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Suspended bridges subjected to moving loads and support motions due to earthquake

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Abstract

The paper deals with the vibration of suspended bridges subjected to the simultaneous action of moving loads and vertical support motions due to earthquake. The basic partial integro-differential equation is applied to the vertical vibration of a suspended beam. The dynamic actions of traffic loads are modelled as a row of equidistant moving forces, while the earthquake is considered by vertical motions of supports. The governing equation is solved first analytically to receive an ordinary differential equation and next numerically. Moreover, the designed world's largest suspended bridge—Messina Bridge—is investigated (central span of length 3.3 km). The paper studies the effect of various lags of the earthquake arrival because the earthquake may appear at any time when the train moves along a large-span bridge. The modified Kobe earthquake records have been applied to calculations. The results indicate that the interaction of both the moving and seismic forces may substantially amplify the response of long-span suspended bridges in the vicinity of the supports and increase with the rising speed of trains.

1. Introduction

The suspended bridges are usually built for long spans and are subjected to dead and moving loads, seismic motions of supports, to wind forces, temperature and many other forces. The dynamic study of suspended bridges was initiated by the collapse of the well-known Tacoma Bridge in the USA. Thereafter, several papers and books appeared that discovered the dangerous effect of wind on this type of structures [1–3]. Besides, the moving loads cause another type of vibrations, see Refs. [4–10]. A lot of references tackled the earthquake itself.

Among the many problems of suspended bridges, the simultaneous dynamic action of moving loads together with the motion of bridge supports due to earthquake remains to be investigated in a sufficient way. The first attempts were done in Refs. [11,12], where only the routine types of suspension bridges (spans about

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500 m) were taken into account. The conclusions of these papers showed that the maximum bridge response depends on the occurrence time of support motion with respect to the arrival time of moving loads on the bridge.

Now, we have a super-long suspended bridge in mind, e.g. the Messina Bridge with a 3.3 km-long central span, which should link mainland Italy with island Sicily. The present paper would like to show the effect of the simultaneous dynamic action of moving loads together with the motion of bridge supports due to earthquakes. For the first time, only the vertical vibrations of the suspended bridge are considered. The subject is very broad and the field for further investigations has been opened (e.g. spatial vibration due to horizontal forces, stress wave propagation, stochastic, probability, reliability questions, etc.).

2. Basic assumptions

The present study applies the simplified model of a suspended beam [3,9] to the dynamic analysis of a long-span suspension bridge subjected to moving loads and support excitations due to earthquake. It is assumed:

(1) Linear deflection theory of the suspended Bernoulli–Euler beam with single span and constant horizontal component of the cable tension. Many of the actual bridges are constructed with more than one cable. In the present simplified idealization, the cables are tightened together in the vertical plane.
(2) The vertical vibrations of the bridge are considered only. They are excited by a sequence of equidistant moving forces, P, and the vertical support motions due to earthquake, $a(t)$, see Fig. 1.
(3) The vertical motions exciting the supports of the bridge are synchronous due to the rapid transmission of P-waves caused by earthquake, where P-waves denote another phenomenon than moving forces P mentioned above. (A more general case—a time shift in support motions is studied in Ref. [12].)
(4) The deflection shape of the tensile cable is a parabola of the second degree in the equilibrium position.
(5) The cable tension $T+\Delta T$ is, in fact, doubled tension—total tension of two cables normally existing in suspension bridges (including the Messina Bridge).
(6) The effect of inertia of car bodies for train-type loads will be neglected, as it is negligible for the response of the large-span bridges [9].

3. Formulation

The theoretical model of a suspended bridge is schematically drawn in Fig. 1, where a sequence of identical forces P (lumped loads) with equal distances d is moving along the beam at a constant speed v. The suspended bridge is composed of a uniform beam and a cable that is anchored at the tip points of two undeformable pylons. The beam is simply supported at rigid pylons. Thus, the partial differential equation describes the dynamic behaviour of the bridge, see Ref. [9] (in what follows, the primes denote the derivatives with respect to the length coordinate x and dots with respect to time t):

$$m\frac{\partial^2 u(x,t)}{\partial t^2}+c\frac{\partial u(x,t)}{\partial t}+EI\frac{\partial^4 u(x,t)}{\partial x^4}-T\frac{\partial^2 u(x,t)}{\partial x^2}-\Delta T\frac{\mathrm{d}^2 y(x)}{\mathrm{d}x^2}=p(x,t), \tag{1}$$

