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Direct transfer functions and path blocking in a discrete mechanical system

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Abstract

In this work some issues concerning the prediction capabilities of the global transfer direct transfer (GTDT) method when blocking transmission paths in a mechanical system are addressed. The formalism of the GTDT approach, which requires no force determination, is first applied to a discrete system made of a combination of springs, dampers and masses. Then, a path blocking analysis is carried out for the cases of setting the displacement of any mass to zero and of removing some system elements. Differences with the more standard force transmission path analysis (TPA) are outlined.

1. Introduction

There exist different options to try to reduce the vibration or acoustic pressure levels in a given vibroacoustical system. Think, for example, in the case of an airplane's cabin where the noise spectrum at the pilot location is to be diminished. If it was possible to measure the operational forces acting on the airplane, force transmission path analysis (TPA) methods could become a very useful tool because they could serve to rank the influence of the various paths to the interior noise (see e.g., Refs. [1–8]). Then, one could calculate how the suppression or reduction of a given force will decrease the interior noise. On the other hand, if it was not possible to modify the forces acting on the airplane, an alternative to reduce the interior noise would be to diminish the radiation of the cabin panels, for instance, by modifying their physical properties. In such a case, it would prove very interesting to factorise the noise at the pilot location in terms of the contributions of the various panel vibrations. If one was able to determine how a modification in the panel characteristics may diminish its overall level of vibration, the factorisation will serve to calculate the consequent reduction of interior noise level. This type of factorisation among responses can be found, for instance, by means of the global transfer direct transfer (GTDT) TPA method (see e.g., Refs. [2,9–13]) or, alternatively, by means of path blocking (covering) techniques (see e.g., Ref. [14]).

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The force TPA and GTDT approaches can be classified as two-step methods because they involve two sets of measurements: the first set is carried out with the system stationary and serves to characterise it, while the second set of measurements is performed with the system running under operational conditions. However, and as a general tendency, it could be roughly said that the GTDT method offers much simpler sets of measurements because no force determination is required, although it yields more intricate data postprocess. As opposite, the force TPA method accounts for more cumbersome measurements but easier postprocess. This point will be discussed to some extent in this paper. Finally, it is worthwhile to mention that alternative approaches to transfer path analysis, or modifications to the force TPA and GTDT methods, exist. For instance, one-step methods aiming at finding force and/or response transmission paths solely from operational force and response measurements (see e.g., Refs. [15–18]). Further literature on these and related subjects can be found in the insofar cited references, among others.

Recently, a detailed analysis of some discrete mechanical systems consisting of springs, dampers and masses using several force transmission path methods has been carried out [1]. In this paper, the formalism of the GTDT approach is analytically applied to one of these systems. On one hand, some points concerning the possible predictive capabilities of the method when blocking a transmission path will be addressed. Two cases will be considered, path blocking by setting to zero the displacement of a system mass and path blocking by means of removing some system elements. As quoted in Ref. [1], very few works and information on path blocking approaches are available. On the other hand, the simplicity of the example also proves very useful to explain pedagogically some of the key ideas of the GTDT method, as well as to complement some previous work on it. Concerning the former, it will be shown that the factorisation among responses can be performed using easily measurable quantities, such as the transfer functions among the displacements of the masses, or the operational displacement vector. Measuring these quantities does not require the blocking of any mass or to remove any part of the system, as happens in the force TPA approach [1]. Concerning the second aim, the connectivity role of the so-called direct transfer matrix and some factorisation issues will be made apparent for the considered discrete mechanical system. Transmission paths will be identified with direct transfer functions rather than with a set of physical elements, in the line of what is done in Refs. [2,12,13].

Some of the above issues were previously analysed for some continuum systems involving, for instance, the bending wave propagation in an Euler–Bernouilli beam or the free field acoustic wave propagation in a two-dimensional space (see Refs. [12,13]). Although using a different terminology, the role of the direct transfer function matrix eigenvalues and eigenvectors when coupling plates by means of springs was addressed in Ref. [11]. On the other hand, an application of the GTDT approach to discrete systems was performed in Ref. [13] considering the discretisation of the Helmholtz equation by means of several finite element and finite difference numerical schemes. Instead, and as previously mentioned, a more *standard* discrete mechanical system has been used in this work.

The paper is organised as follows. In Section 2, the mechanical system to be studied, the problem to be solved and some measurable quantities that can be easily obtained in practical industrial cases are presented. These quantities will be the starting point of the analysis. In Section 3, it is shown how to obtain the direct transfer functions among the displacements of the masses from these easily measurable quantities. An alternative way to obtain direct transfer functions for the herein developed analytic case is also provided. In Section 4, the factorisation of any displacement of a mass in terms of the displacement due to the external force directly acting on it, plus the contributions from the displacements of the remaining masses is presented. Some considerations on the predictive capabilities of the method when blocking some paths in the original analysed system are given in Section 5. In Section 6, a numerical example is carried out to elucidate some of the previously exposed concepts. Conclusions are finally drawn in Section 7.

2. Problem statement

2.1. Equations of motion

Consider a 4 dof (degrees of freedom) discrete mechanical system consisting of two masses m_1 and m_4, respectively connected to ground by springs and dampers characterised by (k_{g1}, c_{g1}) and (k_{g4}, c_{g4}). Masses m_1

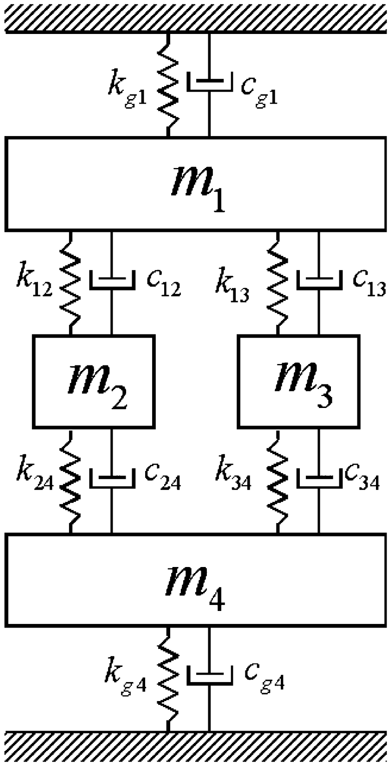

Fig. 1. Analysed 4 dof mechanical system.

and m_4 are also connected one to another by means of two parallel configurations of springs $(k_{12}, k_{24}, k_{13}, k_{34})$, dampers $(c_{12}, c_{24}, c_{13}, c_{34})$ and masses (m_2, m_3) according to the distribution in Fig. 1.

Assuming a time harmonic behaviour of radian frequency ω, the equation of motion of the system in the frequency domain is given by

$$(\overline{\mathbf{K}} - \omega^2\mathbf{M})\mathbf{X} = \mathbf{F}, \tag{1}$$

where $\mathbf{X} = (X_1, X_2, X_3, X_4)^\top$ is the vector of displacements of the masses and $\mathbf{F} = (F_1, F_2, F_3, F_4)^\top$ is the vector containing the forces acting on them. $\mathbf{M}$ is the mass matrix given by

$$\mathbf{M} := \mathrm{diag}(m_1, m_2, m_3, m_4), \tag{2}$$

while $\overline{\mathbf{K}}$ is the complex stiffness matrix built from the storage stiffness, $\mathbf{K}$, and the viscous damping, $\mathbf{C}$, matrices

$$\overline{\mathbf{K}} := \mathbf{K} + \mathrm{i}\omega\mathbf{C}. \tag{3}$$

$\overline{\mathbf{K}}$ is explicitly given by

$$\overline{\mathbf{K}} = \begin{pmatrix} \overline{k}_1 & -\overline{k}_{12} & -\overline{k}_{13} & 0 \\ -\overline{k}_{12} & \overline{k}_2 & 0 & -\overline{k}_{24} \\ -\overline{k}_{13} & 0 & \overline{k}_3 & -\overline{k}_{34} \\ 0 & -\overline{k}_{24} & -\overline{k}_{34} & \overline{k}_4 \end{pmatrix} \tag{4}$$

with $\overline{k}_1 := \overline{k}_{12} + \overline{k}_{13} + \overline{k}_{g1}$, $\overline{k}_4 := \overline{k}_{24} + \overline{k}_{34} + \overline{k}_{g4}$, $\overline{k}_2 := \overline{k}_{12} + \overline{k}_{24}$ and $\overline{k}_3 := \overline{k}_{13} + \overline{k}_{34}$.

A dynamic stiffness matrix $\mathbf{Z}^{\mathrm{SPR}}$ can be defined by

$$\mathbf{Z}^{\mathrm{SPR}} := \overline{\mathbf{K}} - \omega^2\mathbf{M}, \tag{5}$$

so that the equation of motion, Eq. (1), can be rewritten as

$$\mathbf{Z}^{\mathrm{SPR}}\mathbf{X} = \mathbf{F}. \tag{6}$$

Note that while $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ are constant matrices, $\overline{\mathbf{K}}$, $\mathbf{Z}^{\mathrm{SPR}}$, $\mathbf{X}$ and $\mathbf{F}$ are frequency dependent quantities, although this is not shown explicitly in the herein used notation.

2.2. Easily measurable quantities: the global transfer matrix, $\mathbf{T}^G$ and the operational displacement vector, $\mathbf{X}^{\mathrm{op}}$

In a practical industrial case, involving a complex system, the dynamic stiffness matrix will not be known a priori. However, its inverse (dynamic compliance matrix) can be determined by means of experiments. Let us denote it by $\mathbf{H}^{\mathrm{SPR}} := (\mathbf{Z}^{\mathrm{SPR}})^{-1}$. It then follows from Eq. (6) that

$$\mathbf{X} = \mathbf{H}^{\mathrm{SPR}}\mathbf{F}. \tag{7}$$

Equations analogous to Eq. (7) are at the basis of force TPA methods (see e.g., Refs. [1–8] and references therein). However, and as mentioned in the Introduction, one of the aims in this paper is to avoid the explicit use of forces. Instead of working with $\mathbf{H}^{\mathrm{SPR}}$, it is then expected to just work with responses (displacements, velocities, accelerations, acoustic pressure, etc.) (see Refs. [12,13]). This will avoid the necessity to perform demounting tests and to measure, or indirectly determine, the operational forces, which may be rather complicate in many cases.

An easily measurable matrix that will serve the above purposes is the *global transfer* matrix, or *transmissibility* matrix, $\mathbf{T}^G$. The global transfer function between two masses m_i and m_j is given by the quotient between the displacement of m_j and the displacement of m_i, when a unit force F_i is applied at m_i and there is no restriction on the displacements of the remaining masses. Hence,

$$T^G_{ij} = X_j/X_i. \tag{8}$$

Note that Eq. (8) coincides with the usual concept of transfer function, the adjective global arising from the fact that the measured displacements X_i and X_j are not only due to the response of masses m_i, m_j to F_i, but to the response of the whole system to F_i. Note also that T^G_{ij} in Eq. (8) also stands for a velocity/velocity or acceleration/acceleration transfer function given that a cancelled factor $\partial^n_t \leftrightarrow (\mathrm{i}\omega)^n$, with $n = 1, 2$, would appear both in the numerator and denominator of Eq. (8) (∂^n_t stands for the n-th-order partial time derivative).

