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Vibration of annular plate concentrators with conical cross-section

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Abstract

In this paper, the torsional vibration and radial vibration of isotropic metal thin annular plates with conical cross-section are deduced by using Bessel functions; and according to electro-mechanical analogy, the electro-mechanical equivalent circuits are obtained. Based on the electro-mechanical equivalent circuits, the resonance frequency equations and the expressions of angular and radial displacement amplitude magnifications are derived, and the resonance frequencies and magnifications are solved by analytical method. The relationships between torsional and radial resonance frequencies and geometrical dimensions are discussed; and the relationships between angular and radial displacement amplitude magnifications and the ratio of outer radius over inner radius are analyzed. Modal shapes are analyzed by finite element method (ANSYS). It is shown that the values calculated by using the theory of this paper are in good agreement with the results simulated by ANSYS. The method can serve as benchmark values for researchers to validate other variable thickness thin annular plates.

1. Introduction

A number of plates with different shapes, sizes, thickness variations and boundary conditions play an important role in many fields, such as aerospace, marine, mechanical, petroleum industry, transport and electronic engineering. Along with the development of ultrasonic technology, annular plates are widely used in more and more applications. For example, thin annular plates are used to concentrate energy, magnify displacement/velocity and obtain high power in ultrasonic metal and plastic welding, ultrasonic machining and ultrasonic cold tube-drawing. In underwater acoustics and vibration control, radial vibration and torsional vibration are widely used, and a number of papers have dealt with natural frequencies of annular plates by using variety of methods in order to investigate the effect of elastic foundation. The vibration of a plate supported laterally by an elastic foundation has been discussed in Leissa's celebrated book [1]. Leissa deduces that the effect of a full Winkler foundation merely increases the square of the natural frequency of the plate by a constant. The variable thickness plates have

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Nomenclature

a	inner radius
b	outer radius
$c_r = \sqrt{E/\rho(1-\sigma^2)}$	longitudinal wave velocity
$c_t = \sqrt{E/2(1+\sigma)\rho}$	shear wave velocity
E	Young's modulus
f_r	radial resonance frequency
f_t	torsional resonance frequency
F_a	radial force at inner surface
F_b	radial force at outer surface
$G = E/2(1+\sigma)$	shear modulus
h_a	thickness at inner surface
h_b	thickness at outer surface
$k_r = \omega/c_r$	wavenumber of radial vibrations
$k_t = \omega/c_t$	wavenumber of torsional vibrations
M_a	external moment at inner surface
M_b	external moment at outer surface
M_r	radial displacement amplitude magnification
M_t	angular displacement amplitude magnification
S_a	area of inner surface
S_b	area of outer surface
T_r	radial stress
T_t	tangential stress
v	radial velocity
v_a	radial velocity at inner surface
v_b	radial velocity at outer surface
ξ	radial displacement
ρ	material density
σ	Poisson's ratio
ϕ	angular displacement
φ	torsional angular velocity
φ_a	torsional angular velocity at inner surface
φ_b	torsional angular velocity at outer surface
ω	angular frequency

been studied by several methods. Many researchers have used Ritz method to solve the problem approximately [2–6]. The radial and torsional vibrations of thin annular plates can be solved by using transfer matrix [7–8]. The differential quadrature (DQ) and harmonic differential quadrature (HDQ) can be used to solve the vibrations of variable thickness plates or uniform thickness plates [9–12]. It is noted that it is difficult to find exact analytical solutions for common variable thickness annular plates; however, in some special cases the analytical solutions can be found, such as step type, exponential type and conical (or linear) type, but those methods are too complex to calculate the resonance frequency [13–17].

In this paper, electro-mechanical equivalent circuits are introduced to analyze the free vibration of thin annular plates with conical cross-section. Based on the equivalent circuits, the analytical resonance frequencies of torsional and radial vibrations and the analytical results of angular and radial displacement amplitude magnifications are derived. The relationships between resonance frequencies and geometrical dimensions are discussed, and the relationships between angular and radial displacement amplitude magnifications and geometrical dimensions are discussed, too. The method of this paper is simpler than those mentioned above, and the result is in good agreement with the result from FEM. The method can be also used to analyze other variable thickness annular plates.

