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Vibration analysis of multiple-stepped beams with the composite element model

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Abstract

A new approach to analyze the free and forced vibrations of beams with multiple cross-section steps is proposed using a composite element method in this paper. The results are compared to receptance function method and classical Rayleigh–Ritz method and finite element results. The natural frequencies obtained from the composite element method are found to be in close agreement with other methods mentioned above and finite element method. The forced vibration responses of the stepped beam are also calculated from the composite element method and compared with those obtained from the finite element method. Time histories from both methods are found to match very well. This indicates the correctness of the proposed method for vibration analyses of the stepped beam. The proposed method can be extended easily to deal with beams consisting of an arbitrary number of nonuniform segments.

1. Introduction

Beams are fundamental models for the structural elements of many engineering applications and have been studied extensively. There are many examples of structures that may be modeled with beam-like elements, for instance, long span bridges [1], tall buildings [2], and robot arms [3].

The vibration of Euler–Bernoulli beams with one step change in cross-section has been well studied. Jang and Bert [4,5] derived the frequency equations for combinations of classical end supports as fourth-order determinants equated to zero. Taleb and Suppiger [6] derived the frequency equation of a stepped beam with simple support using the Cauchy iteration method. Balasubramanian and Subramanian [7,8] investigated the performance of a four-degree-of-freedom per node element in the vibration analysis of a stepped cantilever. Popplewell and Daqing Chang [9] used the "force mode functions" to improve the global Rayleigh–Ritz method of convergence by introducing discontinuities into the second and third derivatives of the assumed deflection. Krishnan et al. [10] discussed the difficulties of a finite difference analysis for a pinned–pinned stepped beam. De Rosa [11] studied the vibration of a stepped beam with elastic end supports. Recently,

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Koplow et al. [12] presented closed form solutions for the dynamic response of Euler–Bernoulli beams with step changes in cross-section.

There are also some works on the vibration of beams with more than one step change in cross-section. Gorman [13] analyzed exactly the vibrating properties of one-step beams with several boundary condition combinations and symmetrical two-step beams on identical end supports. Bapat and Bapat [14] proposed the transfer matrix approach for beams with n-steps but provided no numerical results. Ju et al. [15] used a first-order shear deformation theory and the corresponding finite element formulation to study the free vibrations of clamped–clamped and cantilevered beams with one or two step changes in cross-section. Lee and Bergman [16] used the dynamic flexibility method to derive the frequency equation of a beam with n-step changes in cross-section. Naguleswaran [17] studied a beam with up to three steps with 45 combinations of classical and elastic end supports using the frequency equation method of Jang and Bert [4,5]. Jaworski and Dowell [18] carried out a study for the free vibration of a cantilevered beam with multiple steps and compared the results of several theoretical methods with experiment.

In this paper, a new method is presented to analyze the free and forced vibrations of beams with either a single step change or multiple step changes using the composite element method (CEM) [19,20]. The correctness and accuracy of the proposed method are verified by some examples in the existing literatures. The principal advantage of the proposed method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam. Moreover, the presented work can easily be extended to beams with an arbitrary number of nonuniform segments.

2. Theory

2.1. Brief introduction to the CEM

The composite element is a relatively new tool for finite element modeling. This method is basically a combination of the conventional finite element method (FEM) and the highly precise classical theory (CT). In the CEM, the displacement field is written as the sum of the finite element displacement and the shape functions from the CT. The displacement field of the CEM can be expressed as

$$u_{\mathrm{CEM}}(x,t) = u_{\mathrm{FEM}}(x,t) + u_{\mathrm{CT}}(x,t), \tag{1}$$

where $u_{\mathrm{FEM}}(x,t)$ and $u_{\mathrm{CT}}(x,t)$ are the individual displacement fields from the FEM and CT, respectively.

Taking a planar beam element as an example, the first term of the CEM displacement field can be expressed as the product of the shape function vector $N(x)$ and the nodal displacement vector q

$$u_{\mathrm{FEM}}(x,t) = N(x)q(t), \tag{2}$$

where $q(t) = [v_1(t), \theta_1(t), v_2(t), \theta_2(t)]^{\mathrm{T}}$ and 'v' and 'θ' represent the transverse and rotational displacements, respectively, and

$$\begin{aligned} N(x) &= \left[1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3, \frac{x}{L} - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3, 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3, \left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\right], \\ &= [N_1(x), N_2(x), N_3(x), N_4(x)]. \end{aligned} \tag{3}$$

The second term $u_{\mathrm{CT}}(x,t)$ is obtained by the multiplication of the analytical mode shapes with a vector of N coefficients c (also called the c degrees-of-freedom or c-coordinates)

$$u_{\mathrm{CT}}(x,t) = \sum_{i=1}^{N} \varphi_i(x) c_i(t), \tag{4}$$

where φ_i ($i = 1,2,\ldots,N$) is the analytical shape function of the beam. Different analytical shape functions are used according to the boundary conditions of the beam.