As an analytical example is being studied, it is possible to obtain $\mathbf{T}^G$ from $\mathbf{H}^{\mathrm{SPR}}$ because $T^G_{ij} = X_j/X_i = (X_j/F_i)/(X_i/F_i) = H^{\mathrm{SPR}}_{ji}/H^{\mathrm{SPR}}_{ii}$ (this relation is not true for measured force and global transfer functions). Then, $\mathbf{T}^G$ for the mechanical system in Fig. 1 becomes

$$\mathbf{T}^G = \begin{pmatrix} 1 & H^{\mathrm{SPR}}_{21}/H^{\mathrm{SPR}}_{11} & H^{\mathrm{SPR}}_{31}/H^{\mathrm{SPR}}_{11} & H^{\mathrm{SPR}}_{41}/H^{\mathrm{SPR}}_{11} \\ H^{\mathrm{SPR}}_{12}/H^{\mathrm{SPR}}_{22} & 1 & H^{\mathrm{SPR}}_{32}/H^{\mathrm{SPR}}_{22} & H^{\mathrm{SPR}}_{42}/H^{\mathrm{SPR}}_{22} \\ H^{\mathrm{SPR}}_{13}/H^{\mathrm{SPR}}_{33} & H^{\mathrm{SPR}}_{23}/H^{\mathrm{SPR}}_{33} & 1 & H^{\mathrm{SPR}}_{43}/H^{\mathrm{SPR}}_{33} \\ H^{\mathrm{SPR}}_{14}/H^{\mathrm{SPR}}_{44} & H^{\mathrm{SPR}}_{24}/H^{\mathrm{SPR}}_{44} & H^{\mathrm{SPR}}_{34}/H^{\mathrm{SPR}}_{44} & 1 \end{pmatrix}. \tag{9}$$

The terms H^{SPR}_{ij} in Eq. (9) are provided in Appendix A.

Assume now that the system is working under certain operational conditions i.e., an operational force $\mathbf{F}^{\mathrm{op}} = (F^{\mathrm{op}}_1, F^{\mathrm{op}}_2, F^{\mathrm{op}}_3, F^{\mathrm{op}}_4)^\top$ is acting on the system leading to the displacements $\mathbf{X}^{\mathrm{op}} = (X^{\mathrm{op}}_1, X^{\mathrm{op}}_2, X^{\mathrm{op}}_3, X^{\mathrm{op}}_4)^\top$. Operational forces are rather difficult quantities to obtain in practice, while it is straightforward to measure the operational system responses. Consequently, the sole use of $\mathbf{X}^{\mathrm{op}}$ is desired in the herein presented approach.

The type of problems that are expected to be solved following the GTDT formulation can be summarised in the next two questions for the mechanical system in Fig. 1.

- With the only use of easily measurable quantities such as the global transfer matrix, $\mathbf{T}^G$, and the operational displacement vector, $\mathbf{X}^{\mathrm{op}}$, is it possible to factorise the displacement, X^{op}_i, of any mass, m_i, in terms of the displacements of the remaining masses and the displacement solely due to the external force acting on m_i?

- From the knowledge of the global transfer matrix, $\mathbf{T}^G$, and the operational displacement vector, $\mathbf{X}^{\text{op}}$, is it possible to predict the new operational displacement vector when some transmission paths in the original system are somehow blocked?

It will be shown in subsequent sections that the application of the GTDT method of TPA gives an affirmative answer to the first question. In what concerns the second question, it will be shown that the answer can be affirmative in some cases (when the modification consists of blocking the displacement of a mass) but in others some further measurements will be required (when the modification involves changing the properties or removing some spring and damper elements).

3. The direct transfer matrix, $\mathbf{T}^D$

3.1. Obtaining $\mathbf{T}^D$ from $\mathbf{T}^G$

In order to obtain the desired displacement factorisation, it is necessary to introduce the concept of *direct transfer function*. Its definition is next stated for the particular case addressed in this paper (see Refs. [2,12] for general definitions). The direct transfer function, T^D_{ij} between two masses m_i and m_j, with $i \neq j$, is given by the displacement of m_j divided by the displacement of m_i, when a unit force acts on m_i and all remaining displacements of masses are kept blocked, i.e., $T^D_{ij} := X_j/X_i$, with $X_k = 0$, $\forall k \neq i,j$. On the other hand, the direct transfer function from a mass m_i to itself, $T^D_{ii} := X'_i/X^{\text{ext}}_i$, is given by the quotient between the displacement X'_i of m_i when a unit force is applied on it and the displacements of all remaining masses are blocked ($X_k = 0$, $\forall k \neq i$), and the displacement X^{ext}_i of m_i when a unit force is applied on it, and there is no restriction on the displacements of all other masses.

Considering all direct transfer functions among masses, the direct transfer matrix, $\mathbf{T}^D$, can be build. This matrix can be obtained from the measurable global transfer function, $\mathbf{T}^G$. It follows that (see Refs. [2,12,13])

$$1/T^D_{ii} = [\mathbf{T}^G]^{-1}_{ii}, \tag{10a}$$

$$T^D_{ij}/T^D_{jj} = -[\mathbf{T}^G]^{-1}_{ij}. \tag{10b}$$

In almost any case of practical interest, the inversion of the matrix $\mathbf{T}^G$ has to be carried out. This may pose some numerical stability problems that can be addressed employing specific regularisation and/or resampling techniques [7,8,19,20]. Even when dealing with analytical examples, the cumbersome work of inverting $\mathbf{T}^G$ has usually to be performed [12,13]. Fortunately, this is not the case for the mechanical system presented here because in this particular case $\mathbf{Z}^{\text{SPR}}$ is known a priori. In fact, $\mathbf{T}^G$ can be rewritten as

$$\mathbf{T}^G = \text{diag}(1/H^{\text{SPR}}_{11}, 1/H^{\text{SPR}}_{22}, 1/H^{\text{SPR}}_{33}, 1/H^{\text{SPR}}_{44})\mathbf{H}^{\text{SPR}}, \tag{11}$$

where the symmetry of $\mathbf{H}^{\text{SPR}}$ has been used (see Appendix A). The inverse $[\mathbf{T}^G]^{-1}$ can then be easily computed as

$$\begin{aligned}[\mathbf{T}^G]^{-1} &= [\mathbf{H}^{\text{SPR}}]^{-1}\,\text{diag}(1/H^{\text{SPR}}_{11}, 1/H^{\text{SPR}}_{22}, 1/H^{\text{SPR}}_{33}, 1/H^{\text{SPR}}_{44})^{-1} \\ &= \mathbf{Z}^{\text{SPR}}\,\text{diag}(H^{\text{SPR}}_{11}, H^{\text{SPR}}_{22}, H^{\text{SPR}}_{33}, H^{\text{SPR}}_{44})\end{aligned} \tag{12}$$

given that all terms in the second line of Eq. (12) are already known. From Eqs. (5), (A.1)–(A.4), (A.11), (10a) and (12) it follows that the diagonal elements of the direct transfer matrix, $\mathbf{T}^D$, are given by

$$T^D_{ii} = 1/[(\overline{k}_i - \omega^2 m_i)H^{\text{SPR}}_{ii}] \quad \forall i = 1\ldots4. \tag{13}$$

On the other hand, from Eqs. (5), (A.5)–(A.10), (10b), (12) and (13) the off-diagonal elements of $\mathbf{T}^D$ (T^D_{ij}, $\forall i,j = 1\ldots4$, $i \neq j$) become

$$T^D_{ij} = \frac{\overline{k}_{ij}}{(\overline{k}_j - \omega^2 m_j)} \quad \text{for } (i,j) \neq \{(1,4),(4,1),(2,3),(3,2)\}, \tag{14a}$$

$$T^D_{14} = T^D_{41} = T^D_{23} = T^D_{32} = 0 \tag{14b}$$

with $\overline{k}_{ij} \equiv \overline{k}_{ji}$. The key point of matrix $\mathbf{T}^D$ is the connectivity or path information it contains. For instance, $T^D_{14} = 0$ expresses the logical fact that m_4 cannot move when m_1 is vibrating if m_2 and m_3 are kept fixed i.e., $X_2 = X_3 = 0$ (see Fig. 1). Analogously, $T^D_{32} = 0$ expresses the fact that m_2 will not move when m_3 is doing so, if m_1 and m_4 remain blocked. Further considerations on the concepts of transmission path and connectivity are provided in Appendix B.

It should be noticed that the connectivity or path information contained in $\mathbf{T}^D$ was already available in the equation of motion, Eq. (6), characterised by matrix $\mathbf{Z}^{\mathrm{SPR}}$ ($T^D_{ij} = -T^D_{jj}\mathbf{Z}^{\mathrm{SPR}}_{ij}H^{\mathrm{SPR}}_{jj}$). However, and as previously pointed out, $\mathbf{Z}^{\mathrm{SPR}}$ will not be known a priori in most practical cases. Moreover, if it is desired to obtain $\mathbf{Z}^{\mathrm{SPR}}$ by means of experiments (as in some force TPA approaches) the difficulty of having to block the masses to do so and to demount several parts of the system has to be faced [1,3,4]). On the contrary, using the above procedure the connectivity or path information contained in $\mathbf{T}^D$ and $\mathbf{Z}^{\mathrm{SPR}}$ is obtained from the simply measurable global transfer function, $\mathbf{T}^G$.

3.2. An alternative procedure to obtain $\mathbf{T}^D$

The direct transfer functions in Eqs. (13) and (14) can be alternatively obtained for the present analytical example solving the appropriate equations of motion according to the T^D_{ij} and T^D_{ii} definitions.

First consider the off-diagonal elements of $\mathbf{T}^D$ and choose an arbitrary element such as T^D_{12}, i.e., the direct transfer function between the m_1 displacement and the m_2 displacement. Remember that, by definition, T^D_{12} (from 1 to 2) stands for the displacement of m_2 divided by the displacement of m_1, when a unit force acts on m_1 and all remaining displacements of masses are blocked. This will correspond to suppressing rows $\{3,4\}$, as well as columns $\{3,4\}$, in Eq. (6) and then solving for $\mathbf{X} = (X_1, X_2)^\top$ and $\mathbf{F} = (1,0)^\top$. This yields the reduced system of equations:

$$\begin{pmatrix} \overline{k}_1 - \omega^2 m_1 & -\overline{k}_{12} \\ -\overline{k}_{12} & \overline{k}_2 - \omega^2 m_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{15}$$

with solution

$$X_1 = \frac{(\overline{k}_2 - \omega^2 m_2)}{(\overline{k}_1 - \omega^2 m_1)(\overline{k}_2 - \omega^2 m_2) - \overline{k}^2_{12}}, \tag{16a}$$

$$X_2 = \frac{\overline{k}_{12}}{(\overline{k}_1 - \omega^2 m_1)(\overline{k}_2 - \omega^2 m_2) - \overline{k}^2_{12}}. \tag{16b}$$

The direct transfer function is then given by $T^D_{12} = X_2/X_1$, from which the first expression in Eq. (14a) for $(i,j) = (1,2)$ is recovered.

All the remaining direct transfer functions in Eq. (14) can be easily recovered following the above procedure. For instance, to find the element T^D_{34} simply remove rows and columns $\{1,2\}$ in Eq. (6) and then solve the resulting system for $\mathbf{X} = (X_3, X_4)^\top$ and $\mathbf{F} = (1,0)^\top$. To find the element T^D_{42}, it is only necessary to eliminate rows and columns $\{1,3\}$ in Eq. (6) and then solve for $\mathbf{X} = (X_2, X_4)^\top$, $\mathbf{F} = (0,1)^\top$ and so on.

