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Axially symmetric vibrations of fluid-filled poroelastic circular cylindrical shells

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Abstract

Employing Biot's theory of wave propagation in liquid saturated porous media, axially symmetric vibrations of fluid-filled and empty poroelastic circular cylindrical shells of infinite extent are investigated for different wall-thicknesses. Let the poroelastic cylindrical shells are homogeneous and isotropic. The frequency equation of axially symmetric vibrations each for a pervious and an impervious surface is derived. Particular cases when the fluid is absent are considered both for pervious and impervious surfaces. The frequency equation of axially symmetric vibrations propagating in a fluid-filled and an empty poroelastic bore, each for a pervious and an impervious surface is derived as a limiting case when ratio of thickness to inner radius tends to infinity as the outer radius tends to infinity. Cut-off frequencies when the wavenumber is zero are obtained for fluid-filled and empty poroelastic cylindrical shells both for pervious and impervious surfaces. When the wavenumber is zero, the frequency equation of axially symmetric shear vibrations is independent of nature of surface, i.e., pervious or impervious and also it is independent of presence of fluid in the poroelastic cylindrical shell. Non-dimensional phase velocity for propagating modes is computed as a function of ratio of thickness to wavelength in absence of dissipation. These results are presented graphically for two types of poroelastic materials and then discussed. In general, the phase velocity of an empty poroelastic cylindrical shell is higher than that of a fluid-filled poroelastic cylindrical shell.

The phase velocity of a fluid-filled bore is higher than that of an empty poroelastic bore. Previous results are shown as a special case of present investigation. Results of purely elastic solid are obtained.

1. Introduction

Gazis [1,2] discussed the propagation of free harmonic waves along a hollow elastic circular cylinder of infinite extent and presented numerical results. Kumar [3] studied the axially symmetric vibrations of fluid-filled and empty elastic circular cylindrical shells of various wall-thicknesses. Employing Biot's theory [4], Tajuddin and Sarma [5] studied the torsional vibrations of poroelastic cylinders. Sharma and Gogna [6] solved the problem of elastic wave propagation in a cylindrical bore in a poroelastic solid and derived the frequency equations for empty and fluid-filled bores. Cui et al. [7] and Abousleiman and Cui [8] presented poroelastic solutions in an inclined borehole and transversely isotropic well-bore cylinders. Malla Reddy and Tajuddin [9]

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studied the plane-strain vibrations of thick-walled hollow poroelastic cylinders. Wisse et al. [10,11] presented the experimental results of guided wave modes in porous cylinders and extended the classical theory of wave propagation in elastic cylinders to poroelastic mandrel modes. Kanj et al. [12] presented poromechanics of anisotropic hollow cylinders. Kanj and Abousleiman [13] presented poromechanical solutions of Lamé problem and discussed different cases in detail. Chao et al. [14] studied the shock-induced borehole waves in porous formations. Vashishth and Khurana [15] presented the solutions of elastic wave propagation along a cylindrical borehole in an anisotropic poroelastic solid and derived frequency equations for empty and fluid-filled boreholes. Farhang et al. [16] investigated the wave propagation in transversely isotropic cylinders. Tajuddin and Ahmed Shah [17,18] studied the circumferential waves and torsional vibrations of infinite hollow poroelastic cylinders in presence of dissipation.

In the present analysis, the axially symmetric vibrations of fluid-filled and empty poroelastic circular cylindrical shells of infinite extent are investigated employing Biot's [4] theory. Biot's model consists of an elastic matrix permeated by a network of interconnected spaces saturated with liquid. The frequency equation of such vibrations is derived each for a pervious surface and an impervious surface. The frequency equation of axially symmetric vibrations propagating in a fluid-filled and an empty poroelastic bore is obtained as a limiting case, when the ratio of thickness to inner radius approaches to infinity as the outer radius tends to infinity with finite inner radius. These frequency equations are derived for both pervious and impervious surfaces. Cut-off frequencies when the wavenumber is zero are obtained for fluid-filled and empty poroelastic cylindrical shells, both for pervious and impervious surfaces. For zero wavenumber, the frequency equation of axially symmetric shear vibrations is independent of nature of surface as well as presence of fluid in the poroelastic shell. This frequency equation is discussed for limiting values of ratio of thickness to inner radius h/r_1, when these values are too small and too large. When $h/r_1 \to 0$, it gives the theoretical frequencies of thin poroelastic cylindrical shell and when $h/r_1 \to \infty$ for $r_1 \to 0$, the modes of a poroelastic solid cylinder are obtained. Non-dimensional phase velocity for propagating modes is computed in absence of dissipation for fluid-filled and empty poroelastic cylindrical shells and poroelastic bore each for a pervious and an impervious surface. The cut-off frequency as a function of h/r_1 is determined. The results are presented graphically for two types of poroelastic materials and then discussed. Results of some previous works are shown as a particular case of the present investigation. By ignoring the liquid effects, and after rearrangement of terms, results of purely elastic solid are shown as a particular case considered by Kumar [3].

2. Governing equations

The equations of motion of *a* homogeneous, isotropic poroelastic solid [4] in presence of dissipation b are

$$N\nabla^2\vec{u} + (A+N)\nabla e + Q\nabla \in = \frac{\partial^2}{\partial t^2}(\rho_{11}\vec{u} + \rho_{12}\vec{U}) + b\frac{\partial}{\partial t}(\vec{u} - \vec{U}),$$
$$Q\nabla e + R\nabla \in = \frac{\partial^2}{\partial t^2}(\rho_{12}\vec{u} + \rho_{22}\vec{U}) - b\frac{\partial}{\partial t}(\vec{u} - \vec{U}), \tag{1}$$

where ∇^2 is the Laplacian, $\mathbf{u} = (u, v, w)$ and $\mathbf{U} = (U, V, W)$ are displacements of solid and liquid, respectively, e and $\in$ are the dilatations of solid and liquid; A, N, Q, R are all poroelastic constants and ρ_{ij} $(i,j = 1, 2)$ are the mass coefficients following Biot [4]. The poroelastic constants A, N correspond to familiar Lamé constants in purely elastic solid. The coefficient N represents the shear modulus of the solid. The coefficient R is a measure of the pressure required on the liquid to force a certain amount of the liquid into the aggregate while total volume remains constant. The coefficient Q represents the coupling between the volume change of the solid to that of liquid.

The equation of motion for a homogeneous, isotropic, inviscid elastic fluid is

$$\nabla^2\Phi = \frac{1}{V_f^2}\frac{\partial^2\Phi}{\partial t^2}, \tag{2}$$

where Φ is displacement potential function and V_f is the velocity of sound in the fluid. The displacement of fluid is $\mathbf{u}_f = (u_f, v_f, w_f)$.

The stresses σ_{ij} and the liquid pressure s of the poroelastic solid are

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\in)\delta_{ij} \quad (i,j = 1,2,3),$$
$$s = Qe + R\in, \tag{3}$$

where δ_{ij} is the well-known Kronecker delta function.

The fluid pressure P_f is given by

$$P_f = -\rho_f \frac{\partial^2 \Phi}{\partial t^2}. \tag{4}$$

In Eq. (4), ρ_f is the density of the fluid.

