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Reduction of uncertainty effect on damage identification using feedback control

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Abstract

This paper presents a study of the effect of model uncertainty on damage detection when feedback controllers, such as passive controllers, are incorporated into the structure. The use of feedback control techniques can generate additional modal parameters of closed-loop systems for structural damage identification. One major issue of the application of feedback control techniques is the effect of model uncertainty, such as boundary condition variation, on structural damage detection. The objective of this research is to develop control methodologies to reduce the effect of model uncertainty on structural damage detection.

1. Introduction

Reliable and efficient techniques for structural health monitoring of large structures, such as aircraft, are essential to safe operation, cost reduction, and failure prevention. Currently, there are various structural health monitoring technologies under development, including, for example, vibration-based methods [1–4], ultrasonic guided wave-based methods [5], and electro mechanical impedance-based methods. The conventional ultrasonic non-destructive evaluation methods have been applied in the engineering community for many decades. However, ultrasonic inspection over large areas of thin-wall structures, such as airframes, can be time-consuming and expensive to apply. On the other hand, vibration-based methods utilize the structural dynamic responses that are globally sensitive to damage rather than localized conventional techniques such as ultrasonic and eddy current methods.

Vibration-based methods use the changes of vibration characteristics for damage identification. In general, vibration-based methods require the measurements of natural frequencies and mode shapes [1,3]. However, mode shape measurements require a large number of sensors. One major issue and limitation of the frequency-shift-based method, which requires only the measured natural frequencies, is that the number of reliable identified natural frequencies is usually much smaller than the degrees of freedom required to accurately identify the damage. To solve this problem, a virtual passive control technique for structural health monitoring [4] was recently developed. This passive control technique uses only the existing real-time control

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systems to alter natural frequencies and no additional physical elements are attached to the system. Another limitation of the frequency-shift-based method is that the natural frequencies can be relatively insensitive to some minor damage. The feedback control can be designed to enhance the sensitivity of natural frequencies to structural damage [6], such as joint damage, for the improvement of damage identification.

When feedback control is applied to get additional data, the effect of model uncertainty and measurement noise on the identified natural frequencies of the closed-loop system is crucial for successful damage identification [7]. This paper presents an investigation of the effect of model uncertainty, boundary condition variation, on natural frequency changes for the considered damage when feedback controllers are incorporated into the structure. The study uses a correlation approach [3], which is based on the comparison of the identified parameter change and the change of the analytical model due to damage. Only a small number of sensors are required for the correlation approach for damage identification [8]. This transfer function correlation approach has been successfully applied to damage identification of a flexible beam experiment with only one sensor [9]. The previous study shows that the performance of structural health monitoring is sensitive to the designed controller [8]. Methodologies of optimal control design are developed to reduce the effect of model uncertainty on damage identification. The simulation based on the finite element model of a cantilevered Euler's beam is used to demonstrate the proposed approach.

2. Correlation approach

A brief description of correlation approach is given in this section. In this paper, the study is based on the analysis of the natural frequencies of the closed-loop system. The natural frequency vector of the first n structural modes of the closed-loop system is defined as

$$\boldsymbol{\omega} = [\,\omega_1 \quad \omega_2 \quad \cdots \quad \omega_n\,], \tag{1}$$

where ω_j is the jth structural natural frequency of the closed-loop system. The changes of the natural frequency vectors for the ith referred damage case, such as the stiffness loss of the ith element, are defined as

$$\Delta\boldsymbol{\omega}^i = \boldsymbol{\omega}^i - \boldsymbol{\omega}, \quad i = 1, \ldots, m, \tag{2}$$

where $\boldsymbol{\omega}^i$ are the natural frequency vectors of the ith referred damage case. The correlations between the tested system with the weighted change vector, $\Delta\boldsymbol{\omega}_W$, which represents the difference of the identified parameters between the tested system and the healthy system, and the ith damage case with the weighted change vector $\Delta\boldsymbol{\omega}^i{}_W$ are defined as

$$C_i = \frac{\Delta\boldsymbol{\omega}_W(\Delta\boldsymbol{\omega}^i_W)^{\mathrm{T}}}{|\Delta\boldsymbol{\omega}_W||\Delta\boldsymbol{\omega}^i_W|}, \tag{3}$$

where the weight of each element is the standard deviation of the corresponding element due to all the considered m damage cases [3]. The correlation C_i represents the cosine between two vectors. The value of the correlation C_i is between -1 and 1. When the absolute value of C_i is close to 1, it indicates that the ith element is a damage candidate [8].

