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Vibrational dynamic materials and composites

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Abstract

In this paper, the concept of dynamic materials is briefly outlined and exemplified following the work of the author published during the last decade. Then, a special kind of kinetic dynamic materials is introduced, the so-called 'vibrational dynamics' material with vibrational composites being its special case. A parametrically produced, vibrational dynamic material definition is introduced. Among such materials can also be a range of known systems, which can be considered to be capable of changing their properties under the action of vibration. As a case in point, an unusual dynamic material is considered, in the form of a pipe conveying a pulsating fluid.

1. Introduction

The concept and the idea of creating dynamic materials were formulated by K.A. Lurie and the author of this paper in Refs. [1,2]. Reviews of the development of this idea can be found in Refs. [3–5]. *Dynamic materials* are defined as media whose material parameters (density, stiffness, dissipative characteristics, self-induction, capacity, etc.) change in both space and time [2]. Two kinds of such materials can be distinguished. The first kind is referred to as the *activated dynamic materials*. Such materials are obtained by changing the material parameters of different regions of the medium in the absence of the relative motion. The materials of the second kind are called *kinetic dynamic materials*. These are obtained by means of endowing the whole system or some of its regions with some prescribed relative motion.

This paper introduces a special kind of kinetic dynamic materials, which are generalized as *vibrational dynamic materials*. Such materials are defined as materials whose quasi-static properties change essentially under relatively high-frequency excitation. The theoretical basis for the realization of vibrational dynamic materials is *vibrational rheology* [3,6], which is the part of rheology that considers relatively slow changes in the rheological properties of bodies caused by high-frequency vibrations. An informal definition of vibrational rheology might be that it is a rheology for an observer who "does not see" fast forces acting on the material and fast motions.

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2. The concept of vibrational materials

To introduce vibrational materials, a one-dimensional rod is considered made of a nonlinear material whose stress–strain relation is given as

$$\sigma = f(\varepsilon, \dot{\varepsilon}). \tag{1}$$

According to the main idea of vibrational mechanics [3], when the rod is acted upon by a high-frequency excitation with a frequency Ω, the stress σ and the strain ε can be represented as

$$\sigma = \Sigma(t) + \sigma_1(t, \Omega t), \quad \varepsilon = E(t) + \varepsilon_1(t, \Omega t), \tag{2}$$

where t is the "slow time", $\tau = \Omega t$ is the "fast time", Σ and E are the slow components of the deformation field (may be constant), and σ_1 and ε_1 are the fast components of the deformation field. The mean values of these fields are zero:

$$\langle \sigma_1 \rangle = 0, \quad \langle \varepsilon_1 \rangle = 0. \tag{3}$$

The angular brackets in Eq. (3) and thereafter imply the following integration:

$$\langle \cdots \rangle = \int_0^{2\pi} \ldots \, \mathrm{d}\tau. \tag{4}$$

As follows from Eqs. (2) and (3), the average values of the complete stress and strain fields are given as

$$\langle \sigma \rangle = \Sigma(t), \quad \langle \varepsilon \rangle = E(t). \tag{5}$$

Substituting Eq. (2) into Eq. (1) and averaging the result, the following equation for the mean values is obtained:

$$\Sigma = F(E, \dot{E}), \quad F(E, \dot{E}) = \langle f(E(t) + \varepsilon_1(t, \tau), \dot{E}(t) + \dot{\varepsilon}_1(t, \tau)) \rangle. \tag{6}$$

This equation is the vibro-rheological characteristic of the material of the rod. From (6) we see that the ordinary material can be recovered when the fast actions are removed. Specific form of this characteristic is determined by the fast component of the strain, $\varepsilon_1(t,\tau)$.

The simplest case is when $\varepsilon_1(t,\tau)$ is prescribed kinematically, for example as

$$\varepsilon_1(t, \tau) = B(t) \cos(\Omega t + \beta(t)), \tag{7}$$

where $B(t)$ and $\beta(t)$ are slowly changing amplitude and phase, respectively.

A more complicated case of "dynamic" excitation is dealt with when $\varepsilon_1(t,\tau)$ should be found as a solution of equation of "fast" dynamic.