3. Solution of the problem

Let (r, θ, z) be the cylindrical polar coordinates. Consider a homogeneous, isotropic, infinite poroelastic cylindrical shell filled with inviscid elastic fluid. Let the inner and outer radii be r_1 and r_2 respectively so that the thickness of poroelastic shell is $h\,[=(r_2-r_1)>0]$. The axis of the poroelastic shell is in the direction of z-axis. Then for axially symmetric vibrations, the displacement of solid $\mathbf{u} = (u, 0, w)$ which can be readily be evaluated from field Eq. (1) is

(a) when $|kV_1| < \omega$,

$$u = \left\{ C_1 \frac{\partial}{\partial r} J_0(\alpha_1 r) + C_2 \frac{\partial}{\partial r} Y_0(\alpha_1 r) + C_3 \frac{\partial}{\partial r} J_0(\alpha_2 r) + C_4 \frac{\partial}{\partial r} Y_0(\alpha_2 r) + A_1[-\mathrm{i}kJ_1(\alpha_3 r)] + B_1[-\mathrm{i}kY_1(\alpha_3 r)] \right\} \mathrm{e}^{\mathrm{i}(kz+\omega t)},$$

$$w = \left\{ C_1[\mathrm{i}kJ_0(\alpha_1 r)] + C_2[\mathrm{i}kY_0(\alpha_1 r)] + C_3[\mathrm{i}kJ_0(\alpha_2 r)] + C_4[\mathrm{i}kY_0(\alpha_2 r)] + A_1\left[\frac{\partial}{\partial r} J_1(\alpha_3 r) + \frac{1}{r} J_1(\alpha_3 r)\right] + B_1\left[\frac{\partial}{\partial r} Y_1(\alpha_3 r) + \frac{1}{r} Y_1(\alpha_3 r)\right] \right\} \mathrm{e}^{\mathrm{i}(kz+\omega t)}. \tag{5}$$

In Eq. (5), ω is the frequency of wave, k is wavenumber, C_1, C_2, C_3, C_4, A_1, and B_1 are constants, $J_0(x)$, $Y_0(x)$ are Bessel functions of first and second kind each of order zero, $J_1(x)$, $Y_1(x)$ are Bessel functions of first and second kind each or order one. Here i is complex unity or $\mathrm{i}^2 = -1$ and

$$\alpha_1^2 = \frac{\omega^2}{V_1^2} - k^2, \quad \alpha_2^2 = \frac{\omega^2}{V_2^2} - k^2, \quad \alpha_3^2 = \frac{\omega^2}{V_3^2} - k^2, \tag{6}$$

where $V_i\,(i = 1, 2)$ are dilatational wave velocities of first and second kind, respectively, V_3 is shear wave velocity.

The displacement of fluid column $\mathbf{u}_f = (u_f, 0, w_f)$ for axially symmetric vibrations is

$$u_f = A_f \frac{\partial}{\partial r} J_0(\alpha_f r)\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \quad v_f = 0,$$
$$w_f = A_f \mathrm{i}kJ_0(\alpha_f r)\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \tag{7}$$

where A_f is constant and

$$\alpha_f^2 = \frac{\omega^2}{V_f^2} - k^2. \tag{8}$$

With the help of displacement potential function, fluid pressure is given by

$$P_f = A_f \omega^2 \rho_f J_0(\alpha_f r)\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \quad \text{when } |kV_f| < \omega. \tag{9}$$

(b) When $|kV_3|<\omega<|kV_1|$:
In this case, the solid displacement is obtained from Eq. (5) after replacing Bessel functions $J_1(\alpha_1 r)$, $Y_1(\alpha_1 r)$, $J_1(\alpha_2 r)$, $Y_1(\alpha_2 r)$ by modified Bessel functions $I_1(\alpha_1 r)$, $K_1(\alpha_1 r)$, $I_1(\alpha_2 r)$, $K_1(\alpha_2 r)$, respectively. Similarly, the Bessel functions of second order are changed accordingly keeping $J_1(\alpha_3 r)$, $Y_1(\alpha_3 r)$ and $J_2(\alpha_3 r)$, $Y_2(\alpha_3 r)$ same.

(c) When $\omega<|kV_1|$:
In this case, the solid displacement is obtained from Eq. (5) after replacing Bessel functions of first and second kind by modified Bessel functions of first and second kind, respectively.

Substituting the displacement function u and w from Eq. (5), fluid pressure from Eq. (9), into Eq. (3) together with Eq. (7), the relevant displacement, liquid pressure and stresses are

$$\begin{aligned}\sigma_{rr}+s+P_f=[&C_1M_{11}(r)+C_2M_{12}(r)+C_3M_{13}(r)+C_4M_{14}(r)\\&+A_1M_{15}(r)+B_1M_{16}(r)+A_fM_{17}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)},\end{aligned} \tag{10}$$

$$\begin{aligned}\sigma_{rz}=[&C_1M_{21}(r)+C_2M_{22}(r)+C_3M_{23}(r)+C_4M_{24}(r)\\&+A_1M_{25}(r)+B_1M_{26}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)},\end{aligned} \tag{11}$$

$$s=[C_1M_{31}(r)+C_2M_{32}(r)+C_3M_{33}(r)+C_4M_{34}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \tag{12}$$

$$\frac{\partial s}{\partial r}=[C_1N_{31}(r)+C_2N_{32}(r)+C_3N_{33}(r)+C_4N_{34}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \tag{13}$$

$$\begin{aligned}u-u_f=[&C_1M_{41}(r)+C_2M_{42}(r)+C_3M_{43}(r)+C_4M_{44}(r)\\&+A_1M_{45}(r)+B_1M_{46}(r)+A_fM_{47}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)},\end{aligned} \tag{14}$$

$$\sigma_{rr}+s=[C_1M_{51}(r)+C_2M_{52}(r)+C_3M_{53}(r)+C_4M_{54}(r)+A_1M_{55}(r)+B_1M_{56}(r)]\,\mathrm{e}^{\mathrm{i}(kz+\omega t)}, \tag{15}$$

where C_1, C_2, C_3, C_4, A_1, B_1, A_f are all constants and the coefficients $M_{ij}(r)$, $N_{ij}(r)$ are

$$M_{11}(r)=\{[(Q+R)\delta_1^2-(A+Q)]k^2+[(Q+R)\delta_1^2-(P+Q)]\alpha_1^2\}J_0(\alpha_1 r)+\frac{2N\alpha_1}{r}J_1(\alpha_1 r),$$

$$M_{12}(r)=\{[(Q+R)\delta_1^2-(A+Q)]k^2+[(Q+R)\delta_1^2-(P+Q)]\alpha_1^2\}Y_0(\alpha_1 r)+\frac{2N\alpha_1}{r}Y_1(\alpha_1 r),$$

$$M_{13}(r)=\{[(Q+R)\delta_2^2-(A+Q)]k^2+[(Q+R)\delta_2^2-(P+Q)]\alpha_2^2\}J_0(\alpha_2 r)+\frac{2N\alpha_2}{r}J_1(\alpha_2 r),$$

$$M_{14}(r)=\{[(Q+R)\delta_2^2-(A+Q)]k^2+[(Q+R)\delta_2^2-(P+Q)]\alpha_2^2\}Y_0(\alpha_2 r)+\frac{2N\alpha_2}{r}Y_1(\alpha_2 r),$$

$$M_{15}(r)=-2\mathrm{i}kN\alpha_3J_0(\alpha_3 r)+\frac{2\mathrm{i}kN}{r}J_1(\alpha_3 r),$$
$$M_{16}(r)=-2\mathrm{i}kN\alpha_3Y_0(\alpha_3 r)+\frac{2\mathrm{i}kN}{r}Y_1(\alpha_3 r),$$

$$M_{17}(r)=\omega^2\rho_fJ_0(\alpha_f r),\quad M_{21}(r)=-2\mathrm{i}kN\alpha_1J_1(\alpha_1 r),$$
$$M_{22}(r)=-2\mathrm{i}kN\alpha_1Y_1(\alpha_1 r),$$