2. Torsional vibrations and radial vibrations of annular plates with conical cross-section

2.1. Torsional vibrations

The cross-section of the annular plate is illustrated in Fig. 1, in this figure, r and Z are radial and axial coordinates, and the others are defined above. The cylindrical coordinate is used in the analysis, considering the annular plate to be a thin annular plate, its thickness is much less than its radius, i.e. $h_b \ll r_b$, the stress and the strain are independent of the axial coordinate, and the stress of the axial coordinate can be ignored, so the vibration of the thin annular plate can be assumed to be axisymmetric, and the wave equation of torsional vibrations of annular plates with variable thickness is expressed as follows [8]:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \left(\frac{1}{r} + \frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}r}\right)\frac{\mathrm{d}\phi}{\mathrm{d}r} - \left(\frac{1}{r^2} + \frac{1}{rh}\frac{\mathrm{d}h}{\mathrm{d}r}\right)\phi = \frac{1}{c_t^2}\frac{\partial^2\phi}{\partial t^2} \quad (1)$$

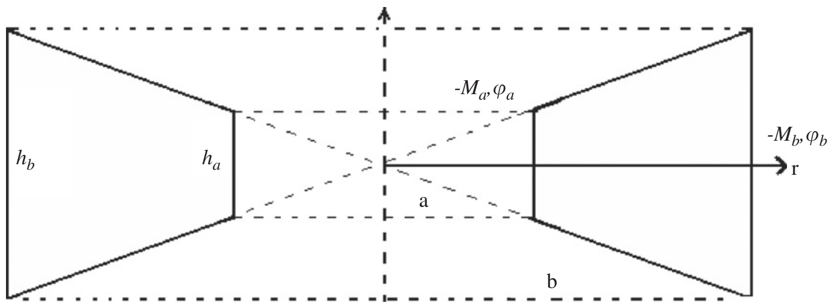

Fig. 1. A sketch map of an annular plate with conical section.

where $h = h(r)$ is the thickness of the annular plate. And the tangential stress of the plate is

$$T_t = G \cdot r \cdot \frac{\partial \phi}{\partial r} \tag{2}$$

If the thickness function can be expressed as $h(r) = h_0 \cdot r$, where h_0 is a constant; in other words, the cross-section of the annular plate is conical. For harmonic vibration, substituting the angular displacement component $\phi = \phi_0 \cdot \exp(\mathrm{j}\omega t)$ into Eq. (1) yields

$$\frac{\mathrm{d}^2\phi_0}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi_0}{\mathrm{d}r} + \left(k_t^2 - \frac{2}{r^2}\right)\phi_0 = 0 \tag{3}$$

where ϕ_0 is the angular displacement amplitude. It is obvious that Eq. (3) is Bessel equation of order $\frac{3}{2}$. Using the expression $\phi = \phi_0 \cdot \exp(\mathrm{j}\omega t)$ and the solution of Eq. (3), the following expression can be obtained:

$$\phi = r^{-1/2}[AJ_{3/2}(k_t r) + BY_{3/2}(k_t r)]\exp(\mathrm{j}\omega t) \tag{4}$$

where A and B are constants, $J_{3/2}(k_u r)$ and $Y_{3/2}(k_u r)$ are the first kind and the second kind Bessel functions of order $\frac{3}{2}$, respectively. From Eq. (4), the angular velocity can be obtained as

$$\varphi = \frac{\partial \phi}{\partial t} = \mathrm{j}\omega \cdot r^{-1/2}[AJ_{3/2}(k_t r) + BY_{3/2}(k_t r)]\exp(\mathrm{j}\omega t) \tag{5}$$