3. A nonlinearly elastic material

In this section, a material is considered, the stress field in which is independent of the stress rate. The following polynomial form of the stress–strain relation is assumed [7]:

$$\sigma = a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4 + \cdots, \tag{8}$$

where a_n are constants. Substituting Eqs. (2) and (6) into Eq. (8) and performing the averaging, the following expression is obtained for the vibro-rheological characteristic of the corresponding dynamic material:

$$\Sigma = A_0 + A_1E + A_2E^2 + A_3E^3 + A_4E^4 + \cdots, \tag{9}$$

where

$$\begin{aligned} A_0 &= \frac{1}{2}a_2B^2 + \frac{3}{8}a_4B^4 + \cdots, \quad A_1 = a_1 + \frac{3}{8}a_3B^2 + \cdots, \\ A_2 &= a_2 + 3a_4B^2 + \cdots, \quad A_3 = a_3 + \cdots, \quad A_4 = a_4 + \cdots. \end{aligned} \tag{10}$$

Eq. (9) can be easily obtained taking into account that $\langle \cos \Omega t \rangle = 0$, $\langle \cos^2 \Omega t \rangle = 1/2 \ldots$.

Hence, the effective modulus of elasticity E_V^* of the vibrational dynamic material under small deformations,

$$E_V^* = E^* + \frac{3}{8}a_3B^2 + \cdots, \tag{11}$$

differs from the corresponding modulus of the original material $E^* = a_1$. The dynamic material is stiffer if $a_3 > 0$ and is softer if $a_3 > 0$. It is remarkable that in this case, as a result of the action of vibration, there appears not only the transformation of the elastic characteristic of the material, but also the additional stress $\Sigma_0 = A_0$.

4. On some other materials

Similarly to the purely elastic material discussed in the previous section, it is possible to obtain the vibro-rheological characteristics of other materials, some of which are considered below.

Positional-viscous material (see Ref. [3] for details). For such material, in which the dissipation coefficient depends on deformation, the original and vibro-rheological stress–strain relations take the following form:

$$\begin{aligned} \sigma &= (a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4 + \cdots)\dot{\varepsilon}, \\ \Sigma &= (A_0 + A_1E + A_2E^2 + A_3E^3 + A_4E^4 + \cdots)\dot{E}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} &A_0 = a_0 + \frac{1}{2}a_2B^2 + \frac{3}{8}a_4B^4 + \cdots, \quad A_1 = a_1 + \frac{3}{8}a_3B^2 + \cdots, \\ &A_2 = a_2 + 3a_4B^2 + \cdots, \quad A_3 = a_3 + \cdots, \quad A_4 = a_4 + \cdots. \end{aligned} \tag{13}$$

If all a_n are positive, then vibration leads to the increase of dissipation in the material. If some of a_n are negative, the dissipation may decrease. Moreover, as a result of vibration, the dissipation may become "negative" and self-excited oscillations of the material may occur, see Ref. [3].

Nonlinearly viscous material is described in Ref. [8]. For such material, in which the dissipation coefficient depends on velocity of deformation, the original and vibro-rheological stress–strain relations are of the form:

$$\begin{aligned} \sigma &= (a_0 + a_2\dot{\varepsilon}^2 + a_4\dot{\varepsilon}^4 + \cdots)\dot{\varepsilon}, \\ \Sigma &= (A_0 + A_2\dot{E}^2 + A_4\dot{E}_4 + \cdots)\dot{E}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} &A_0 = a_0 + \frac{3}{2}a_2(B\Omega)^2 + \frac{15}{8}\frac{15}{8}a_4(B\Omega)^4 + \cdots, \\ &A_2 = a_2 + 5a_4(B\Omega)^3 + \cdots, \quad A_4 = a_4 + \cdots. \end{aligned} \tag{15}$$

Just like in the previous case, the effect of vibration on dissipation in the material depends greatly on the signs of the coefficients a_n.