Like the FEM, the CEM can be refined using the h-refinement technique by increasing the number of finite elements. Moreover, it can also be refined through the c-refinement method, by increasing the number of shape functions. In this paper, we apply the c-refinement from the CEM, where the beam needs only to be discretized into one element. This will reduce the total number of degrees-of-freedom in the FEM.

The displacement field of the CEM for the Euler–Bernoulli beam element can be written from Eqs. (1) to (4) as

$$u_{\mathrm{CEM}}(x,t) = S(x)Q(t), \tag{5}$$

where $S(x) = [N_1(x), N_2(x), N_3(x), N_4(x), \phi_1(x), \phi_2(x), \ldots, \phi_N(x)]$ is the generalized shape function of the CEM, $Q(t) = [v_1(t), \theta_1(t), v_2(t), \theta_2(t), c_1(t), c_2(t), \ldots, c_N(t)]^{\mathrm{T}}$ is the vector of generalized displacements, and N is the number of shape functions used from the CT.

2.2. Free vibration of beams from the CEM

As shown in Fig. 1, the height of the beam $d(x)$ with n step changes in cross-section is expressed as

$$d(x) = \begin{cases} d_1, & 0 \leqslant x < L_1 \\ d_2, & L_1 \leqslant x \leqslant L_2 \\ \vdots & \\ d_n, & L_{n-1} \leqslant x \leqslant L_n \end{cases}. \tag{6}$$

It is assumed that the beam has aligned neutral axis, the flexibility of the beam $EI(x)$ can be expressed as

$$EI(x) = \begin{cases} \dfrac{wd_1^3}{12}, & 0 \leqslant x < L_1 \\ \dfrac{wd_2^3}{12}, & L_1 \leqslant x \leqslant L_2 \\ \vdots & \\ \dfrac{wd_n^3}{12}, & L_{n-1} \leqslant x \leqslant L_n \end{cases}, \tag{7}$$

where w is the width of the beam, which is assumed to be a constant in this paper. For the stepped beam with misaligned neutral axes, the expression of $EI(x)$ cannot be expressed simply as shown in Eq. (7).

The beam mass per unit length is

$$m(x) = \begin{cases} \rho w d_1, & 0 \leqslant x < L_1 \\ \rho w d_2, & L_1 \leqslant x \leqslant L_2 \\ \vdots & \\ \rho w d_n, & L_{n-1} \leqslant x \leqslant L_n \end{cases}, \tag{8}$$

where ρ is the mass density of the beam.

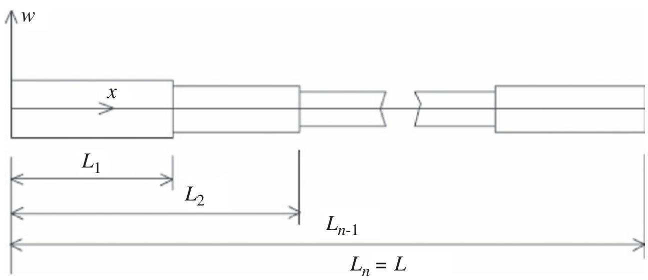

Fig. 1. Sketch of the stepped free–free beam with n segments.

The elemental stiffness matrix of the stepped beam can be obtained from the following equation:

$$\mathbf{K_e} = \int_0^L \frac{d^2S^T}{dx^2} EI(x) \frac{d^2S}{dx^2}\,dx, \\ = \begin{bmatrix} [k_{qq}] & [k_{qc}] \\ [k_{cq}] & [k_{cc}] \end{bmatrix}, \tag{9}$$

where the submatrix $[k_{qq}]$ corresponds to the element stiffness matrix from the FEM for the stepped beam; the submatrix $[k_{qc}]$ corresponds to the coupling terms of the q-dofs and the c-dofs; submatrix $[k_{cq}]$ is a transpose matrix of $[k_{qc}]$, and the submatrix $[k_{cc}]$ corresponds to the c-dofs and is a diagonal matrix.