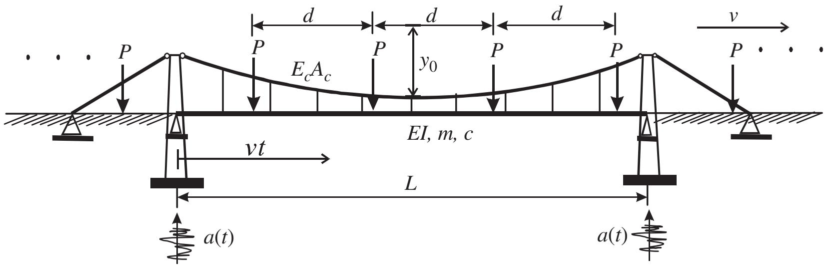

Fig. 1. A single-span suspended beam subjected to moving loads P and support motions $a(t)$.

where m is the mass per unit length of the beam, $u(x,t)$ are the vertical displacements of the beam, EI is the flexure rigidity of the beam, c is the damping constant of the beam, $p(x, t)$ is the load per unit length caused by the moving loads P , see Refs. [14,15], T is the horizontal tension in the cable, and ΔT is the horizontal component of the incremental force in the suspended cable due to external loadings [13]. The horizontal tension in the cable due to the dead load is given by

$$T = -\frac{mg}{y} = \frac{mgL^2}{8y_0} \tag{2}$$

and the sag function of the cable is

$$y(x) = 4y_0[x/L - (x/L)^2], \tag{3}$$

where y_0 is the cable sag at mid span, see Fig. 1.

To express the incremental force ΔT in the cable, the following equations and symbols for a parabolic cable shape will be adopted in this study (g is the gravity constant, L is the central span length of the beam, E_cA_c is the axial rigidity of the cable):

Let us consider the original length $\mathrm{d}s_0 = \sqrt{[(\mathrm{d}x)^2 + (\mathrm{d}y)^2]}$ and deformed length $\mathrm{d}s = \sqrt{[(\mathrm{d}x)^2 + (\mathrm{d}y + \mathrm{d}u)^2]}$ of an infinitesimal element of the cable shown in Fig. 2. By Hook's law for the deformed cable element due to the force increase ΔT, it yields [13]

$$\frac{\Delta T(\mathrm{d}s_0/\mathrm{d}x)}{E_cA_c} \approx \frac{\mathrm{d}s - \mathrm{d}s_0}{\mathrm{d}s_0} \approx \frac{\mathrm{d}y}{\mathrm{d}s_0}\frac{\mathrm{d}u}{\mathrm{d}s_0} = \left(\frac{\mathrm{d}x}{\mathrm{d}s_0}\right)^2 \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}u}{\mathrm{d}x}. \tag{4}$$

Considering the boundary conditions for the cable with two-hinged ends under vertical support motions and multiplying Eq. (4) by $(\mathrm{d}s_0/\mathrm{d}x)^2$, one can integrate this equation from 0 to L to obtain

$$\frac{\Delta T}{E_cA_c}\int_0^L \left(\frac{\mathrm{d}s_0}{\mathrm{d}x}\right)^3 \mathrm{d}x = \int_0^L y'u'\mathrm{d}x = y'u\big|_0^L - y''\int_0^L u\mathrm{d}x. \tag{5}$$

Then, the force increment ΔT in Eq. (4) is computed as

$$\begin{aligned}\Delta T &= \frac{E_cA_c}{L_c}\int_0^L y'u'\mathrm{d}x = \frac{E_cA_c}{L_c}\left[y'u\big|_0^L - y''\int_0^L u\mathrm{d}x\right] \\ &= \frac{E_cA_c}{L_c}\left[-\frac{4y_0}{L}(u_0 + u_L) + \frac{8y_0}{L^2}\int_0^L u\mathrm{d}x\right],\end{aligned} \tag{6}$$

where L_c(= the effective length of the cable) is received as

$$L_c = \int_0^L \left(\frac{\mathrm{d}s_0}{\mathrm{d}x}\right)^3 \mathrm{d}x = \int_0^L \left(\sqrt{1 + y'^2}\right)^3 \mathrm{d}x. \tag{7}$$

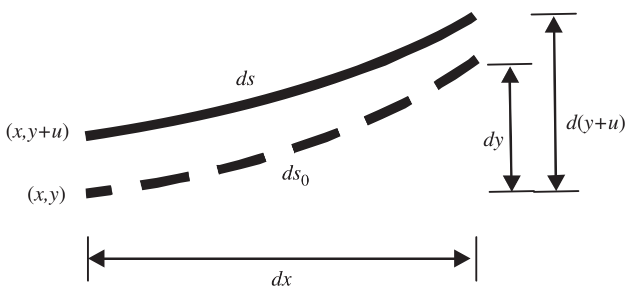

Fig. 2. An infinitesimal element of the cable.

