration frequency of the bladed-disk system, so as to have an obvious effect on the vibration characteristic of the bladed-disk. This also indicates that there exists important internal relations between the blade mistuning and the coupling of the blades and the disk.

4. Conclusions

Based on the established mechanical model, the innovative theoretical analysis method and experiment measurements, research on the coupling vibration and mistuning effects of a bladed-disk in an aeroengine has been successfully carried out. Because blade mistuning in an aeroengine is unavoidable, both the rational explanations for the phenomenon of producing the excessive vibration response of a single blade are given, and technology methods on how to decrease the maximal response level by changing the misting distributing of blades on a disk.

The main conclusions obtained in this study are as follows:

(1) Because of blade mistuning, the natural frequency of a mistuned bladed-disk is denser and has a wider scatterance, and this indicates that more resonance may occur in the mistuned bladed-disk.
(2) Mode shape of the mistuned bladed-disk possesses the character of excessive large amplitudes in few blades, and these blades will potentially become the most hapless.
(3) Resonance range of the mistuned bladed-disk is much larger than that of the tuned bladed-disk, and the dispersion degree of natural frequency for mistuned blades has a direct effect on the width of the resonance bandwidth.
(4) The random mistuned distribution with a small frequency difference should have a small vibration response and be easy to be realized.
(5) When the exciting frequency is close to some order natural frequency of a bladed-disk, letting this order frequency to be tuned as possible, this can effectively reduce the vibration response level.

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