According to the boundary conditions of angular velocities $\partial\phi/\partial t|_{r=a} = \varphi_a$ and $\partial\phi/\partial t|_{r=b} = -\varphi_b$, the expressions of A and B can be obtained

$$\begin{cases} A = -\dfrac{1}{\mathrm{j}\omega}\dfrac{b^{1/2}Y_a\varphi_b + a^{1/2}Y_b\varphi_a}{J_bY_a - J_aY_b}\exp(-\mathrm{j}\omega t) \\ B = \dfrac{1}{\mathrm{j}\omega}\dfrac{b^{1/2}J_a\varphi_b + a^{1/2}J_b\varphi_a}{J_bY_a - J_aY_a}\exp(-\mathrm{j}\omega t) \end{cases} \tag{6}$$

where $J_a = J_{3/2}(k_t a)$, $J_b = J_{3/2}(k_t b)$, $Y_a = Y_{3/2}(k_t a)$, $Y_b = Y_{3/2}(k_t b)$. According to the boundary conditions of moments $M|_{r=a} = a \cdot T_t|_{r=a} \cdot S_a = -M_a$, $M|_{r=b} = b \cdot T_t|_{r=b} \cdot S_b = -M_b$, M_a, M_b as functions of φ_a, φ_b can be obtained

$$\begin{cases} M_a = -\dfrac{G}{2\mathrm{j}\omega} \cdot S_a \cdot a\left[-1 + \dfrac{2a(Y'_aJ_b - J'_aY_b)}{J_bY_a - J_aY_b}\right]\varphi_a - \dfrac{G}{\mathrm{j}\omega} \cdot S_a \cdot a\left[\dfrac{a^{1/2}b^{1/2}(J_aY'_a - J'_aY_a)}{J_bY_a - J_aY_b}\right]\varphi_b \\ M_b = -\dfrac{G}{2\mathrm{j}\omega} \cdot S_b \cdot b\left[1 + \dfrac{2b(Y'_bJ_a - J'_bY_a)}{J_bY_a - J_aY_b}\right]\varphi_b - \dfrac{G}{\mathrm{j}\omega} \cdot S_b \cdot b\left[\dfrac{a^{1/2}b^{1/2}(J_bY'_b - J'_bY_b)}{J_bY_a - J_aY_b}\right]\varphi_a \end{cases} \tag{7}$$

where $S_a = 2\pi a h_a$, $S_b = 2\pi b h_b$, J'_a, J'_b, Y'_a, Y'_b are the differential of J_a, J_b, Y_a, Y_b, respectively; their expressions are $J'_a = (\mathrm{d}/\mathrm{d}r)[J_{3/2}(k_t r)]|_{r=a}$, $J'_b = (\mathrm{d}/\mathrm{d}r)[J_{3/2}(k_t r)]|_{r=b}$, $Y'_a = (\mathrm{d}/\mathrm{d}r)[Y_{3/2}(k_t r)]|_{r=a}$, $Y'_b = (\mathrm{d}/\mathrm{d}r)[Y_{3/2}(k_t r)]|_{r=b}$. Introducing two expressions $M'_a = 2M_ak_t(J_bY_a - J_aY_b)/\rho c_t S_a$, $M'_b = 2M_bk_t\,(J_bY_a - J_aY_b)/\rho c_t S_b$, according to the characteristics of Bessel functions, $a(J_a Y'_a - J'_a Y_a) = b(J_b Y'_b - J'_b Y_b) = \pi/2$ can be obtained, so Eqs. (7) can be rewritten as

$$\begin{cases} M'_b = (Z_{1t} + Z_{3t})\varphi_b + Z_{3t}\varphi_a \\ M'_a = (Z_{2t} + Z_{3t})\varphi_a + Z_{3t}\varphi_b \end{cases} \tag{8}$$

where Z_{1t}, Z_{2t}, Z_{3t} are three mechanical impedances of torsional vibrations.

$$\begin{cases} Z_{1t} = \mathrm{j}b(J_b Y_a - J_a Y_b) + \mathrm{j}2b^2(Y'_b J_a - J'_b Y_a) - Z_{3t} \\ Z_{2t} = -\mathrm{j}a(J_b Y_a - J_a Y_b) + \mathrm{j}2a^2(Y'_a J_b - J'_a Y_b) - Z_{3t} \\ Z_{3t} = \mathrm{j}\pi a^{1/2} b^{1/2} \end{cases} \tag{9}$$

The physical concepts of electrical quantities and mechanical quantities are different from each other, but some of them have the same mathematical expressions. Electro-mechanical analogy is used to deal with the problem of mechanics and acoustics by the method that is used in electricity [18]; as a result, the problem would be simpler to deal with. For example, the electro-mechanical equivalent circuit of ultrasonic transducer is obtained via electro-mechanical analogy; the method can be used to obtain the resonance frequency of ultrasonic transducer as well as the resonance frequency of circuit. According to the theory of electro-mechanical analogy, the electro-mechanical equivalent circuit of the annular plate with conical cross-section can be obtained as in Fig. 2.