The elemental consistent mass matrix can be expressed as

$$\mathbf{M_e} = \int_0^L S(x)^T m(x) S(x)\,dx, \\ = \begin{bmatrix} [m_{qq}] & [m_{qc}] \\ [m_{cq}] & [m_{cc}] \end{bmatrix}, \tag{10}$$

where the submatrix $[m_{qq}]$ corresponds to the elemental mass matrix from the FEM for the stepped beam; the submatrix $[m_{qc}]$ corresponds to the coupling terms of the q-dofs and the c-dofs; submatrix $[m_{cq}]$ is a transpose matrix of $[m_{qc}]$, and the submatrix $[m_{cc}]$ corresponds to the c-dofs and is a diagonal matrix.

After introducing the boundary conditions, this can be performed by setting the associated degrees-of-freedom in the systematic stiffness matrix $\mathbf{K}$ to be a large number, say, 10^{12}, the governing equation for free vibration of the beam can be expressed as

$$(\mathbf{K} - \omega^2\mathbf{M})V = 0, \tag{11}$$

where $\mathbf{K}$ and $\mathbf{M}$ are system stiffness and mass matrices, respectively, ω is the circular frequency, from which and the natural frequencies are identified. The ith normalized mode shapes of the stepped beam can be expressed as

$$\Psi_i = \sum_{i=1}^{4} N_i V_i + \sum_{i=1}^{N} \varphi_i V_{i+4}. \tag{12}$$

2.3. Forced vibration analysis

The equation of motion of the forced vibration of the beam with n steps when expressed in terms of the CEM is

$$\mathbf{M}\ddot{Q} + \mathbf{C}\dot{Q} + \mathbf{K}Q = f(t), \tag{13}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the system mass and stiffness matrices, which are the same as those shown in Eq. (11), $\mathbf{C}$ is the damping matrix which represents a Rayleigh damping model,

$$\mathbf{C} = a_1\mathbf{M} + a_2\mathbf{K}, \tag{14}$$

where a_1 and a_2 are constants to be determined from two modal damping ratios. For an external force $F(t)$ acting at the location x_F from the left support, the generalized force vector $f(t)$ can be expressed as

$$f(t) = [N_1(x_F)\ \ N_2(x_F)\ \ N_3(x_F)\ \ N_4(x_F)\ \ \phi_1(x_F)\ \ \cdots\ \ \phi_n(x_F)]^T F(t). \tag{15}$$

The generalized acceleration $\ddot{Q}$, velocity $\dot{Q}$ and displacement Q of the stepped beam can be obtained from Eq. (13) by direct integration. The physical acceleration $\ddot{u}(x,t)$ is obtained from

$$\ddot{u}(x,t) = [S(x)]^T \ddot{Q}. \tag{16}$$

The physical velocity and displacement can be obtained in a similar way, i.e.

$$\dot{u}(x,t) = [S(x)]^{\mathrm{T}}\dot{Q}, \tag{17a}$$

$$u(x,t) = [S(x)]^{\mathrm{T}}Q. \tag{17b}$$

3. Applications

3.1. Case study 1: a free–free beam with a single step

The free vibration of the free–free beam studied in Koplow et al. [12] is restudied using the method proposed in the present paper and the results are compared with those in Koplow et al. [12].

Fig. 2 shows the geometry of the beam under study. The material has a mass density of $\rho = 2830\,\mathrm{kg/m^3}$, and a Young's modulus of $E = 71.7\,\mathrm{GPa}$. Table 1 shows the convergence in the CEM when different terms of c-dofs are used. When 350 numbers of c-dofs are used, the first three natural frequencies are converged. Fig. 3 shows the first three mode shapes of the stepped beam with 50 and 350 terms of c-dofs. From this figure one can see that the mode shapes change very little as the number of c-dofs increases. The first three natural frequencies of the beam are 291.9, 1176.2 and 1795.7 Hz, respectively. The calculated natural frequencies from the CEM are very close to the experimental values in Koplow et al. [12] when the test is measured at location A in Fig. 2, which are 291, 1165 and 1771 Hz, respectively. The relative errors between the CEM and the experimental values of the three natural frequencies are 0.31, 0.96 and 1.39 percent, respectively. This shows the accuracy of the proposed method.

