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Damped model updating using complex updating parameters

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Abstract

Most of the model updating techniques do not employ damping matrices and hence cannot be used for the accurate prediction of complex frequency response functions (FRFs) and complex mode shapes. In this paper, the response function method (RFM) is extended to deal with the complexity of FRF and modal data using complex updating parameters. In the proposed model updating procedure, the finite element model is updated in such a way that the updated model reflects general damping in the experimental model by considering the updating parameters as complex. The effectiveness of the proposed finite element updating procedure is demonstrated by numerical examples as well as by actual laboratory experiments. First, a study is performed using numerical simulation based on a fixed–fixed beam structure with structural damping, viscous damping and structural and viscous damping models. Various levels of damping and noise are assumed in the data. The numerical study is followed by a case involving actual measured data for the case of an F-shaped test structure. The updated results have shown that the complex updating finite element model updating procedure can be used to derive an accurate model of the system. This is illustrated by matching of the complex FRFs obtained from the updated model with that of experimental data.

1. Introduction

Accurate mathematical models are required to describe the vibration characteristics of structures, which can be subsequently used for designing purposes to limit the negative effects of vibrations. The finite element (FE) method [1] is the most widely used method for accurate modeling. It is well known that FE predictions are often called into question when they are in conflict with test results [2,3]. Inaccuracies in the FE model and errors in results predicted by them can arise due to the use of incorrect modeling of boundary conditions, incorrect modeling of joints, difficulties in modeling of damping, etc. This has led to the development of model updating techniques, which aim at reducing the inaccuracies present in an analytical model in the light of measured dynamic test data. A number of model updating methods have been proposed in recent years as shown in the surveys by Imregun and Visser [4] and Mottershead and Friswell [5], and details of these can be found in the text by Friswell and Mottershead [6]. Model updating methods can be broadly classified into direct methods (these methods are essentially non-iterative) and the iterative methods. A significant number of

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methods [7–9] that were first to emerge belonged to the direct category. Although, these methods are computationally cheaper and reproduce the measured modal data exactly, they violate structural connectivity and updated structural matrices are difficult to interpret. On the other hand, iterative methods provide wide choices of updating parameters, structural connectivity can be easily maintained and corrections suggested in the selected parameters can be physically interpreted. Iterative methods use either eigen-data or frequency response function (FRF) data. Collins et al. [10] used the eigen-data sensitivity for analytical model updating in an iterative framework. Lin and Ewins [11] proposed the response function method (RFM), which used measured FRF data to update an analytical model. Comparison of the RFM and the inverse eigen-sensitivity methods neglecting damping was done with an objective to study the accuracy with which they predicted the corrections required in the FE model [12]. Modak et al. [13] proposed an updating method in which updating of the undamped FE model is done by imposing constraint on natural frequencies and mode shapes. Most of the updating methods neglect the damping. Therefore, these methods can be used up to the point of predicting accurately the natural frequencies and real modes in the measured region. But these cannot be used for complex FRFs and complex modes. All structures exhibit some form of damping, but despite a large literature on damping, it still remains one of the least well-understood aspects of general vibration analysis. A commonly used model originated by Rayleigh [14] assumes that instantaneous generalized velocities are the only variables. The Taylor expansion then leads to a model that encapsulates damping behavior in a dissipation matrix, directly analogous to the mass and stiffness matrices. Material damping can arise from a variety of micro-structural mechanisms [15], but for small strains, it is often adequate to represent it through an equivalent linear model of the material. Damping can then be taken into account via the viscoelastic correspondence principle, which leads to the concept of complex moduli. Maia et al. [16] have emphasized the need for development of identification methodologies for general damping models and indicated several difficulties that might arise. Pilkey [17] and Pilkey et al. [18] described two types of procedures, direct and iterative, for computation of the system-damping matrix. Adhikari and Woodhouse [19] identified the damping of the system as viscous damping. However, viscous damping is by no means the only damping model. Adhikari and Woodhouse [20] identified a non-viscous damping model using an exponentially decaying relaxation function.

Most of the above methods either identify a modal damping matrix or construct a physical damping matrix using experimentally identified modal data. The identified damping matrices may not be very accurate or represent general damping models, which can be incorporated in the FE model updating methodology to predict accurately the complex FRFs and complex mode shapes.

Some research efforts have also been made to update the damping matrices. Imregun et al. [21,22] conducted several studies using simulated and experimental data to gauge the effectiveness of RFM and extended the RFM to update the proportional damping matrix by updating the coefficients of the damping matrix. Yong and Zhenguo [23] proposed a two-step model updating procedure for lightly damped structures using neural networks. In the first step, mass and stiffness are updated using natural and antiresonance frequencies. In the second step, damping ratios are updated. Lin and Zhu [24] extended RFM to update damping coefficients in addition to mass and stiffness matrices.