In applications, the load mechanical impedance of the output terminals of the concentrator is difficult to determine, so the frequency equation where the load mechanical impedance is ignored should be considered. In the above analysis, the mechanical loss in the plate is ignored, in the case of no load mechanical impedance, according to Fig. 2, the torsional frequency equation can be obtained as follows:

$$Z_{1t} + \frac{Z_{2t} Z_{3t}}{Z_{2t} + Z_{3t}} = 0 \tag{10}$$

The solution of Eq. (10) is k_t, considered the expressions of k_t and c_t, the resonance frequency can be obtained

$$f_t(n) = \frac{k_t(n)}{2\pi}\sqrt{\frac{E}{2(1+\sigma)\rho}} \tag{11}$$

where n is a positive integer, while $n = 1$, the resonance frequency is defined as fundamental resonance frequency. It can be seen from Eq. (10) and the expressions Z_{1t}, Z_{2t}, Z_{3t} that when the material parameters E, ρ, σ are given, k_t is related to the inner radius a and outer radius b, so the resonance frequency is also related to a and b.

Another important parameter of the torsional vibration annular plate concentrator is the angular displacement amplitude magnification. According to Fig. 2, we have

$$M_t = \left|\frac{\phi_a}{\phi_b}\right| = \left|\frac{\varphi_a}{\varphi_b}\right| = \left|\frac{Z_{3t}}{Z_{2t} + Z_{3t}}\right| \tag{12}$$

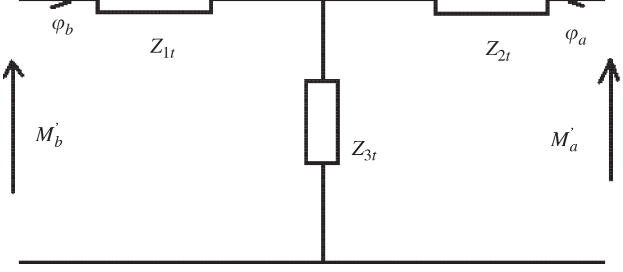

Fig. 2. Equivalent circuit of annular thin plate with conical cross-section.

When the material parameters E, ρ, σ and geometrical radiuses a, b are given, k_t can be computed from Eq. (10), then substituting k_t and E, ρ, σ, a, b into Eq. (12), the magnification M_t can be computed. From further numerical calculation, it can be found that the magnification M_t is merely related to the ratio of outer radius over inner radius.

2.2. Radial vibrations

The radial vibration can be assumed to be axisymmetric too. In Fig. 1, the variables M_a, M_b, φ_a, φ_b are replaced by F_a, F_b, v_a, v_b, respectively. The wave equation of radial vibrations of annular plates with variable thickness is as follows:

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}r^2}+\left(\frac{1}{r}+\frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}r}\right)\frac{\mathrm{d}\xi}{\mathrm{d}r}-\left(\frac{1}{r^2}-\frac{\sigma}{rh}\frac{\mathrm{d}h}{\mathrm{d}r}\right)\xi=\frac{1}{c_r^2}\frac{\partial^2\xi}{\partial t^2} \tag{13}$$

And the radial stress of the plate is

$$T_r=\frac{E}{1-\sigma^2}\left(\frac{\mathrm{d}\xi}{\mathrm{d}r}+\sigma\frac{\xi}{r}\right) \tag{14}$$

If the cross-section of the annular plate is conical, Eq. (13) can be rewritten as

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}\xi}{\mathrm{d}r}-\frac{1-\sigma}{r^2}\xi=\frac{1}{c_r^2}\frac{\partial^2\xi}{\partial t^2} \tag{15}$$