$$M_{23}(r)=-2\mathrm{i}kN\alpha_2J_1(\alpha_2 r),\quad M_{24}(r)=-2\mathrm{i}kN\alpha_2Y_1(\alpha_2 r),$$

$$M_{25}(r)=N(k^2-\alpha_3^2)J_1(\alpha_3 r),\quad M_{26}(r)=N(k^2-\alpha_3^2)Y_1(\alpha_3 r),\quad M_{27}(r)=0,$$

$$M_{31}(r) = (R\delta_1^2 - Q)(\alpha_1^2 + k^2)J_0(\alpha_1 r), \quad M_{32}(r) = (R\delta_1^2 - Q)(\alpha_1^2 + k^2)Y_0(\alpha_1 r),$$

$$M_{33}(r) = (R\delta_2^2 - Q)(\alpha_2^2 + k^2)J_0(\alpha_2 r), \quad M_{34}(r) = (R\delta_2^2 - Q)(\alpha_2^2 + k^2)Y_0(\alpha_2 r),$$

$$M_{35}(r) = 0, \quad M_{36}(r) = 0, \quad M_{37}(r) = 0, \quad M_{41}(r) = -\alpha_1 J_1(\alpha_1 r),$$
$$M_{42}(r) = -\alpha_1 Y_1(\alpha_1 r),$$

$$M_{43}(r) = -\alpha_2 J_1(\alpha_2 r), \quad M_{44}(r) = -\alpha_2 Y_1(\alpha_2 r), \quad M_{45}(r) = -\mathrm{i}kJ_1(\alpha_3 r),$$

$$M_{46}(r) = -\mathrm{i}kY_1(\alpha_3 r), \quad M_{47}(r) = \alpha_f J_1(\alpha_f r),$$

$$M_{51}(r) = M_{11}(r), \quad M_{52}(r) = M_{12}(r), \quad M_{53}(r) = M_{13}(r), \quad M_{54}(r) = M_{14}(r),$$

$$M_{55}(r) = M_{15}(r), \quad M_{56}(r) = M_{16}(r), \quad M_{57}(r) = 0,$$

$$M_{6j}(r) = M_{2j}(r), \quad M_{7j}(r) = M_{3j}(r); \quad j = 1, 2, 3, 4, 5, 6, 7,$$

$$N_{31}(r) = (R\delta_1^2 - Q)(\alpha_1^3 + k^2\alpha_1)J_1(\alpha_1 r), \quad N_{32}(r) = (R\delta_1^2 - Q)(\alpha_1^3 + k^2\alpha_1)Y_1(\alpha_1 r),$$

$$N_{33}(r) = (R\delta_2^2 - Q)(\alpha_2^3 + k^2\alpha_2)J_1(\alpha_2 r), \quad N_{34}(r) = (R\delta_2^2 - Q)(\alpha_2^3 + k^2\alpha_2)Y_1(\alpha_2 r),$$

$$N_{35}(r) = 0, \quad N_{36}(r) = 0, \quad N_{37}(r) = 0. \tag{16}$$

In Eq. (16), δ_1^2 and δ_2^2 are

$$\delta_i^2 = \frac{1}{(RK_{12} - QK_{22})}[(RK_{11} - QK_{12}) - V_i^{-2}(PR - Q^2)] \quad (i = 1, 2), \tag{17}$$

where $P = A + 2N$ and

$$K_{11} = \rho_{11} - \frac{\mathrm{i}b}{\omega}, \quad K_{12} = \rho_{12} + \frac{\mathrm{i}b}{\omega}, \quad K_{22} = \rho_{22} - \frac{\mathrm{i}b}{\omega}. \tag{18}$$

4. Frequency equation

The outer surface of the poroelastic cylindrical shell is assumed to be free from stress. At the interface of solid and fluid, the radial displacement is continuous. Thus, the boundary conditions for a fluid-filled poroelastic circular cylindrical shell, in case of a pervious surface are

$$\sigma_{rr} + s + P_f = 0, \quad \sigma_{rz} = 0, \quad s = 0, \quad u - u_f = 0; \quad \text{at } r = r_1,$$
$$\sigma_{rr} + s = 0, \quad \sigma_{rz} = 0, \quad s = 0; \quad \text{at } r = r_2. \tag{19}$$

The boundary conditions for a fluid-filled poroelastic circular cylindrical shell, in case of an impervious surface are

$$\sigma_{rr} + s + P_f = 0, \quad \sigma_{rz} = 0, \quad \frac{\partial s}{\partial r} = 0, \quad u - u_f = 0; \quad \text{at } r = r_1,$$
$$\sigma_{rr} + s = 0, \quad \sigma_{rz} = 0, \quad \frac{\partial s}{\partial r} = 0; \quad \text{at } r = r_2. \tag{20}$$

Substitution of Eqs. (10)–(12), (14) and (15) into Eq. (19) result in a system of seven homogeneous algebraic equations in seven constants C_1, C_2, C_3, C_4, A_1, B_1 and A_f. For a non-trivial solution, the determinant of the coefficients must vanish. By eliminating these constants, the frequency equation of axially symmetric vibrations of a fluid-filled poroelastic circular cylindrical shell in case of a pervious surface is

$$|A_{ij}| = 0, \quad i,j = 1, 2, \ldots, 7. \tag{21}$$

In Eq. (21), the elements A_{ij} are

$$A_{ij} = M_{ij}(r_1), \quad i = 1,2,3,4 \quad \text{and} \quad j = 1,2,3,4,5,6,7,$$
$$A_{ij} = M_{ij}(r_2), \quad i = 5,6,7 \quad \text{and} \quad j = 1,2,3,4,5,6,7, \tag{22}$$

where $M_{ij}(r)$ are defined in Eq. (16).

Arguing on similar lines, Eqs. (10), (11), (13)–(15) together with Eq. (20) yield the frequency equation of axially symmetric vibrations of a fluid-filled poroelastic circular cylindrical shell of infinite extent in case of an impervious surface to be

$$|B_{ij}| = 0, \quad i,j = 1,2,\ldots,7, \tag{23}$$

where the elements B_{ij} are

$$B_{ij} = A_{ij}, \quad i = 1,2,4,5,6 \quad \text{and} \quad j = 1,2,3,4,5,6,7,$$
$$B_{3j} = N_{3j}(r_1), \quad j = 1,2,3,4,5,6,7,$$
$$B_{7j} = N_{3j}(r_2), \quad j = 1,2,3,4,5,6,7, \tag{24}$$

where $M_{ij}(r)$ and $N_{ij}(r)$ are defined in Eq. (16).

By eliminating liquid effects from frequency equation of pervious surface Eq. (21), that is, setting $b \to 0$, $\rho_{12} \to 0$, $\rho_{22} \to 0$, $(A - Q^2/R) \to \lambda$, $N \to \mu$, $Q \to 0$ and $R \to 0$ the results of purely elastic solid are obtained as a special case considered by Kumar [3]. The frequency equation of an impervious surface Eq. (23) has no counterpart in purely elastic solid.

4.1. Frequency equation for an empty poroelastic cylindrical shell

When the fluid density is zero, that is, $\rho_f = 0$ the fluid-filled poroelastic cylindrical shell will become an empty poroelastic cylindrical shell. Thus, the frequency equation of pervious surface Eq. (21) reduce to

$$|A_{ij}| = 0, \quad i = 1,2,3,5,6,7 \quad \text{and} \quad j = 1,2,3,4,5,6, \tag{25}$$

where the elements A_{ij} are defined in Eq. (22) for $\rho_f = 0$.

