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High-order angular response beamformer for vector sensors

Joseph A. Clark*

Spectrum Technology, Inc., 1522 North Taylor Street, Arlington, VA 22207, USA

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Abstract

A linear processing scheme for computing higher-order angular response modes of a vector sensor is described. Examples of modal response beampatterns are presented. The response modes form (in principle) a complete, orthonormal set that can be transformed into steerable sets of one or more directive beams. The linear processing scheme facilitates calibration of vector sensor measurement systems. The angular resolution that can be achieved with the new processing scheme is predicted to be $(155/N_m)$ degrees, where N_m is the highest order of computed response mode, for the higher orders. The number of higher-order response modes appears to be limited only by the computational power available.
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1. Introduction

Underwater acoustic vector sensors combine a triaxial arrangement of motion sensing devices (such as accelerometers) with a pressure sensing hydrophone in a neutrally buoyant package smaller than half a wavelength [1]. The sensors are used alone or in arrays to detect and localize sources of sound [2,3]. Analytical models of vector sensor measurement systems have been developed to evaluate their detection [4,5] and localization [6,7] performance.

Modal beam processing was recently introduced as a processing scheme for spherical [8] and circular [9,10] arrays of scalar sensors (microphones). Rather than directly beamforming an array of signals, a two-step linear process is used: (1) spherical or cylindrical modal beams are formed. (2) The modal beams are combined to form one or more computationally steerable directive beams. The number of modal beams that can be formed from microphone array data is limited to a few lower modes, however, by the number of sensors in the array and by the radius of the array.

We have investigated the possibility of adapting the modal processing scheme to vector sensor measurement systems [11,12]. A vector sensor can be electrically steered in any direction around its origin or simultaneously in multiple directions. Thus we are able to generate data with a single vector sensor equivalent to the data generated by the circular or spherical arrays of microphones noted above. We found that for the case of vector sensors, there are no apparent physical restrictions on the number of higher-order modal and directive beams that can be formed.

*Corresponding author. Tel.: +1 301 227 4637.
E-mail address: clarkja3@verizon.net

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Alternative nonlinear processing schemes for localizing underwater sound sources with high angular resolution have been reported in the literature [13–15]. The linear processing scheme described here has the advantage over nonlinear schemes that high angular resolution can be achieved without affecting the calibration of a measurement system.

In this communication the elementary response modes of a vector sensor are reviewed in order to introduce the terminology used to describe the higher response modes. Then the higher-order modes are described. The angular response modes of a vector sensor are introduced in terms of their modal beampatterns. Directive beampatterns obtained by combining sets of modal beams are also presented. Procedures for steering and forming multiple directive beams are described. The localization performance of vector sensor measurement systems incorporating the new response modes is evaluated and a formula predicting their performance is presented. For simplicity, a two-dimensional (2-D), deterministic approach is taken in this communication and only measurement systems composed of a single vector sensor are considered.

2. Review of elementary response modes

The amplitude response (b) of a vector sensor measurement system to an arbitrary wavefield can be expressed as the matrix product of a steering vector (**w**) and a data vector (**d**) [2]. The vector elements (d_i) account for signals from each element sensor (i) in the (four element) vector sensor package. The intensity response (I) of a measurement system is $I = |b|^2$. The intensity response of a simulated measurement system is presented in the following figures as relative intensity levels (IL(ϕ)), where $\mathrm{IL} = 10\log_{10}(I/I_0)$ and (I_0) is the maximum value of ($I(\phi)$), a function of the look directions (ϕ). IL(ϕ) corresponds to a beampattern characterizing the angular response of a vector sensor.

In the elementary case of a system composed of just one omni directional hydrophone, the intensity response forms a circular pattern as shown by the dashed circle in Fig. 1(a). This pattern is directly identifiable as a zero-order modal beam (b_0). It is equivalent to the radiation pattern produced by a monopole point source.

The measurement system response of a particle motion sensor (such as an accelerometer) to a plane wave incident at an angle (ϕ_p) forms a first-order modal beampattern (a dipole pattern). This pattern is also shown in Fig. 1(a) for the case of a wave incident at $\phi_p = 0°$ and an accelerometer oriented in the same direction.