The proposed method is an FRF-based method in which updating parameters are considered as complex to overcome the problem of complex modal and FRF data and the updated model reflects general damping in the experimental data. In order to demonstrate the effectiveness of the proposed complex updating parameter procedure, first a study is performed using numerical simulation based on a fixed–fixed beam structure with structural damping, viscous damping and structural and viscous damping models. This is followed by a study involving actual measured data for the case of an F-shaped test structure, which resembles the skeleton of a drilling machine.

2. Basic theory

The proposed method is a development of RFM given by Lin and Ewins [11], which is an iterative method and uses measured FRF data directly without requiring any modal extraction. In the proposed method, it is assumed that initially there is no damping in the analytical model, which results in real FRFs whereas experimental FRFs are complex because of the presence of damping. Following identities relating the dynamic stiffness matrix $[Z]$ and the receptance FRF matrix $[\alpha]$ can be written for the analytical model as well as the actual structure, respectively:

$$[Z_A^R][\alpha_A^R] = [I] \quad \text{(initially)} \tag{1}$$

$$[Z_X^C][\alpha_X^C] = [I] \tag{2}$$

where subscripts A and X denote analytical (like an FE model) and experimental models, respectively, and superscripts R and C represent real and complex values. Expressing $[Z_X]$ in Eq. (2) as $[Z_A^R]+[\Delta Z]$ and then subtracting Eq. (1) from it, the following matrix equation is obtained:

$$[\Delta Z^R][\alpha_X^C] = [Z_A^R]([\alpha_A^R] - [\alpha_X^C]) \tag{3}$$

Pre-multiplying the above equation by $[\alpha_A]$ and using Eq. (1) give

$$[\alpha_A^R][\Delta Z^R][\alpha_X^C] = [\alpha_A^R] - [\alpha_X^C] \tag{4}$$

If only the jth column of an experimentally measured FRF matrix $\{\alpha_X\}_j$, is available then the above equation is reduced to

$$[\alpha_A^R][\Delta Z^R]\{\alpha_X^C\}_j = \{\alpha_A^R\}_j - \{\alpha_X^C\}_j \tag{5}$$

which is the basic relationship of the RFM. With this method, a physical variables-based updating parameter formulation is used. Linearizing $[\Delta Z]$ with respect to $\{p\}$, $\{p\} = \{p_1, p_2, \ldots, p_{\mathrm{nu}}\}^t$ being the vector of physical variables associated with individual or group of FEs, gives

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial [Z]}{\partial p_i} \Delta p_i \right) \tag{6}$$

where nu is the total number of updating parameters. $[Z]$ in Eq. (6) is replaced by $[K]-\omega^2[M]$. Dividing and multiplying Eq. (6) by p_i and then writing u_i in place of $\Delta p_i/p_i$. We can write Eq. (6) as

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial([K] - \omega^2[M])}{\partial p_i} p_i \right) u_i \tag{7}$$

Thus $\{u\} = \{u_1, u_2, \ldots, u_{\mathrm{nu}}\}^t$ is the unknown vector of fractional correction factors to be determined during updating. Eq. (5), after making the substitution for $[\Delta Z]$ from Eq. (7), can be written at various frequency points chosen from the frequency range considered. The resulting equations can be framed in the following matrix form:

$$[S(\omega)^C]_{(n\times nf)\times(\mathrm{nu})}\{u^C\}_{\mathrm{nu}\times 1} = \{\Delta\alpha(\omega)^C\}_{(n\times nf)\times 1} \tag{8}$$

Here $[S]$ is the sensitivity matrix. The fractional correction factors $\{u\}$ obtained using Eq. (8) are complex. As correction factors are complex, this results in complex updating parameters. The updated version of the analytical FE model is built using this set of complex parameters of physical variables. The parameter vector of physical variables $\{p\}$ is considered to be complex in the form

$$\{p\}^C = \{p\}^R + j\{p\}^I \tag{9}$$

The real part of the complex updating parameter represents change in physical variable. The imaginary part relates to damping of the system. If the updating parameters result in updating only the stiffness matrix of the FE model, then the identified damping matrix is proportional to the stiffness matrix at the elemental level whereas if updating parameters result in updating both mass and stiffness matrices of the FE model, then the identified damping matrix is proportional to both mass and stiffness matrix at the elemental level. Practically, the FRFs are available only at a few degrees of freedom, which means FRF data are incomplete. In the present study, the coordinate incompleteness has been dealt by using analytically generated FRFs. This has been done by replacing the responses of unmeasured coordinates by analytical counterparts. The process is repeated in an iterative way until convergence is obtained. At the end of the first iteration, the correction factor would have an updated physical variable given by

$$p_1^C = (1 + u_1^C)p_0^R \tag{10}$$

Initially, parameter of the physical variable is real and after the first iteration the updated parameter becomes complex. Similarly at the end of the second iteration, the updated physical variable value will be

$$p_2^C = (1 + u_2^C)p_1^C \tag{11}$$

The performance is judged on the basis of the accuracy with which the FRFs predicted by the updated FE model match the simulated experimental FRFs or the experimental FRFs.