Elastic materials in which loading and unloading conditions generate different elastic modules can also be considered based on Ref. [1]. The simplest kind of such material, according to Ref. [9], is an elastic material, whose original and vibro-rheological relations look as [1,9]:

$$\sigma = \begin{cases} E_+\varepsilon & \text{if } \varepsilon > 0, \\ E_-\varepsilon & \text{if } \varepsilon < 0, \end{cases} \tag{16}$$

$$\Sigma = \begin{cases} E_+E & \text{if } E > B, \\ E_-E & \text{if } E < -B, \\ ((E_+ + E_-)/2 + (E_+ - E_-)\arcsin(E/B)/\pi) & \\ \quad +(B/\pi)(E_+ - E_-)\sqrt{1 - (E/B)} & \text{if } |E| < B. \end{cases} \tag{17}$$

The dependence given by Eq. (17) can be represented as

$$\Sigma(E) = \Sigma(0) + \Sigma_1(E); \quad \Sigma(0) = \frac{B}{\pi}(E_+ - E_1), \ \ \Sigma_1(0) = 0, \tag{18}$$

where $\Sigma(0)$ and $\Sigma_1(E)$ can be easily obtained from expression (17).

This representation shows that the resulting dynamic material has a "smooth" elastic characteristic (if $|E| < B$) in contrast to the piecewise character of the original one. Besides, additional stresses appear as a result of vibration. Elastic plastic material can be easily considered in the same manner [10,11].

5. An introduction to vibrational dynamic composites

Consider again a rod made of a nonlinear material. Assume that a standing plane dilatation wave of the length L is excited in the rod. Then, the amplitude $B_1(x)$ of the high frequency component of deformation (see Eq. (7)) will be a periodic function of x (the co-ordinate along the rod) with the period L, and expression for deformation will be written as $\varepsilon_1(\tau,x) = B_1(x)\cos\Omega t$. As a result, the vibro-rheological characteristic of the dynamic material described by Eq. (5) will also vary periodically along the rod, i.e. a *vibrational dynamic composite* will be obtained. In particular, in case of a nonlinearly elastic material, the modulus of elasticity of the vibrational dynamic composite under small deformations will be the following L-periodic function of x:

$$E_V^*(x) = E_V^*(x+L) = E^* + \frac{3}{8}a_3 B_1^2(x) + \cdots. \tag{19}$$

Elastic waves of dilatation, if sufficiently long relative to L, would propagate in such material with the velocity which can be larger or smaller than velocity $c = (E^*/\rho)^{1/2}$ depending on the sign of the a_3 coefficient.

If the wave amplitude $B_1(x)$ can be presented as

$$B_1(x) = B_0 \cos(2\pi x/L), \quad B_0 = \text{const}, \tag{20}$$

then the following expressions for the dynamic elasticity modulus in the dynamic composite would be realized:

$$E_V^*(x) = E_V^*(x+L/2) = E^* + \frac{3}{8}a_3 B_0^2 \cos^2(2\pi x/L) + \cdots. \tag{21}$$

The generalization to more complicated materials, including electromagnetic materials, for example, presents no special problem.

6. Parametrically produced vibrational dynamic material. Example: a tube conveying a pulsating fluid

Vibrational dynamic material can be created by means of high-frequency parametric excitation of original linear or nonlinear elastic material. In this sense, a simple example of a structure made of dynamic material is a rod exposed to the parametrical excitation. As is well-known [12], such an excitation produces stabilization effect—so-called Indian magic rope impressively exemplifies this phenomenon. As is shown in Ref. [13], the bending stiffness of a rod increases due to vertical vibrations of the bottom end in certain conditions. Therefore, the rod acquires stable straight vertical shape.

A pipe conveying a pulsating fluid may also be regarded as an unusual example of a parametrically produced dynamic material. The destabilizing effect of the flow, both stationary and pulsating, is well known. In the stationary case it is displayed by reducing to zero the main frequency of free oscillations of the pipe with the fluid, provided that both ends of the pipe are immovable. If the fluid is pulsating, the destabilization may occur also via parametric resonance.

Paidoussis and Sundarajan [14] have discovered that the pulsations of the flow at certain frequencies and amplitudes may stabilize the pipe which had been unstable in the absence of the pulsations. This effect has the same physical foundation as in the case of the magic Indian rope [13]. In the following, this stabilization phenomenon is analyzed using the method of direct separation of motions. It is shown that under certain conditions, the pulsations increase the dynamic stiffness of the pipe with fluid, thereby stabilizing it.

