Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 322 (2009) 29–38
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Hybrid wave/mode active control of bending vibrations in beams based on the advanced Timoshenko theory

C. Mei*

Department of Mechanical Engineering, The University of Michigan-Dearborn, 4901 Evergreen Road, Dearborn, MI 48128, USA

Received 28 June 2007; received in revised form 22 August 2008; accepted 6 November 2008
Handling Editor: S. Bolton
Available online 24 December 2008

Abstract

A hybrid approach to active control of bending vibrations in beams based on the advanced Timoshenko theory is described in this paper. It combines elements of both wave and mode approaches to active control and is an attempt to improve on the performance of these approaches individually. As is well known that the classical Euler–Bernoulli beam model considers only the lateral inertia and the elastic forces caused by bending deflections, and the effects of rotary inertia and shear distortion are neglected. As a result, the theory is not valid for higher frequencies, typically when the transverse dimensions are not negligible with respect to the wavelength. In the proposed hybrid approach based on the advanced Timoshenko model, wave control is first applied at one or more points in the structure. It is designed on the basis of the local behavior of the structure and is intended to either absorb vibrational energy or add damping, especially at higher frequencies. Then modal control is applied, being designed on the basis of the modified global equations of motion of the structure-plus-wave-controller. Because the higher order modes are relatively well damped, hybrid control improves the model accuracy and the robustness of the system and gives better broadband vibration attenuation performance. Numerical results are presented.

1. Introduction

Due to demands for mechanical structures to be lighter and faster, there has been increasing interests in active vibration control in recent years. In this paper, a hybrid approach consisting of complementary wave- and mode-based control is described. The design is based on the advanced Timoshenko theory.

Vibrations can be described in a number of ways, with the most common descriptions being in terms of modes and in terms of wave motion. From modal standpoint, the response of a structure is the summation of the modes of that structure [1]. The contribution from each mode depends on the global properties of the structure, namely, the material, dimension, and boundary conditions. The vibrations of elastic structures such as strings, beams, and plates can also be described in terms of waves propagating and attenuating in waveguides. Such waves are reflected and transmitted when incident upon discontinuities [2–4]. Correspondingly there are two

*Tel.: +1 313 593 5369; fax: +1 313 593 3851.
E-mail address: cmei@umich.edu

doi:10.1016/j.jsv.2008.11.003

main approaches to active vibration control in which the control is designed in terms of either modal or wave behavior.

In modal active vibration control, the aim is to control the characteristics of the modes of vibration [5–7], i.e., their damping factors, natural frequencies or mode shapes. Active wave control aims to control the distribution of energy in the structure by either reducing the transmission of waves from one part of the structure to another (i.e., isolating one part of the structure from another) or by absorbing the energy carried by the waves (i.e., adding damping). Active wave vibration control is most often feedforward. In which the disturbance is detected, and a control force applied somewhere downstream to produce a destructive signal to cancel the incoming wave or to absorb the energy associated with it. Feedforward wave control has been applied in controlling both single wave type [8–11] and multiple wave types [12–15] of structural waves. Wave control can also be feedback [16–18].

Both modal and wave active control have advantages and disadvantages. The advantages of modal methods include the generality of the approach and its global nature; the disadvantages include the modal uncertainties at high frequencies, and observation and control spillover due to unavoidable modal truncation in real time implementation. There is a trade-off between robustness and model accuracy with accurate modeling of the structure being essential to successful control system design. Wave designs are based on the local properties of the structure and are inherently much less sensitive to system properties and, therefore, more robust than global models of structures, especially at higher frequencies. However, it does not consider global motion: the global behavior can adversely affect the amount of control achieved.

A hybrid approach was developed for actively controlling bending vibrations in an Euler–Bernoulli beam in [19,20] for improving the performance of modal and wave control approaches individually. However, at higher frequencies (typically when the transverse dimensions are not negligible with respect to the wavelength), the effects of rotary inertia and shear distortion, which are neglected in the Euler–Bernoulli beam model, must be taken into account. Rayleigh [21] introduced the effect of rotatory inertia and Timoshenko [22,23] extended it to include the effect of transverse shear deformation.

In this paper, a hybrid controller is designed to control bending vibrations in beams based on the advanced Timoshenko theory. In the proposed hybrid approach, wave control is first designed and is targeted at higher frequencies. Two control strategies are presented, one is to optimally absorb the vibrational energy, and the other is to add optimal damping to the structure. Modal control is then designed for the lower modes of the structure based on the modified equations of motion of the structure-plus-wave-controller. Because the higher order modes are now well controlled, the hybrid control improves the model accuracy and the robustness of the system, and hence gives better broadband vibration attenuation performance. Numerical results are presented.