Using the relation $y'' = -8y_0/L^2$ for a parabolic cable in Eq. (3) and the force increment ΔT in Eq. (6), Eq. (1) can be transformed into the following partial integro-differential equation:

$$m\frac{\partial^2 u(x,t)}{\partial t^2} + c\frac{\partial u(x,t)}{\partial t} + EI\frac{\partial^4 u(x,t)}{\partial x^4} - T\frac{\partial^2 u(x,t)}{\partial x^2} + A\int_0^L u(x,t)\mathrm{d}x = p(x,t) + \frac{AL}{2}[u(L,0) + u(0,t)], \tag{8}$$

where

$$A = \left(\frac{8y_0}{L^2}\right)^2 \frac{E_c A_c}{L_c}. \tag{9}$$

The train loading in Eq. (8) is expressed as [14,15]:

$$p(x,t) = P\sum_{k=1}^{N}\{\delta[x - v(t - t_k)][H(t - t_k) - H(t - t_k - L/v)]\}. \tag{10}$$

Here $\delta(x)$ is the Dirac function, $H(t)$ is the Heaviside unit step functions, [$H(t) = 0$ for $t<0$, $H(t) = 1$ for $t\geqslant 0$], N is the Nth moving force acting on the beam, and $t_k = (k-1)d/v$ is the arriving time of the kth load on the beam.

The boundary conditions of the suspended beam under uniform vertical support movements are expressed as

$$\begin{aligned} u(0,t) = u(L,t) = a(t), \\ EIu''(0,t) = EIu''(L,t) = 0, \end{aligned} \tag{11}$$

where $a(t)$ represents the synchronous vertical displacement at both supports due to earthquake, see Fig. 1. The initial conditions are zero, when the first force enters the bridge:

$$u(x,0) = \dot{u}(x,0) = 0. \tag{12}$$

4. Analysis

To solve the partial integro-differential equation (8) with non-homogeneous time-dependent boundary conditions (11), we decompose the total displacement into the sum of the quasi-static deflection $U(x,t)$ due to the support motions applied statically and the additional natural deformations $u_n(x,t)$ caused by the dynamic effect of inertia and damping forces [16,17]:

$$u(x,t) = U(x,t) + u_n(x,t). \tag{13}$$

The substitution of Eq. (13) into Eq. (11) gives

(a) the boundary conditions for natural deformation :

$$u_n(0,t) = u_n(L,t) = 0, \quad EIu_n''(0,t) = EIu_n''(L,t) = 0 \tag{14}$$

(b) the boundary conditions for quasi-static deflection :

$$U(0,t) = U(L,t) = a(t), \quad EIU''(0,t) = EIU''(L,t) = 0. \tag{15}$$

The following shape functions satisfy the homogeneous boundary conditions (14).

$$u_n(x,t) = \sum_{n=1}^{\infty} q_n(t)\sin\frac{n\pi x}{L}, \tag{16}$$

where $q_n(t)$ is the generalized coordinate associated with the n-shape function of the beam. On the other hand, the quasi-static deflection $U(x,t)$ must satisfy the boundary conditions (15) and the following equation:

$$EI\frac{\partial^4 U(x,t)}{\partial x^4} - T\frac{\partial^2 U(x,t)}{\partial x^2} + A\int_0^L U(x,t)\mathrm{d}x = 0. \tag{17}$$

To solve the quasi-static deflection $U(x,t)$ in Eq. (17) with the non-homogeneous boundary conditions (15), one can move the integral term to the right-hand side. Eq. (17) can be transformed into

$$\frac{\partial^4 U(x,t)}{\partial x^4} - \lambda^2\frac{\partial^2 U(x,t)}{\partial x^2} = -\frac{A}{EI}\int_0^L U(x,t)\mathrm{d}x, \tag{18}$$

where $\lambda = \sqrt{T/EI}$.