The diagonal terms of $\mathbf{T}^D$ in Eq. (13) can be recovered in a similar way. Pay attention, for instance, to the T^D_{11} element. According to the definition of the direct transfer function from a subsystem to itself, T^D_{11} is the quotient between the displacement of m_1, when a unit force is applied on it and the displacements of all remaining masses are fixed ($X_i = 0,\ \forall i \neq 1$), and the displacement of m_1 when a unit force is applied on it and there is no restriction on the displacements of other masses. Denoting the vector displacement in the first situation by $\mathbf{X}'$ and the vector displacement in the second situation by $\mathbf{X}^{\mathrm{ext}}$, $\mathbf{X}'$ can be found by suppressing rows and columns $\{2,3,4\}$ in Eq. (6), which yields the immediate solution

$$X'_1 = \frac{1}{(\overline{k}_1 - \omega^2 m_1)}. \tag{17}$$

On the other hand, $\mathbf{X}^{\text{ext}}$ can be found from the solution of Eq. (6) with $\mathbf{X} \equiv \mathbf{X}^{\text{ext}} = (X_1^{\text{ext}}, X_2^{\text{ext}}, X_3^{\text{ext}}, X_4^{\text{ext}})^\top$ and $\mathbf{F} = (1,0,0,0)^\top$. Given that $\mathbf{H}^{\text{SPR}} := (\mathbf{Z}^{\text{SPR}})^{-1}$, the solution becomes

$$\mathbf{X}^{\text{ext}} = (H_{11}^{\text{SPR}}, H_{21}^{\text{SPR}}, H_{31}^{\text{SPR}}, H_{41}^{\text{SPR}})^\top. \tag{18}$$

The direct transfer function is then given by $T_{11}^D = X_1'/X_1^{\text{ext}}$, from which the result already found in Eq. (13) for $i = 1$ is recovered. The remaining transfer functions T_{ii}^D, $\forall i \neq 1$, can be derived straightforwardly in a similar way.

4. Factorisation under operational conditions

4.1. Displacement of masses coherent factorisation

In the previous section it has been shown how the direct transfer matrix, $\mathbf{T}^D$, can be obtained from the measurable global transfer matrix, $\mathbf{T}^G$. It will be next shown how to use $\mathbf{T}^D$ to factorise the displacement of any mass in terms of the displacement due to the force acting on it plus the displacements due to other subsystem contributions. To do so, suppose that an operational force $\mathbf{F}^{\text{op}} = (F_1^{\text{op}}, F_2^{\text{op}}, F_3^{\text{op}}, F_4^{\text{op}})^\top$ is applied to the system resulting into the displacements $\mathbf{X}^{\text{op}} = (X_1^{\text{op}}, X_2^{\text{op}}, X_3^{\text{op}}, X_4^{\text{op}})^\top$. It can then be proved that the following factorisation holds [2,12,13]:

$$\mathbf{X}^{\text{op}} = (\operatorname{dev} \mathbf{T}^D)^\top \mathbf{X}^{\text{op}} + \mathbf{\Lambda}^D \mathbf{X}^{\text{op,ext}}, \tag{19}$$

where $\operatorname{dev} \mathbf{T}^D$ stands for a matrix containing the off-diagonal elements of $\mathbf{T}^D$, $(\operatorname{dev} T_{ij}^D := T_{ij}^D(1-\delta_{ij}))$, $\mathbf{\Lambda}^D$ contains the diagonal terms of $\mathbf{T}^D$ $(\Lambda_{ij}^D := T_{ij}^D \delta_{ij})$ and $\mathbf{X}^{\text{op,ext}}$ is the operational external displacement vector (δ_{ij} stands for Kronecker's delta). The vector $\mathbf{X}^{\text{op,ext}}$ contains the displacement of each mass that is solely due to the external force acting on it (not to forces acting on the remaining masses). Obviously, $\mathbf{X}^{\text{op,ext}}$ cannot be directly measured but it can be obtained from the measurable quantities $\mathbf{T}^G$ and $\mathbf{X}^{\text{op}}$,

$$\mathbf{X}^{\text{op,ext}} = [(\mathbf{T}^G)^\top]^{-1} \mathbf{X}^{\text{op}}. \tag{20}$$

Consequently, the decomposition in Eq. (19) has been obtained with the sole use of the global transfer matrix $\mathbf{T}^G$ and the operational displacement vector $\mathbf{X}^{\text{op}}$.

For an easier comprehension of the meaning of Eq. (19), it will be expanded for a particular mass, e.g., m_1. This yields

$$\begin{aligned} X_1^{\text{op}} &= T_{21}^D X_2^{\text{op}} + T_{31}^D X_3^{\text{op}} + T_{11}^D X_1^{\text{op,ext}} \\ &= T_{21}^D X_2^{\text{op}} + T_{31}^D X_3^{\text{op}} + X_1'^{\text{op}}, \end{aligned} \tag{21}$$

where $T_{41}^D = 0$ has been used. Eq. (21) states that the operational displacement of m_1, X_1^{op}, can be factorised as a contribution due to the displacement of mass m_2, plus a contribution due to the displacement of mass m_3, plus a contribution due to the displacement induced by the external force applied on m_1. Note that $X_1^{\text{op,ext}}$ represents the displacement of m_1 due to the external force acting on it plus the contributions of all other mass responses to this force. Hence, $X_1'^{\text{op}} = T_{11}^D X_1^{\text{op,ext}}$ represents the fraction of $X_1^{\text{op,ext}}$ that is only due to the former (see e.g., Refs. [12,13]). Obviously, there is no contribution from m_4 because m_1 and m_4 are not directly connected. The influence of the m_4 displacement onto the displacement of m_1 takes place through m_2 and m_3.

The type of factorisation among responses given by Eq. (19) can be obtained experimentally in industrial applications using a covering (path blocking) technique (see e.g., Ref. [14]) to obtain the direct transfer matrix. For instance, in the case of the airplane's cabin example mentioned in the Introduction, the various panel vibration/vibration direct transfer functions as well as the direct transfer functions from the panel vibrations to the acoustic pressure at the pilot's location can be found by completely covering all cabin panels and then uncovering them by pairs to obtain $\mathbf{T}^D$. Once operational data have been measured, a factorisation of Eq. (19) type can be performed to obtain the various panel contributions to interior noise in real operation conditions. Unfortunately, this is a rather lengthy and cumbersome procedure that, in addition, becomes intrusive as it may alter the original (non-covered) vibration of the cabin. This is so because covering requires the use of heavy mass-loaded material, which is difficult to link to the panels without influencing their behaviour. However, it has been shown from the herein presented discrete system, together with previous work on the subject [2,10,12,13], that it should be possible to obtain the same information of the covering approach, without altering the structure vibration and using shorter and easier sets of measurements.

4.2. A brief comment on the energetic factorisation

For completeness, a brief comment will be made in this subsection concerning the displacement factorisation in the energetic case. Suppose that every F_i in a general equation of motion, $\mathbf{ZX} = \mathbf{F}$ or $\mathbf{X} = \mathbf{HF}$, is of white noise type so that any pair (F_i, F_j) is uncorrelated, i.e., $\langle F_i^* F_j \rangle = |F_i|^2 \delta_{ij}$. Here, the bracket denotes an averaging over a frequency bandwidth, $\Delta\omega$, so that for any function, y, its average becomes $\langle y \rangle := 1/\Delta\omega \int_{\Delta\omega} y \, \mathrm{d}\omega$ and its modulus $|y| := (1/\Delta\omega \int_{\Delta\omega} y^* y \, \mathrm{d}\omega)^{1/2}$.

Any displacement in $\mathbf{X} = \mathbf{HF}$ can be decomposed as

$$X_i = \sum_{j=1}^{N} H_{ij} F_j. \tag{22}$$

Multiplying X_i by its complex conjugate, averaging over frequency and taking into account that forces are uncorrelated yields

$$\langle X_i^* X_i \rangle = |X_i|^2 = \sum_{j=1}^{N} |H_{ij}|^2 |F_j|^2. \tag{23}$$

An analogous procedure to the one developed in the previous sections for the coherent case can be followed defining $|T_{ij}^G|^2 := |H_{ij}|^2 / |H_{ii}|^2$ to arrive at a decomposition similar to the ones in Eqs. (19)–(21) for the energetic approach

$$|X_i^{\mathrm{op}}|^2 = \sum_{j \neq i}^{N} |T_{ji}^D|^2 |X_j^{\mathrm{op}}|^2 + |X_i'^{\mathrm{op}}|^2. \tag{24}$$

The factorisations in Eqs. (23) and (24) are commonly used for problems in the mid-high frequency range.

5. Analytic analysis of path blocking

5.1. Blocking the displacement of a mass

Once having a factorisation like Eq. (19), one may wonder about its usefulness. The first obvious answer is that it can serve to locate vibration and/or acoustic problems in a mechanical system. Imagine, for instance, that the factor $T_{21}^D X_2^{\mathrm{op}}$ in Eq. (21) largely exceeds the other contributions to the operational displacement of m_1, X_1^{op}. Then, it could be conjectured that by setting $T_{21}^D X_2^{\mathrm{op}} = 0$ the displacement of m_1 should diminish considerably. There are two options to do so, either by setting $X_2^{\mathrm{op}} = 0$ or by making $T_{21}^D = 0$. The first possibility corresponds to blocking the composed path (see Appendix B for definition) connecting m_1 and m_4 by means of blocking the displacement of mass m_2. The second possibility corresponds to blocking the elemental path (see Appendix B for definition) linking m_2 and m_1. The first case will be analysed in this subsection and the second case in the following one.

In the case of blocking the displacement of a mass, the GTDT method retains full prediction power in the sense that if one is able to compute the new system direct transfer matrix, $\widetilde{\mathbf{T}}^D$, resulting from this blocking, then the new system operational displacements, $\widetilde{\mathbf{X}}^{\mathrm{op}}$, can be readily obtained (note that a tilde will be used hereafter to differentiate the new system variables from the original system ones). This can be done in the following way.

The first step consists in building the matrix $\widetilde{\mathbf{T}}^D$. This is quite a straightforward task for the herein analysed 4 dof mechanical system. All $\widetilde{T}_{ij}^D$ involving m_2 will automatically vanish because m_2 cannot be excited to produce any other displacement ($\widetilde{T}_{2j}^D = 0, \ \forall j$) and the excitation of any mass will produce no response on m_2 ($\widetilde{T}_{i2}^D = 0, \ \forall i$). Hence, blocking m_2 logically yields a reduced 3 dof system (see Fig. 2). On the other hand,

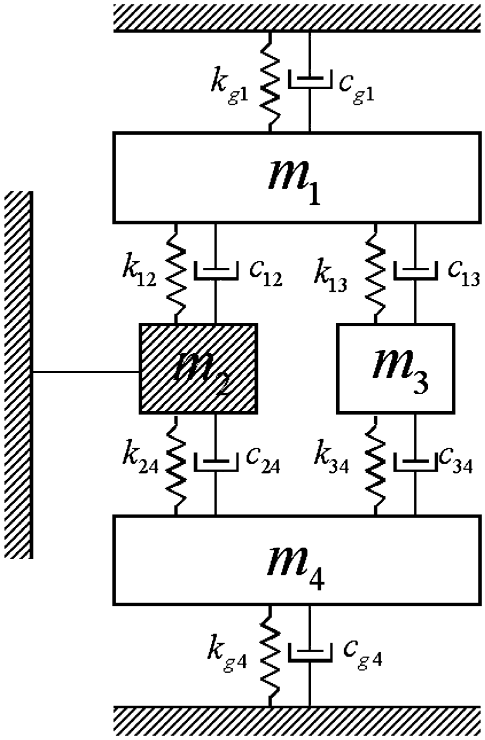

Fig. 2. Blocking of paths: blocking mass m_2.

note that in accordance with the definition of direct transfer function, the remaining direct transfer functions will not change as they do not involve m_2, i.e.,

$$\widetilde{T}^D_{ij} = T^D_{ij} \quad \forall i = 1, 3, 4, \ j = 1, 3, 4, \ i \neq j. \tag{25}$$