2. Hybrid active vibration control

In the absence of damping, the equations of motion of a Timoshenko beam are [2]

$$GA\kappa\left[\frac{\partial\psi(x,t)}{\partial x}-\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=f(x,t), \tag{1a}$$

$$EI\frac{\partial^2\psi(x,t)}{\partial x^2}+GA\kappa\left[\frac{\partial y(x,t)}{\partial x}-\psi(x,t)\right]-\rho I\frac{\partial^2\psi(x,t)}{\partial t^2}=0, \tag{1b}$$

where x is the position along the beam axis, t the time, $y(x,t)$ the transverse deflection of the center line of the beam, $\psi(x,t)$ the slope due to bending, I the area moment of inertia of cross section, A the cross-sectional area, and E, G and ρ are the Young's modulus, shear modulus and mass density, respectively. $f(x,t)$ is the externally applied force. It can be seen that Eqs. (1a) and (1b) are coupled through the slope and the transverse deflection of the structure.

The equations of motion corresponding to free vibrations can be written as [7]

$$L(\mathbf{W}_n)=\omega_n^2 M(\mathbf{W}_n), \tag{2}$$

where ω_n are the natural frequencies and

$$L=\begin{bmatrix} -\kappa AG\dfrac{\partial^2}{\partial x^2} & \kappa AG\dfrac{\partial}{\partial x} \\ -\kappa AG\dfrac{\partial}{\partial x} & -EI\dfrac{\partial^2}{\partial x^2}+\kappa AG \end{bmatrix},\quad M=\begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix} \text{ and } \mathbf{W}_n=\begin{bmatrix} Y_n \\ \Psi_n \end{bmatrix}.$$

It was found that the operators L and M are self-adjoint for classical boundary conditions (that is, clamped, free, and simply-supported boundaries) [24]. The self-adjointness of a system has an important implication, that is, if a system is self-adjoint, then the eigenfunctions are orthogonal [7]. The orthogonal condition is as follows [24]:

$$\int_0^L \mathbf{W}_n^{\mathrm{T}} M \mathbf{W}_m \,\mathrm{d}x=\begin{cases} 0 & \text{for } m=n, \\ \text{A non-zero constant} & \text{for } m\neq n. \end{cases} \tag{3}$$

The responses can be expressed as a sum of an infinite number of modal components as

$$y(x,t)=\sum_{n=1}^{\infty} Y_n(x)q_n(t) \quad \text{and} \quad \psi(x,t)=\sum_{n=1}^{\infty} \Psi_n(x)q_n(t), \tag{4}$$

where $Y_n(x)$ and $\Psi_n(x)$ are mode shapes and $q_n(t)$ $(i=1,2,\ldots,\infty)$ modal coordinates of the system. The equations of motion of the forced vibrations can then be written in terms of the modal coordinates as

$$\ddot{q}_n(t)+\omega_n^2 q_n(t)=f_n(t),\quad f_n(t)=\int f(x,t)Y_n(x)\,\mathrm{d}x,\quad n=1,2,\ldots, \tag{5}$$

where $f_n(t)$ is the nth modal force.

Assuming time harmonic motion, with time dependence suppressed, the solutions to Eq. (2) have been found as [25]

$$\begin{bmatrix} Y_n(x) \\ \psi_n(x) \end{bmatrix}=\begin{bmatrix} C_1^n\,\mathrm{e}^{-\mathrm{i}k_1^n x}+C_2^n\,\mathrm{e}^{\mathrm{i}k_1^n x}+C_3^n\,\mathrm{e}^{-k_2^n x}+C_4^n\,\mathrm{e}^{k_2^n x} \\ -\mathrm{i}P_nC_1^n\,\mathrm{e}^{-\mathrm{i}k_1^n x}+\mathrm{i}P_nC_2^n\,\mathrm{e}^{\mathrm{i}k_1^n x}-N_nC_3^n\,\mathrm{e}^{-k_2^n x}+N_nC_4^n\,\mathrm{e}^{k_2^n x} \end{bmatrix}, \tag{6}$$

where

$$P_n=k_1^n\left(1-\frac{\omega_n^2}{(k_1^n)^2C_s^2}\right),\quad N_n=k_2^n\left(1+\frac{\omega_n^2}{(k_2^n)^2C_s^2}\right),$$

$$k_1^n=\left\{\frac{1}{2}\left[\left(\frac{1}{C_s}\right)^2+\left(\frac{C_r}{C_b}\right)^2\right]\omega_n^2+\sqrt{\frac{\omega_n^2}{C_b^2}+\frac{1}{4}\left[\left(\frac{1}{C_s}\right)^2-\left(\frac{C_r}{C_b}\right)^2\right]^2\omega_n^4}\right\}^{1/2},$$

$$k_2^n=\left\{\left|\frac{1}{2}\left[\left(\frac{1}{C_s}\right)^2+\left(\frac{C_r}{C_b}\right)^2\right]\omega_n^2-\sqrt{\frac{\omega_n^2}{C_b^2}+\frac{1}{4}\left[\left(\frac{1}{C_s}\right)^2-\left(\frac{C_r}{C_b}\right)^2\right]^2\omega_n^4}\right|\right\}^{1/2},$$

$$C_b=\sqrt{\frac{EI}{\rho A}},\quad C_s=\sqrt{\frac{GA\kappa}{\rho A}},\quad C_r=\sqrt{\frac{\rho I}{\rho A}}.$$