Eq. (25) is the frequency equation of axially symmetric vibrations of an empty poroelastic cylindrical shell in case of a pervious surface.

Similarly, the frequency equation of axially symmetric vibrations of an empty poroelastic cylindrical shell for an impervious surface is

$$|B_{ij}| = 0, \quad i = 1,2,3,5,6,7 \quad \text{and} \quad j = 1,2,3,4,5,6. \tag{26}$$

In Eq. (26), the elements B_{ij} are defined in Eq. (24) for $\rho_f = 0$.

4.2. Cut-off frequencies

The frequencies obtained by equating wavenumber to zero are referred to as the cut-off frequencies. Thus for $k = 0$, the frequency equation of pervious surface Eq. (21) reduce to the product of two determinants as

$$D_1 D_2 = 0, \tag{27}$$

where D_1 and D_2 are

$$D_1 = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{17} \\ A_{31} & A_{32} & A_{33} & A_{34} & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{47} \\ A_{51} & A_{52} & A_{53} & A_{54} & 0 \\ A_{71} & A_{72} & A_{73} & A_{74} & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} A_{25} & A_{26} \\ A_{65} & A_{66} \end{bmatrix}. \tag{28}$$

The elements A_{ij} of D_1 and D_2 are defined in Eq. (22) are now evaluated for $k = 0$. From Eq. (27) it is clear that either $D_1 = 0$ or $D_2 = 0$ and these two equations give the cut-off frequencies of axially symmetric vibrations. The frequency equation

$$D_1 = 0, \tag{29}$$

gives the cut-off frequencies of axially symmetric vibrations of fluid-filled poroelastic cylindrical shells for a pervious surface while the frequency equation

$$D_2 = 0, \tag{30}$$

does not depend on fluid parameters, and it give the cut-off frequencies of axially symmetric shear vibrations which are independent of presence of fluid in the poroelastic cylindrical shell.

Similarly, the frequency equation of an impervious surface Eq. (23), when $k = 0$ is reduced to the product of two determinants

$$D_3 D_4 = 0, \tag{31}$$

where D_3 and D_4 are

$$D_3 = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{17} \\ B_{31} & B_{32} & B_{33} & B_{34} & 0 \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{47} \\ B_{51} & B_{52} & B_{53} & B_{54} & 0 \\ B_{71} & B_{72} & B_{73} & B_{74} & 0 \end{bmatrix}, \quad D_4 = \begin{bmatrix} B_{25} & B_{26} \\ B_{65} & B_{66} \end{bmatrix}. \tag{32}$$

The elements B_{ij} appearing in D_3 and D_4 are defined in Eq. (24) are now evaluated for $k = 0$. From Eq. (31) it is clear that either $D_3 = 0$ or $D_4 = 0$. The equation

$$D_3 = 0, \tag{33}$$

corresponds to the cut-off frequencies of axially symmetric vibrations of a fluid-filled poroelastic cylindrical shell in case of an impervious surface, while the equation

$$D_4 = 0, \tag{34}$$

yield the cut-off frequencies independent of presence of fluid. Also it is seen that Eqs. (30) and (34) are same by virtue of Eq. (24). Hence Eq. (30) is independent of nature of surface, that is, pervious or impervious surface. Therefore, the cut-off frequencies given by Eq. (30) are independent of presence of fluid in the poroelastic cylindrical shell and nature of surface. By using recurrence relations for Bessel functions [19], Eq. (30) is simplified to

$$J_0'(\alpha_3 r_1) Y_0'(\alpha_3 r_2) - J_0'(\alpha_3 r_2) Y_0'(\alpha_3 r_1) = 0, \tag{35}$$

where α_3 is defined in Eq. (6) for $k = 0$.

Eq. (35) is the frequency equation of axially symmetric shear vibrations. It is same for fluid-filled and empty poroelastic cylindrical shells as well as for pervious and impervious surfaces. By eliminating liquid effects from Eq. (35), the results of purely elastic solid discussed by Gazis [1] are obtained as a special case. The frequency Eq. (35) is discussed for limiting values of h/r_1, when these values are too small and too large.

(a) For thin poroelastic cylindrical shell

When $h/r_1 \ll 1$, under the verifiable assumption of non-zero $\alpha_3 h$ it is seen that $\alpha_3 r_1 \gg 1$ and $\alpha_3 r_2 \gg 1$. Using well-known Hankel–Kirchhoff asymptotic approximations for Bessel functions [19],

$$J_0'(x) \approx \sqrt{\frac{2}{\pi x}} \left[-\sin\left(x - \frac{\pi}{4}\right) - \frac{3}{8x} \cos\left(x - \frac{3\pi}{4}\right) \right],$$
$$Y_0'(x) \approx \sqrt{\frac{2}{\pi x}} \left[\cos\left(x - \frac{\pi}{4}\right) - \frac{3}{8x} \sin\left(x - \frac{\pi}{4}\right) \right],$$

the frequency equation of axially symmetric shear vibrations Eq. (35) is reduced to

$$\sin(\alpha_3 h) - \frac{3\alpha_3 h}{8\alpha_3^2 r_1 r_2}\cos(\alpha_3 h) \approx 0. \tag{36}$$

Eq. (36) is the frequency equation of axially symmetric shear vibrations of a thin poroelastic cylindrical shell. When $\alpha_3 r_1 \to \infty$, $\alpha_3 r_2 \to \infty$ Eq. (36) is reduced to

$$\sin(\alpha_3 h) = 0 \quad \Rightarrow \quad \alpha_3 h = \pi q, \quad q = 1, 2, 3, \ldots$$

so that

$$\omega = \frac{q\pi V_3}{h}, \quad q = 1, 2, 3, \ldots \tag{37}$$

which are the theoretical frequencies of poroelastic plate of thickness h. Moreover, near the origin $h/r_1 = 0$, and assuming

$$\alpha_3 h = q\pi + \eta^*, \quad \eta^* \ll 1, \tag{38}$$

then using the frequency equation of axially symmetric shear vibrations of thin poroelastic cylindrical shell, that is, Eq. (36) yields

$$\eta^* = \frac{3}{8q\pi}\left(\frac{h}{r_1}\right)^2, \quad q = 1, 2, 3, \ldots \tag{39}$$

On substituting Eq. (39) into Eq. (38) and using Eq. (37), we can write

$$\omega = \frac{V_3 q\pi}{h}\left[1 + \frac{3}{8(q\pi)^2}\left(\frac{h}{r_1}\right)^2\right], \quad q = 1, 2, 3, \ldots \tag{40}$$

which are the theoretical frequencies of axially symmetric shear vibrations of a poroelastic plate of thickness h near the origin.

(b) For poroelastic solid cylinder

When $h/r_1 \to \infty$ as $r_1 \to 0$ and h finite, the frequency Eq. (35) tends asymptotically to

$$J_0'(\alpha_3 h) = 0, \tag{41}$$

which is the frequency equation of axially symmetric shear vibrations of a poroelastic solid cylinder of radius h. In Eq. (41), by ignoring the liquid effects, the results of purely elastic solid are obtained as a special case considered by Gazis [1].