3. Optimal control design

The changes of the natural frequencies due to structural damage vary as boundary condition changes. The natural frequency of the first n structural modes of the closed-loop system can be expressed as

$$\boldsymbol{\omega}(\mathbf{c},\mathbf{b},\mathbf{z}) = \left[\,\omega_1(\mathbf{c},\mathbf{b},\mathbf{z}) \quad \omega_2(\mathbf{c},\mathbf{b},\mathbf{z}) \quad \cdots \quad \omega_n(\mathbf{c},\mathbf{b},\mathbf{z})\,\right], \tag{4}$$

with

$$\mathbf{c} = \left[\,c_1 \quad \cdots \quad c_{nc}\,\right], \quad \mathbf{b} = \left[\,b_1 \quad \cdots \quad b_{nb}\,\right], \quad \mathbf{z} = \left[\,z_1 \quad \cdots \quad z_m\,\right], \tag{5}$$

where $\omega_i(\mathbf{c},\mathbf{b},\mathbf{z})$ is the natural frequency of the ith structural modes of the closed-loop system with the designed controller $\mathbf{c}$, boundary condition variable vector $\mathbf{b}$, and damage variable vector $\mathbf{z}$. The problem of concern here is to find an "optimal" controller that can minimize the changes of $\Delta\boldsymbol{\omega}^i$ due to the boundary change from condition (1) to condition (2). For the considered m damage cases, the cost function can be defined as

$$J = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{|\Delta\omega_i^j(\mathbf{c}, \mathbf{b}^{(2)}) - \Delta\omega_i^j(\mathbf{c}, \mathbf{b}^{(1)})|}{\sigma_i}, \quad (6)$$

where $\Delta\omega_i^j(\mathbf{c},\mathbf{b})$ is the change of the ith natural frequency due to the jth damage case, and σ_i is the weight used in correlation approach, the standard deviation of the ith natural frequency due to all the damaged cases for the open-loop system. The problem is to find an optimal controller $\mathbf{c}$ to minimize the cost function J with boundary condition vector changing from $\mathbf{b}^{(1)}$ to $\mathbf{b}^{(2)}$.

4. Discussion of results

The finite element model of a cantilevered aluminum Euler's beam, as shown in Fig. 1, is used in the study. For structural damage, we consider the stiffness loss of 15 elements with equal lengths from the fixed end to the end of a spring with a varied spring constant k_d (N/m), which represents the uncertain boundary condition. The designed controller is an attached system with two degrees of freedom as shown in Fig. 1.

This passive controller has four designed variables, two masses and two springs. In the control design, these variables are constrained in the specified ranges to limit physical conditions, for example the maximum of each mass is lower than 20% of beam mass. The *Matlab* program *fmins*, which uses the Nelder–Mead simplex (direct search) method, is applied to find a solution to minimize the cost function in Eq. (6). Fig. 2 shows the natural frequencies of the first three modes as functions of spring constant ($k_d \geqslant 5$) for the open-loop system without passive controller. The natural frequency of the first mode increases 1.9% as k_d changes from 5 to 10^5; the natural frequency of the second mode increases 7.48%; and the natural frequency of the third mode increases 18.8%. The natural frequencies of these three modes have negligible variations as k_d changes from 10^3 to 10^5. Fig. 3 shows the changes of natural frequencies of the first three structural modes due to 10% stiffness loss of each element for five different k_d cases, $k_d = 5, 10, 20, 100$, or 10^5. For all the damage cases, the first mode natural frequency change has little variation as k_d increases from 5 to 10^5. The second mode natural frequency change has some variation, and the third mode natural frequency change for some damage cases is significant as k_d varies. In this example, there is only one boundary condition variable k_d. For the open-loop system without controller, the change of the ith natural frequency due to 10% stiffness loss of the jth element is computed as $\Delta\omega_i^j(k_d)$, as a function of k_d. For the closed-loop system with controller $\mathbf{c}$, the change of the ith natural frequency due to 10% stiffness loss of the jth element is computed as $\Delta\omega_i^j(\mathbf{c}, k_d)$, as a function of $\mathbf{c}$ and k_d. For the comparison, a cost function for the open-loop system is defined as a function of k_d

$$J_0(k_d) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{|\Delta\omega_i^j(k_d) - \Delta\omega_i^j(5)|}{\sigma_i}. \quad (7)$$