3. Case study of a fixed–fixed beam structure using simulated data

A simulated study on a fixed–fixed beam is conducted to evaluate the effectiveness of the proposed method. The dimensions of the beam are 910 mm × 50 × 5 mm. The beam is modeled using 30, two-noded beam elements with nodes at the two ends being fixed as shown in Fig. 1. The displacements in the y-direction and the rotation about the z-axis are taken as two degrees of freedom at each node. Analytical data of the FRFs are obtained with known discrepancies in the thickness of all the FEs as shown in Table 1. The simulated complex FRF data, which are treated as experimental data, are obtained by generating a damped FE model with various types of damping. These are as follows:

1. Experimental FRFs are obtained when only proportional viscous damping is present in the system.
2. Experimental FRFs are obtained when only proportional structural damping is present in the system.
3. Experimental FRFs are obtained when proportional viscous and structural damping are present in the system.
4. Experimental FRFs are obtained when non-proportional viscous damping is present in the system.

Damping considered for all the cases is proportional damping being proportional to both mass $[M]$ and stiffness $[K]$ matrices. In case of viscous damping, the viscous damping matrix $[C]$ is given by

$$[C] = \alpha_V[M] + \beta_V[K] \tag{12}$$

where C is the viscous damping matrix and α_V and β_V are the damping constants for viscous damping.

In case of structural damping, structural damping matrix $[D]$ is given by

$$[D] = \alpha_S[M] + \beta_S[K] \tag{13}$$

where D is the structural damping matrix and α_S and β_S are the damping constants for structural damping.

In all the three test cases described above, three different sub-cases are considered depending on the amount of damping present in the system. These are lightly, moderately and highly damped. Table 2 shows the values

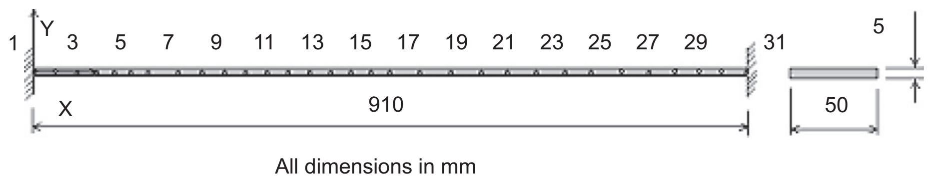

Fig. 1. Beam structure and its FE mesh.

Table 1
Discrepancies between the finite element and the simulated experimental model for the case of fixed–fixed beam structure

Element number	3	5	11	16	25	29	All other elements
% Deviation in thickness	+20	+40	25	+40	+30	+30	+20

Table 2
Damping constants values for modeling different levels of damping in the system

Levels of damping	Viscous damping		Structural damping	
	α_V	β_V	α_S	β_S
Lightly damped	0.000001	0.0001	0.001	0.001
Moderately damped	0.00001	0.0001	0.01	0.01
Highly damped	0.00005	0.0001	0.1	0.1

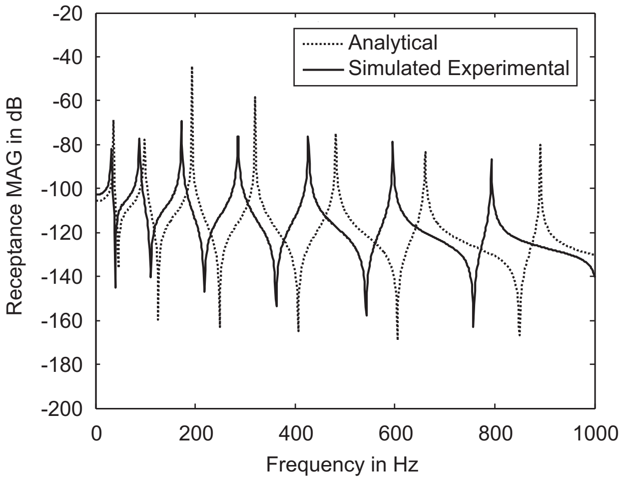

Fig. 2. Overlay of simulated experimental FRF and analytical FRF for the case of noise-free simulated experimental data of lightly structural damped system.

of damping constants for the cases of lightly, moderately and highly damped systems for viscous and structural damping. Thickness of each element of the beam structure is considered as complex updating parameters. So for the beam structure, 30 updating parameters are taken. The proposed method has been evaluated for the cases of incomplete and noisy data. The real-life measured data are always incomplete, as it is not practical to measure all the coordinates specified in the analytical FE model and always contains some measurement noise. Incompleteness is considered by assuming that only lateral degrees of freedom, at all the 29 nodes, are measured. These have been referred to as 50% incomplete data. Different levels of noise, i.e. noise free, 1% noise, 2% noise and 3% noise in simulated experimental data, are considered. The frequency range from 0 to 1000 Hz is considered for the updating procedure. The performance of the proposed method is judged on the basis of accuracy with which the FRFs obtained by the updated model match with the simulated experimental FRFs using a quality index referred to as percentage average error in FRF (AEFRF), which is calculated as

$$\mathrm{AEFRF} = \frac{100}{n_f}\sum_{j=1}^{n_f} \mathrm{abs}\left(\frac{([\alpha(f_j)])_A - ([\alpha(f_j)])_X}{([\alpha(f_j)])_X}\right) \tag{14}$$

where n_f is the frequency range to be considered (in present case 0–1000 Hz).