3.2. Case study 2: a simply supported beam with three steps

In this case, a simply supported beam with three steps is used for a free vibration analysis. The geometry of the beam is shown in Fig. 4. The parameters of the beam are $\rho = 2830\,\mathrm{kg/m^3}$ and $E = 34\,\mathrm{GPa}$. In the CEM model, the beam is discretized into one element and with 200 of c-dofs; in total, there are 204 dofs in the CEM.

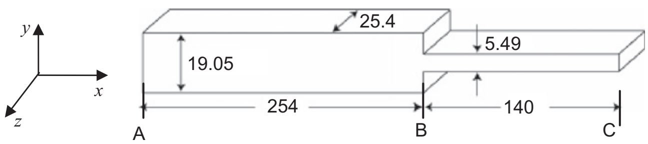

Fig. 2. Sketch of the stepped free–free beam in Ref. [12]. Dimension in millimeter.

Table 1
Convergence of the natural frequencies from the FEM for case study 1.

No. of c-dofs	Natural frequencies (Hz)		
	f_1	f_2	f_3
50	301.6	1185.4	1836.1
100	296.4	1180.8	1814.1
150	294.7	1179.1	1807.0
200	293.7	1178.1	1803.1
250	293.0	1177.4	1800.4
300	292.5	1176.9	1798.4
310	292.1	1176.6	1796.8
320	291.9	1176.2	1795.8
350	291.9	1176.2	1795.7

Note: f_1, f_2 and f_3 represent the first three natural frequencies, respectively.

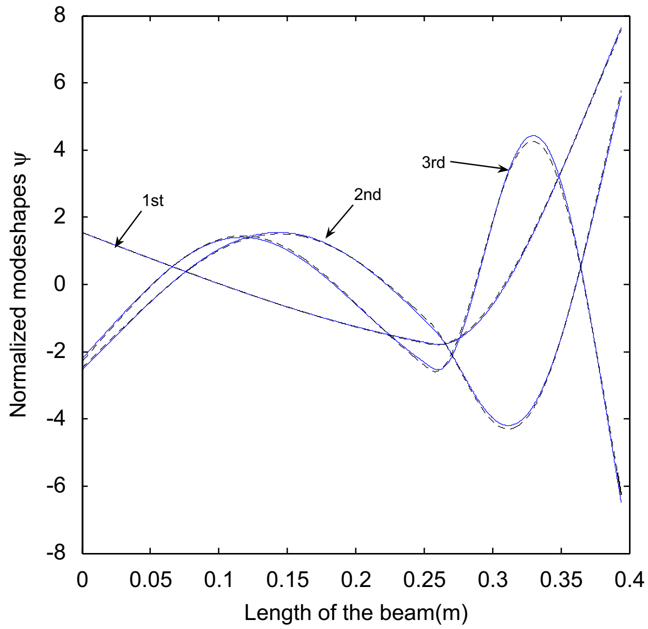

Fig. 3. Comparison on the normalized mode shapes of the stepped beam with different *c*-dofs (- solid: 50 *c*-dofs, - - dashed: 350 *c*-dofs).

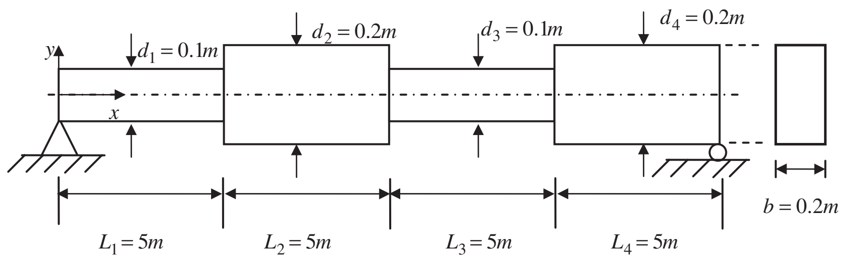

Fig. 4. Sketch of the stepped simply supported beam (dimensions are not scaled).

The first 10 natural frequencies of the beam are listed in Table 2, and the associated mass normalized mode shapes of the first six modes are shown in Fig. 5. In order to check the accuracy of the CEM, finite element of the beam is also built using Matlab finite element package [21]. The beam is discretized into 160 Euler–Bernoulli beam elements, where each node has two dofs, and has a total of 322 dofs. The first 10 natural frequencies of the beam from the finite element model are also listed in Table 2 for comparison. The natural frequencies from two methods are in good agreement with a maximum difference of 0.107 Hz; this indicates the effectiveness and accuracy of the proposed method. The first five mode shapes are also shown in Fig. 5. From this figure, one can see that the mode shapes from CEM and FEM match each very well. The first five mode shapes of the uniform beam and stepped beam are shown in Fig. 6, demonstrating that changes in the mode shape of the beam due to the geometrical discontinuity are significant.