The nth mode dependant coefficients C_i^n $(i=1,2,3,4)$ can be found with boundary conditions given. For example, for a cantilever beam, the boundary conditions are

(1) At $x=0$ (clamped end)

$$y(0,t)=0 \quad \text{and} \quad \psi(0,t)=0; \tag{7}$$

(2) At $x = L$ (free end)

$$GA\kappa\left[\frac{\partial y(x,t)}{\partial x} - \psi(x,t)\right]_{x=L} = 0 \quad \text{and} \quad EI\frac{\partial\psi(x,t)}{\partial x}\bigg|_{x=L} = 0. \tag{8}$$

Eq. (7) gives

$$C_1^n + C_2^n + C_3^n + C_4^n = 0,$$
$$-\mathrm{i}P_n C_1^n + \mathrm{i}P_n C_2^n - N_n C_3^n + N_n C_4^n = 0. \tag{9}$$

Eq. (8) gives

$$-\mathrm{i}C_1^n(k_1^n - P_n)\,\mathrm{e}^{-\mathrm{i}k_1^n L} + \mathrm{i}C_2^n(k_1^n - P_n)\,\mathrm{e}^{\mathrm{i}k_1^n L} - C_3^n(k_2^n - N_n)\,\mathrm{e}^{-k_2^n L} + C_4^n(k_2^n - N_n)\,\mathrm{e}^{k_2^n L} = 0,$$
$$-k_1^n P_n C_1^n\,\mathrm{e}^{-\mathrm{i}k_1^n L} - k_1^n P_n C_2^n\,\mathrm{e}^{\mathrm{i}k_1^n L} + N_n k_2^n C_3^n\,\mathrm{e}^{-k_2^n L} + N_n k_2^n C_4^n\,\mathrm{e}^{k_2^n L} = 0. \tag{10}$$

Eqs. (9) and (10) can be rewritten in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -\mathrm{i}P_n & \mathrm{i}P_n & -N_n & N_n \\ -\mathrm{i}(k_1^n - P_n)\,\mathrm{e}^{-\mathrm{i}k_1^n L} & \mathrm{i}(k_1^n - P_n)\,\mathrm{e}^{\mathrm{i}k_1^n L} & -(k_2^n - N_n)\,\mathrm{e}^{-k_2^n L} & (k_2^n - N_n)\,\mathrm{e}^{k_2^n L} \\ -k_1^n P_n\mathrm{e}^{-\mathrm{i}k_1^n L} & -k_1^n P_n\,\mathrm{e}^{\mathrm{i}k_1^n L} & N_n k_2^n\,\mathrm{e}^{-k_2^n L} & N_n k_2^n\,\mathrm{e}^{k_2^n L} \end{bmatrix} \begin{bmatrix} C_1^n \\ C_2^n \\ C_3^n \\ C_4^n \end{bmatrix} = 0. \tag{11}$$

It can be seen that there exist infinite number sets of C_i^n ($i = 1,2,3,4$) that satisfy Eq. (11). Deleting one row in Eq. (11) allows the rest of the coefficients to be solved in terms of one chosen coefficient. For example, deleting the first row and solving the coefficients in terms of C_1^n, one has

$$\begin{bmatrix} C_2^n \\ C_3^n \\ C_4^n \end{bmatrix} = \begin{bmatrix} \mathrm{i}P_n & -N_n & N_n \\ \mathrm{i}(k_1^n - P_n)\,\mathrm{e}^{\mathrm{i}k_1^n L} & -(k_2^n - N_n)\,\mathrm{e}^{-k_2^n L} & (k_2^n - N_n)\,\mathrm{e}^{k_2^n L} \\ -k_1^n P_n\,\mathrm{e}^{\mathrm{i}k_1^n L} & N_n k_2^n\,\mathrm{e}^{-k_2^n L} & N_n k_2^n\,\mathrm{e}^{k_2^n L} \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{i}P_n \\ \mathrm{i}(k_1^n - P_n)\,\mathrm{e}^{-\mathrm{i}k_1^n L} \\ k_1^n P_n\,\mathrm{e}^{-\mathrm{i}k_1^n L} \end{bmatrix} C_1^n. \tag{12}$$

C_1^n is chosen in such a way that it allows the mode shape to be mass normalized, that is, to satisfy

$$\int_0^L (\rho A\,Y_m(x)Y_n(x) + \rho I\psi_m(x)\psi_n(x))\,\mathrm{d}x = \delta_{mn}, \quad m,n = 1,2,3,\ldots, \tag{13}$$

where δ_{mn} is the Dirac delta function.

The control design is described as follows. It shall be pointed out that in principle, there is no restriction on which wave and modal control design approaches are used. In this paper, however, the particular case considered is that of collocated, point force/sensor feedback wave control combined with modal control designed using pole placement.