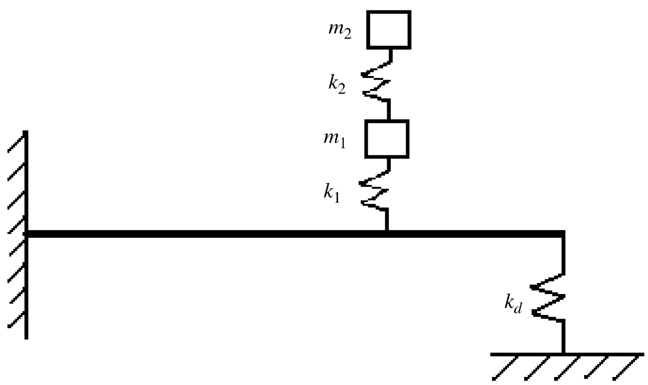

Fig. 1. Cantilevered Euler's beam with an attached passive controller.

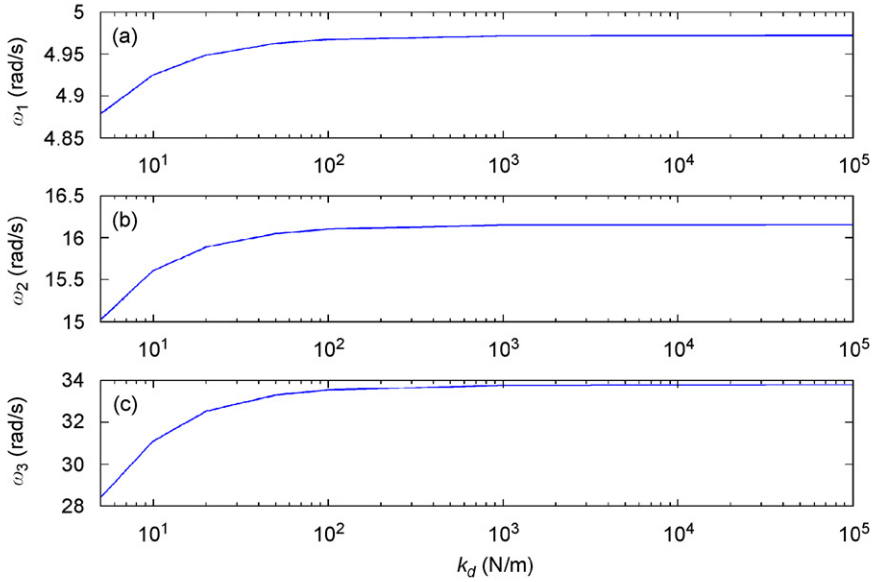

Fig. 2. Natural frequencies of the first three modes as functions of k_d: (a) 1st mode; (b) 2nd mode; and (c) 3rd mode.