The frequency equation of axially symmetric vibrations of an empty poroelastic cylindrical shell for a pervious surface (25), when $k = 0$ is reduced to

$$D_5 D_2 = 0, \tag{42}$$

where D_2 is given in Eq. (28) and D_5 is

$$D_5 = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{51} & A_{52} & A_{53} & A_{54} \\ A_{71} & A_{72} & A_{73} & A_{74} \end{bmatrix}. \tag{43}$$

The elements A_{ij} of D_5 defined in Eq. (22) are evaluated for $\rho_f = 0$ and $k = 0$. From Eq. (42) it is clear that either $D_5 = 0$ or $D_2 = 0$. The case of $D_2 = 0$ is discussed in Eq. (30). The frequency equation

$$D_5 = 0, \tag{44}$$

give the cut-off frequencies of axially symmetric vibrations of an empty poroelastic cylindrical shell for a pervious surface. Eq. (44) corresponds to axially symmetric extensional waves, discussed by Malla Reddy and Tajuddin [9] for a pervious surface. From Eq. (42), it can be seen that the extensional and shear waves are uncoupled in case of axially symmetric vibrations for a pervious surface.
Similarly, the frequency equation of axially symmetric vibrations of an empty poroelastic cylindrical shell for an impervious surface Eq. (26), when $k = 0$ is reduced to

$$D_6 D_4 = 0, \tag{45}$$

where D_4 is given in Eq. (32) and D_6 is

$$D_6 = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{51} & B_{52} & B_{53} & B_{54} \\ B_{71} & B_{72} & B_{73} & B_{74} \end{bmatrix}. \tag{46}$$

The elements B_{ij} of D_6 defined in Eq. (24) are evaluated for $\rho_f = 0$ and $k = 0$. From Eq. (45) it is clear that either $D_6 = 0$ or $D_4 = 0$. The case of $D_4 = 0$ is discussed in Eq. (34). The case of

$$D_6 = 0, \tag{47}$$

corresponds to axially symmetric extensional waves, discussed by Malla Reddy and Tajuddin [9], for an impervious surface. From Eq. (45), it can be seen that the extensional and shear waves are uncoupled in case of axially symmetric vibrations for an impervious surface.

4.3. Frequency equation for poroelastic bore

When the outer radius of the poroelastic circular cylindrical shell approaches to infinity, i.e., $r_2 \to \infty$, the thickness of the shell tends to infinity with finite inner radius. In this case the poroelastic shell reduce to a circular poroelastic bore of radius r_1 in an infinite poroelastic solid. Under this condition, the frequency equation of axially symmetric vibrations of a fluid-filled poroelastic cylindrical shell for a pervious surface Eq. (21) reduce to

$$|P_{ij}| = 0, \quad i,j = 1,2,3,4, \tag{48}$$

where the elements P_{ij} are

$$P_{11} = \{[(Q+R)\delta_1^2 - (A+Q)]k^2 + [-(Q+R)\delta_1^2 + (P+Q)]\alpha_1^2\}K_0(\alpha_1 r_1) - \frac{2N\alpha_1}{r_1}K_1(\alpha_1 r_1),$$

$$P_{12} = \{[(Q+R)\delta_2^2 - (A+Q)]k^2 + [-(Q+R)\delta_2^2 + (P+Q)]\alpha_2^2\}K_0(\alpha_2 r_1) - \frac{2N\alpha_2}{r_1}K_1(\alpha_2 r_1),$$

$$P_{13} = -2N\mathrm{i}k\alpha_3 K_0(\alpha_3 r_1) - \frac{2N\mathrm{i}k}{r_1}K_1(\alpha_3 r_1), \quad P_{14} = \omega^2\rho_f J_0(\alpha_f r_1),$$

$$P_{21} = -2N\mathrm{i}k\alpha_1 K_1(\alpha_1 r_1), \quad P_{22} = -2N\mathrm{i}k\alpha_2 K_1(\alpha_2 r_1),$$
$$P_{23} = -N(k^2 + \alpha_3^2)K_1(\alpha_3 r_1), \quad P_{24} = 0,$$

$$P_{31} = (R\delta_1^2 - Q)(k^2 - \alpha_1^2)K_0(\alpha_1 r_1), \quad P_{32} = (R\delta_2^2 - Q)(k^2 - \alpha_2^2)K_0(\alpha_2 r_1),$$
$$P_{33} = 0, \quad P_{34} = 0,$$
$$P_{41} = \alpha_1 K_1(\alpha_1 r_1), \quad P_{42} = \alpha_2 K_1(\alpha_2 r_1), \quad P_{43} = -\mathrm{i}kK_1(\alpha_3 r_1), \quad P_{44} = \alpha_f J_1(\alpha_f r_1). \tag{49}$$

In Eq. (49), J_0, J_1 are Bessel functions of first kind of order zero and one; K_0, K_1 are modified Bessel functions of second kind of order zero and one.

Similarly, frequency Eq. (23) when $r_2 \to \infty$ and finite r_1, reduce to

$$|Q_{ij}| = 0, \quad i,j = 1,2,3,4, \tag{50}$$

where

$$Q_{ij} = P_{ij}, \quad i = 1,2,4, \quad j = 1,2,3,4 \quad \text{and}$$
$$Q_{31} = (R\delta_1^2 - Q)(k^2\alpha_1 - \alpha_1^3)K_1(\alpha_1 r_1),$$
$$Q_{32} = (R\delta_2^2 - Q)(k^2\alpha_2 - \alpha_2^3)K_1(\alpha_2 r_1), \quad Q_{43} = 0, \quad Q_{44} = 0. \tag{51}$$

Eqs. (48) and (50) are the frequency equations of axially symmetric vibrations of a fluid-filled circular poroelastic bore for a pervious and an impervious surface, respectively.

When $\rho_f = 0$, the fluid-filled poroelastic bore becomes an empty poroelastic bore. Thus for $\rho_f = 0$, Eq. (48) reduce to

$$|P_{ij}| = 0, \quad i,j = 1,2,3, \tag{52}$$

where P_{ij} are defined in Eq. (49) for $\rho_f = 0$.

Similarly, frequency Eq. (50) for $\rho_f = 0$ reduce to

$$|Q_{ij}| = 0, \quad i,j = 1,2,3, \tag{53}$$

where Q_{ij} are defined in Eq. (51) for $\rho_f = 0$.

Eqs. (52) and (53) are the frequency equations of axially symmetric vibrations of an empty poroelastic bore for a pervious and an impervious surface, respectively.

The cut-off frequencies of fluid-filled and empty poroelastic bores for pervious and impervious surfaces are obtained in a similar way as obtained in case of poroelastic cylindrical shells. The cut-off frequencies of poroelastic bore, after simplification [19], are given by the equation

$$K_0'(\alpha_3 r_1) = 0. \tag{54}$$

Eq. (54) is independent of nature of surface, that is, pervious or impervious and it is independent of presence of fluid in the poroelastic bore.