2.1. Wave control

The collocated feedback control is applied at some point along a structural element as shown in Fig. 1. In the frequency domain, the wave control force is written as

$$F(\omega) = -H_y(\omega)Y(\omega). \tag{14}$$

The control is dynamically identical to spring attachments with dynamic translational and rotational stiffnesses K_T and K_R, which are normally frequency dependent and complex. This gives rise to reflected and transmitted propagating waves as shown in Fig. 1. The amplitudes of these waves are given in terms of reflection and transmission coefficients r and t, where

$$\mathbf{t} = \mathbf{I} + \mu\mathbf{c} + \eta\mathbf{d}, \quad \mathbf{r} = \mu\mathbf{c} - \eta\mathbf{d}, \tag{15}$$

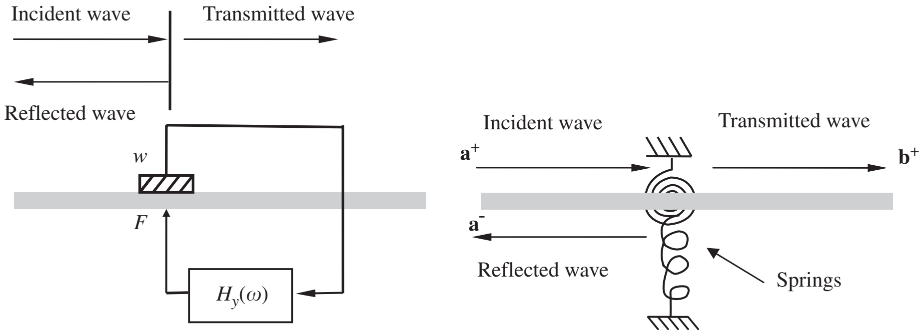

Fig. 1. Collocated feedback control.

where **I** is the identity matrix and

$$\mathbf{c} = \begin{bmatrix} \mathrm{i}N & \mathrm{i}N \\ P & P \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} -\mathrm{i}P & -N \\ \mathrm{i}P & N \end{bmatrix},$$

$$\mu = \frac{K_T GA\kappa}{2(-Pk_2 + Nk_1) + (-P - \mathrm{i}N)K_T GA\kappa}, \quad \eta = \frac{K_R EI}{2(-Pk_1 - Nk_2) + (-N + \mathrm{i}P)K_R EI}.$$

The controller can be designed in a number of ways as mentioned earlier. Two example design strategies are presented, one is to add optimal damping to the structure, and the other is to optimally absorb the vibrational energy.

When designed to add optimal damping to the structure, the controller in frequency domain is in the form of

$$H_y(\omega) = \mathrm{i}\omega C(\omega). \tag{16a}$$

The objective of the control is to absorb as much of the incident energy as possible through adding optimal damping. The optimal control gain is found by assuming a wave incident on one side of the control location and then by designing the control gain $C(\omega)$ so as to maximize the absorbed incoming energy, in other words, to minimize the power reflected and transmitted per unit incident power $|r|^2 + |t|^2$. The optimal control gain is obtained through self-written scripts using Matlab Symbolic Math Toolbox, and it is found as $C(\omega) = 2GA\kappa(-k_1N + k_2P)/(\omega\sqrt{P^2 + N^2})$.

When designed to optimally absorb vibration energy, in frequency domain, the controller is in the form of

$$H_y(\omega) = C_1(\omega) + \mathrm{i}\omega C_2(\omega). \tag{16b}$$

Following a similar procedure as described above, the optimal energy absorbing control gains are found to be $C_1(\omega) = 2GA\kappa P(k_1N - k_2P)/(P^2 + N^2)$ and $C_2(\omega) = 2GA\kappa N(k_1N - k_2P)/(\omega(P^2 + N^2))$.

In both cases, the design criteria used is to minimize the reflected and transmitted vibrational energy, by assuming a single propagating wave is incident on the control location and neglecting near-field waves.

The optimal controllers of Eqs. (16a) and (16b) are non-causal. Hence, a causal controller that approximates the ideal controller must be found for real time implementation. A causal controller can be obtained using a digital FIR (finite impulse response) controller through truncating the non-causal part of the controller; it can also be obtained by tuning the optimal controller to be optimum at some targeted frequency. Both approaches are illustrated in details in the next section.

2.2. Modal control

Modal control is now implemented by wrapping state-space control around the wave controlled structure. In general, an approximation for the wave control is required. Two approximation approaches are used in this paper, one is to use FIR filter, and the other is to tune to controller to be optimal at a certain frequency.