First, the case of experimental data obtained using structural damping with different levels of damping and noise is considered. Fig. 2 shows the overlay of analytical FRF, which is undamped, and simulated experimental FRF (noise free) obtained considering light structural damping in the experimental data. It can be observed that the analytical FRF ($5y5y$) and simulated experimental FRF ($5y5y$) do not match with each other. $5y5y$ represents excitation at node 5 and response at node 5 both in the y-direction. After updating, simulated experimental FRF and updated FRF match exactly as shown in Fig. 3. The AEFRF is 0% and the

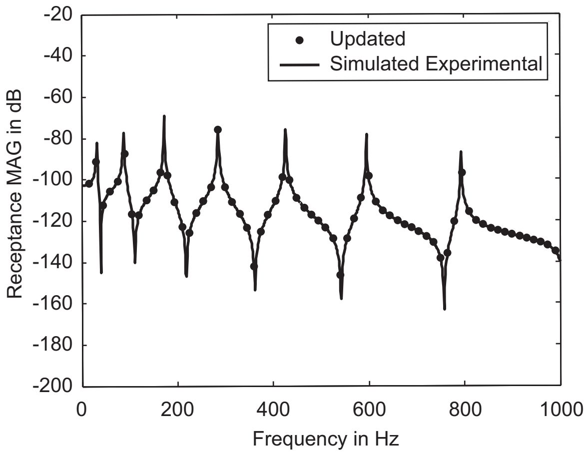

Fig. 3. Overlay of simulated experimental FRF and updated FRF for the case of noise-free simulated experimental data of a lightly structural damped system.

final value of the real part of thickness is close to 5 mm (the original thickness) and the imaginary part is used to construct damping matrices. The real and imaginary parts of the elemental thickness after updating are given in Fig. 4. Similarly, the proposed algorithm is evaluated using different noise levels and structural damping levels. Table 3 represents various values of AEFRF obtained for different cases and it is found to be in acceptable limits. Some typical results are discussed here. For high viscous damping case with 3% noise, the AEFRF is 0.16%, which is an acceptable value. Fig. 5 shows the overlays of analytical undamped FRF and simulated experimental FRF with 3% noise, which is obtained from a high viscous damping-based system and Fig. 6 shows the overlay of the updated FRF with simulated experimental FRF. It can be observed that updated FRF matches well the simulated experimental FRF. An investigation has also been carried out when structural and viscous dampings are present in simulated experimental data for general damping purpose. It has been found that the proposed algorithm is working well for those cases also. The AEFRF for the case 3% noise simulated experimental data obtained from the high viscous and structural damping system is 0.25%. The overlay of updated FRF and simulated experimental FRF matches well, as shown in Fig. 7.

A simulated experimental case of non-proportional damping is generated by considering, for one of the elements (i.e. the 10th element), the contribution proportional to stiffness being reduced to half. For the global damping matrix, this leads to non-proportional damping. The damping matrix satisfies the non-Rayleigh style damping as given by the equation

$$CM^{-1}K \neq KM^{-1}C \tag{15}$$

A mesh of the non-proportional damping matrix is shown in Fig. 8. The overlay of updated FRF and simulated experimental FRF, obtained from non-proportional viscous damping in the system, matches well as shown in Fig. 9.

4. Case study of the F-shaped structure using experimental FRF data

The proposed method is now evaluated for the case of an F-shaped structure, as shown in Fig. 10, using experimental data. The F-shaped structure has been constructed by bolting the two beam members horizontally to a vertical beam member, which, in turn, has been welded to the base plate at the bottom. All the beam members have a square cross-section with 37.7 mm as one of its side. The FE model of the F-structure is built, as shown in Fig. 11, using 48 two-dimensional frame elements (two translational degrees of freedom in the x and y directions and one rotational degree of freedom) to model in-plane dynamics. In the