Consequently, it only becomes necessary to compute the direct transfer functions, $\widetilde{T}^D_{ii}$, to obtain the new matrix $\widetilde{\mathbf{T}}^D$. Remember that $\widetilde{T}^D_{ii} = \widetilde{X}'_i/\widetilde{X}^{\text{ext}}_i$. Again, taking into account the T^D_{ii} definition it follows that $\widetilde{X}'_i = X'_i$ (this will be a crucial point). This is so because X'_i represents the displacement of mass m_i when m_i is excited and all remaining masses m_j, $\forall j \neq i$, are blocked, while $\widetilde{X}'_i$ represents the same for the new system where, in particular, m_2 is already blocked. Hence, both quantities are identical and it is only necessary to compute $\widetilde{X}^{\text{ext}}_i$ to obtain $\widetilde{T}^D_{ii}$. The first term, $\widetilde{X}^{\text{ext}}_1$, corresponding to $\widetilde{T}^D_{11}$ can be found solving the equations of motion of the reduced system

$$\begin{pmatrix} \overline{k}_1 - \omega^2 m_1 & -\overline{k}_{13} & 0 \\ -\overline{k}_{13} & \overline{k}_3 - \omega^2 m_3 & -\overline{k}_{34} \\ 0 & -\overline{k}_{34} & \overline{k}_4 - \omega^2 m_4 \end{pmatrix} \begin{pmatrix} X^{\text{ext}}_1 \\ X^{\text{ext}}_3 \\ X^{\text{ext}}_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{26}$$

Eq. (26) has the solution $\mathbf{X}^{\text{ext}} = (\widetilde{H}^{\text{SPR}}_{11}, \widetilde{H}^{\text{SPR}}_{21}, \widetilde{H}^{\text{SPR}}_{31})^\top$ but $\widetilde{H}^{\text{SPR}}_{11}$ (see Appendix A) is the only term we will need in this case. Following the procedure in Section 3.2 and taking into account the above considerations yields

$$\widetilde{T}^D_{11} = \frac{\widetilde{X}'_1}{\widetilde{X}^{\text{ext}}_1} = \frac{X_1 y'}{\widetilde{X}^{\text{ext}}_1} = \frac{1}{(\overline{k}_1 - \omega^2 m_1)\widetilde{H}^{\text{SPR}}_{11}}. \tag{27}$$

To compute the remaining direct transfer functions from the displacement of a mass to itself, Eq. (26) is solved with force vectors $\mathbf{F} = (0, 1, 0)^\top$ and $\mathbf{F} = (0, 0, 1)^\top$. Proceeding in an analogous way to what has been done for $\widetilde{T}^D_{11}$, the following expressions can be derived:

$$\widetilde{T}^D_{33} = \frac{\widetilde{X}'_3}{\widetilde{X}^{\rm ext}_3} = \frac{X'_3}{\widetilde{X}^{\rm ext}_3} = \frac{1}{(\overline{k}_3 - \omega^2 m_3)\widetilde{H}^{\rm SPR}_{22}}, \tag{28a}$$

$$\widetilde{T}^D_{44} = \frac{\widetilde{X}'_4}{\widetilde{X}^{\rm ext}_4} = \frac{X'_4}{\widetilde{X}^{\rm ext}_4} = \frac{1}{(\overline{k}_4 - \omega^2 m_4)\widetilde{H}^{\rm SPR}_{33}}. \tag{28b}$$

From Eqs. (25), (27), (28) and (A.12)–(A.15) all elements in the new system direct transfer matrix, $\widetilde{\mathbf{T}}^D$, are available.

Once having $\widetilde{\mathbf{T}}^D$, it is possible to compute the new global transfer function matrix, $\widetilde{\mathbf{T}}^G$, using the following equation corresponding to the inverse of the relations in Eq. (10) (see Refs. [12,13]),

$$\widetilde{\mathbf{T}}^G = -\widetilde{\mathbf{\Lambda}}^D(\text{dev}\,\widetilde{\mathbf{T}}^D - \mathbf{I})^{-1} \tag{29}$$

with $\mathbf{I}$ standing for the identity matrix.

Next, use can be made of the abovementioned fact that $\widetilde{X}'_i = X'_i$ for the direct transfer functions, T^D_{ii}. This is an important relation because it allows to obtain the external operational displacements of the new system in terms of the old ones

$$\widetilde{X}^{\rm op,ext}_i = \frac{T^D_{ii}}{\widetilde{T}^D_{ii}} X^{\rm op,ext}_i \qquad \forall i. \tag{30}$$

Having computed $\widetilde{\mathbf{X}}^{\rm op,ext}$, Eq. (20) can be reverted to obtain the operational displacement vector of the new system

$$\widetilde{\mathbf{X}}^{\rm op} = (\widetilde{\mathbf{T}}^G)^\top \widetilde{\mathbf{X}}^{\rm op,ext}. \tag{31}$$

Finally, from $\widetilde{\mathbf{T}}^D$, $\widetilde{\mathbf{X}}^{\rm op,ext}$ and $\widetilde{\mathbf{X}}^{\rm op}$, a factorisation identical to Eq. (19) can be performed for the new modified system

$$\widetilde{\mathbf{X}}^{\rm op} = (\text{dev}\,\widetilde{\mathbf{T}}^D)^\top \widetilde{\mathbf{X}}^{\rm op} + \widetilde{\mathbf{\Lambda}}^D \widetilde{\mathbf{X}}^{\rm op,ext}. \tag{32}$$

It has then been checked that when modifying the original system by means of a mass blocking, it is possible to obtain the same amount of information, Eqs. (31) and (32), for the new system than for the old one. This has been done without the necessity of making any new measurement. Moreover, it is clear from the above argumentation that this is a general result which can be applied to any mechanical system analysed by means of the GTDT approach (the key point lies on the fact that $\widetilde{X}'_i = X'_i$). Whenever modifications involve blocking dof from the original system, full predictability can be achieved if one is able to compute the new system direct transfer matrix.

5.2. Removing springs and dampers

The possibility of making $T^D_{21}X^{\rm op}_2 = 0$ by blocking the elemental path from m_1 to m_4 is next considered. This case corresponds to setting $T^D_{21} = 0$. It follows from Eq. (14b) that it suffices to take $\overline{k}_{12} = 0$ ($k_{12} = c_{12} = 0$) to do so. Observe that this corresponds to the SF experimental disconnect option in Ref. [1], see Fig. 3.

The new direct transfer function matrix, $\widehat{\mathbf{T}}^D$, can be easily found in this case by setting ($\overline{k}_{12} = 0$) in Eqs. (13) and (14)

$$\widehat{\mathbf{T}}^D = \mathbf{T}^D(\overline{k}_{12} = 0), \tag{33}$$

where use has been made of a wide hat to distinguish this matrix from the blocked mass one, $\widetilde{\mathbf{T}}^D$ (tilde), and from the original one, $\mathbf{T}^D$. Note that according to the direct transfer function definition, when setting $T^D_{21} = 0$ it could happen that some direct transfer functions involving m_1 or m_2 become modified. However, direct transfer functions involving masses different from m_1 or m_2 will remain unaltered. Observe from Eq. (13) that while $\widehat{T}^D_{34} = T^D_{34}$ and $\widehat{T}^D_{43} = T^D_{43}$, as expected, some other functions have changed $\widehat{T}^D_{31} \neq T^D_{31}$ and $\widehat{T}^D_{42} \neq T^D_{42}$.

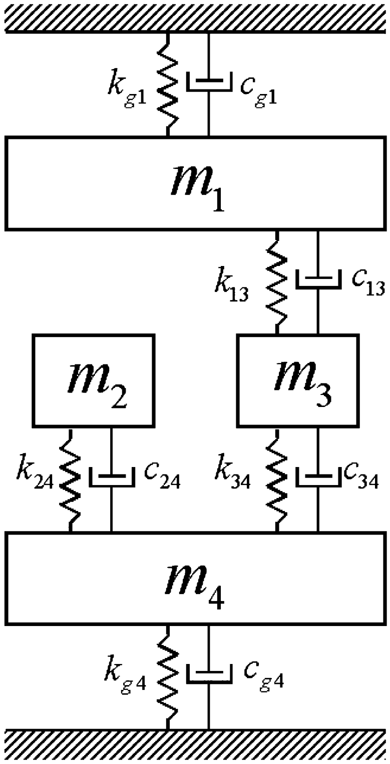

Fig. 3. Blocking of paths: removing spring k_{1S} and damper c_{1S}.

In what concerns direct transfer functions, T^D_{ii}, they all become modified since the response of the whole system is involved in their denominator.

The procedure in the previous section shall be now followed. From $\widehat{\mathbf{T}}^D$, the new global transfer function matrix (Eq. (29)) can be computed

$$\widehat{\mathbf{T}}^G = -\widehat{\mathbf{\Lambda}}^D(\mathrm{dev}\,\widehat{\mathbf{T}}^D - \mathbf{I})^{-1}. \tag{34}$$

Next, one has to obtain the new external operational displacement vector, $\widehat{\mathbf{X}}^{\mathrm{op,ext}}$. Unfortunately this turns out to be more difficult in this situation than for the mass blocking case because now $\widehat{X}'_i \neq X'_i$. This is so because $\widehat{X}'_i$ and X'_i are measured with all remaining masses m_j, $j \neq i$, blocked, but in this case the $\widehat{X}'_i$ measurement is done with the configuration in Fig. 4a ($\overline{k}_{12} = 0$), while the X'_i measurement corresponds to Fig. 4b ($\overline{k}_{12} \neq 0$). However, there is still an option to retain full prediction power but at the price of using the diagonal terms of the dynamic compliance matrix. From Eq. (11) it follows that

$$\begin{aligned}[(\mathbf{T}^G)^\top]^{-1} &= [\mathbf{H}^{\mathrm{SPR}}\,\mathrm{diag}(1/H^{\mathrm{SPR}}_{11}, 1/H^{\mathrm{SPR}}_{22}, 1/H^{\mathrm{SPR}}_{33}, 1/H^{\mathrm{SPR}}_{44})]^{-1} \\ &= \mathrm{diag}(H^{\mathrm{SPR}}_{11}, H^{\mathrm{SPR}}_{22}, H^{\mathrm{SPR}}_{33}, H^{\mathrm{SPR}}_{44})[\mathbf{H}^{\mathrm{SPR}}]^{-1}\end{aligned} \tag{35}$$

and from Eq. (20) and taking into account that $\mathbf{X}^{\mathrm{op}} = \mathbf{H}^{\mathrm{SPR}}\mathbf{F}^{\mathrm{op}}$, the external operational displacement vector of the original system, $\mathbf{X}^{\mathrm{op,ext}}$, can be rewritten as

$$\mathbf{X}^{\mathrm{op,ext}} = \mathrm{diag}(H^{\mathrm{SPR}}_{11}, H^{\mathrm{SPR}}_{22}, H^{\mathrm{SPR}}_{33}, H^{\mathrm{SPR}}_{44})\mathbf{F}^{\mathrm{op}}. \tag{36}$$

Analogously, for the new modified system it follows that

$$\widehat{\mathbf{X}}^{\mathrm{op,ext}} = \mathrm{diag}(\widehat{H}^{\mathrm{SPR}}_{11}, \widehat{H}^{\mathrm{SPR}}_{22}, \widehat{H}^{\mathrm{SPR}}_{33}, \widehat{H}^{\mathrm{SPR}}_{44})\mathbf{F}^{\mathrm{op}}. \tag{37}$$

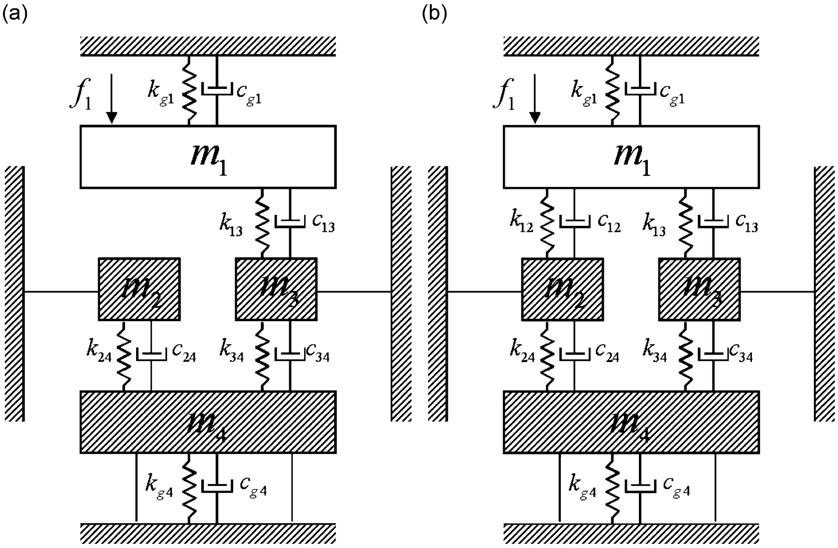

Fig. 4. (a) Configuration used to determine $\widehat{X}'_1$, (b) configuration used to determine X'_1.