2.2.1. FIR approximation to optimal damping wave controller

First, let us consider modal control design of the structure modified by the optimal damping wave controller. In this case, a causal controller is obtained using a digital FIR controller through truncating the non-causal part of the controller. To enable modal control design using the modified equation of motion, a polynomial in ω is used to approximate the controller $\hat{H}_y(\omega)$ in the low frequency region, where modal control is to be most effective. The real and imaginary parts are approximated over this frequency range using a weighted least-squares procedure as

$$\hat{H}_y(\omega) = \hat{H}_{\mathrm{real}} + \hat{H}_{\mathrm{imaginary}}, \quad \hat{H}_{\mathrm{real}} = a + b\omega^2, \quad \hat{H}_{\mathrm{imaginary}} = c\omega. \tag{17a}$$

The polynomials are chosen to be second-order or lower so that the order of the system is not changed. For a point wave control force at x_c, and a controller defined by Eq. (17a), it follows that the wave control force is approximated in this low frequency region by

$$\hat{f}_c(x,t) = -[aw(x,t) - b\ddot{w}(x,t) + c\dot{w}(x,t)]\delta(x - x_c). \tag{18a}$$

The modal forces become

$$f_n(t) = \int f(x,t)Y_n(x)\,\mathrm{d}x + \int \hat{f}_c(x,t)Y_n(x)\,\mathrm{d}x, \quad n = 1,2,\ldots\,. \tag{19}$$

The equations of motion can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}, \tag{20}$$

where $\mathbf{f}(t) = [f_1(t)\ f_2(t)\ \ldots\ f_n(t)]^{\mathrm{T}}$ and $\mathbf{q}(t) = [q_1(t)\ q_2(t)\ldots q_n(t)]^{\mathrm{T}}$ are the external force and modal force vectors, and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping and stiffness matrices given by

$$M_{ij} = \delta_{ij} - b\phi_i(x_c)\phi_j(x_c), \quad C_{ij} = c\phi_i(x_c)\phi_j(x_c), \quad K_{ij} = \omega_i^2\delta_{ij} + a\phi_i(x_c)\phi_j(x_c), \tag{21a}$$

It is noted that the wave control force $\hat{f}_c(x,t)$ now couples the uncontrolled modes. Furthermore, Eq. (20) is normally non-self-adjoint, so that the new modes are complex.

The state vector $\mathbf{X}(t) = [\mathbf{q}^{\mathrm{T}}(t) : \dot{\mathbf{q}}^{\mathrm{T}}(t)]^{\mathrm{T}}$ is now introduced. The states involve the uncontrolled modal coordinates of the undamped structure. Eq. (20) is then rewritten in state-space form as

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{f}(t), \tag{22}$$

where the coefficient matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}. \tag{23}$$

Pole placement approach is adopted for the modal control design [7].

2.2.2. Tuned approximation to optimal energy absorbing wave controller

In tuned wave control the aim is to tune the controller so that it is equal to the optimal controller at some specific frequency ω_d. The controller is then

$$H_y(\omega_d) = C_1(\omega_d) + \mathrm{i}\omega C_2(\omega_d). \tag{17b}$$

In the time domain, this corresponds to proportional plus derivative (PD) feedback control (i.e., a combined spring–damper). This is of course a classical control methodology; the control gains, however, is calculated from a wave perspective to be optimal at some desired tuned frequency ω_d. For a point wave control force at x_c, and a controller defined by Eq. (17b), it follows that the wave control force is approximated in the low frequency region by

$$\hat{f}_c(x,t) = -C_1(\omega_d)w(x,t)\delta(x - x_c) - C_2(\omega_d)\dot{w}(x,t)\delta(x - x_c). \tag{18b}$$

Table 1
The first six natural frequencies of the system.

Mode number	1	2	3	4	5	6
Frequency (Hz)	4.46	27.76	77.76	152.31	251.71	375.91

Table 2
The locations of the disturbance, the controllers, and the response point.

	Disturbance	Wave controller	Modal controllers		Response point
Location	0.1L	0.2L	(1/6)L	(5/12)L	0.7L

The rest of the design follows the same procedures as described in Section 2.2.1, the only difference being the coefficients of Eq. (20), which are now given as

$$\mathbf{M} = \mathbf{I}, \quad C_{ij} = C_2(\omega_d)\phi_i(x_c)\phi_j(x_c), \quad C_{ij} = \omega_i^2\delta_{ij} + C_1(\omega_d)\phi_i(x_c)\phi_j(x_c). \tag{21b}$$

3. Numerical example

In this section, some numerical results are presented for a cantilever beam. The physical properties of the beam as well as the locations of excitation/control forces and sensors are chosen to be the same as that of [20]. The physical parameters are as follows: Young's modulus E is 190 GN/m^2, shear modulus G 77.5 GN/m^2, Poisson's ratio v 0.29, mass density ρ 7800 mg/m^3, and the width b, thickness h, and length L of the beam being 0.04, 0.002, and 0.6 m, respectively. Shear coefficient κ is related to Poisson's ratio v by $\kappa = 10(1+v)/(12+11v)$ [26]. The lowest six natural frequencies are given in Table 1. A disturbance is applied at one point, wave control is applied at a second point and modal control is applied at two further points as given in Table 2. The positions of these points are chosen so as to avoid the nodes of the first six modes. Numerical results show the response at a position $x = 0.7L$ per unit disturbance force. The controlled and uncontrolled frequency responses are compared for the cases of wave control alone, modal control alone, and hybrid wave/mode control. In the FIR approximated wave controller, the filter contains 20 terms. The choice of the FIR filter length is a compromise between the accuracy of the controller and the calculation time for the control output. In the tuned approximation of the wave controller, the controller is tuned to be optimal at its third natural frequency.