3.3. Case study 3: a cantilever beam with several steps

In this case, the cantilever beam studied by Jaworski and Dowell [18] is restudied to further check the accuracy and effectiveness of the proposed method. Fig. 7 shows the dimensions of the beam under study. The parameters of the beam under study are: $E = 60.6\,\text{GPa}$ and $\rho = 2664\,\text{kg/m}^3$. In the CEM model of the beam, the beam is discretized into one element and 350 terms of *c*-dofs are used in the calculation. The first and second flapwise (out-of-plane) bending mode frequencies are calculated to be 10.758 and 67.553 Hz, and the

Table 2
Comparison on the natural frequencies from CEM and FEM for a simply supported beam for case study 2.

Mode no.	Natural frequencies (Hz)		
	From CEM	From FEM	Difference
1	0.433	0.432	0.001
2	1.799	1.796	0.003
3	4.411	4.399	0.012
4	9.522	9.510	0.012
5	13.246	13.219	0.027
6	19.301	19.291	0.010
7	25.721	25.670	0.051
8	34.959	34.881	0.078
9	43.174	43.067	0.107
10	55.473	55.467	0.006

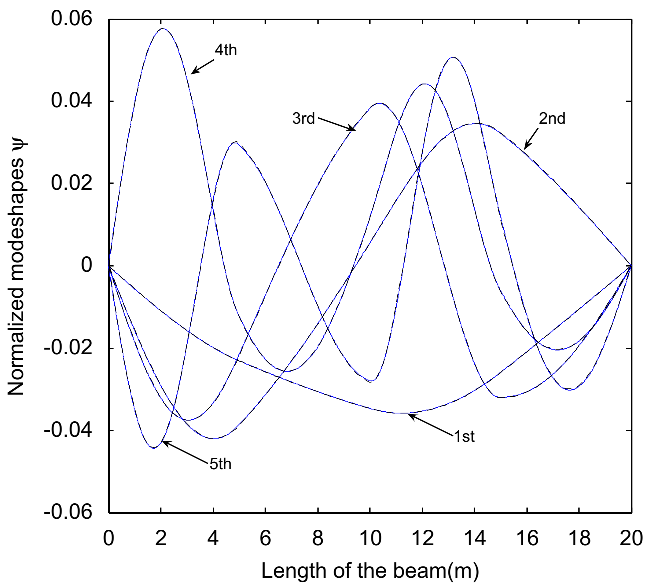

Fig. 5. Comparison between the FEM and CEM of the normalized mode shapes for the stepped beam in case study 2 (- solid: CEM; - - dashed: FEM).

first chordwise (in-plane) bending mode frequency is 54.699 Hz. The results from the CEM agree well with the theoretical results of Jaworski and Dowell [18] using Euler–Bernoulli theory, as shown in Table 3.

3.4. Case study 4: forced vibration analysis of a cantilever beam with two steps

In this study case, the forced vibration analysis for the stepped is investigated. The dynamic responses of the beam under external force are obtained from the CEM and the results are compared with those from the FEM. Fig. 8 shows the cantilever beam under study. The parameters of the beam under study are $E = 69.6\,\text{GPa}$ and $\rho = 2700\,\text{kg/m}^3$. A sinusoidal external force is assumed to act at free end of the beam with a magnitude of 1 N and at a frequency of 10 Hz. The time step is 5 ms in calculating the dynamic response. The Rayleigh damping model is adopted in the calculation with 0.01 and 0.02 as the first two modal damping ratios. In the CEM model, the beam is discretized into one element and 350 *c*-dofs are used in the calculation