For harmonic vibration, we have

$$\xi=[Cf(r)+Dg(r)]\exp(\mathrm{j}\omega t) \tag{16}$$

where C, D are constants, $f(r)$, $g(r)$ are two introduced functions, their expressions are

$$\begin{cases} f(r)=r^{-1/2}J_\alpha(kr)\\ g(r)=r^{-1/2}Y_\alpha(kr)\end{cases} \tag{17}$$

$J_\alpha(kr)$, $Y_\alpha(kr)$ are the first kind and the second kind Bessel functions, respectively. $\alpha=\sqrt{\frac{5}{4}-\sigma^2}$. From Eq. (16), the radial velocity can be obtained as

$$v=\frac{\partial\xi}{\partial t}=\mathrm{j}\omega[Cf(r)+Dg(r)]\exp(\mathrm{j}\omega t) \tag{18}$$

According to the boundary conditions of radial velocities $\partial\xi/\partial r|_{r=a}=v_a$, $\partial\xi/\partial r|_{r=b}=-v_b$ and radial forces $T_r|_{r=a}\cdot S_a=-F_a$, $T_r|_{r=b}\cdot S_b=-F_b$, F_a, F_b as functions of v_a, v_b can be obtained as follows:

$$\begin{cases} F'_b=(Z_{1r}+Z_{3r})v_b+Z_{3r}v_a\\ F'_a=(Z_{2r}+Z_{3r})v_a+Z_{3r}v_b\end{cases} \tag{19}$$

where Z_{1r}, Z_{2r}, Z_{3r} are three mechanical impedances of radial vibration and F'_a, F'_b are two introduced functions

$$\begin{cases} F'_a=\dfrac{F_ak_r}{\rho c_rS_a[G(a)f(a)-F(a)g(a)]}\\ F'_b=\dfrac{F_bk_r}{\rho c_rS_b[G(b)f(b)-F(b)g(b)]}\end{cases} \tag{20}$$

$$\begin{cases} Z_{1r} = \dfrac{F(b)[g(a)-g(b)] - G(b)[f(a)-f(b)]}{g(b)F(b)-f(b)G(b)} Z_{3r} \\ Z_{2r} = \dfrac{F(a)[g(b)-g(a)] - G(a)[f(b)-f(a)]}{g(a)F(a)-f(a)G(a)} Z_{3r} \\ Z_{3r} = \mathrm{j}\dfrac{1}{f(a)g(b)-f(b)g(a)} \end{cases} \tag{21}$$

where $f(a) = f(r)|_{r=a}$, $f(b) = f(r)|_{r=b}$, $g(a) = g(r)|_{r=a}$, $g(b) = g(r)|_{r=b}$, $F(a) = F(r)|_{r=a}$, $F(b) = F(r)|_{r=b}$, $G(a) = G(r)|_{r=a}$, $G(b) = G(r)|_{r=b}$, $F(r)$, $G(r)$ are two introduced functions, the expressions are

$$\begin{cases} F(r) = \dfrac{\mathrm{d}f(r)}{\mathrm{d}r} + \dfrac{\sigma}{r} f(r) \\ G(r) = \dfrac{\mathrm{d}g(r)}{\mathrm{d}r} + \dfrac{\sigma}{r} g(r) \end{cases} \tag{22}$$

According to the theory of electro-mechanical analogy, the electro-mechanical equivalent circuit of radial vibrations can be obtained as the electro-mechanical equivalent circuit of torsional vibrations in Fig. 2, but the variables M'_a, M'_b, φ_a, φ_b, Z_{1t}, Z_{2t}, Z_{3t} are replaced by F'_a, F'_b, v_a, v_b, Z_{1r}, Z_{2r}, Z_{3r}, respectively. Then, the radial frequency equation can be obtained as

$$Z_{1r} + \frac{Z_{2r}Z_{3r}}{Z_{2r} + Z_{3r}} = 0 \tag{23}$$

The solution of Eq. (23) is k_r, considered the expressions of k_r, c_r, the resonance frequency can be obtained as

$$f_r(n) = \frac{k_r(n)}{2\pi}\sqrt{\frac{E}{\rho(1-\sigma^2)}} \tag{24}$$

where n is a positive integer, while $n = 1$, the resonance frequency is defined as fundamental frequency. k_r is also related to the inner radius a and outer radius b when the material parameters are given.