If the two modal responses noted above are linearly summed (as amplitudes) and then squared, the directive pattern shown in Fig. 1(b) (a cardioid pattern) is produced. While the angular response represented by this pattern is quite broad, a direction of maximum response can be identified in the zero degree direction. In order to maintain calibration of a measurement system utilizing this directive beam pattern, the two modal amplitude responses must each be normalized by a factor of (0.5). Signal energy is then conserved. However, normalization factors are suppressed in the following presentation to simplify the equations.

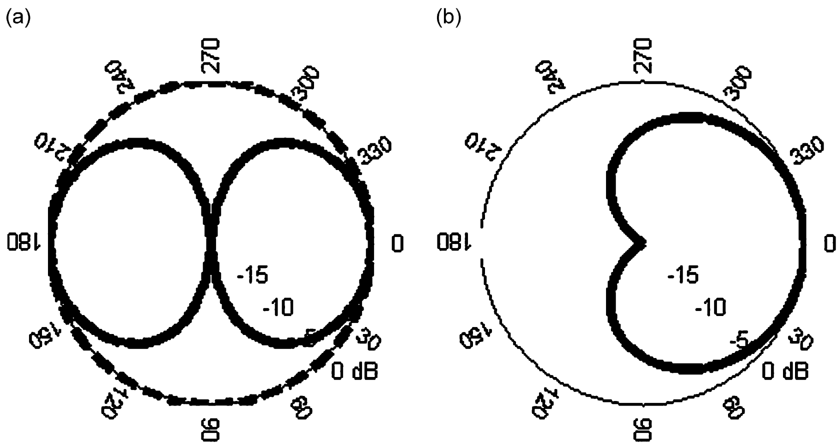

Fig. 1. Intensity response of a vector sensor to a plane wave incident from 0°: (a) $n = 0$ modal beam (dashed line), $n = 1$ modal beam (solid line) and (b) directive beam formed by sum of modal beams.

The localization performance of a radiated noise measurement system incorporating a vector sensor can be evaluated from the directive beampattern. One commonly used measure of the localization performance is the beamwidth (BW) between points that correspond to half the power measured in the direction of maximum response (3 dB down points). The BW computed by this measure for the beampattern in Fig. 1(b) is 131°.

A vector sensor includes three collocated accelerometers oriented in orthogonal directions. These accelerometer signals are combined by an appropriate steering vector (**w**) to orient the response pattern in any desired direction. The elementary steering vector takes the form $\mathbf{w}_1 = [\cos\phi, \sin\phi]$ for the 2-D case illustrated in the figures. The amplitude response of this first-order angular response mode as a function of direction is therefore:

$$b_1(\phi) = d_x \cos\phi + d_y \sin\phi, \tag{1}$$

where (d_x) and (d_y) represent signals from the two accelerometers oriented in the x and y directions, respectively.

The ability to steer the response of the first-order directive beam in any direction is demonstrated in Fig. 2. The two modes presented in Fig. 1(a) are shown again separately in Fig. 2(a). However, the first-order (dipole) response is now steered to 330°. The combined modes are shown in Fig. 2(b). The directive (cardioid) beam is observed to also be oriented in the 330° look direction.

3. Higher-order vector sensor angular response modes

Higher-order response modes are generated by modifying the steering vector (**w**). An nth order steering vector of the form $\mathbf{w}_n = [\cos n\phi, \sin n\phi]$ generates an nth order modal amplitude response of the form:

$$b_n = d_x \cos n\phi + d_y \sin n\phi. \tag{2}$$

Summation of higher-order response modes into steerable directive beams requires an additional phase angle coefficient. This coefficient must be obtained from a priori information about the source locations or the desired look direction. The coefficient modifies the amplitude response given by Eq. (2). The modified form is

$$b_n = d_x \cos(n\phi - (n-1)\phi_p) + d_y \sin(n\phi - (n-1)\phi_p), \tag{3}$$

where (ϕ_p)is the direction of an incident wave or the desired look direction. If there is just one incident wave, (ϕ_p) is determined from the directive (cardioid) pattern (Fig. 2(b)) formed by the elementary vector sensor modes. (The case of multiple incident waves is discussed below.) The additional phase angle coefficient ($-(n-1)\phi_p$) included in Eq. (3) can be physically interpreted as a rotation applied to each of the higher-order modal beam patterns that causes one of the modal beam lobes in each modal beam pattern to superpose exactly with a lobe in the other patterns in the desired look direction rather than in the zero degree reference