The equation that governs small flexural vibrations of the pipe can be written as

$$EI\frac{\partial^4 u}{\partial z^4}+m_f v^2\frac{\partial^2 u}{\partial z^2}+2m_f v\frac{\partial^2 u}{\partial z\partial t}+2m_f\frac{\mathrm{d}v}{\mathrm{d}t}\frac{\partial u}{\partial z}+m\frac{\partial^2 u}{\partial t^2}=0, \tag{22}$$

where z is the co-ordinate along the pipe, $u(z,t)$ is the flexural deflection of the pipe, EI is its bending stiffness, m_f is the mass of the fluid per unit of length, m is the corresponding mass of the pipe with the fluid,

$$v=V+A\Omega\sin\Omega t=V(1+\mu\sin\Omega t) \tag{23}$$

is the velocity of the flow, with the amplitude of the pulsation velocity $A\Omega$ assumed to be small relative to the mean velocity V.

In accordance with the method of direct separation of motions it is assumed that

$$u=U(t)+\psi(t,\Omega t), \tag{24}$$

where U is the slow component of the deflection, ψ is the fast one, t is the slow time, and $\tau=\Omega t$ is the fast time. Here it is assumed that ψ is periodical with respect to τ, with a period 2π, and has zero average:

$$\langle\psi\rangle=0. \tag{25}$$

Substituting Eqs. (23) and (24) into Eq. (22), and performing the separation of motions, the following equations can be derived for the slow and fast motions:

$$\begin{aligned}&EI\frac{\partial^4 U}{\partial z^4}+m_f V^2\frac{\partial^2 U}{\partial z^2}+2m_f V\frac{\partial^2 U}{\partial z\partial t}+m\frac{\partial^2 U}{\partial t^2}+2\mu m_f V^2\left\langle\frac{\partial^2\psi}{\partial z^2}\sin\Omega t\right\rangle\\&\quad+2\mu m_f V\left\langle\frac{\partial^2\psi}{\partial z\partial t}\sin\Omega t\right\rangle+2\mu m_f V\Omega\left\langle\frac{\partial\psi}{\partial z}\cos\Omega t\right\rangle=0,\end{aligned} \tag{26}$$

$$\begin{aligned}&EI\frac{\partial^4\psi}{\partial z^4}+2V^2 m_f\mu\left(\frac{\partial^2\psi}{\partial z^2}\sin\Omega t-\left\langle\frac{\partial^2\psi}{\partial z^2}\sin\Omega t\right\rangle\right)+2Vm_f\mu\left(\frac{\partial^2\psi}{\partial z\partial t}\sin\Omega t-\left\langle\frac{\partial^2\psi}{\partial z\partial t}\sin\Omega t\right\rangle\right)\\&\quad+m_f V^2\frac{\partial^2\psi}{\partial z^2}+2m_f V^2\mu\frac{\partial^2 U}{\partial z^2}\sin\Omega t+2m_f V\frac{\partial^2\psi}{\partial z\partial t}+2m_f V\mu\frac{\partial^2 U}{\partial z\partial t}\sin\Omega t+m\frac{\partial^2\psi}{\partial t^2}\\&\quad+2m_f V\mu\Omega\left[\left(\frac{\partial U}{\partial z}+\frac{\partial\psi}{\partial z}\right)\cos\Omega t-\left\langle\frac{\partial\psi}{\partial z}\cos\Omega t\right\rangle\right]=0,\end{aligned} \tag{27}$$

Eq. (26) is reduced by means of averaging the original equation and Eq. (27) by means of the requirement that taking into account Eq. (26) the original equation would be satisfied (note that instead of one unknown function u two functions U and ψ are put, that's why two equations can be satisfied).

One of the main advantages of the method of direct separation of motions is the possibility of a very approximate solution of the equations of fast motions without introducing a large error into the equation of slow motions, see Refs. [3,4,13].