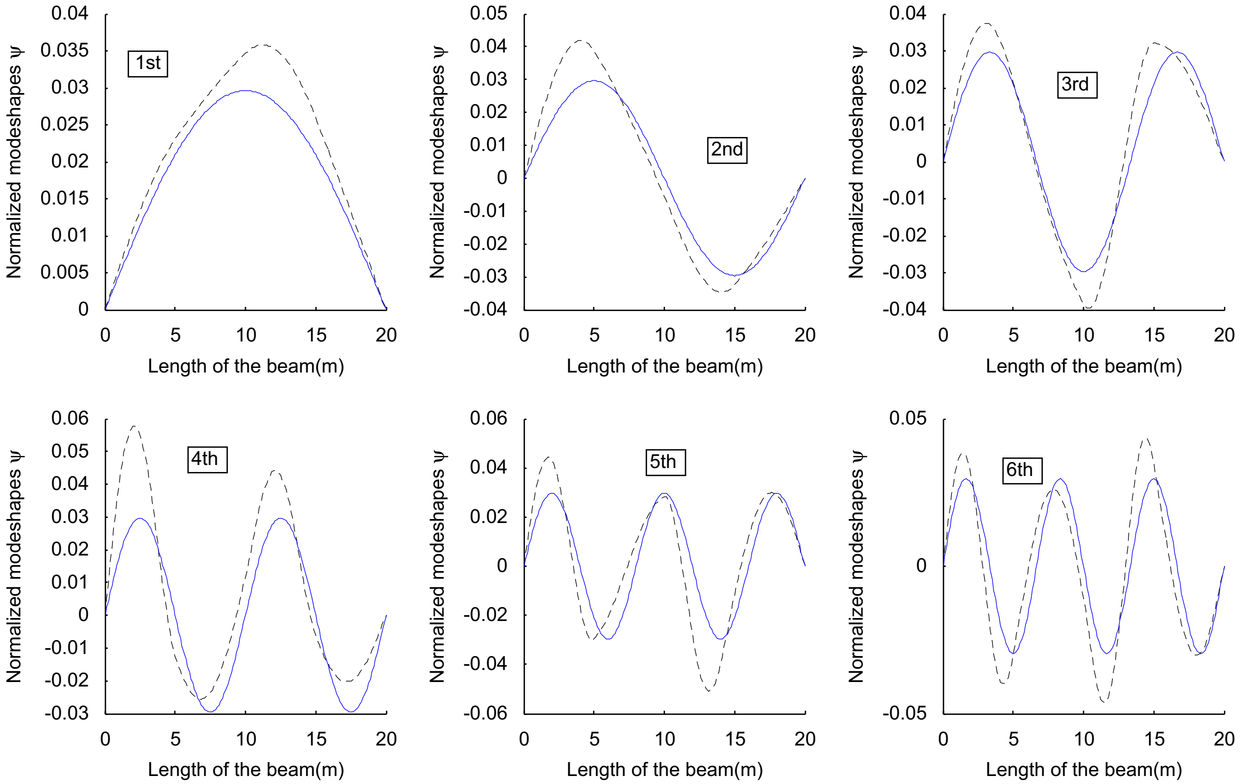

Fig. 6. Comparison of the normalized mode shapes for the stepped and the uniform beams (- solid: uniform beam; - - dashed: stepped beam).

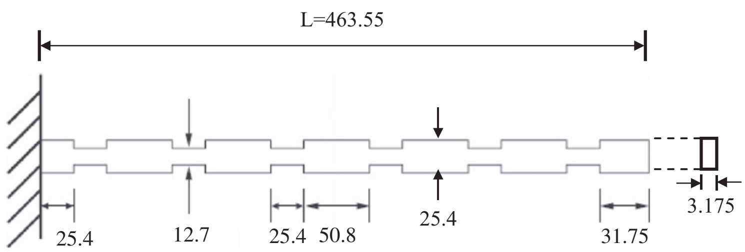

Fig. 7. Cantilever beam of Ref. [15] with up and down steps. Dimension in millimeter.

Table 3
Modal results comparison (Hz) for the stepped beam for case study 3.

Mode	Present	Rayleigh–Ritz [18]	CMA [18]	ANSYS [18]				Experiment [18]
	Euler	Euler	Euler	Euler	Timoshenko	2D Shell	3D Solid	
ω_{1B}	10.758	10.752	10.816	10.775	10.745	10.44	10.46	10.63
ω_{2B}	67.553	67.429	67.463	67.469	67.456	65.54	65.70	66.75
ω_{1C}	54.699	54.795	54.985	54.469	54.429	49.62	49.83	49.38

Note: ω_{1B}, ω_{2B} are the first and second out-of-plane bending mode frequencies, respectively.
ω_{1C} denotes the first in-plane bending mode frequency.
CMA represents component modal analysis.