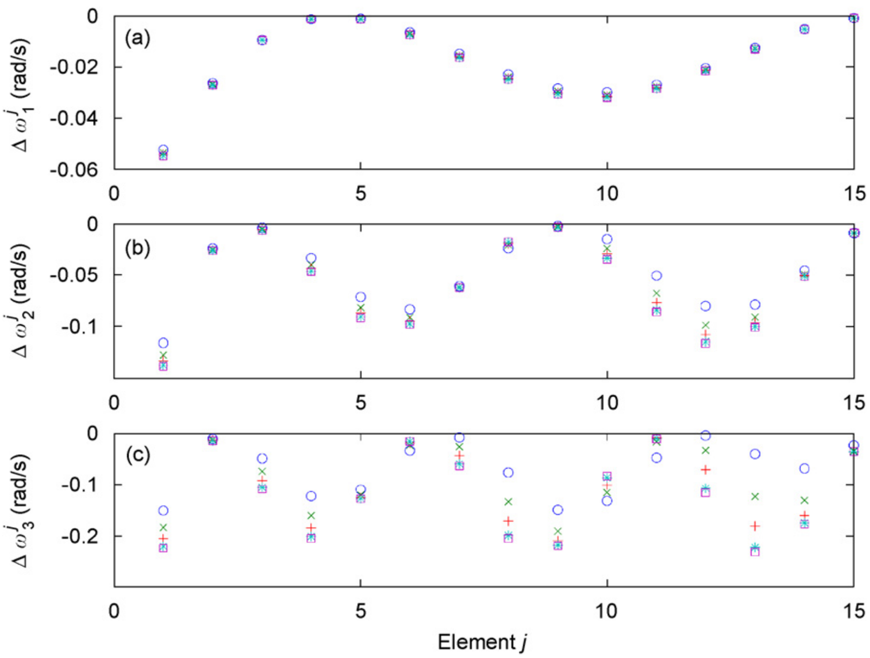

Fig. 3. Natural frequency changes of the first three modes due to 10% stiffness loss of each element: (a) 1st mode; (b) 2nd mode; and (c) 3rd mode. o—$k_d = 5$; x—$k_d = 10$; +—$k_d = 20$; *—$k_d = 100$; and □—$k_d = 10^5$.

The cost function of the closed-loop system with controller **c** is defined as

$$J_c(k_d) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{|\Delta\omega_i^j(\mathbf{c}, k_d) - \Delta\omega_i^j(\mathbf{c}, 5)|}{\sigma_i}. \tag{8}$$

The optimal control problem is to find a controller to minimize this cost function. The ratio between the cost function of the closed-loop system with the optimal controller and the cost function of the open-loop system is defined as

$$r(k_d) = \frac{J_c(k_d)}{J_0(k_d)}. \tag{9}$$

This ratio represents an index of performance improvement. For each of the 15 locations (15 elements) on the beam, an optimal controller with two degrees of freedom, shown in Fig. 1, is designed. Table 1 shows the results of four cases with optimal controllers at four different locations S. The cost function J_0, which represents an index of the change due to boundary condition variation, increases as k_d increases, and it changes little when k_d varies from 10^3 to 10^5. The design with the optimal controller at location 12 gives the best performance, and this controller reduces the effect due to boundary condition uncertainty to less than 35% of the uncertainty effect of the open-loop system for various k_d cases. Fig. 4 shows the parameter change $\Delta\omega_i^j(\mathbf{c}, k_d)$ of the closed-loop system with the optimal controller at location 12. Fig. 5 shows the changes of natural frequency due to k_d uncertainty, $\Delta\omega_i^j(10^5)$–$\Delta\omega_i^j(5)$ and $\Delta\omega_i^j(\mathbf{c},10^5)$–$\Delta\omega_i^j(\mathbf{c},5)$, for both open-loop and closed-loop systems where k_d changes from 5 to 10^5. The change of k_d has little effect on the natural frequency of the first mode, and the controller has little influence on this mode. The natural frequency of the second mode shows some changes due to the variation of k_d, and the controller reduces the effect from boundary condition variation. The uncertainty of k_d has a significant effect on the natural frequency of the third mode, and the optimal controller significantly reduces the effect of boundary condition uncertainty.

Table 1
Cost functions and ratios for various boundary condition k_d (N/m)

	$k_d = 10$	$k_d = 20$	$k_d = 50$	$k_d = 100$	$k_d = 10^3$	$k_d = 10^5$
$J_0(k_d)$	7.7219	12.2908	14.8168	15.5843	16.2378	16.3075
$r(k_d)$, $S = 1$	0.6743	0.5781	0.5830	0.5796	0.5747	0.5741
$r(k_d)$, $S = 4$	0.4727	0.4453	0.4388	0.4376	0.4369	0.4368
$r(k_d)$, $S = 7$	0.5377	0.4470	0.4122	0.4039	0.3973	0.3966
$r(k_d)$, $S = 12$	0.3495	0.3201	0.3140	0.3131	0.3127	0.3127