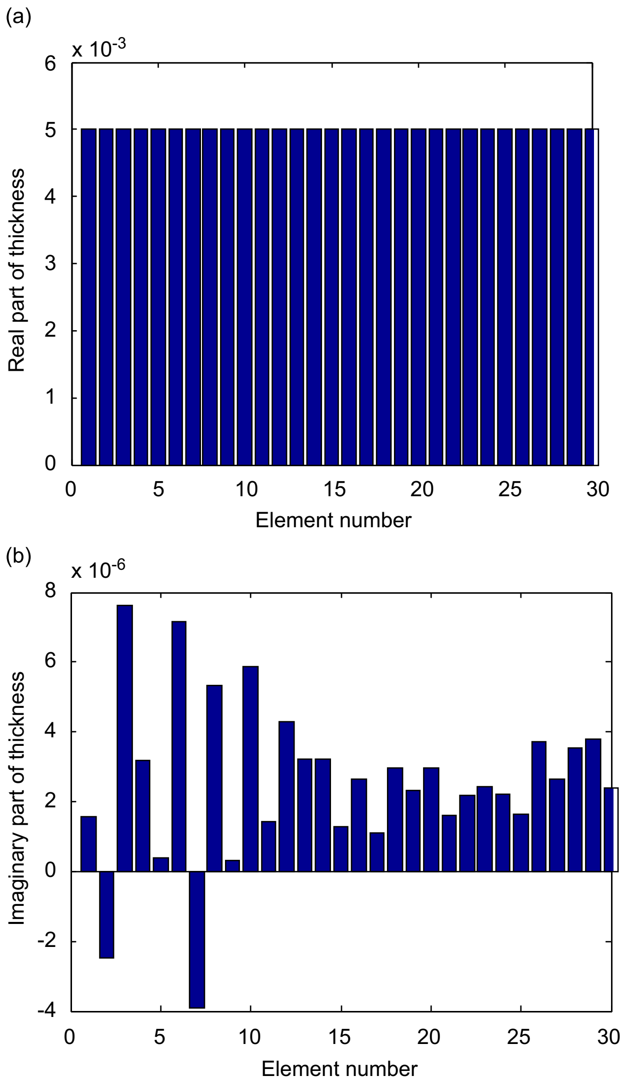

Fig. 4. Thickness after updating: (a) real part, (b) imaginary part for noise-free simulated experimental data of a lightly structural damped system.

Table 3
AEFRF (%) after updating for different cases using simulating experimental data

	AEFRF (%) after FE model updating			
	0% Noise	1% Noise	2% Noise	3% Noise
1. Viscously damped				
Lightly damped	0.0080	0.1037	0.1410	0.2208
Moderately damped	0.1469	0.1614	0.1677	0.2607
Highly damped	0.2813	0.3595	0.3851	0.3972
2. Structurally damped				
Lightly damped	0.000	0.0361	0.0733	0.1231
Moderately damped	0.0095	0.0472	0.1027	0.1412
Highly damped	0.0501	0.0618	0.1133	0.1613
3. Viscously and structurally damped				
Highly damped	0.0751	0.0827	0.1469	0.2472

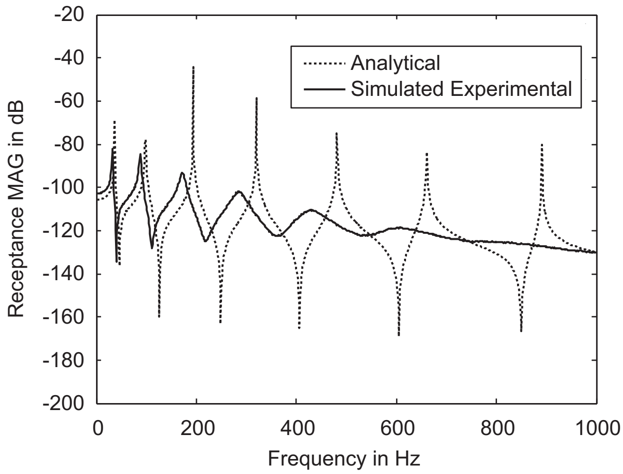

Fig. 5. Overlay of simulated experimental FRF and analytical FRF for the case of 3% noise in simulated experimental data of a highly viscous damped system.

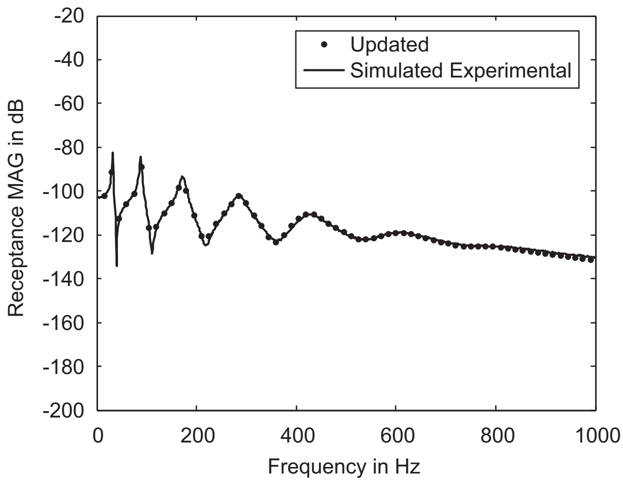

Fig. 6. Overlay of simulated experimental FRF and updated FRF for the case of 3% noise in simulated experimental data of a highly viscous damped system.