The radial displacement amplitude magnification can be also obtained from the equivalent circuit.

$$M_r = \left|\frac{\xi_a}{\xi_b}\right| = \left|\frac{v_a}{v_b}\right| = \left|\frac{Z_{3r}}{Z_{2r} + Z_{3r}}\right| \tag{25}$$

From Eq. (23), k_r is computed, and then substituting k_r into Eq. (25), the magnification M_r can be computed. From further numerical calculation, it can be found that M_r is also merely related to the ratio of b over a.

It should be point out, the velocities (φ and v) and displacements (ϕ and ξ) of the vibrations are far less than the sound velocities (c_t and c_r) and corresponding wavelengths, so the method of this paper is not suitable for nonlinear vibration problem of solid mechanics. In order to analyze the relationships between the resonance frequencies and magnifications and the geometrical dimensions, the resonance frequencies and magnifications are computed by using Eqs. (10)–(12) and Eqs. (23)–(25) when the inner radius and outer radius are fixed. The metal annular plate material used here is steel, its material parameters are as follows: $\rho = 7800\,\mathrm{kg/m^3}$, $E = 2.09 \times 10^{11}\,\mathrm{N/m^2}$, $\sigma = 0.28$.

The relationships between the resonance frequencies and magnifications and the geometrical radiuses can be obtained as the following figures.

Figs. 3 and 4 are theoretical relationships between the torsional and radial fundamental resonance frequencies and the radiuses, respectively. It can be seen that when the inner radius is fixed, the torsional resonance frequency and the radial resonance frequency are decreased when the outer radius is increased. As the outer radius increased, the torsional resonance frequencies' curves and the radial resonance frequencies' curves of the annular plates with different inner radiuses are accord with each other. When the outer radius is fixed, the bigger the inner radius, the bigger the torsional resonance frequency; but for radial vibration, it is not so, the bigger the inner radius, the smaller the radial resonance frequency. It should be pointed out that the intersections or coincidences of the curves cannot appear because the resonance frequencies are also affected

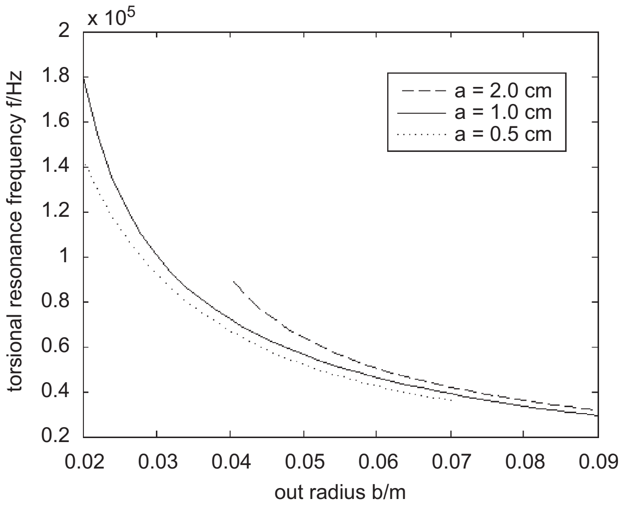

Fig. 3. Theoretical relationship between the torsional resonance frequency and radius.

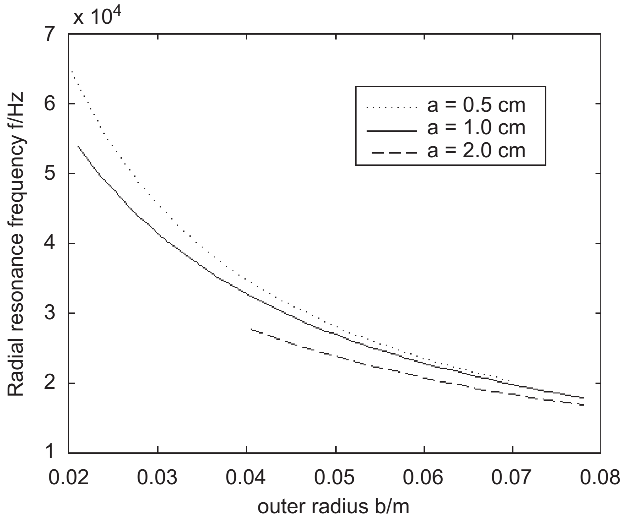

Fig. 4. Theoretical relationship between the radial resonance frequency and radius.

by the inner radius, but the primary factor is the outer radius. The reason for this is that the torsional stiffness and radial stiffness are decreased when the inner radius is decreased while the outer radius is kept constant.