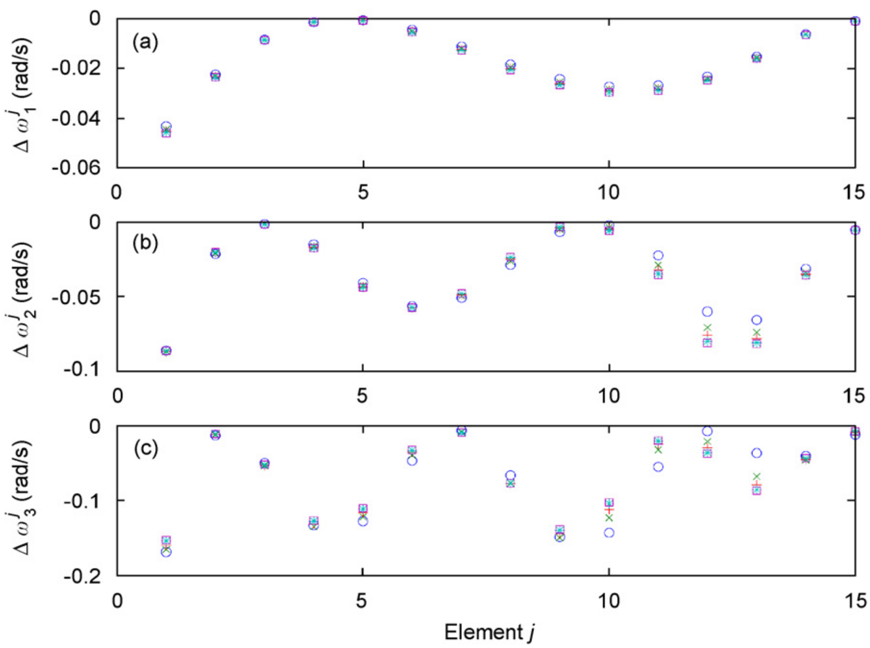

Fig. 4. Natural frequency changes of the first three structure modes of closed-loop system due to 10% stiffness: (a) 1st mode; (b) 2nd mode; and (c) 3rd mode. o—$k_d = 5$; x—$k_d = 10$; +—$k_d = 20$; *—$k_d = 100$; and □—$k_d = 10^5$.

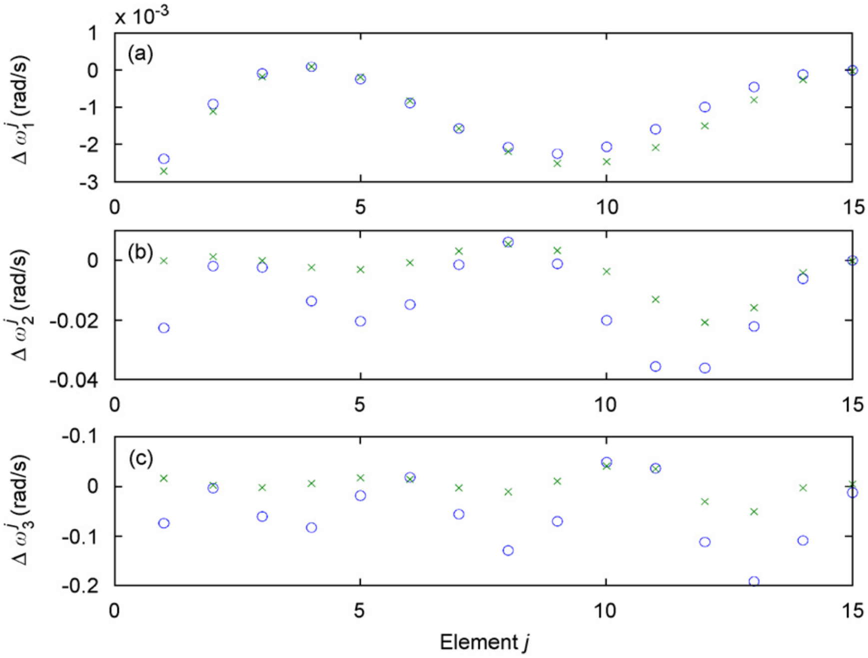

Fig. 5. Natural frequency changes due to k_d variation from 5 to 10^5: (a) 1st mode; (b) 2nd mode; and (c) 3rd mode. o—open-loop system; and x—closed-loop system.