5. Non-dimensionalization of frequency equation

For propagating modes in a non-dissipative medium, the wavenumber k is real. The phase velocity C is the ratio of frequency to wavenumber, that is, $C = \omega/k$. To analyze the frequency equations of axially symmetric vibrations of fluid-filled and empty poroelastic cylindrical shells in cases of pervious and impervious surfaces, it is convenient to introduce the following non-dimensional variables:

$$a_1 = PH^{-1}, \quad a_2 = QH^{-1}, \quad a_3 = RH^{-1}, \quad a_4 = NH^{-1}, \quad m_{11} = \rho_{11}\rho^{-1}, \quad m_{12} = \rho_{12}\rho^{-1},$$
$$\tilde{x} = (V_0 V_1^{-1})^2, \quad \tilde{y} = (V_0 V_2^{-1})^2, \quad \tilde{z} = (V_0 V_3^{-1})^2, \quad \delta = hL^{-1}, \quad m_{22} = \rho_{22}\rho^{-1},$$
$$t = \rho_f \rho^{-1}, \quad m = V_f V_3^{-1}, \quad \Omega = \omega h C_0^{-1}, \quad \xi = C V_f^{-1}, \quad \zeta = C C_0^{-1}, \quad d = DL^{-1}, \tag{55}$$

where Ω is non-dimensional frequency, ξ is non-dimensional phase velocity of fluid-filled poroelastic cylindrical shells, ζ is non-dimensional phase velocity of empty poroelastic cylindrical shells, $H = P + 2Q + R$, $\rho = \rho_{11} + 2\rho_{12} + \rho_{22}$, C_0 and V_0 are the reference velocities ($C_0^2 = N/\rho$, $V_0^2 = H/\rho$), C [$=\omega/k$] is phase

Table 1

Material	Parameter								
	a_1	a_2	a_3	a_4	m_{11}	m_{12}	m_{22}	$x^{\sim}$	$y^{\sim}$	$z^{\sim}$
I	0.843	0.065	0.028	0.234	0.901	−0.001	0.101	0.999	4.763	3.851
II	0.960	0.006	0.028	0.412	0.877	0	0.123	0.913	4.347	2.129

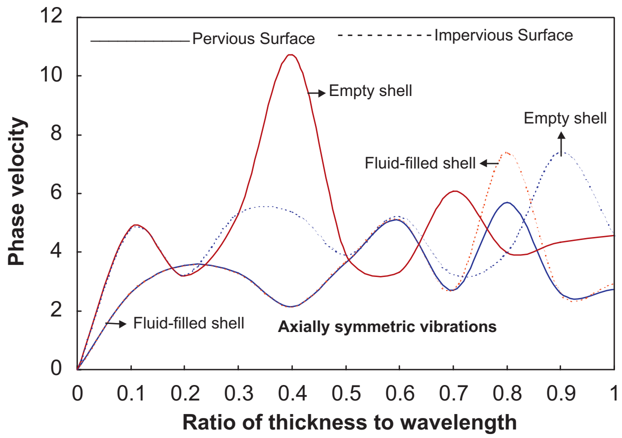

Fig. 1. Phase velocity as a function of wavelength (material-I, thin shell).

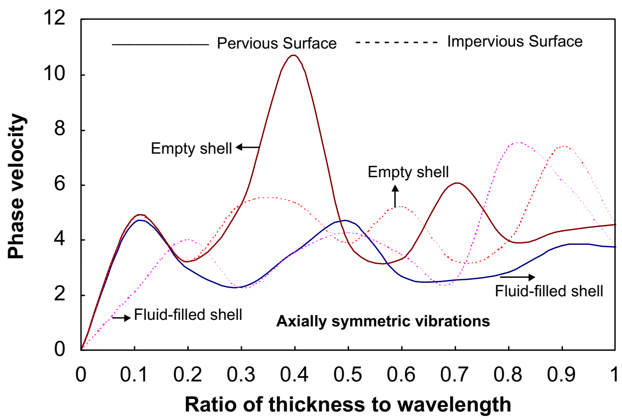

Fig. 2. Phase velocity as a function of wavelength (material-I, moderately thick shell).

velocity of fluid-filled or empty poroelastic cylindrical shells, h is the thickness of the poroelastic cylindrical shell, D is diameter of the bore and L is wavelength. Let

$$g = \frac{r_2}{r_1}, \quad \text{so that } \frac{h}{r_1} = (g-1). \tag{56}$$

6. Results and discussion

Two types of poroelastic materials are considered to carry out the computational work, one is sandstone saturated with kerosene, say material-I [20], the other one is sandstone saturated with water, material-II [21], whose non-dimensional physical parameters are given in Table 1.

For a given poroelastic material, frequency Eqs. (21) and (23), when non-dimensionalized using Eqs. (55) and (56), constitute a relation between non-dimensional phase velocity ξ and ratio of thickness to wavelength

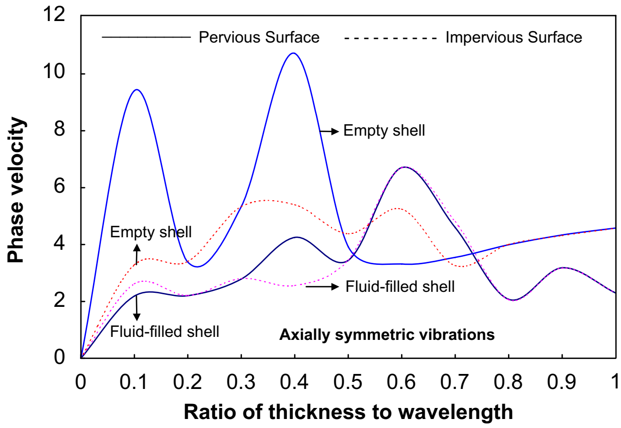

Fig. 3. Phase velocity as a function of wavelength (material-I, thick shell).

δ ($=h/L$) for fixed values of g. Different values of g, viz., 1.034, 1.286 and 3 are taken for numerical computation. These values of g represent thin poroelastic cylindrical shell, moderately thick poroelastic cylindrical shell and thick poroelastic cylindrical shell, respectively. The values of δ lie in [0, 1]. Non-dimensional phase velocity ξ is determined for different values of δ and for fixed values of g, each for a pervious and an impervious surface. For fluid-filled poroelastic cylindrical shells, the values of m and t are taken as $m = 1.5$ and $t = 0.4$. The non-dimensional form of Eqs. (21), (23), (25), (26), etc., are solved numerically to compute either the phase velocity or the frequency, following the analysis of Gazis [2].

The phase velocity of axially symmetric vibrations of fluid-filled and empty poroelastic cylindrical shells is presented in Figs. 1–3 for material-I each for a pervious and an impervious surface. Fig. 1 shows the phase velocity of fluid-filled and empty thin poroelastic cylindrical shells. From Fig. 1 it is clear that the phase velocity of a fluid-filled shell for an impervious surface is almost same to that of a pervious surface and it is slightly higher than that of a pervious surface in $0.7<\delta<0.9$. The variation of phase velocity for an empty shell is similar to that of a fluid-filled shell. In general, the phase velocity of an empty poroelastic cylindrical shell is higher than that of a fluid-filled shell. Therefore, it is inferred that the presence of fluid in the thin poroelastic cylindrical shell decreases the phase velocity each for a pervious and an impervious surface.

The phase velocity for moderately thick shell in case of material-I is presented in Fig. 2 each for a pervious and an impervious surface. The phase velocity of an impervious surface is higher, in general, than that of a pervious surface for a fluid-filled poroelastic cylindrical shell. The phase velocity varies in a similar way for a moderately thick empty poroelastic cylindrical shell. The phase velocity for an empty shell is higher than that of a fluid-filled shell, both for pervious and impervious surfaces. Also, as the thickness of a fluid-filled shell increase, there is a small increase in the phase velocity each for a pervious and an impervious surface. In case of empty poroelastic cylindrical shells, the increase in thickness does not have significant effect on the phase velocity both for pervious and impervious surfaces. Therefore, it can be inferred that the phase velocity is affected with the increase of thickness for a fluid-filled shell but not for an empty shell.

Fig. 3 shows the phase velocity of fluid-filled and empty thick poroelastic shells in case of material-I each for a pervious and an impervious surface. It is seen from Fig. 3 that the phase velocity of a pervious surface is same to that of an impervious surface for a fluid-filled shell. Similar variation of phase velocity is observed for an empty poroelastic shell. Again, in case of poroelastic thick shells, the phase velocity of empty poroelastic shells is higher than that of fluid-filled shells. The phase velocity of a fluid-filled shell is affected by the thickness of the shell while it does not effect an empty poroelastic shell.