Extracting $\mathbf{F}^{\mathrm{op}}$ from Eq. (36) and inserting it in Eq. (37) allows to obtain the external operational displacement vector of the new system in terms of the original one

$$\widehat{\mathbf{X}}^{\mathrm{op,ext}} = \mathrm{diag}\left(\frac{\widehat{H}_{11}^{\mathrm{SPR}}}{H_{11}^{\mathrm{SPR}}}, \frac{\widehat{H}_{22}^{\mathrm{SPR}}}{H_{22}^{\mathrm{SPR}}}, \frac{\widehat{H}_{33}^{\mathrm{SPR}}}{H_{33}^{\mathrm{SPR}}}, \frac{\widehat{H}_{44}^{\mathrm{SPR}}}{H_{44}^{\mathrm{SPR}}}\right)\mathbf{X}^{\mathrm{op,ext}}. \tag{38}$$

Finally, the new operational displacement vector and its factorisation corresponding to Eqs. (31) and (32) can be found using Eq. (38)

$$\widehat{\mathbf{X}}^{\mathrm{op}} = (\widehat{\mathbf{T}}^{G})^{\top}\widehat{\mathbf{X}}^{\mathrm{op,ext}}, \tag{39}$$

$$\widehat{\mathbf{X}}^{\mathrm{op}} = (\mathrm{dev}\widehat{\mathbf{T}}^{D})^{\top}\widehat{\mathbf{X}}^{\mathrm{op}} + \widehat{\mathbf{\Lambda}}^{D}\widehat{\mathbf{X}}^{\mathrm{op,ext}}. \tag{40}$$

Hence, it has been shown that it is possible to retain full power prediction in this case but at the price of having to obtain the diagonal terms of the system compliance matrix. This may still be an advantage when comparing with the force TPA approach because no operational forces are to be measured and it is not necessary to obtain the whole compliance matrix. On the other hand, the TPA approach can handle modifications in a more easier way because it directly works with the external forces acting on the system.

5.3. Discussion

Whether to perform in practice a force TPA analysis or a GTDT factorisation may not only depend on the type of desired results, but on many other factors such as the demounting possibilities of the analysed machinery, the available time for measurements, etc. In any case, once the mechanical system under study has been characterised by either one approach or the other, and a problem detected, the most difficult task probably consists in computing the new modified system dynamic compliance matrix and the new direct transfer matrix.

In the previous section, it has been shown how the latter can be done for the simple analytic 4 dof discrete system, but in practical industrial cases things may be much more involved. However, let us perform some rather speculative comments on the implementation of the GTDT approach in these situations, given that occasionally, some simplifying hypotheses could be made leading to acceptable results. This could be the case, specially when working in the mid-high frequency range. For instance, in the airplane cabin example referred throughout the paper, it can be assumed that all cabin panels are independently mounted on a quite rigid structure. Changing the physical properties of one of the panels (e.g., a window) may result in a considerable modification of the direct transfer function between the window vibration and the interior acoustic pressure. However, these changes may affect to a much lesser extent the direct transfer functions among the window vibration and its neighbour panels. Hence, it could be assumed that only one, or a few, direct transfer functions change in this situation. This has seemed to be the case for some tested applications [21], although these kind of approximations may strongly vary from one particular problem to another.

6. Numerical example

In what follows, a numerical experiment will be presented to elucidate some of the previously presented concepts. The mass, stiffness and damping values used in the numerical computations are given in Table 1. Throughout the simulations it has been considered that a constant external operational force is acting on m_4 i.e., $\mathbf{F}^{\mathrm{op}} = (0, 0, 0, 10)^{\top}$. Results will be presented for the frequency range [0 5] Hz.

In Fig. 5, the values $20\log_{10}(|T_{ij}^{G,D}|)$ have been plotted for some global and direct transfer functions corresponding to the original mechanical system in Fig. 1. In Fig. 5a, T_{11}^{G} and T_{11}^{D} are shown. Logically $20\log_{10}(T_{11}^{G}) = 0$ because $T_{11}^{G} = 1$, while T_{11}^{D} is a completely different shaped function. Fig. 5b contains the transfer functions from m_2 to m_3. As these masses are not directly connected, $T_{23}^{D} = 0$ and its logarithm is at $-\infty$. On the contrary, T_{23}^{G} is a well-defined function. In Figs. 5c and d plots are given for $20\log_{10}(T_{13}^{G,D})$ and $20\log_{10}(T_{42}^{G,D})$, which correspond to directly connected masses. Concerning the location of peaks and dips in the global and direct transfer functions see the explanation in Ref. [12].

In Fig. 6, 10 times the logarithms of the squared moduli of various displacement factorisations according to Eq. (19) are provided. Two clearly distinguishable zones can be detected in the displacement spectra of all subplots: a first zone approximately ranging from 0 to 2 Hz, where the resonant behaviour of the system dominates, and a second non-resonant zone beyond 2 Hz. The displacement, X_1^{op}, of mass m_1 is shown in Fig. 6a. As there is no force acting on m_1 and this mass is not directly connected to m_4, there are only contributions from masses m_2, $T_{21}^{D}X_2^{\mathrm{op}}$, and m_3, $T_{31}^{D}X_3^{\mathrm{op}}$. In the resonant zone both contributions alternate and have a similar degree of influence to the overall X_1^{op} displacement. Note also that the moduli of these contributions can surpass the overall displacement modulus, which is given by the modulus of their displacement coherent summation (the system relative phases have not been plotted for the sake of brevity). On the other hand, it can be observed that X_1^{op} is mainly produced by the contribution of mass m_2 in the non-resonant zone.

A similar analysis can be performed for the displacements of masses m_2, m_3 and m_4. In Fig. 6b it can be seen, as expected, that only masses m_1 and m_4 contribute to the overall displacement, X_2^{op}, of mass m_2. In the non-resonant zone, the displacement is wholly due to the contribution of mass m_4 (remind that m_4 is the mass where the external operational force is being applied). In Fig. 6c a very close situation is observed. The displacement, X_3^{op}, of m_3 has only contributions from masses m_1 and m_4 and in the non-resonant zone the displacement level is plenty produced by the sole contribution of m_4. Finally, in Fig. 6d we present the results for mass m_4 that will have contributions from m_2, m_3 and from the external force acting on it. It can be observed that the latter, $X_4^{\prime\mathrm{op}}$, fully accounts for the displacement level in the non-resonant frequency range.

The possibility of blocking some paths in order to diminish the displacement of mass m_1 is next considered. As commented and seen from Fig. 6a, X_1^{op} is almost wholly produced by the vibration of m_2 in the

Table 1
Mass, stiffness and damping values used in the numerical example.

Mass	(kg)	Stiffness	($\mathrm{N\,m^{-1}}$)	Damping	($\mathrm{kg\,s^{-1}}$)
m_1	2.5	k_{g1}	30	c_{g1}	0.01
m_2	1.5	k_{12}	50	c_{12}	0.01
m_3	1.1	k_{24}	50	c_{24}	0.01
m_4	2.5	k_{13}	25	c_{13}	0.1
		k_{34}	15	c_{34}	0.1
		k_{g4}	23	c_{g4}	0.01

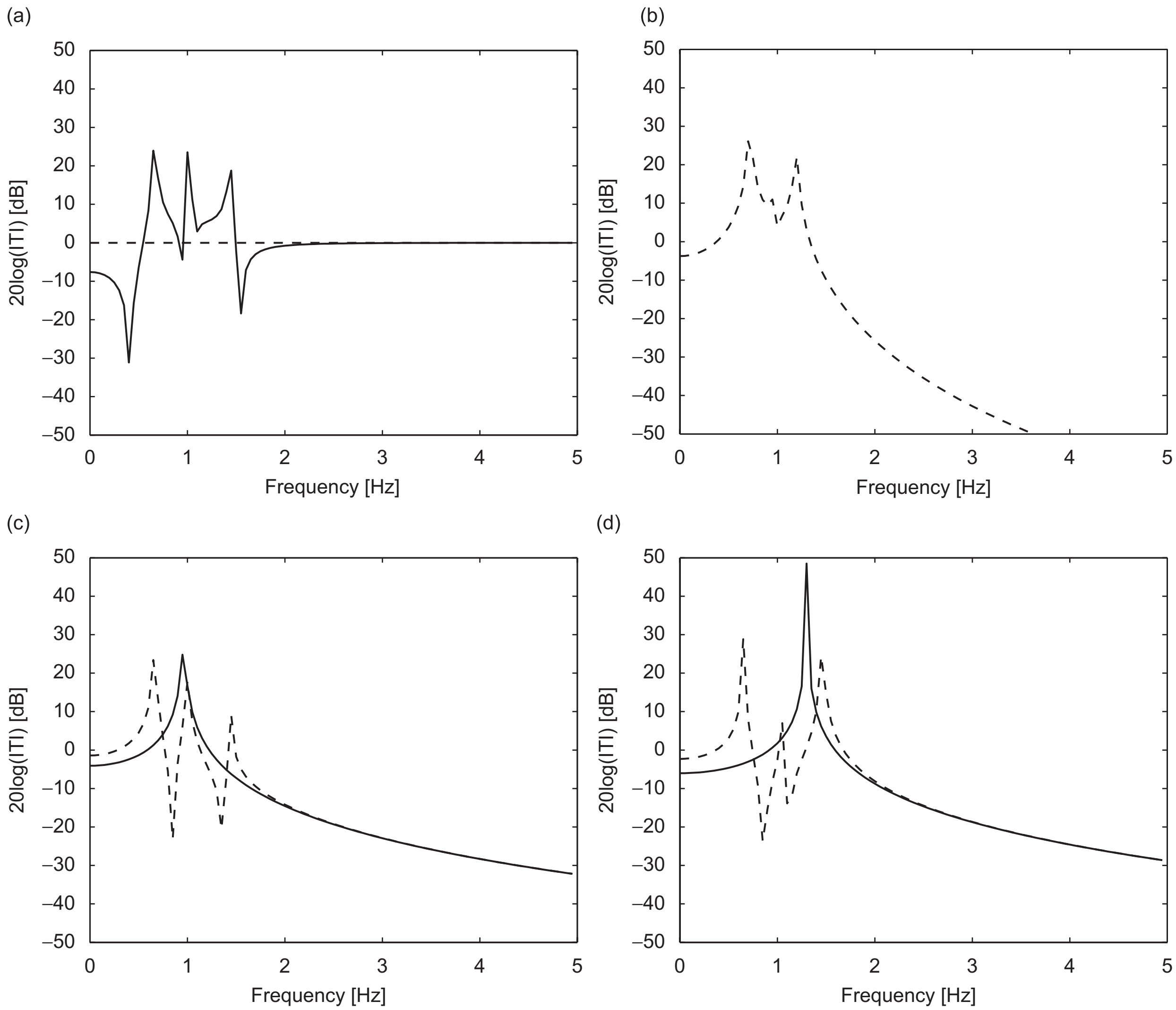

Fig. 5. (a) T_{11}^{D} continuous, T_{11}^{G} dashed, (b) T_{23}^{D} continuous, T_{23}^{G} dashed, (c) T_{13}^{D} continuous, T_{13}^{G} dashed, (d) T_{42}^{D} continuous, T_{42}^{G} dashed.

non-resonant range. Hence, the two possibilities of making $T_{21}^{D}X_{2}^{\mathrm{op}}=0$ analysed in Section 5 have been simulated. First, the results of performing a mass blocking by setting to zero the displacement of mass m_2 ($X_2^{\mathrm{op}}=0$) will be presented and, second, the results of removing the spring k_{12} and damper c_{12} ($T_{21}^{D}=0$) will be shown.