F-shaped structure, there are three joints, which are modeled by taking coincident nodes at each of them. Thus, the two nodes that are geometrically coincident are taken as joint nodes instead of one node. A horizontal, a vertical and a torsional spring couple the two nodes at each of such a coincident pair. The stiffnesses of these springs are K_x, K_y and K_t, respectively. The modal test is performed by exciting the structure with an impact hammer at 16 locations and response is measured at one location using accelerometer as shown in Fig. 12. The FRFs so acquired are analyzed using a global curve fitting technique available in ICATS [25] to obtain experimental sets of modes in the range of 0–1000 Hz.

The correlation between the analytical and the experimental set of modal data is now performed using MAC, which is calculated for the pair of ith analytical and jth measured mode shape as

$$\mathrm{MAC}(\{\phi_A\}^i, \{\phi_X\}^j) = \frac{(\{\phi_A\}^{i\mathrm{T}}\{\phi_X\}^j)^2}{(\{\phi_A\}^{iT}\{\phi_A\}^j)(\{\phi_X\}^{i\mathrm{T}}\{\phi_X\}^j)} \tag{16}$$

A comparison of the corresponding experimental and analytical natural frequencies, the percentage difference between them and the corresponding MAC value for first five modes are given in Table 4.

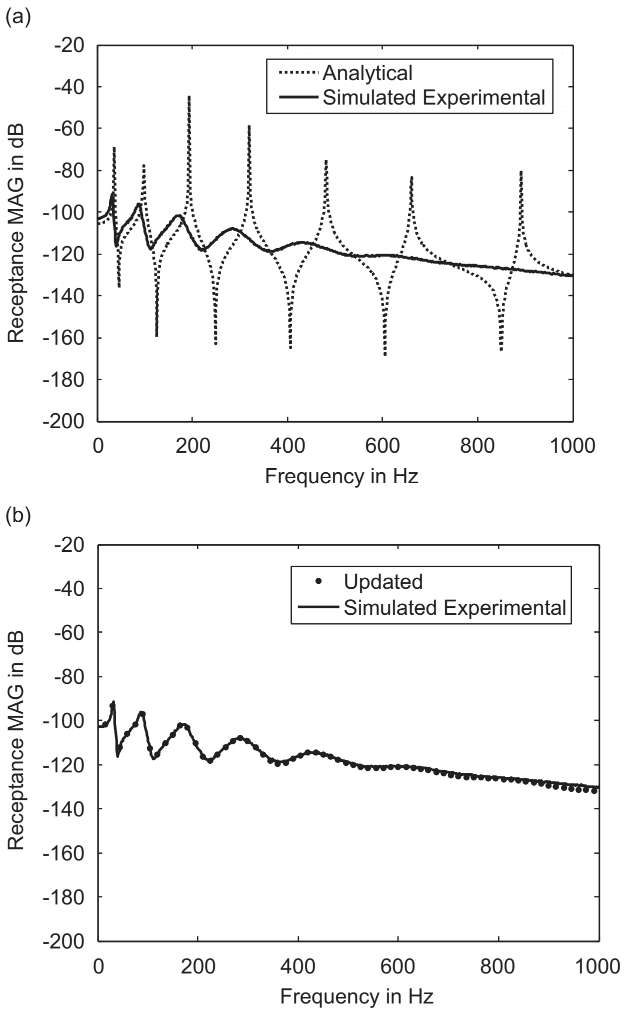

Fig. 7. Overlay of (a) simulated experimental FRF and analytical FRF (b) simulated experimental FRF and updated FRF for the case of 3% noise in simulated experimental data of a highly viscous and structural damped system.

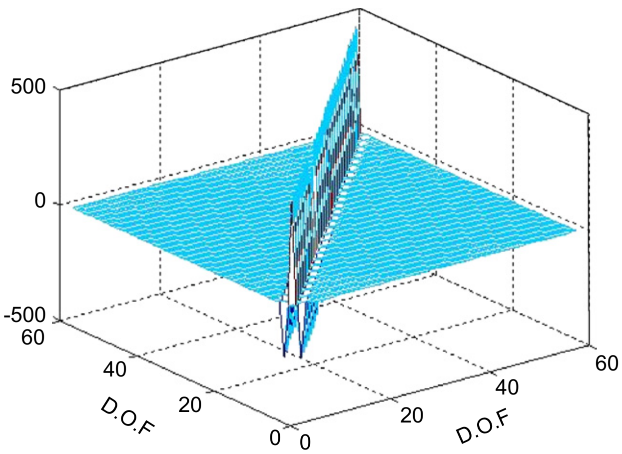

Fig. 8. Mesh of the damping matrix.