Fig. 2(a) shows the frequency responses of the structure after application of the optimal damping wave controller alone and Fig. 2(b) shows the frequency responses of the structure after application of the optimal energy absorbing wave controller alone. Without control, clear, sharp resonances can be observed. The wave controllers are seen to add damping to the structure, and the amount of damping varies from mode to mode. They also change the natural frequencies somewhat.

Modal control can be designed to control any modes provided the knowledge of the responses of those modes is adequate. Here the controller is designed to increase the damping factors of the first two modes of the structure to 0.5 by using pole placement approach. Fig. 3 shows the frequency responses before and after modal control. The lowest two modes are clearly well controlled, while the remainders are virtually unaffected. Sharp resonances associated with the uncontrolled modes, therefore, still exist. Including more modes in the control design can alleviate this problem, but only at the cost of increased model complexity.

Fig. 4(a) shows the frequency responses after the application of hybrid control with optimal damping wave control; and Fig. 4(b) shows the frequency responses after the application of hybrid control with optimal energy absorbing wave control. The hybrid controls clearly add substantial damping, giving improvement over modal control alone. They also give broadband control due to the energy-absorbing nature of the wave controllers, reducing the effects of higher resonances and spillover.

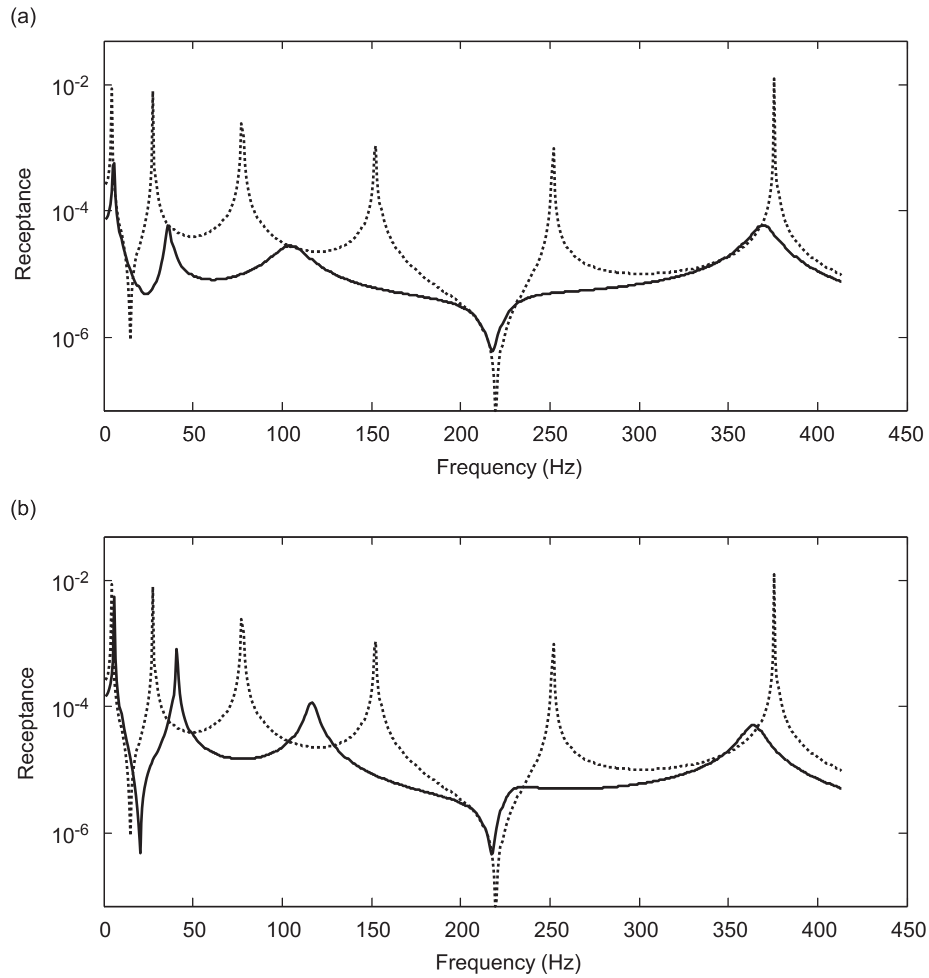

Fig. 2. Frequency responses before (…) and after (___) wave control: (a) optimal damper; (b) optimal energy absorber.