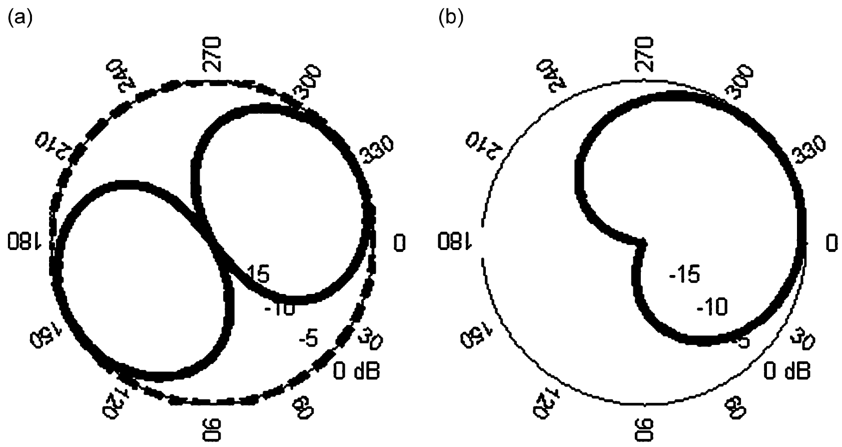

Fig. 2. Intensity response of a vector sensor to a plane wave incident from 330°: (a) $n = 0$ modal beam (dashed line), $n = 1$ modal beam (solid line) and (b) directive beam formed by sum of modal beams.

direction. It does not change the magnitude of the beam patterns. The beampattern corresponding to the amplitude response expressed by Eq. (3) is shown in Fig. 3(a) for the case of a second-order (quadrupole) response mode and a wave incident from 330°.

The result obtained by summing over the $n = 0$, 1 and 2 modal response modes is shown in Fig. 3(b). It is observed that the second-order directive (hypercardioid) beampattern has a strong maximum in the same look direction as in Fig. 2(b). The BW of the second-order directive beampattern is 71.7°.

An extension of the modal beam generation scheme to 10th order is shown in Fig. 4. A 10th-order modal pattern is shown in Fig. 4(a) and the directive beam formed by a combination of the first ten orders is presented in Fig. 4(b). The case of a wave incident from 330° is again chosen. The BW of the 10th-order directive beam is 15.5°.

If more than one significant incident plane wave is present in the wavefield, the multiple directions can be approximately determined by a singular value decomposition of the data matrix formed by the outer product of the data vector with itself. A description of this procedure for microphone arrays is given in Ref. [10]. (The option of scanning a highly directive beam to locate sources is also available).

The modal beam-based vector sensor processing scheme for the case of more than one source is illustrated in Fig. 5. In this case a summation was performed over a complete set of modal responses oriented in the 0°

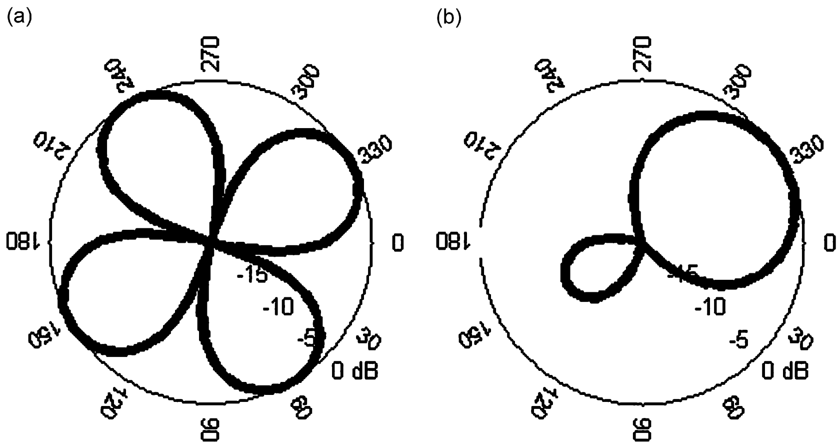

Fig. 3. Intensity response of a vector sensor to a plane wave incident from 330°: (a) $n = 2$ modal beam and (b) directive beam formed by sum of $n = 0$, 1 and 2 modal beams.