Then, solving the differential equation in terms of $U(x,t)$ with respect to x leads to the following general solution:

$$U(x,t) = d_0 \cosh \lambda x + d_1 \sinh \lambda x + c_0 + c_1\frac{x}{L} + \frac{Ax^2}{2T}\int_0^L U\mathrm{d}x. \tag{19}$$

Here, c_0, c_1, d_0, and d_1 are undetermined constants. By introducing the non-homogeneous boundary conditions in Eq. (15) into Eq. (19), the solution of the quasi-static deflection $U(x,t)$ can be obtained as

$$U(x,t) = a(t) + \frac{a(t)G(x)}{B}, \tag{20}$$

where

$$G(x) = 1 + \frac{\lambda^2 x(x-L)}{2} - \frac{\cosh \lambda(x-L/2)}{\cosh(\lambda L/2)}, \tag{21}$$

$$B = \frac{T\lambda^2}{AL} + \frac{(\lambda L)^2}{12} + \frac{\tanh(\lambda L/2)}{\lambda L/2} - 1. \tag{22}$$

5. Solution

Though the quasi-static solution $U(x,t)$ has been obtained in Eq. (18), the natural deformations $u_n(x,t)$ need to be solved by Galerkin's approach. Substituting Eq. (13) into Eq. (8) and moving the term with $U(x,t)$ to the right-hand side yields

$$\begin{aligned} m\ddot{u}_n + c\dot{u}_n + EIu''''_n - Tu''_n + A\int_0^L u_n\mathrm{d}x \\ = p(x,t) - (m\ddot{U} + c\dot{U}) + ALa(t). \end{aligned} \tag{23}$$

Galerkin's method (i.e., the multiplication of both sides of Eq. (23) with respect to the variation of the shape functions, see Eqs. (13), (16) and (20)), and then the integration over the beam length L yields

$$\begin{aligned} m\ddot{q}_n(t) + c\dot{q}_n(t) + \left[\left(\frac{n\pi}{L}\right)^4 EI + \left(\frac{n\pi}{L}\right)^2 T\right]q_n(t) + \frac{2AL}{n\pi^2}(1-\cos n\pi)\sum_{k=1}^{\infty}\frac{1}{k}(1-\cos k\pi)q_k(t) \\ = \left[\sum_{k=1}^{N} F_k(\omega_n, v, t)\right] + \frac{2(1-\cos n\pi)}{n\pi} \times \left\{ALa(t) + [m\ddot{a}(t) + \dot{a}(t)c]\left[\frac{1}{B}\frac{(\lambda L/n\pi)^4}{1+(\lambda L/n\pi)^2} - 1\right]\right\}, \end{aligned} \tag{24}$$

where the generalized force $F_k(\omega_n,v,t)$ of the kth moving force is expressed as

$$F_k(\omega_n, v, t) = \frac{2P}{L}\left[\begin{array}{c} \sin \omega_n(t-t_k)H(t-t_k) \\ +(-1)^{n+1} \sin \omega_n(t-t_k-L/v)H(t-t_k-L/v) \end{array}\right], \tag{25}$$

and $\omega_n = n\pi v/L$ represents the driving frequency of the moving loads.

As shown in Eq. (24), the response of the suspended beam depends not only on the support displacements but also on the support velocities and accelerations. Eq. (24) represents a set of coupled differential equations for all assumed modes. The numerical Newmark's differential scheme with $\beta = 0.25$ and $\gamma = 0.5$ [18] is applied to the solution of Eq. (24).

6. Example—Informative calculation of the designed Messina Bridge

The properties of the designed Messina Bridge [19,20] and of moving loads are listed in Tables 1 and 2, see also Fig. 1, $L = 3300\,\text{m}$, $y_0 = 335\,\text{m}$, $L_c = 3577\,\text{m}$. The shaped functions (16) serve approximately for the normal modes in this example. The first two flexural frequencies presented in Table 1 are very close to those proceeding from Refs. [20,21]. Moreover, Figs. 3–5 show the time histories of the vertical acceleration, velocity, and displacement, respectively, recorded during the Kobe earthquake. However, they were reduced to receive the maximum value of allowable ground acceleration $0.1g$ ($= 0.98\,\text{m}\,\text{s}^{-2}$) given in Ref. [19].