The phase velocity of axially symmetric vibrations of fluid-filled and empty poroelastic cylindrical shells for material-II is presented in Figs. 4–6 each for a pervious and an impervious surface. Fig. 4 shows the phase velocity of poroelastic thin cylindrical shell for material-II. From Fig. 4 it is observed that the variation of phase velocity of fluid-filled and empty shells is similar as discussed in case of material-I (Fig. 1). The phase velocity of pervious and impervious surfaces is almost same for a fluid-filled poroelastic thin shell, in general. This is true for empty poroelastic shells also. The phase velocity, in general, is higher for an empty poroelastic

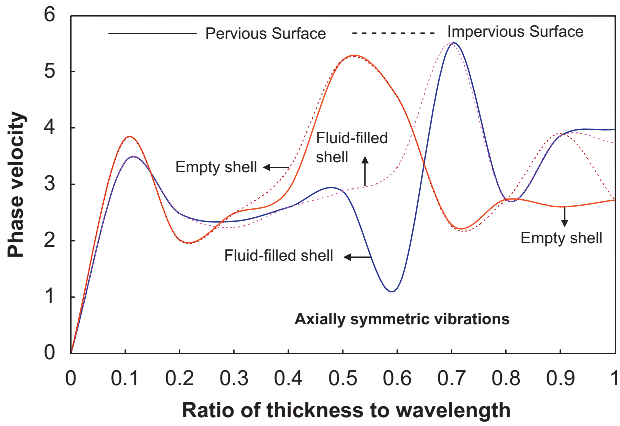

Fig. 4. Phase velocity as a function of wavelength (material-II, thin shell).

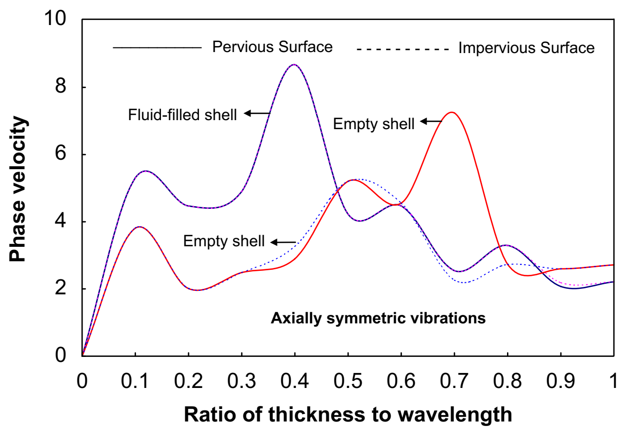

Fig. 5. Phase velocity as a function of wavelength (material-II, moderately thick shell).

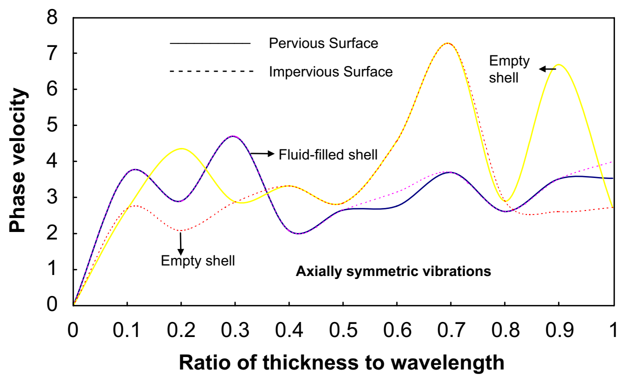

Fig. 6. Phase velocity as a function of wavelength (material-II, thick shell).

shell than that of a fluid-filled poroelastic shell. It is noted that the phase velocity of a fluid-filled poroelastic thin shell in case of material-I is higher than that of material-II each for a pervious and an impervious surface. Thus, it is inferred the presence of mass-coupling parameter increases the phase velocity. The phase velocity for an empty shell in case of material-I is higher than that of material-II, each for a pervious and an impervious surface. The presence of mass-coupling parameter increases the phase velocity of empty poroelastic thin cylindrical shells.

The variation of phase velocity for moderately thick fluid-filled and empty poroelastic cylindrical shells is shown in Fig. 5. It is seen from Fig. 5 that the variation of phase velocity of moderately thick shell is similar to that of poroelastic thin shell both for fluid-filled and empty shells. In general, the phase velocity is increasing with the increase of thickness of poroelastic fluid-filled shells, while the phase velocity of empty shells remain same. The phase velocity of fluid-filled shells in case of material-II is higher than that of material-I for moderately thick shell each for a pervious and impervious surface. Hence, the presence of mass-coupling parameter is reducing the phase velocity of fluid-filled poroelastic shells. For empty moderately thick shells, the phase velocity in case of material-I is higher is higher than that of material-II. Hence for empty poroelastic shells, the presence of mass-coupling parameter is increasing the phase velocity of moderately thick poroelastic shells each for a pervious and an impervious surface.

Fig. 6 shows the phase velocity of fluid-filled and empty poroelastic thick shells each for a pervious and an impervious surface in case of material-II. From Fig. 6 it is clear that the phase velocity of pervious and impervious surfaces is almost same for fluid-filled poroelastic cylindrical thick shells. It is true for empty poroelastic thick shells also. In general, the phase velocity in case of material-II is higher than that of material-I in $0<\delta<0.3$ and beyond $\delta=0.3$ the phase velocity in case of material-I is higher than that of material-II each for a pervious and an impervious surface for fluid-filled shells. The phase velocity of empty thick shell, in case of material-I is higher than that of material-II for pervious and impervious surfaces. Hence, the presence of mass-coupling parameter reduces the phase velocity of empty poroelastic thick shells.

The cut-off frequencies of axially symmetric vibrations of fluid-filled and empty poroelastic cylindrical shells are presented as a function of h/r_1 in Fig. 7 each for a pervious and an impervious surface, in case of material-I. It is seen that as the thickness of a fluid-filled shell increases, the cut-off frequency increases gradually. The cut-off frequency is same for a pervious and an impervious surface for thin and moderately thick poroelastic shells, while the frequency for an impervious surface is higher than that of a pervious surface for poroelastic thick shell. Moreover, the frequency of an empty poroelastic thin shell is higher than that of a fluid-filled shell thin shell. As the thickness increases, the frequency increases for a fluid-filled shell than that of an empty shell each for a pervious and an impervious surface.

Similar phenomenon is observed in case of cut-off frequencies of fluid-filled and empty poroelastic cylindrical shells for material-II shown in Fig. 8. As the thickness of the shell increases, the frequency increases each for a fluid-filled and an empty poroelastic cylindrical shell. The frequency for poroelastic thin and thick fluid-filled shells is higher than that of frequency of moderately thick fluid-filled shell. Frequencies of empty poroelastic shells vary in similar way. Frequency for empty poroelastic thin shell is higher than that of a fluid-filled shell. Hence, presence of fluid in poroelastic thin shell reduces the frequency. The presence of fluid does

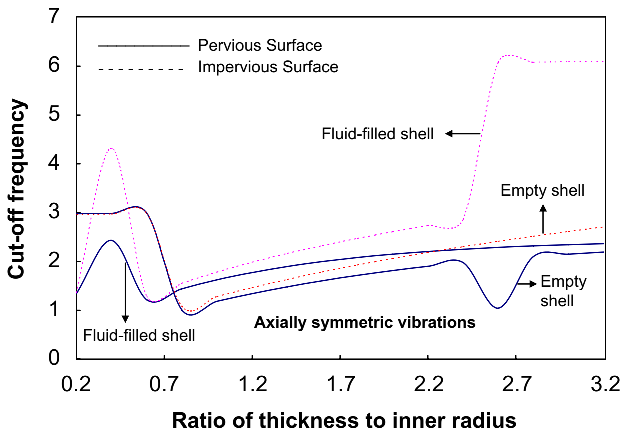

Fig. 7. Frequency as a function of ratio of thickness to inner radius (material-I).