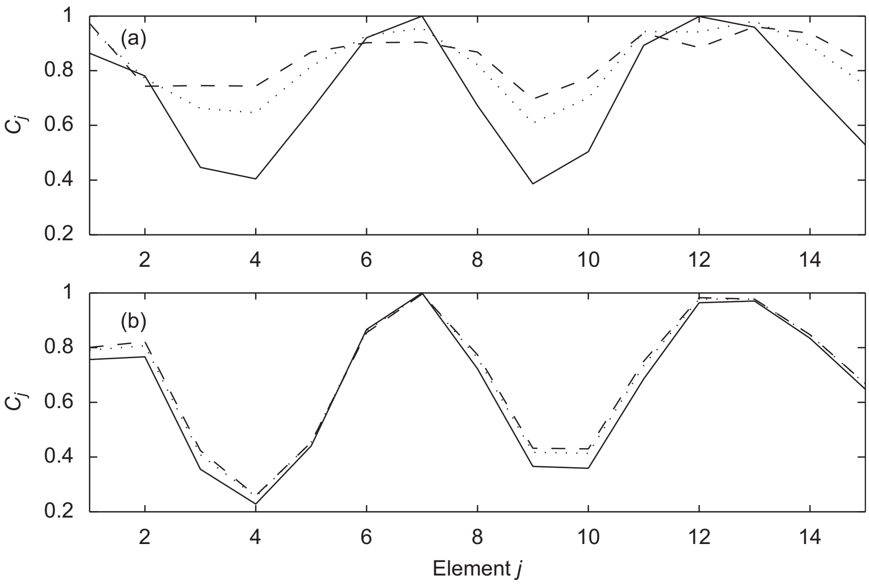

Fig. 6. Correlation of element 7 damage case: (a) open-loop system; and (b) closed-loop system. —— $k_d = 5$; … $k_d = 20$; and - - $k_d = 10^5$.

Next the results based on the correlation analysis are presented. The parameter changes $\Delta\omega_i^j(\mathbf{c},5)$ of the system with $k_d = 5$ are used as the referred ones for computing the correlation in Eq. (3). Figs. 6 and 7 show the results of correlation for the element 7 damage case with various k_d. For the element 7 damage case, the optimal controller at location 12 significantly reduces the effect of k_d variation on the natural frequency of the

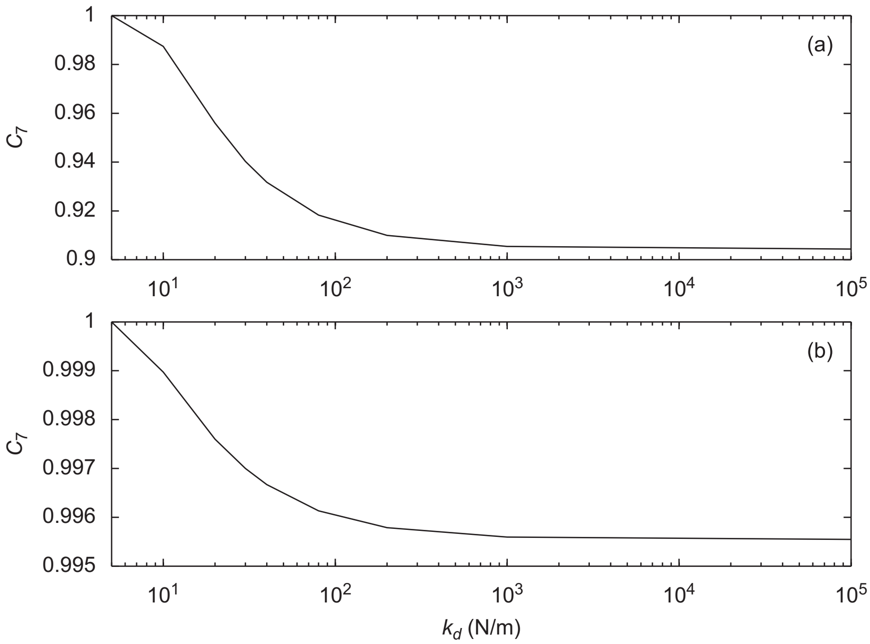

Fig. 7. Correlation C_7 as a function of k_d for element 7 damage case: (a) open-loop system; and (b) closed-loop system.

third mode, as shown in Fig. 5. Fig. 6 shows that the change of k_d has a significant effect on the correlation of the open-loop system, but it has little effect on the correlation of the closed-loop system with the optimal controller. Fig. 7 shows the correlation C_7 as a function of k_d for the element 7 damage case. The correlation C_7 of the open-loop system drops to below 0.92 when k_d is higher than 100, which indicates that element 7 is not a damage candidate. The correlation C_7 of the closed-loop system is always larger than 0.995, which clearly identifies element 7 as a damage candidate.

5. Concluding remarks

This paper presents a study of the effect of boundary condition uncertainty on structural damage detection when feedback controllers are incorporated into the structure. Methodologies are developed to design optimal controllers to reduce the effect of boundary condition uncertainty on natural frequency changes for the considered damage cases. A correlation approach is used to demonstrate that the damage detection can be significantly improved with the use of the optimal controller methodology under boundary condition uncertainty.

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