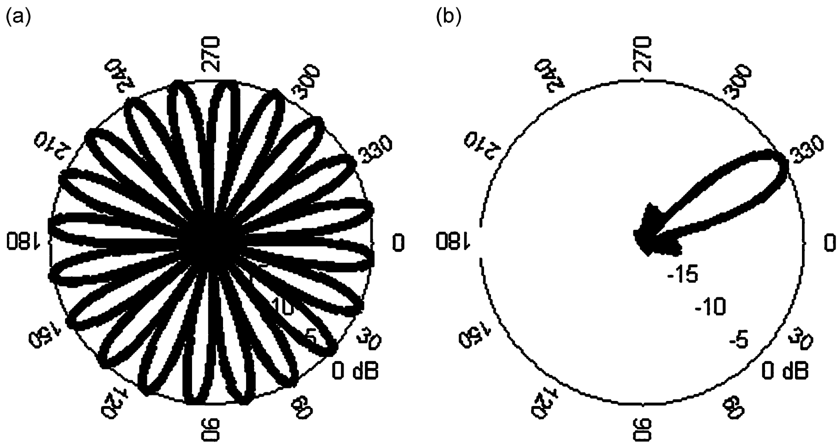

Fig. 4. Intensity response of a vector sensor to a plane wave incident from 330°: (a) $n = 10$ modal beam and (b) directive beam formed by sum of $n = 0$–10 modal beams.

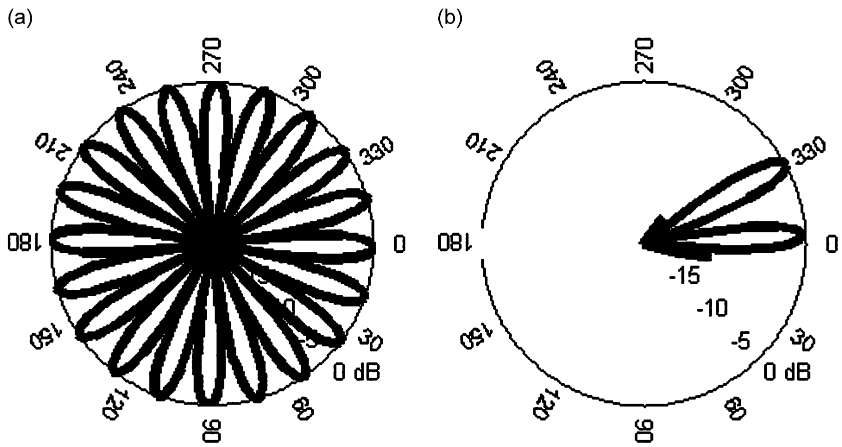

Fig. 5. Intensity response of a vector sensor to a plane waves incident from 0° to 330°: (a) $n = 10$ modal beam and (b) directive beam formed by sum of $n = 0$–10 modal beams.

direction and also a second set oriented in the 330° direction. Fig. 5(a) shows the sum of the two 10th-order modal beampatterns oriented in the two directions. (The directional information is not obvious in the figure.) Fig. 5(b) shows the directive beams formed by the summation of all the modal responses to 10th order. The figure demonstrates that the cumulative result is a beampattern composed of two highly directive beams in the selected directions. (The ability of modal beam-based processing schemes to simultaneously localize multiple sources has previously been demonstrated for microphone array measurement systems [8–10]).

4. Predicted localization performance

The predicted measure of single source localization performance (BW) is a function of the number of beams in the highest modal beampattern. An additional factor accounts for the particular definition of BW chosen. For the 2-D case, the half-power BW of a directive beam of order (N_m) is predicted by the formula: $\mathrm{BW} = 155/N_m$ degrees.

5. Conclusions

While investigating ways to adapt modal beam processing to vector sensor measurements, we have developed a linear scheme for computing higher-order angular response modes of vector sensors. The angular response modes can be identified with modal beampatterns. Each response mode corresponds to an independent measurement of an acoustic field because each mode samples a distinct portion of the total sound field.

Steerable, single and multiple directive beams are formed by linearly summing sets of modal responses up to a chosen highest order (N_m). The highest modal order does not appear to be physically limited. It has been shown that a localization performance measure can be computed from the beampattern of the directive beams. The linear character of this processing scheme facilitates calibration of radiated noise measurement systems utilizing vector sensors.

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