The acceleration of ballasted bridges due to moving loads may produce a significant ballast destabilization and, thus, possible derailment of moving trains, see Refs. [14,15,21–23]. Therefore, the first 16 shape functions are applied to study the acceleration response of the suspended beam at various speeds.

Table 1
Properties of the suspended beam (Messina Bridge)

L (m)	EI (kN m^2)	E_cA_c (kN)	m (t m^{-1})	y_0 (m)	c (N s m^{-1})	ω_1 (rad s^{-1})	ω_2 (rad s^{-1})
3300	8.6×10^7	1.6×10^8	21.6	335	0.78	0.38	0.50

Table 2
Properties of moving loads

P (kN)	d (m)	N [1]
360	18	30

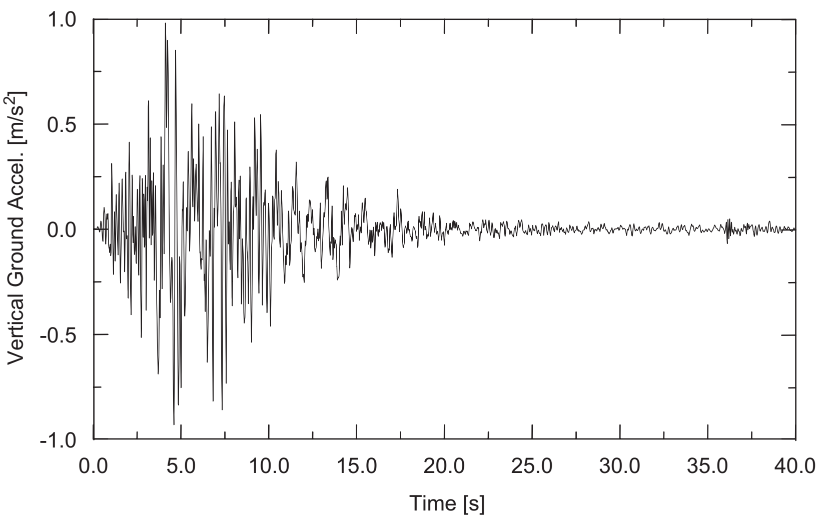

Fig. 3. Time history of the vertical acceleration during the Kobe earthquake, which was reduced to receive the maximum value of ground acceleration 0.1 g, see Ref. [19].

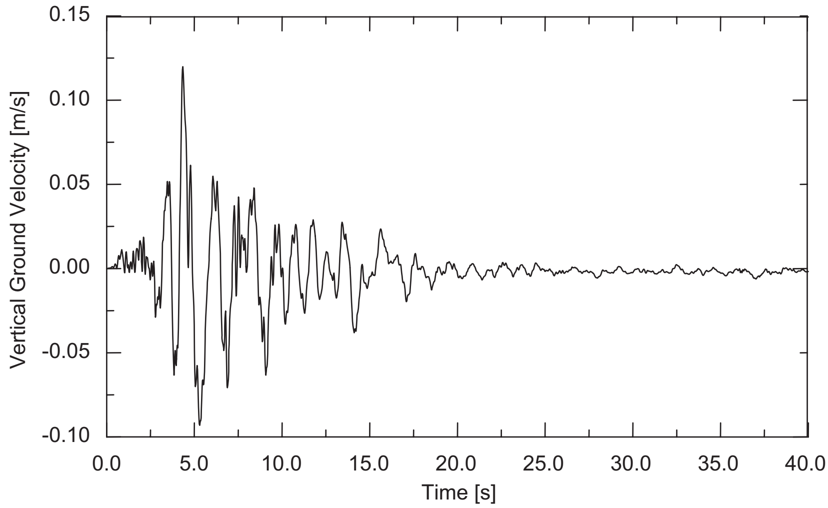

Fig. 4. Time history of the vertical velocity during the Kobe earthquake (reduced to receive the given acceleration, see the note in Fig. 3).