Here, assuming ψ to be small relative to U, Eq. (27) is reduced to (only three terms are accounted for):

$$m\frac{\partial^2\psi}{\partial t^2}+2m_f\mu\left(V^2\frac{\partial^2 U}{\partial z^2}+V\frac{\partial^2 U}{\partial z\partial t}\right)\sin\Omega t+2m_f V\mu\Omega\frac{\partial U}{\partial z}\cos\Omega t=0. \tag{28}$$

The stationary oscillations, described by this equation, are found as

$$\psi=\frac{2m_f\mu}{m\Omega^2}\left(V^2\frac{\partial^2 U}{\partial z^2}+V\frac{\partial^2 U}{\partial z\partial t}\right)\sin\Omega t+\frac{2m_f\mu}{m\Omega^2}\Omega V\frac{\partial U}{\partial z}\cos\Omega t. \tag{29}$$

Substituting this expression into Eq. (26) and performing the averaging, the following equation of slow motion is obtained

$$\left(EI + 2\frac{m_f}{m} m_f (VA)^2\right)\frac{\partial^4 U}{\partial z^4} + 2 m_f V\left(\frac{\partial^2 U}{\partial z \partial t} + \frac{m_f}{m} A^2 \frac{\partial^4 U}{\partial z^3 \partial t}\right) + m_f V^2\left(1 + \frac{2 m_f A\Omega}{mV}\right)\frac{\partial^2 U}{\partial z^2} + m\frac{\partial^2 U}{\partial t^2} = 0. \quad (30)$$

One can clearly see from this equation that the pulsations of the flow lead to the increase of the effective stiffness of the pipe by the amount

$$(EI)_V = 2\frac{m_f^2}{m}(VA)^2, \quad (31)$$

which is, in fact, the explanation of the effect of stabilization of the pipe. Thus the pipe with a pulsating flow can be viewed upon as a vibrational, parametrically produced dynamic material as discussed in previous sections.

It is notable that the additional stiffness $(EI)_v$ is independent of the pulsation frequency Ω. However, if the amplitude of pulsations of the velocity v_p were introduced instead of $A\Omega$ (see Eq. (23)), the effective stiffness would become proportional to Ω^{-2}:

$$(EI)_V = 2\frac{m_f^2}{m}\left(\frac{V v_p}{\Omega}\right)^2. \quad (32)$$

One more obvious conclusion can be drawn from Eq. (31). If the pipe possessed a negligible bending stiffness $EI \approx 0$ (like a cable), the pulsating flow would make of it a beam with an effective bending stiffness.

Note that in Eq. (30) there is also vibration additive to Coriolis acceleration (the second item in the second big brackets). We do not include the analysis of this term because it does not influence the statical stability of the pipe.

It should be emphasized that the solution given here is valid under condition that the frequency of pulsations Ω exceeds sufficiently the natural frequency λ of the pipe with the corresponding stationary flow (as a rule, it is sufficient to have $\Omega \geqslant 3\lambda$). This inequality is based on the examination of many applications [3,4].

Note that if the flow velocity is close to the critical one (which is the most interesting case), the natural frequency of the pipe is low and the above condition is definitely satisfied.

Other applications for using high-frequency parametric effects on specifically design equipment to control their effective dynamic properties are reviewed in Ref. [4], in which an overview of relevant research is given.

The relatively simple analysis summarized within this paper shows that the important results given for the first time by the authors of Ref. [14] can be explained and rationalized straightforwardly by a physically based mathematical interpretation. It is acknowledged that the mathematical modeling in the paper is not complete, but it is noted that it could be usefully supplemented along the lines of recent implementations of the method of direct separation of motions as exemplified in Ref. [15]. The reader's attention is drawn to the potential generality of the concepts of vibrational dynamic materials and vibrational rheology and it is hoped that further applications will emerge in due course.

7. Conclusions

In this paper the idea of dynamic material creation is developed. A special class of such materials was examined and termed vibration dynamic materials. Special cases of such materials are vibration dynamic composites and also parametrically produced vibration materials. Vibro-rheological characteristics determining from the equations of these materials are given. It is shown that vibrational mechanics approach and method of direct separation of motions are effective means for theoretical study of such materials. This is illustrated by the worked example of the tube conveying a pulsating fluid.

Vibro-rheological effects are often explained by recourse to discussions based on the changing physical properties of bodies under influence of vibration. The results shown here help to indicate that such a definition really exists in practice rather than simply being an apparent effect connected with predominance of averaged values. It is noted that such phenomena disappear instantly when the vibrational excitation is removed, whereas the physical material properties remain changed over time.

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