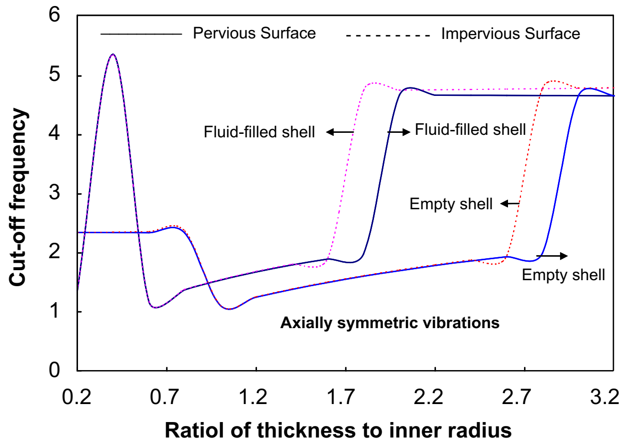

Fig. 8. Frequency as a function of ratio of thickness to inner radius (material-II).

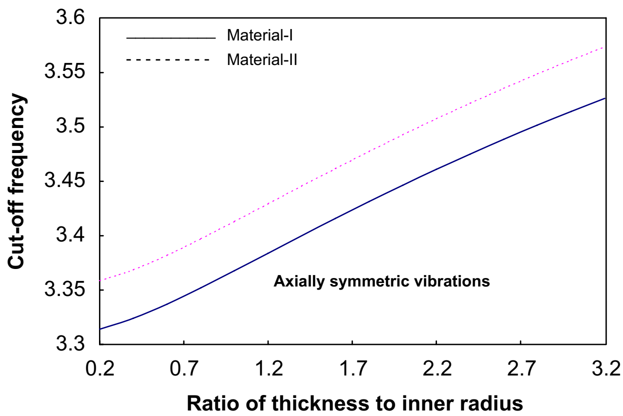

Fig. 9. Frequency as a function of ratio of thickness to inner radius (pervious surface, impervious surface, fluid-filled, empty shells).

not have significant effect on the frequencies of poroelastic moderately thick shells and poroelastic thick shells. The frequency of a fluid-filled poroelastic thick shell is higher for a pervious surface in case of material-II than that of material-I, while the frequency of an impervious surface is less in case of material-II than that of material-I. The frequency for an empty thick shell is higher in case of material-II than that of material-I each for a pervious and an impervious surface.

Frequency Eq. (35) is solved and the non-dimensional frequencies are determined. These frequencies are same for pervious and impervious surfaces, fluid-filled and empty poroelastic cylindrical shells. These common frequencies are shown in Fig. 9 for the considered materials. It is seen from Fig. 9 that the frequency increases with the increase in the thickness of the poroelastic shell. The cut-off frequency in case of material-II is higher than that of material-I. Here, the presence of mass-coupling parameter is reducing the frequency.

The phase velocity of axially symmetric vibrations of fluid-filled and empty bores is presented in Fig. 10 each for a pervious and an impervious surface, in case of material-I. From Fig. 10 it is seen that the phase velocity of pervious and impervious surfaces is same for a fluid-filled bore. Similarly, the phase velocity of pervious and impervious surfaces is same for poroelastic empty bores. The phase velocity of an empty bore is higher than that of a fluid-filled bore each for a pervious and an impervious surface. Hence, the presence of fluid in the poroelastic bore is reducing the phase velocity. The phase velocity of axially symmetric vibrations of fluid-filled and empty bores is presented in Fig. 11 each for a pervious and an impervious surface, in case of material-II. Fig. 11 shows that the variation of phase velocity in case of material-II is similar to that of

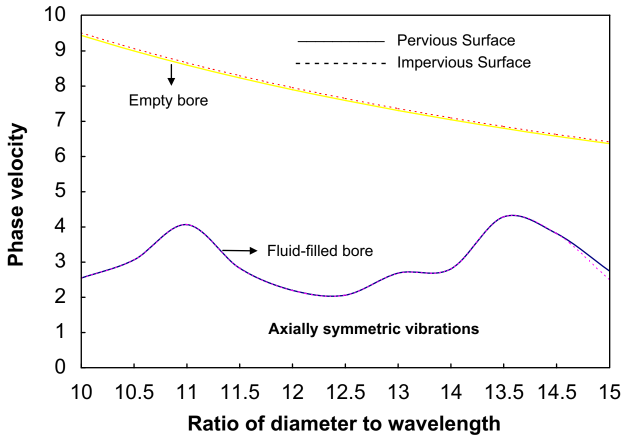

Fig. 10. Phase velocity as a function of ratio of diameter to wavelength (material-I).

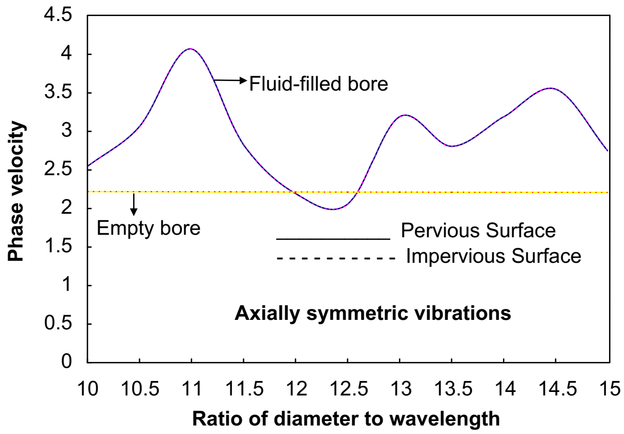

Fig. 11. Phase velocity as a function of ratio of diameter to wavelength (material-II).

material-I. In general, the phase velocity of a fluid-filled poroelastic bore is higher than that of an empty poroelastic bore in case of material-II. For material-II, the presence of fluid in the poroelastic bore is increasing the phase velocity. Hence, the presence of fluid is exhibiting a reverse phenomenon for poroelastic bores of two considered materials. The phase velocity of axially symmetric vibrations of a fluid-filled poroelastic bore in case of materials-I and II is almost same. Hence, the presence of mass-coupling parameter has no significant effect on the phase velocity. The phase velocity of an empty bore in case of material-I is higher than that of material-II. Thus, the presence of mass-coupling parameter is increasing the phase velocity of an empty poroelastic bore.

7. Concluding remarks

The study of axially symmetric vibrations of fluid-filled and empty poroelastic cylindrical shells of infinite extent has lead to following conclusions:

(i) The frequency equation of axially symmetric shear vibrations is independent of nature of surface and presence of fluid in the poroelastic cylindrical shell.

(ii) Thickness has no significant effect on the phase velocity of fluid-filled poroelastic shells.
(iii) In general, the phase velocity of an empty poroelastic cylindrical shell is higher than that of a fluid-filled poroelastic cylindrical shell.
(iv) The phase velocity of pervious and impervious surfaces is same for poroelastic bore, each for material-I and material-II.
(v) The phase velocity of an empty bore is higher than that of a fluid-filled poroelastic bore in case of material-I.
(vi) The phase velocity of a fluid-filled bore is higher than that of an empty poroelastic bore in case of material-II.
(vii) The absence of mass-coupling parameter increases the phase velocity in a fluid-filled poroelastic bore.

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