The results of the factorisation for the mass blocked modified system, Eq. (32), are given in Fig. 7. Remember from Section 5.1 that this situation corresponds to a 3 dof system because $X_2^{\mathrm{op}}=0$ (see Fig. 2). In Fig. 7a the operational displacement of mass m_1 is plotted. Taking into account that there is no external force acting on this mass and that m_1 is only directly connected to m_3, it follows that the m_1 displacement will be entirely due to the contribution of m_3, i.e., $\widetilde{X}_1^{\mathrm{op}}=\widetilde{T}_{31}^{D}\widetilde{X}_3^{\mathrm{op}}$. On the other hand, in Fig. 7b the results for the operational displacement of mass m_3, $\widetilde{X}_3^{\mathrm{op}}$ are shown. This displacement will have contributions from m_1 and m_4 although it can be observed that the latter fully produces the m_3 displacement in the non-resonant frequency range. In Fig. 7c the results for m_4 are provided. In this case the contributions are from the operational external force acting on m_4 and from mass m_3. The former, $\widetilde{X}_4^{\prime\mathrm{op}}$, wholly accounts for the displacement level in the non-resonant zone.

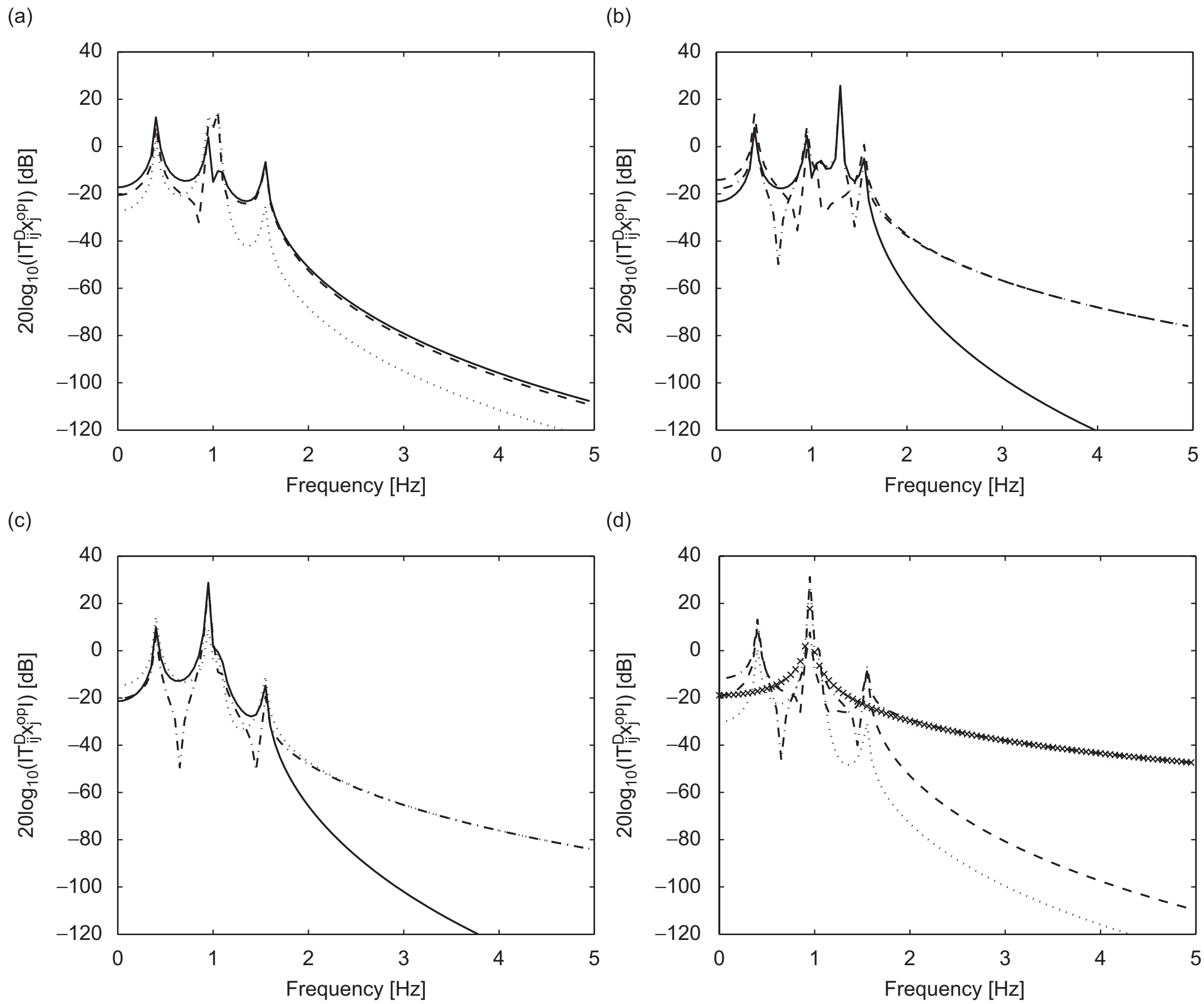

Fig. 6. Original system. (a) X_1^{op} continuous, $T_{21}^D X_2^{\mathrm{op}}$ dashed, $T_{31}^D X_3^{\mathrm{op}}$ dotted; (b) X_2^{op} dashed, $T_{12}^D X_1^{\mathrm{op}}$ continuous, $T_{42}^D X_4^{\mathrm{op}}$ dash-dotted; (c) X_3^{op} dotted, $T_{13}^D X_1^{\mathrm{op}}$ continuous, $T_{43}^D X_4^{\mathrm{op}}$ dash-dotted; (d) X_4^{op} dash-dotted, $T_{24}^D X_2^{\mathrm{op}}$ dashed, $T_{34}^D X_3^{\mathrm{op}}$ dotted, $X_4^{\prime\mathrm{op}}$ ×-shape.

The results of the factorisation when spring k_{12} and damper c_{12} are removed, Eq. (40), are presented in Fig. 8. The results for the operational displacement of mass m_1 can be seen in Fig. 8a. Given that this mass is now only connected to m_3 (see Fig. 3) it follows that $\widehat{X}_1^{\mathrm{op}} = \widehat{T}_{31}^D \widehat{X}_3^{\mathrm{op}}$. Similarly, mass m_2 is only connected to mass m_4 so it will happen that $\widehat{X}_2^{\mathrm{op}} = \widehat{T}_{42}^D \widehat{X}_4^{\mathrm{op}}$ (see Fig. 8b). The displacement of mass m_3 involves a contribution from m_1 and another one from m_4. Again, the latter is fully dominant for the non-resonant frequency range. In Fig. 8d, the operational displacement of mass m_4, $\widehat{X}_4^{\mathrm{op}}$, is given. In this case, contributions arise from masses m_2, m_3 and from the external operational force acting on m_4, $\widehat{X}_4^{\prime\mathrm{op}}$. The later clearly produces the whole displacement level in the non-resonant zone.

Finally, in Fig. 9 a comparison of the various mass operational displacements for the three analysed mechanical systems is presented: the original one, the new mass blocked system and the new system with removed elements. Observing Fig. 9a it can be realised that the objective of reducing the original m_1 operational displacement, X_1^{op}, has been achieved in the non-resonant frequency range. Both methods of path disconnecting yield almost identical displacement reductions at this zone. On the contrary, the situation becomes clearly different in the resonant domain. This is so because the modified systems have different

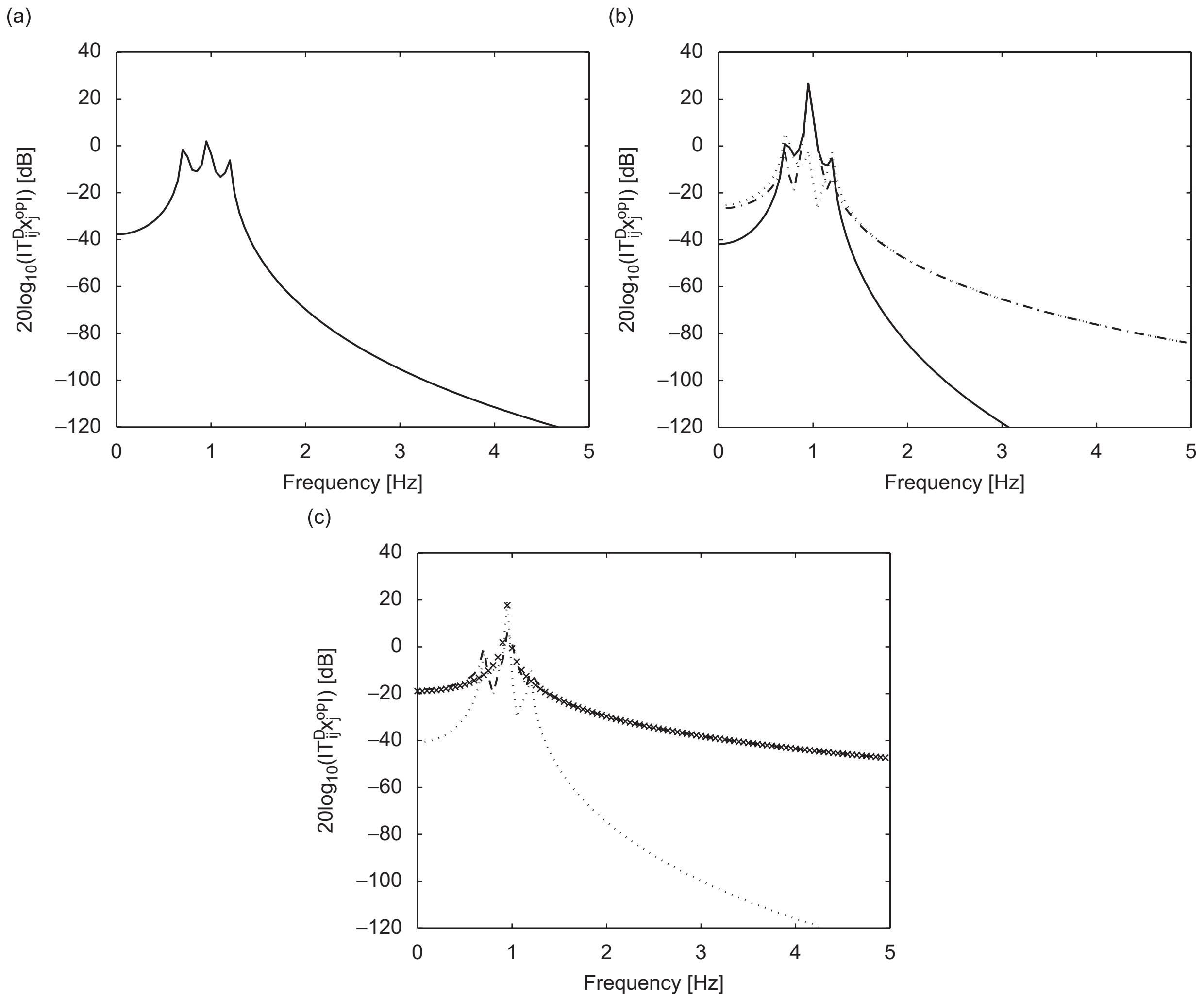

Fig. 7. Blocked mass system. (a) $\widetilde{X}_1^{\mathrm{op}} = \widetilde{T}^D_{31}\widetilde{X}_3^{\mathrm{op}}$ continuous; (b) $\widetilde{X}_3^{\mathrm{op}}$ dotted, $\widetilde{T}^D_{13}\widetilde{X}_1^{\mathrm{op}}$ continuous, $\widetilde{T}^D_{43}\widetilde{X}_4^{\mathrm{op}}$ dash-dotted; (c) $\widetilde{X}_4^{\mathrm{op}}$ dash-dotted, $\widetilde{T}^D_{34}\widetilde{X}_3^{\mathrm{op}}$ dotted, $\widetilde{X}_4'^{\mathrm{op}}$ $\times$-shape.

boundary conditions and hence different eigenfrequencies. Consequently, at some frequencies $\widetilde{X}_1^{\mathrm{op}}$ and $\widehat{X}_1^{\mathrm{op}}$ become lower than the original one, X_1^{op}, but at other frequencies the opposite occurs.