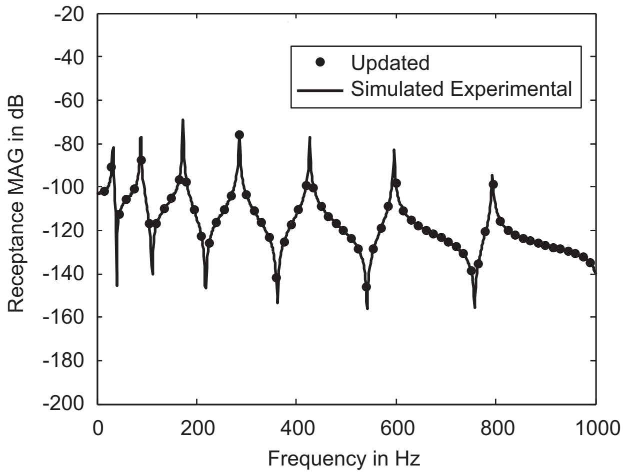

Fig. 9. Overlay of simulated experimental FRF and analytical FRF for the case of noise-free simulated experimental data of the non-proportional lightly viscous damped system.

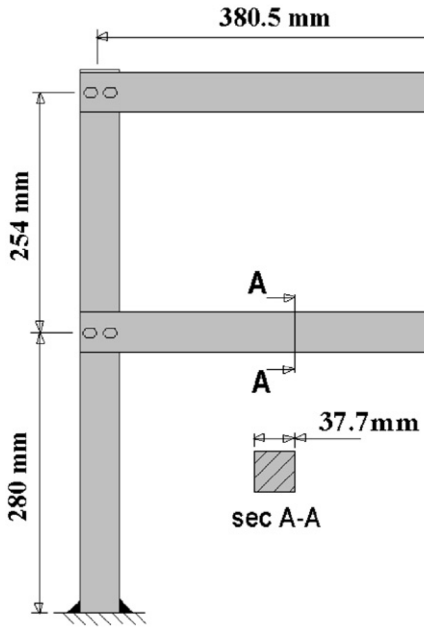

Fig. 10. An F-shaped structure.

An overlay of the measured $14x17x$ FRF and the corresponding FE model FRF is shown in Fig. 13. The FRF $14x17x$ represents excitation at node 14 and response at node 17 both in the x-direction. It is observed that the shape of the FE model FRF curve is similar to the measured curve. It therefore infers that though the FE model is in error but it is, in principle, of updatable quality.

Choice of updating parameters on the basis of engineering judgment about the possible locations of modeling errors in a structure is one of the strategies to ensure that only physical meaningful corrections are made. In case of an F-structure, modeling of stiffness of the joints is expected to be a dominant source of inaccuracy in the FE model, assuming that the values of the material and the geometric parameters are correctly known. Analytical sensitivity analysis of the joint springs shows that the torsional stiffness is the most important variable affecting the natural frequencies. Torsional springs of stiffnesses K_{t1}, K_{t2} and K_{t3}

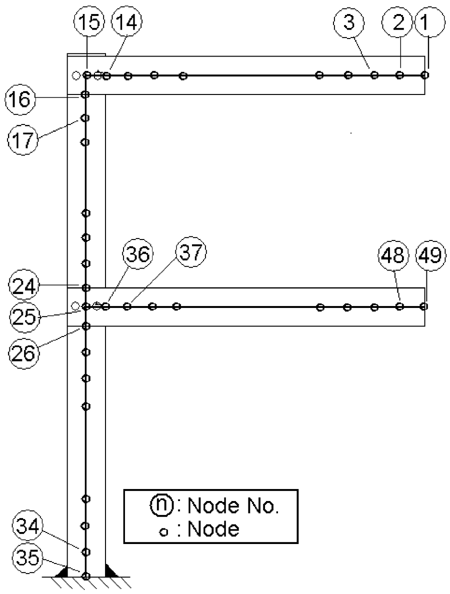

Fig. 11. Initial FE model.

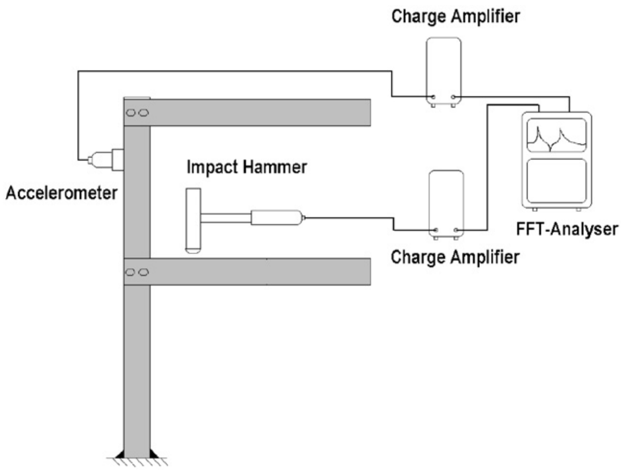

Fig. 12. Instrumentation setup for the modal test using impact excitation.