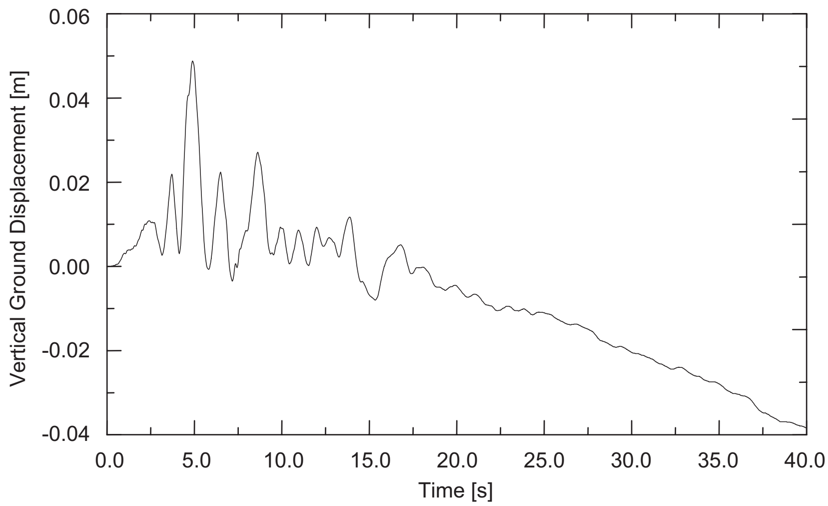

Fig. 5. Time history of the vertical displacement during the Kobe earthquake (reduced to receive the given acceleration, see the note in Fig. 3).

If we do not consider the support motions due to the earthquake, the maximum acceleration of the suspended bridge will monotonically grow along with the increase of travelling speeds of moving loads as shown in Fig. 6.

The earthquake may come at the moment when the train occurs at the start, half, or three quarters of the large span, respectively. Here, t_g represents the time lag of the earthquake start with respect to the face of the train. The following time lags of the earthquake arrival were studied: $vt_g/L = 0$; 0.5; and 0.75. However, the time plots $a_{\max}{-}v{-}x/L$ of the three mentioned cases are similar, so that only the first case is documented in Fig. 7. The earthquake amplifies the bridge response in the vicinity of support ends. Fig. 7 illustrates also that there exist multiple resonant peaks around the mid span of the suspended beam due to the higher modes that are excited by the earthquake excitations. The magnification effect on the suspension bridge is harmful to the structural safety of the bridge and riding comfort of the moving vehicles. As no

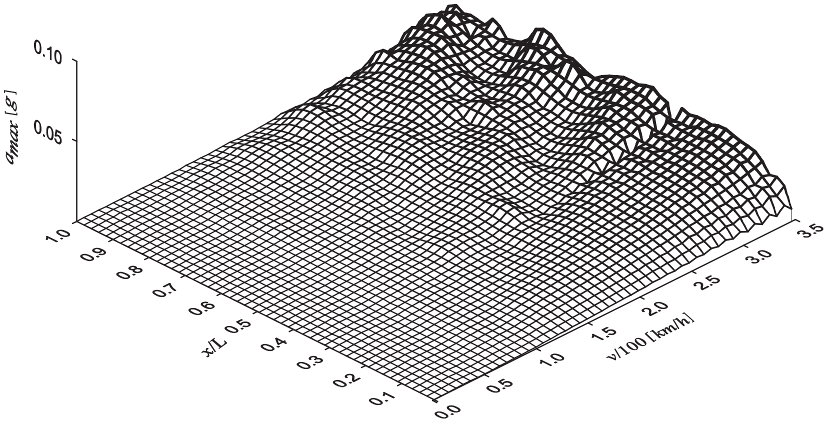

Fig. 6. Plot of $a_{\max}$–v–x/L without earthquake excitation.

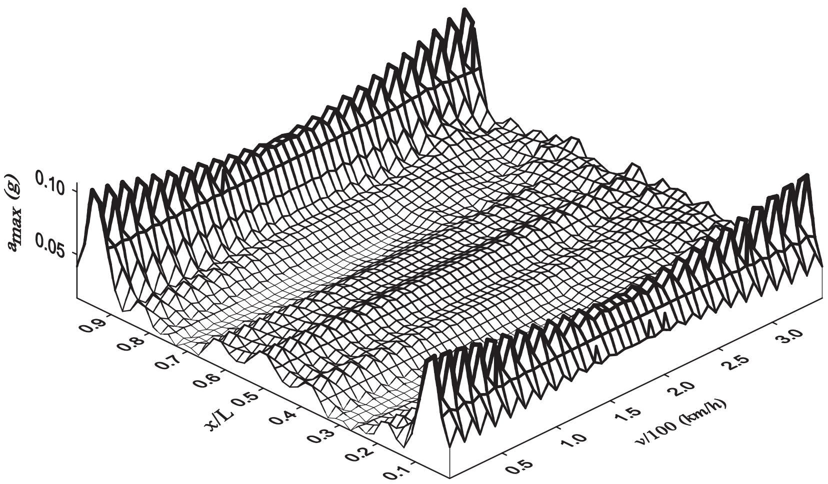

Fig. 7. Plot of $a_{\max}$–v–x/L with an excitation lags of $vt_g/L = 0$.

references have been at disposal, the phenomenon (interaction moving loads with earthquake) needs to be further studied.