In Fig. 9b the results for mass m_2 have been plotted. This mass has been the one fixed in the new blocked mass system so $\widetilde{X}_2^{\mathrm{op}} = 0$. In what concerns the new system with removed elements, its displacement $\widehat{X}_2^{\mathrm{op}}$ differs from the displacement X_2^{op} of the original system in the resonant frequency range, for the reasons explained above, but remains almost unaltered in the non-resonant domain. The results in Fig. 9c correspond to mass m_3 and show similar behaviour. Both, $\widehat{X}_3^{\mathrm{op}}$ and $\widetilde{X}_3^{\mathrm{op}}$, do not present significant differences from X_3^{op} in the non-resonant frequency range. The same holds true for the displacement of the mass m_4 (see Fig. 9d), which again is unaffected by the system modifications in the non-resonant domain.

In summary, it has been shown that either by using the mass blocking technique or by removing some system elements, it is possible to reduce the displacement level of m_1 in the non-resonant frequency range. Moreover, this does not result in a significant variation of the displacements of the masses m_2, m_3 and m_4. In what concerns the resonant frequency range no general tendency can be clearly inferred. However, the above type of analysis could be used to eliminate annoying resonant behaviour at particular frequencies and to predict where the new ones, corresponding to the modified system, will appear.

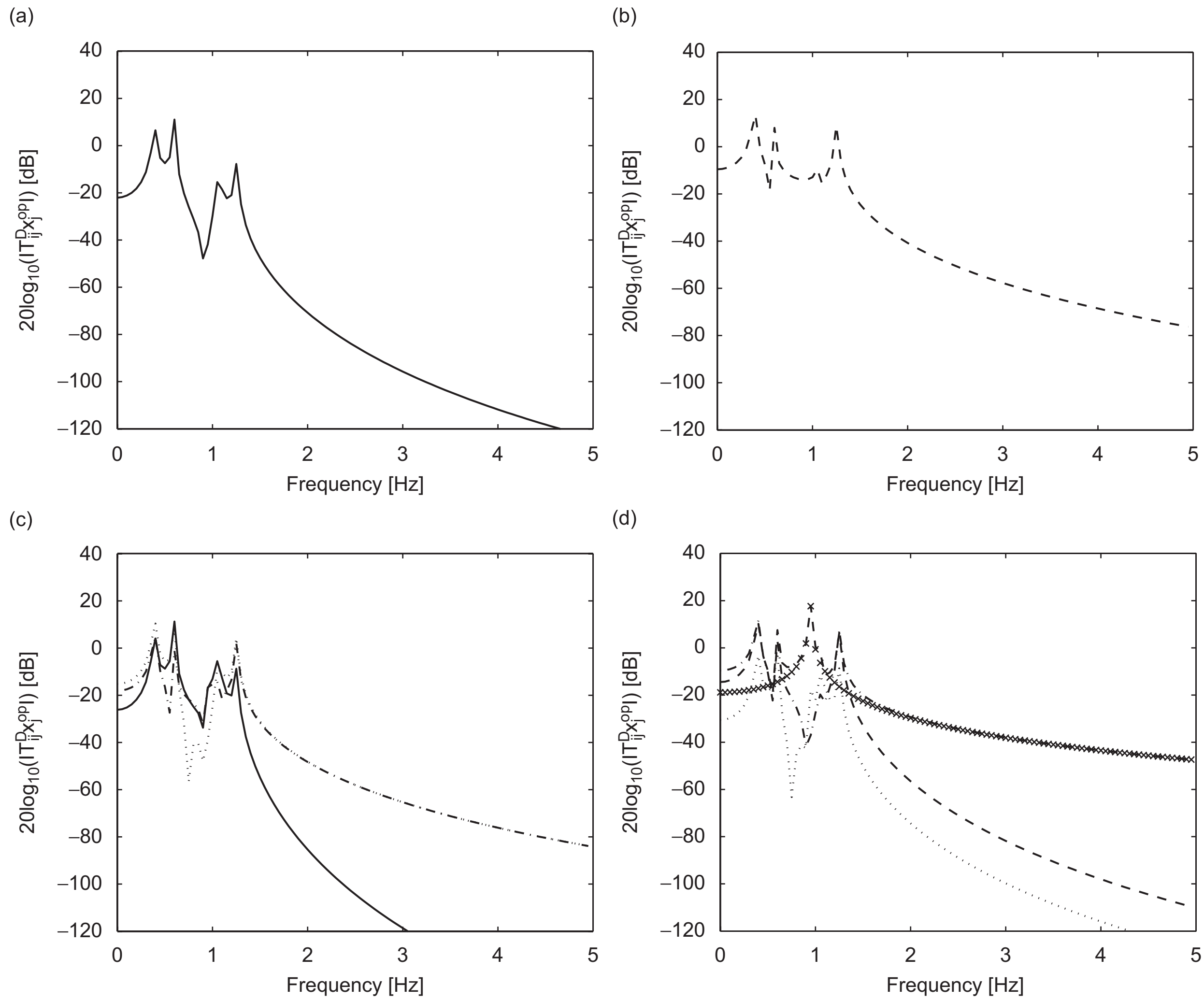

Fig. 8. Removed complex stiffness system. (a) $\widehat{X}_1^{\mathrm{op}} = \widehat{T}^D_{31}\widehat{X}_3^{\mathrm{op}}$ continuous; (b) $\widehat{X}_2^{\mathrm{op}} = \widehat{T}^D_{42}\widehat{X}_4^{\mathrm{op}}$ dashed; (c) $\widehat{X}_3^{\mathrm{op}}$ dotted, $\widehat{T}^D_{13}\widehat{X}_1^{\mathrm{op}}$ continuous, $\widehat{T}^D_{43}\widehat{X}_4^{\mathrm{op}}$ dash-dotted; (d) $\widehat{X}_4^{\mathrm{op}}$ dash-dotted, $\widehat{T}^D_{24}\widehat{X}_2^{\mathrm{op}}$ dashed, $\widehat{T}^D_{34}\widehat{X}_3^{\mathrm{op}}$ dotted, $\widehat{X}_4^{\prime\mathrm{op}}$ $\times$-shape.

7. Conclusions

In this paper use has been made of a simple mechanical discrete system made of springs, dampers and masses to analytically show some features of the global transfer direct transfer (GTDT) method of transmission path analysis (TPA). It has been shown that the displacement of any system mass can be factorised in terms of the displacements of the remaining masses plus the displacement due to the external force acting on it. This factorisation among responses can be performed using very easily measurable quantities, such as the displacement transfer functions among masses and the displacement operational vector. None of this quantities needs any artificial mass blocking or removing any part of the system to be measured. The GTDT approach may then constitute a non-intrusive alternative to covering techniques requiring moreover, much less experimental effort.

The prediction capabilities of the method when blocking some transmission paths in the original mechanical system have been also analysed. It has been shown that in some cases the GTDT approach retains full power prediction but in others some extra measurements may be needed. Although it requires less experimental work than the force TPA approach, the GTDT method postprocess is generally slightly more involved than the

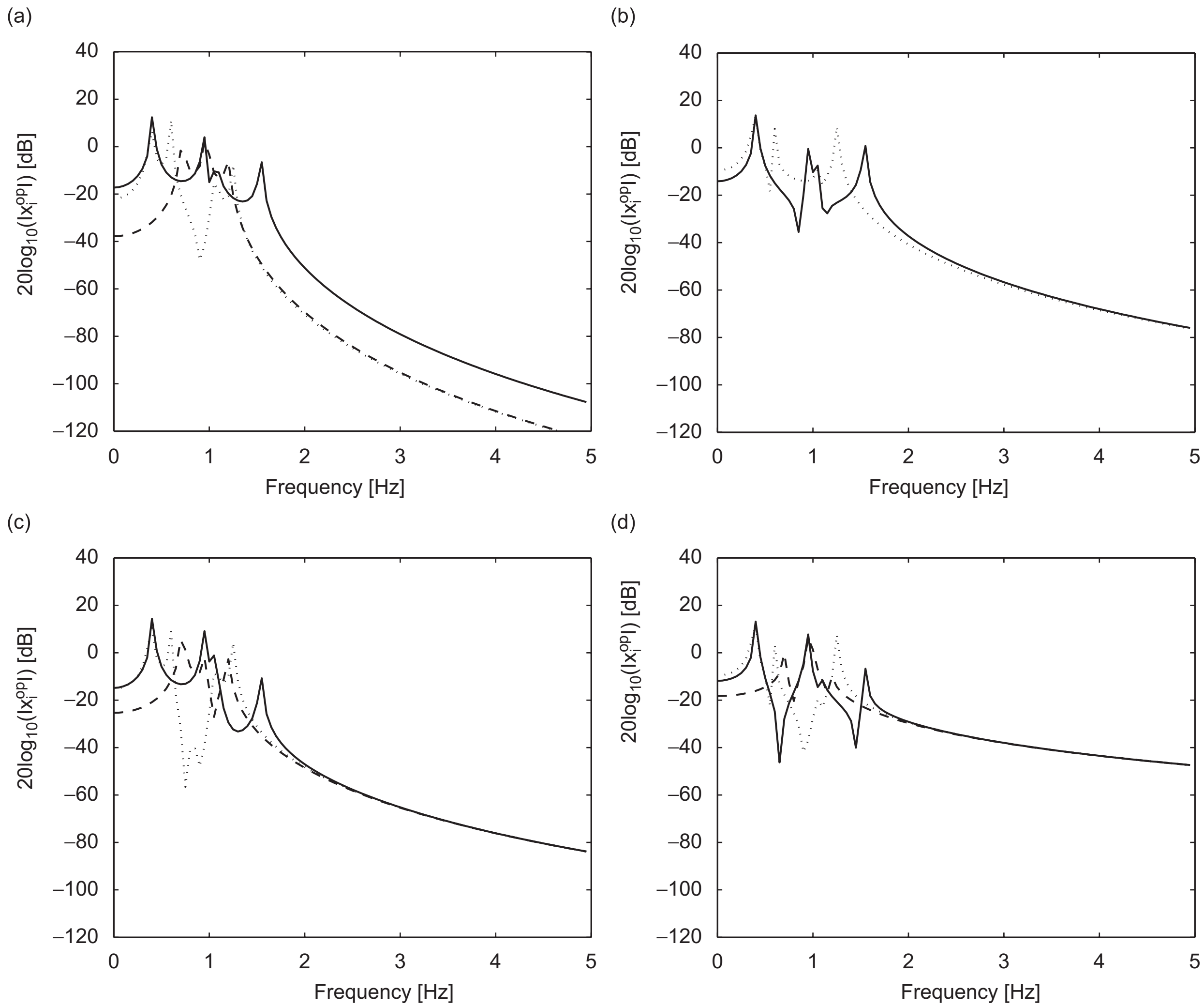

Fig. 9. Comparison among mechanical systems. (a) X_1^{op} continuous, $\widetilde{X}_1^{op}$ dashed, $\widehat{X}_1^{op}$ dotted; (b) X_2^{op} continuous, $\widetilde{X}_2^{op}$ dashed, $\widehat{X}_2^{op}$ dotted; (c) X_3^{op} continuous, $\widetilde{X}_3^{op}$ dashed, $\widehat{X}_3^{op}$ dotted; (d) X_4^{op} continuous, $\widetilde{X}_4^{op}$ dashed, $\widehat{X}_4^{op}$ dotted.

force TPA one. Despite the simplicity of the analysed 4 dof mechanical system, some general predictive tendencies have been inferred. Further cases involving continuum systems as well as practical industrial applications of the method are currently being explored.