Table 4
Correlation of measured and FE model-based modal data for the F-shaped structure before updating

Mode no.	Measured frequency (Hz)	FE-model predictions		MAC value
		Frequency (Hz)	% Error	
1	34.95	43.05	23.17	0.9650
2	104.02	123.67	18.89	0.9364
3	133.96	185.21	38.26	0.9311
4	317.52	385.17	21.30	0.9141
5	980.16	1020.06	4.07	0.6908

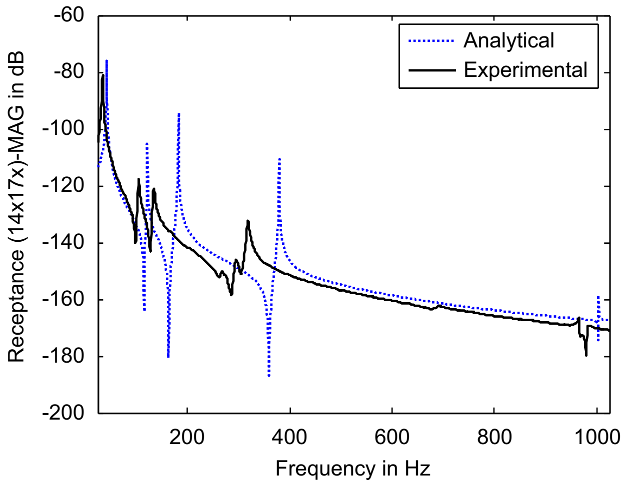

Fig. 13. Overlay of the measured FRF and the corresponding FE model FRF of an F-shaped structure before updating.

Table 5
Values of torsional springs stiffness of each joint after updating of the F-shaped structure

Updating variable	Initial value ($\mathrm{N\,m\,rad^{-1}}$)	Updated values using proposed method ($\mathrm{N\,m\,rad^{-1}}$)	
		Real	Imaginary
K_{t1}	3.28E+06	2.59E+05	4.36E+03
K_{t2}	3.28E+06	2.78E+05	5.1E+03
K_{t3}	3.28E+06	3.1E+05	5.78E+03

Table 6
Correlation between the measured and updated model

Mode no.	Measured frequency (Hz)	Updated model predictions		
		Frequency (Hz)	% Error	MAC
1	34.95	34.25	−2.0	0.9923
2	104.02	100.27	−3.60	0.9693
3	133.96	134.42	0.34	0.9675
4	317.52	313.73	−1.19	0.9423
5	980.16	973.44	−0.68	0.4370

coupling the rotational degrees of freedom of the coincident nodes at the three joints are taken as updating parameters. The other two degrees of freedom of the coincident nodes are taken as rigidly coupled.

The initial and final values of the torsional spring stiffness of each joint are given in Table 5. It is observed that the real values of stiffness of the torsional springs corresponding to three joints are reduced and also values of three springs are not very different from each other while the imaginary values of the stiffness represent damping in the system. A comparison of the correlation between the measured and the updated model natural frequencies is given in Table 6. It is observed from Table 6 that for the proposed method there is a significant reduction in the error in natural frequencies. Fig. 14 shows the overlay of measured and updated FRF. It is noticed that the shape of the updated FRFs is the same as that of measured FRFs. The quantitative index of matching AEFRF of $14x17x$ reduces from 48.03% to 2.97%.

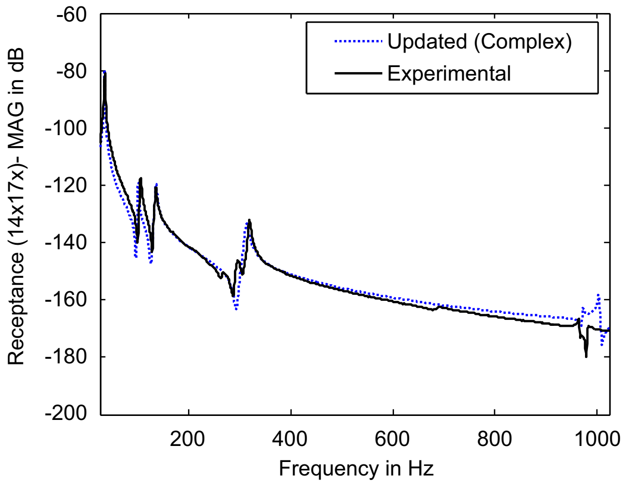

Fig. 14. Overlay of the measured FRF and the corresponding updated model FRF of an F-shaped structure after updating.

5. Conclusions

A new method of FE model updating has been proposed to update the FE model in such a way that the updated model reflects general damping in the experimental model by considering updating parameters as complex. In order to overcome the problem of complex FRF data, the complex updating formulation has been used. The proposed method is a single-step method whereas most of the other FE model updating methods for a damped system are two-step methods. The proposed method addresses the difficulties for updating the damping matrices. The proposed method is working successfully for the cases of simulated numerical data as well as for the experimental data. Simulated numerical case studies based on a fixed–fixed beam structure with structural, viscous and structural and viscous damping models are carried out to assess the effectiveness of the proposed method. The success of these cases has proven the feasibility of the proposed method. The proposed method is then applied to experimental data of the F-shaped structure and joint stiffness is updated also allowing it to be complex during updating. The proposed method is working well for the experimental data of the F-shaped structure.

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