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Discussion

Reply to the comments of Sinniah Ilanko

A.W. Leissa

Department of Mechanical Engineering, Colorado State University, Fort Collins, CO 80523-1374, USA

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The writer has used both the Rayleigh and Ritz methods extensively for half a century to solve eigenvalue problems for continuous systems, especially those involving free vibration and bifurcation buckling. He can recall thinking, as a graduate student more than 50 years ago, when he was first reading Rayleigh's *Theory of Sound*:

> Yes, these are nice methods for obtaining vibration frequencies which *may* be near the actual (i.e. exact) values. But how near are they? Can we be certain that they *are* near the actual values?

When Rayleigh in a few places did add one more term to the approximating displacement function, and went through various types of minimization procedures (all laid out in Ref. [1]), improvements in the frequencies (eigenvalues) and mode shapes (eigenfunctions) did occur. But this always left the nagging question:

> *Now* how close to the actual values are they?

One did not know.

Soon afterwards the writer was shown the Ritz method and, eventually, the two famous articles by Ritz. This was truly impressive. Now one could take as many terms of an *infinite series* of displacement functions to obtain frequencies and mode shapes which had *converged to any degree of accuracy*. If one used a series of displacement functions which is mathematically complete, one could approach the exact values as closely as desired. Thus, while Rayleigh showed marvelous insight in choosing displacement functions to represent the vibratory behavior, and even improved on it in a few cases by adding another term, it was Ritz who developed the systematic approach which guarantees convergence to actual (exact) solutions. These two separate approaches should be properly identified, and not linked together in a single title.

Reference

[1] A.W. Leissa, The historical bases of the Rayleigh and Ritz methods, *Journal of Sound and Vibration* 287 (2005) 961–978.

E-mail address: awleissa@mindspring.com

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