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Appendix A. Dynamic compliance matrix

The elements of the dynamic compliance matrix, H_{ij}^{SPR}, are next given. Note that the dynamic stiffness matrix, $\mathbf{Z}^{SPR}$, is symmetric so $\mathbf{H}^{SPR} = (\mathbf{Z}^{SPR})^{-1}$ will be also symmetric.

The $\mathbf{H}^{\mathrm{SPR}}$ diagonal elements are

$$H_{11}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^6 m_2 m_3 m_4 + \omega^4[m_4(m_2\overline{k}_3 + m_3\overline{k}_2) + m_2 m_3 \overline{k}_4] + \omega^2[m_2(\overline{k}_{34}^2 - \overline{k}_3\overline{k}_4) + m_3(\overline{k}_{24}^2 - \overline{k}_2\overline{k}_4) - m_4\overline{k}_2\overline{k}_3] + \overline{k}_2\overline{k}_3\overline{k}_4 - \overline{k}_2\overline{k}_{34}^2 - \overline{k}_3\overline{k}_{24}^2, \quad (A.1)$$

$$H_{22}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^6 m_1 m_3 m_4 + \omega^4[m_3(m_4\overline{k}_1 + m_1\overline{k}_4) + m_1 m_4 \overline{k}_3] + \omega^2[m_1(\overline{k}_{34}^2 - \overline{k}_3\overline{k}_4) + m_4(\overline{k}_{13}^2 - \overline{k}_3\overline{k}_1) - m_3\overline{k}_4\overline{k}_1] + \overline{k}_3\overline{k}_1\overline{k}_4 - \overline{k}_1\overline{k}_{34}^2 - \overline{k}_4\overline{k}_{13}^2, \quad (A.2)$$

$$H_{33}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^6 m_1 m_2 m_4 + \omega^4[m_2(m_4\overline{k}_1 + m_1\overline{k}_4) + m_1 m_4 \overline{k}_2] + \omega^2[m_1(\overline{k}_{24}^2 - \overline{k}_2\overline{k}_4) + m_4(\overline{k}_{12}^2 - \overline{k}_2\overline{k}_1) - m_2\overline{k}_4\overline{k}_1] + \overline{k}_2\overline{k}_1\overline{k}_4 - \overline{k}_1\overline{k}_{24}^2 - \overline{k}_4\overline{k}_{12}^2, \quad (A.3)$$

$$H_{44}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^6 m_2 m_3 m_1 + \omega^4[m_1(m_2\overline{k}_3 + m_3\overline{k}_2) + m_2 m_3 \overline{k}_1] + \omega^2[m_2(\overline{k}_{13}^2 - \overline{k}_3\overline{k}_1) + m_3(\overline{k}_{12}^2 - \overline{k}_2\overline{k}_1) - m_1\overline{k}_2\overline{k}_3] + \overline{k}_2\overline{k}_3\overline{k}_1 - \overline{k}_2\overline{k}_{13}^2 - \overline{k}_3\overline{k}_{12}^2, \quad (A.4)$$

while the off-diagonal terms are

$$H_{12}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{21}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = \omega^4 m_3 m_4 \overline{k}_{12} - \omega^2 \overline{k}_{12}(m_3\overline{k}_4 + m_4\overline{k}_3) + \overline{k}_{12}(\overline{k}_3\overline{k}_4 - \overline{k}_{34}^2) + \overline{k}_{13}\overline{k}_{24}\overline{k}_{34}, \quad (A.5)$$

$$H_{13}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{31}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = \omega^4 m_2 m_4 \overline{k}_{13} - \omega^2 \overline{k}_{13}(m_2\overline{k}_4 + m_4\overline{k}_2) + \overline{k}_{13}(\overline{k}_2\overline{k}_4 - \overline{k}_{24}^2) + \overline{k}_{12}\overline{k}_{24}\overline{k}_{34}, \quad (A.6)$$

$$H_{14}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{41}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^2(m_2\overline{k}_{13}\overline{k}_{34} + m_3\overline{k}_{12}\overline{k}_{24}) + \overline{k}_2\overline{k}_{13}\overline{k}_{34} + \overline{k}_3\overline{k}_{12}\overline{k}_{34}, \quad (A.7)$$

$$H_{23}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{32}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = -\omega^2(m_1\overline{k}_{24}\overline{k}_{34} + m_4\overline{k}_{12}\overline{k}_{13}) + \overline{k}_1\overline{k}_{24}\overline{k}_{34} + \overline{k}_4\overline{k}_{12}\overline{k}_{13}, \quad (A.8)$$

$$H_{24}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{42}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = \omega^4 m_1 m_3 \overline{k}_{24} - \omega^2 \overline{k}_{24}(m_1\overline{k}_3 + m_3\overline{k}_1) + \overline{k}_{24}(\overline{k}_3\overline{k}_1 - \overline{k}_{13}^2) + \overline{k}_{12}\overline{k}_{13}\overline{k}_{34}, \quad (A.9)$$

$$H_{34}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = H_{43}^{\mathrm{SPR}} \det \mathbf{Z}^{\mathrm{SPR}} = \omega^4 m_1 m_2 \overline{k}_{34} - \omega^2 \overline{k}_{34}(m_1\overline{k}_2 + m_2\overline{k}_1) + \overline{k}_{34}(\overline{k}_2\overline{k}_1 - \overline{k}_{12}^2) + \overline{k}_{12}\overline{k}_{13}\overline{k}_{24} \quad (A.10)$$

and the determinant, $\det \mathbf{Z}^{\mathrm{SPR}}$, is given by

$$\det \mathbf{Z}^{\mathrm{SPR}} = \omega^8 m_1 m_2 m_3 m_4 - \omega^6(m_2 m_3 m_4 \overline{k}_1 + m_2 m_3 m_1 \overline{k}_4 + m_1 m_2 m_4 \overline{k}_3 + m_1 m_3 m_4 \overline{k}_2) + \omega^4[m_1 m_2(\overline{k}_3\overline{k}_4 - \overline{k}_{34}^2) + m_1 m_3(\overline{k}_2\overline{k}_4 - \overline{k}_{24}^2) + m_2 m_4(\overline{k}_3\overline{k}_1 - \overline{k}_{13}^2) + m_3 m_4(\overline{k}_2\overline{k}_1 - \overline{k}_{12}^2) + m_1 m_4 \overline{k}_2\overline{k}_3 + m_2 m_3 \overline{k}_1\overline{k}_4] + \omega^2[m_1(\overline{k}_2\overline{k}_{34}^2 + \overline{k}_3\overline{k}_{24}^2 - \overline{k}_2\overline{k}_3\overline{k}_4) + m_2(\overline{k}_1\overline{k}_{34}^2 + \overline{k}_4\overline{k}_{13}^2 - \overline{k}_3\overline{k}_1\overline{k}_4) + m_3(\overline{k}_1\overline{k}_{24}^2 + \overline{k}_4\overline{k}_{12}^2 - \overline{k}_2\overline{k}_1\overline{k}_4) + m_4(\overline{k}_2\overline{k}_{13}^2 + \overline{k}_3\overline{k}_{12}^2 - \overline{k}_2\overline{k}_3\overline{k}_1)] + (\overline{k}_{12}\overline{k}_{34} - \overline{k}_{13}\overline{k}_{24})^2 + \overline{k}_1\overline{k}_4\overline{k}_2\overline{k}_3 - \overline{k}_1(\overline{k}_2\overline{k}_{34}^2 + \overline{k}_3\overline{k}_{24}^2) - \overline{k}_4(\overline{k}_2\overline{k}_{13}^2 + \overline{k}_3\overline{k}_{12}^2). \quad (A.11)$$

The terms $\widetilde{H}_{ii}^{\mathrm{SPR}}$ appearing in the direct transfer functions of the modified system in Eqs. (25), (27) and (28) are given by

$$\widetilde{H}_{11}^{\mathrm{SPR}} \det \widetilde{\mathbf{Z}}^{\mathrm{SPR}} = \omega^4 m_3 m_4 - \omega^2(m_3\overline{k}_4 + m_4\overline{k}_3) + \overline{k}_3\overline{k}_4 - \overline{k}_{34}^2, \quad (A.12)$$

$$\widetilde{H}_{22}^{\mathrm{SPR}} \det \widetilde{\mathbf{Z}}^{\mathrm{SPR}} = (\omega^2 m_1 - \overline{k}_1)(\omega^2 m_4 - \overline{k}_4), \quad (A.13)$$

$$\widetilde{H}_{33}^{\mathrm{SPR}} \det \widetilde{\mathbf{Z}}^{\mathrm{SPR}} = \omega^4 m_1 m_3 - \omega^2(m_1\overline{k}_3 + m_3\overline{k}_1) + \overline{k}_3\overline{k}_1 - \overline{k}_{13}^2 \quad (A.14)$$

with

$$\begin{aligned}\det \widetilde{\mathbf{Z}}^{\mathrm{SPR}} = & -\omega^6 m_1 m_3 m_4 + \omega^4[m_1(m_3\overline{k}_4 + m_4\overline{k}_3) + m_3 m_4 \overline{k}_1] + \omega^2[m_1(\overline{k}_{34}^2 - \overline{k}_3\overline{k}_4) + m_4(\overline{k}_{13}^2 - \overline{k}_3\overline{k}_1) - m_3\overline{k}_1\overline{k}_4] \\ & + \overline{k}_1\overline{k}_3\overline{k}_4 - \overline{k}_4\overline{k}_{13}^2 - \overline{k}_1\overline{k}_{34}^2. \end{aligned} \tag{A.15}$$

Appendix B. Connectivity and transmission paths

An *elemental* or *direct* path between two masses can be identified with its direct link. Path, P_{12}, is characterised by T_{12}^D while path P_{34} is characterised by T_{34}^D. There is no elemental path connecting e.g., m_4 and m_1 or m_2 and m_3. These masses are connected through the union of various elemental paths, which can be termed as *composed* or *global* paths. For instance, m_4 and m_1 can be linked by $P_{12} \cup P_{24}$, $P_{42} \cup P_{21}$, $P_{13} \cup P_{34}$ or $P_{43} \cup P_{34}$.

From the above path definition it follows that different elemental transmission paths do not necessarily correspond to disjoint sets of physical entities. For example, path P_{21} is characterised by T_{21}^D, whose expression, Eq. (14b), involves the masses, springs and dampers m_1, $\overline{k}_{12}$, $\overline{k}_{13}$ and $\overline{k}_{g1}$. These physical elements are also shared by the path P_{31}, characterised by T_{31}^D. Consequently, the presented path definition may be viewed as a rather intricate way to define transmission paths as one may think that is by far more logical to identify them with sets of physical elements, as done in common practice (see e.g., Ref. [1]). In fact, this may be the case for finite discrete systems such as the simple 4 dof analysed system. However, it is also possible to perform a path analysis between several dof in a single continuous mechanical system instead of a discrete one (e.g., displacement and rotation among various points in a beam [12], acoustic pressure at various points in free field space [13], etc.). In such cases, transmission paths can no longer be identified with physical entities, while the above definition of transmission paths keeps full sense.

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