Moreover, Fig. 8 shows the maximum accelerations of the bridge mid span as a function of the time lag factor of the train (vt_g/L) via the train speeds for various speeds 300 km/h (full line), 200 km/h (dashed line), 100 km/h (dot-dashed line), and 10 km/h (dotted line). The results indicate that the maximum acceleration responses at the mid span of the bridge occur within the time lag factor (vt_g/L) smaller than 0.1 and they will increase along with the rise of train speeds, especially at the zero time lag, i.e., $vt_g/L = 0$. It means that as the uniform earthquake hits the super-long suspension bridge, the train moving with the speed of 300 km/h has run into the bridge and may happen to meet the intensive zone of the earthquake, around 5 s shown in Fig. 3, which may further amplify the mid-span response due to the simultaneous action of the moving train loads and the earthquake excitation.

However, no matter when the earthquake hits the bridge, the maximum acceleration response provides small fluctuations along the beam (i.e. with respect to the x/L coordinate). This is attributed to the fact that the span length of the bridge is so long that the support motions produce only a local influence in the vicinity of supports of the suspended beam. Such a conclusion can be adopted as a critical design state in vehicle–bridge interaction analysis for a train entering the bridge under the action of ground excitations due to earthquake.

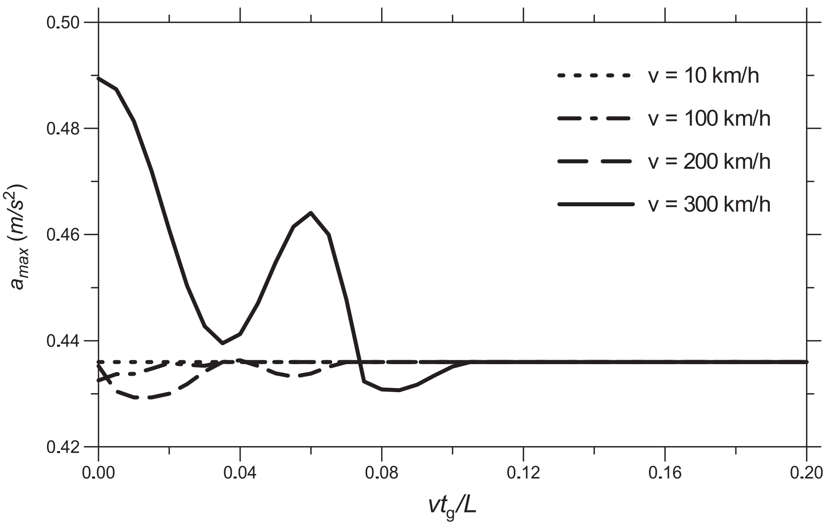

Fig. 8. Maximum accelerations of the bridge mid-span as a function of the time arrival of the earthquake vt_g/L for various speeds.

7. Conclusions

The paper deals with the dynamic response of a suspended beam subjected to the simultaneous action of both the moving loads and vertical support motions due to earthquake. The results indicate that the inclusion of earthquake would amplify the response of the suspended bridge. But the maximum acceleration along the main beam remains almost unchanged even though various time lags of the earthquake exciting the suspended beam have been considered.

It is concluded that the super-long suspension bridge will behave almost insensitive to the occurrence time of earthquakes when the train-type moving loads are passing the bridge.

The dynamic effects are growing with the increasing speed of trains, as regularly on bridges. This phenomenon was observed and confirmed many times, see Refs. [9,14]. Moreover, the dynamic effects appear near the bridge supports as a result of earthquake excitations.

By the way, the low frequencies of long-span bridges are separated from the higher frequencies excited by earthquake and quickly moving trains. Nevertheless, further sides and problems of such important and huge structures must be investigated.

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