4. Concluding remarks

In this paper, a hybrid approach to active control of bending vibrations in beams based on the advanced Timoshenko theory is described. In the proposed approach, local, low-authority wave feedback control, designed on the basis of local wave propagation, is implemented first, the aim being to add optimal damping to the structure in one example, and to optimally absorb vibration energy in the structure in the second example. Since the wave controllers are generally non-causal, two strategies are presented in finding their causal approximations. After the implementation of the wave control, the equation of motion of the system is modified. Global, high-authority modal control is then designed, based on this modified equation, which aims to control the lowest few modes of the system. The hybrid approach exhibits better broadband active vibration control performance than the cases with either modal or wave control alone. There is a reduction in the order of the models required without there being a significant increase in the system representation associated uncertainty. Finally the effects of the unmodeled modes are reduced and the robustness is improved.

While the control design based on the classical Euler–Bernoulli model theory [20] is only applicable to slim beams, the present design based on the advanced Timoshenko model is suitable for deep as well as slim beam elements. Due to the coupling between transverse deflection and bending slope, the analysis and control design based on the current model is more involved. The design is verified by comparing the control performance of a slim beam to that obtained using the Euler–Bernoulli model, good agreement has been reached.

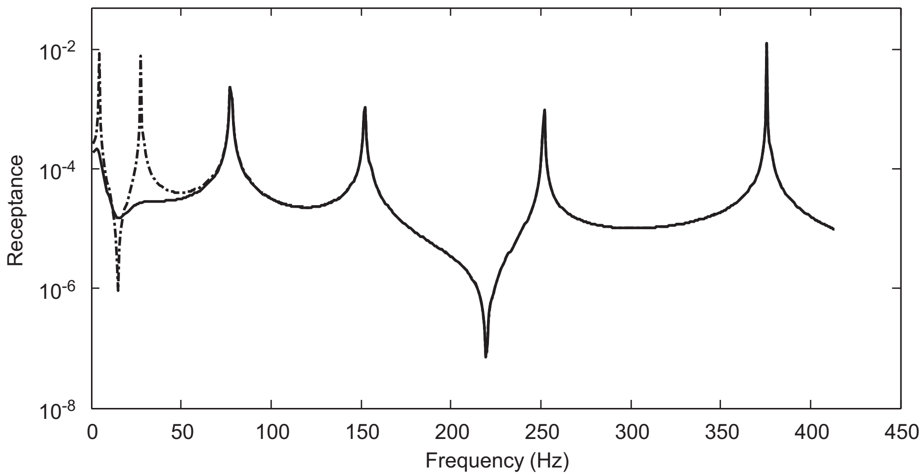

Fig. 3. Frequency responses before (…) and after (___) modal control.