Figs. 5 and 6 are theoretical relationships between the magnifications and the ratio of outer radius over inner radius. It can be seen from Fig. 5 that when the ratio (b/a) is increased, the angular displacement amplitude magnification is increased first and decreased slowly later, the maximum of the magnification and the value of corresponding ratio (b/a) are 7.3 and 4.0, respectively. From Fig. 6, it can be seen that the radial displacement amplitude magnification is decreased when the ratio (b/a) is increased, but it decreased more slowly, in a wide range, the magnification is approximate to 1.0.

3. Theoretical simulation of the resonance frequency of the annular plate with conical cross-section

As far as we know, the software ANSYS is very useful in finding the resonance frequency, so some annular plates are analyzed by the software in order to validate the theory of this paper. The results are listed

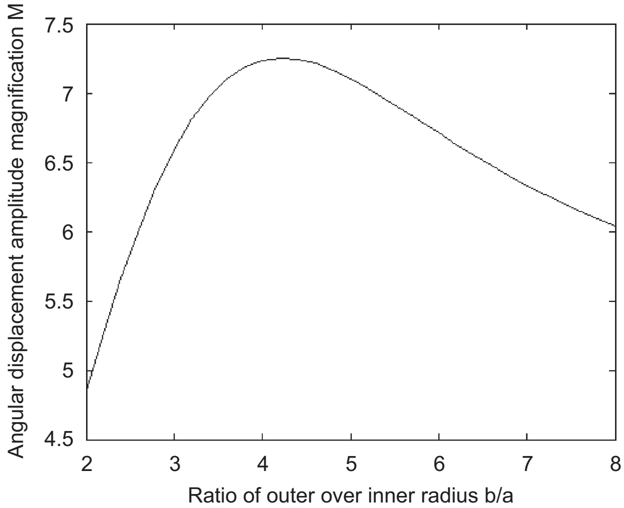

Fig. 5. Theoretical relationship between the angular displacement amplitude magnification and the ratio of outer radius over inner radius.

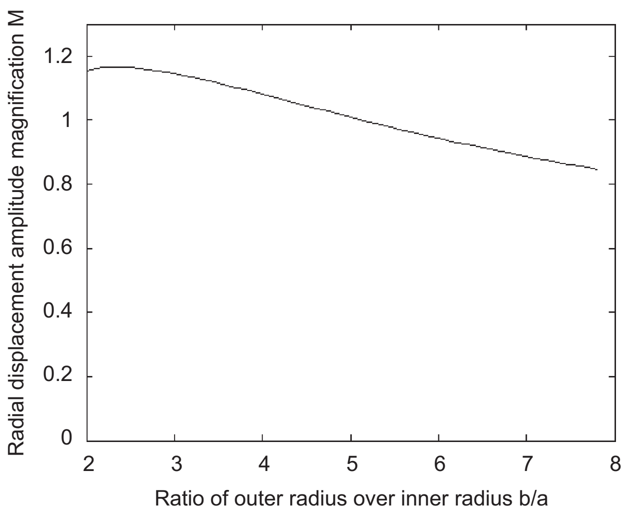

Fig. 6. Theoretical relationship between the radial displacement amplitude magnification and the ratio of outer radius and inner radius.

Table 1
Theoretical and simulated resonance frequencies.