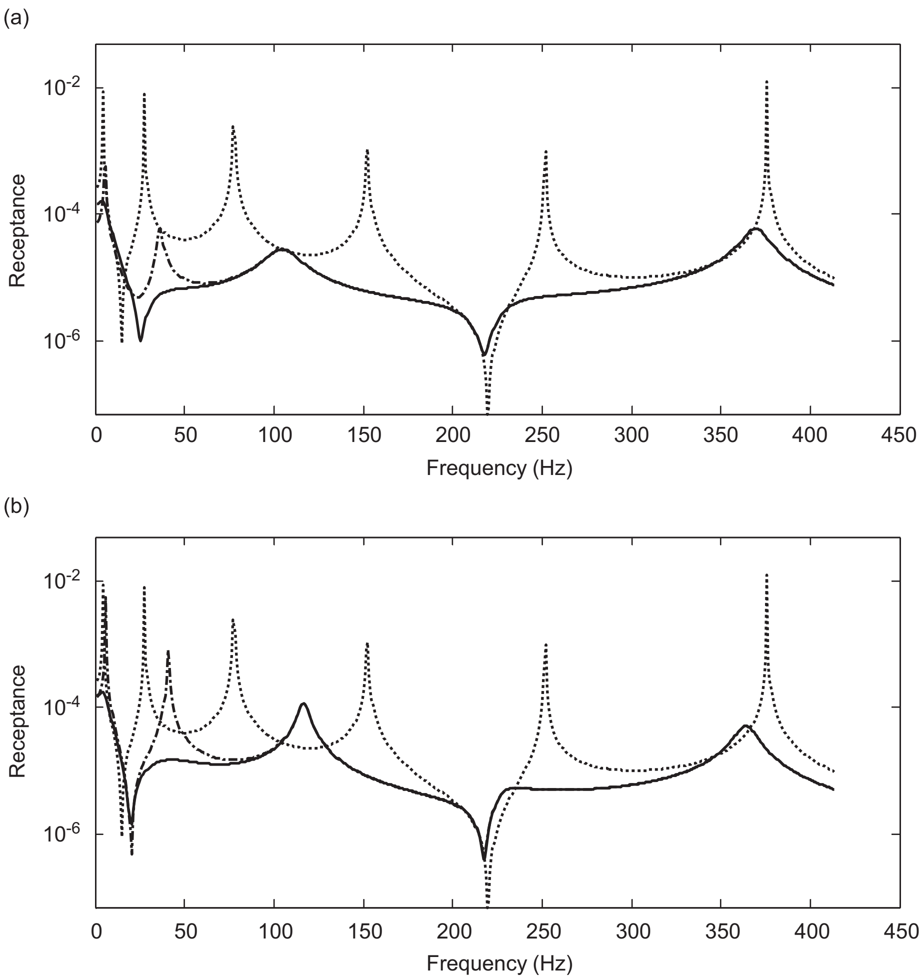

Fig. 4. Hybrid control with causal FIR wave controller; frequency responses before control (…); after wave control (-.-.-.); and after hybrid control (___): (a) with optimal damping wave control; (b) with optimal energy absorbing wave control.

References

[1] L. Meirovitch, *Fundamentals of Vibrations*, McGraw Hill Higher Education, New York, 2001.
[2] K.F. Graff, *Wave Motion in Elastic Solids*, Ohio State University Press, 1975.
[3] L. Cremer, M. Heckl, E.E. Ungar, *Structure-borne Sound*, Springer, Berlin, 1987.
[4] J.F. Doyle, *Wave Propagation in Structures*, Springer, New York, 1989.
[5] L. Meirovitch, H. Baruh, Optimal control of damped flexible gyroscopic systems, *Journal of Guidance and Control* 4 (2) (1981) 157–163.
[6] M.J. Balas, Feedback control of flexible systems, *IEEE Transactions on Automatic Control* AC-23 (4) (1978) 673–679.
[7] L. Meirovitch, *Dynamics and Control of Structures*, Wiley, New York, 1989.
[8] B.R. Mace, Active control of flexural vibrations, *Journal of Sound and Vibration* 114 (2) (1987) 253–270.
[9] J. Lu, M.J. Crocker, P.K. Raju, Active vibration control using wave control concepts, *Journal of Sound and Vibration* 134 (2) (1989) 364–368.
[10] S.J. Elliott, L. Billet, Adaptive control of flexural waves propagating in a beam, *Journal of Sound and Vibration* 163 (2) (1993) 295–310.
[11] J.S. Vipperman, R.A. Burdisso, C.R. Fuller, Active control of broadband structural vibration using the LMS adaptive algorithm, *Journal of Sound and Vibration* 166 (2) (1993) 283–299.
[12] C.R. Fuller, G.P. Gibbs, R.J. Silcox, Simultaneous active control of flexural and extensional waves in beams, *Journal of Intelligence, Material, System and Structure* 1 (1990) 235–247.
[13] R.L. Clark, J. Pan, C.H. Hansen, An experimental study of the active control of multiple-wave types in an elastic beam, *Journal of Acoustic Society of America* 92 (2 Pt. 1) (1992) 871–876.
[14] A.H. Von Flotow, Traveling wave control for large spacecraft structures, *Journal of Guidance* 9 (4) (1985) 462–468.
[15] P. Gardonio, S.J. Elliott, Active control of multiple waves propagating on a one-dimensional system with a scattering termination, *Active* 95 (1995) 115–126.
[16] B.R. Mace, R.W. Jones, Feedback control of flexural waves in beams, *Journal of Structural Control* 3 (1–2) (1996) 89–98.
[17] S.J. Elliott, M.J. Brennan, R.J. Pinnington, Feedback control of flexural waves on a beam, *Proceedings of Inter-noise 93*, Leuven, Belgium, 1993, pp. 843–846.
[18] M.J. Brennan, Active Control of Waves on One-dimensional Structure, PhD Thesis, University of Southampton, 1994.
[19] C. Mei, Hybrid Active Vibration Control of a Distributed Structure, PhD Thesis, University of Auckland, 1998.
[20] C. Mei, B.R. Mace, R.W. Jones, Hybrid wave/mode active vibration control, *Journal of Sound and Vibration* 247 (5) (2001) 765–784.
[21] L. Rayleigh, *Theory of Sound*, Macmillan, New York, 1926.
[22] S.P. Timoshenko, On the correction for shear of the differential equation for transverse vibrations of prismatic bars, *Philosophical Magazine, Series 6* 41 (1921) 744–746.
[23] S.P. Timoshenko, On the transverse vibrations of bars of uniform cross sections, *Philosophical Magazine, Series 6* 43 (1922) 125–131.
[24] S.M. Han, H. Benaroya, T. Wei, Dynamics of transversely vibrating beams using four engineering theories, *Journal of Sound and Vibration* 225 (5) (1999) 935–988.
[25] C. Mei, B.R. Mace, Wave reflection and transmission in Timoshenko beams and wave analysis of Timoshenko beam structures, *ASME Journal of Vibration and Acoustics* 127 (2005) 382–394.
[26] G.R. Cowper, The shear coefficient in Timoshenko's beam theory, *Journal of Applied Mechanics* (1966) 335–340.