No.	a (m)	b (m)	h_a (m)	h_b (m)	f_t (Hz)	f_r (Hz)	f_{nt} (Hz)	f_{nr} (Hz)	Δf_1 (%)	Δf_2 (%)
1	0.01	0.03	0.003	0.009	100,693	40,812	104,464	40,842	3.610	0.074
2	0.01	0.03	0.004	0.012	100,693	40,812	104,812	40,856	3.930	0.108
3	0.01	0.04	0.002	0.008	72,122	32,237	75,620	32,255	4.626	0.056
4	0.005	0.02	0.0025	0.001	144,293	64,455	151,206	64,573	4.572	0.183
5	0.02	0.06	0.006	0.018	50,347	20,406	52,232	20,421	3.609	0.074

in Table 1, in the table, f_t, f_r are, respectively, the theoretical torsional and radial fundamental resonance frequencies that is analyzed in this paper, f_{nt}, f_{nr} are, respectively, the torsional and radial resonance frequencies simulated by ANSYS, $\Delta f_1 = |f_{nt} - f_t|/f_{nt}$, $\Delta f_2 = |f_{nr} - f_r|/f_{nr}$, Δf_1, Δf_2 are the relative errors of theoretical and simulated results. The type of elements is structural-solid-brick 8 nodes 45. Figs. 7 and 8 are

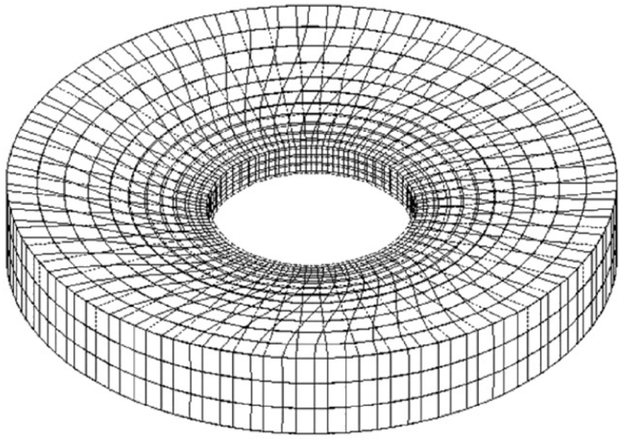

Fig. 7. FEM analysis mode shape for torsional vibration.

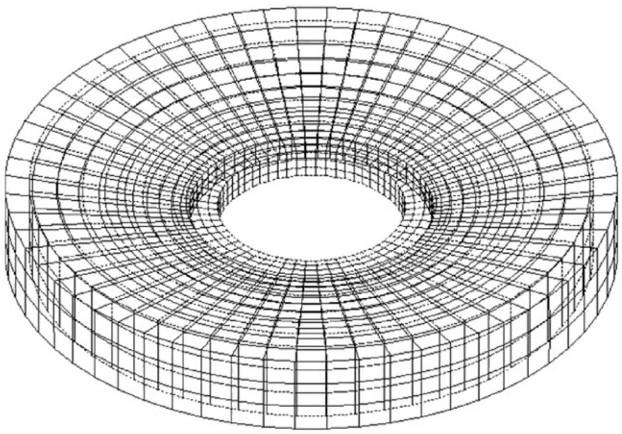

Fig. 8. FEM analysis mode shape for radial vibration.

the simulated modal shapes for the torsional and radial vibration of annular plates. It can be seen that the computed frequencies from the two methods are in good agreement with each other, and the relative errors are very small.

4. Conclusion

In this paper, the torsional and radial vibrations of annular plates with conical cross-section are analyzed by using electro-mechanical equivalent circuit. The relationships between the frequencies and geometrical radiuses are analyzed. When the inner radius is fixed, the torsional and radial resonance frequencies are decreased as the outer radius is increased. Although the frequencies are affected by inner radius, the primary factor is the outer radius. The relationships between the magnifications and the geometrical radius are also analyzed, the magnifications are merely related to the ratio of outer radius over inner radius. The physic concepts of this method are more prominent than others.

In the above analysis, the annular plate with conical cross-section is assumed to be very thin, in other words, its thickness is much less than its radius; and the vibration can be assumed to be axisymmetric. For a thick annular plate, its vibrations are complex; the method of this paper was not suitable. And the method of this paper is not suitable for nonlinear vibration problem of solid mechanics, too.

The frequencies are also analyzed by FEM, the results of the two methods are in a good agreement with each other. Although the magnifications cannot be analyzed by the software ANSYS and difficult to be measured via experiments, it may be useful for finding appropriate ratio (b/a) to concentrate maximal energy. The theory of this paper is helpful for designing annular plates with conical cross-section, and the method is useful for other variable thickness thin annular plates.

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