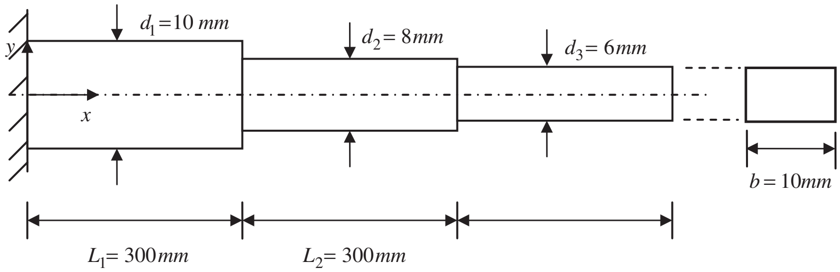

Fig. 8. Sketch of the stepped cantilever beam (dimensions are not scaled).

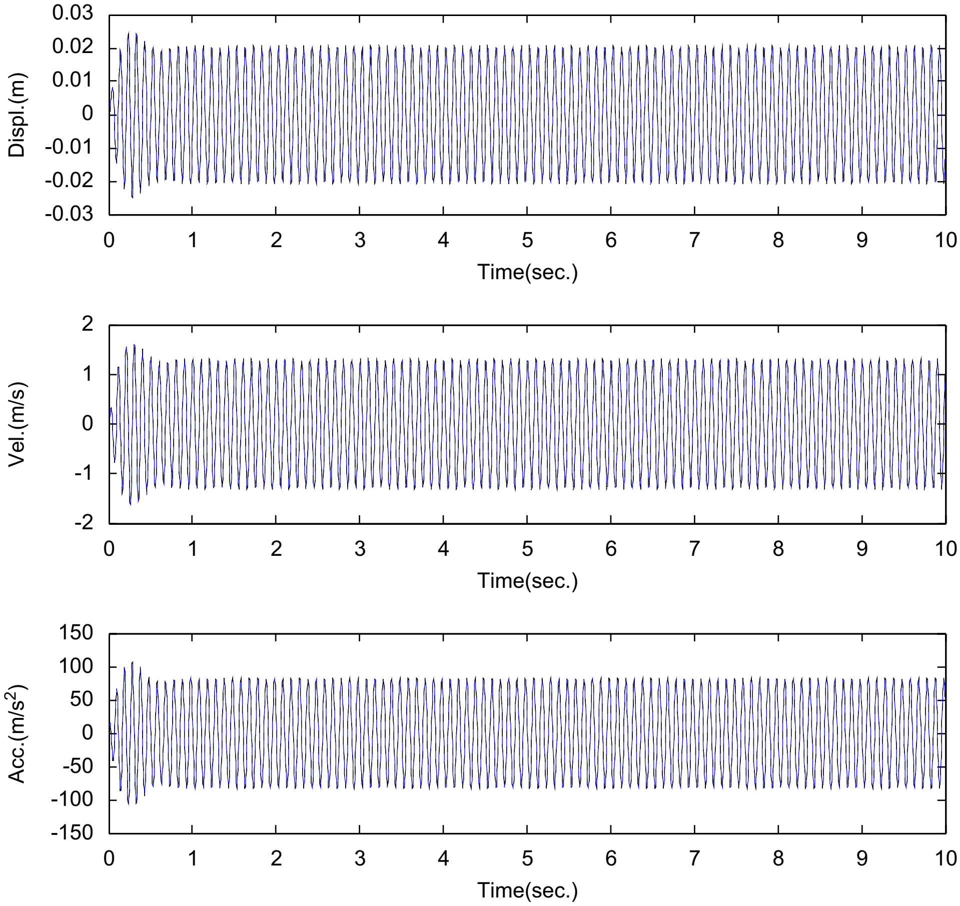

Fig. 9. Forced vibration dynamic response comparison between the CEM and FEM (- solid: CEM; - - dashed: FEM).

of the dynamic responses. Figs. 9(a), (b) and (c) show the displacement response, velocity response and acceleration response at the free end of the beam. In order to check the accuracy of the responses from the CEM, a forced vibration analysis of the beam is carried out using the FEM. The beam is discretized into 90 Euler–Bernoulli beam elements for a total of 182 dofs. The corresponding responses from the FEM and the CEM are compared in Fig. 9. This indicates the accuracy of the proposed method for forced vibration of multiple-stepped beam. Fig. 10 gives a close view between the responses from two methods. From this figure, one can see that the two time histories in every subplot are virtually coincident indicating the excellent agreement between the time histories.

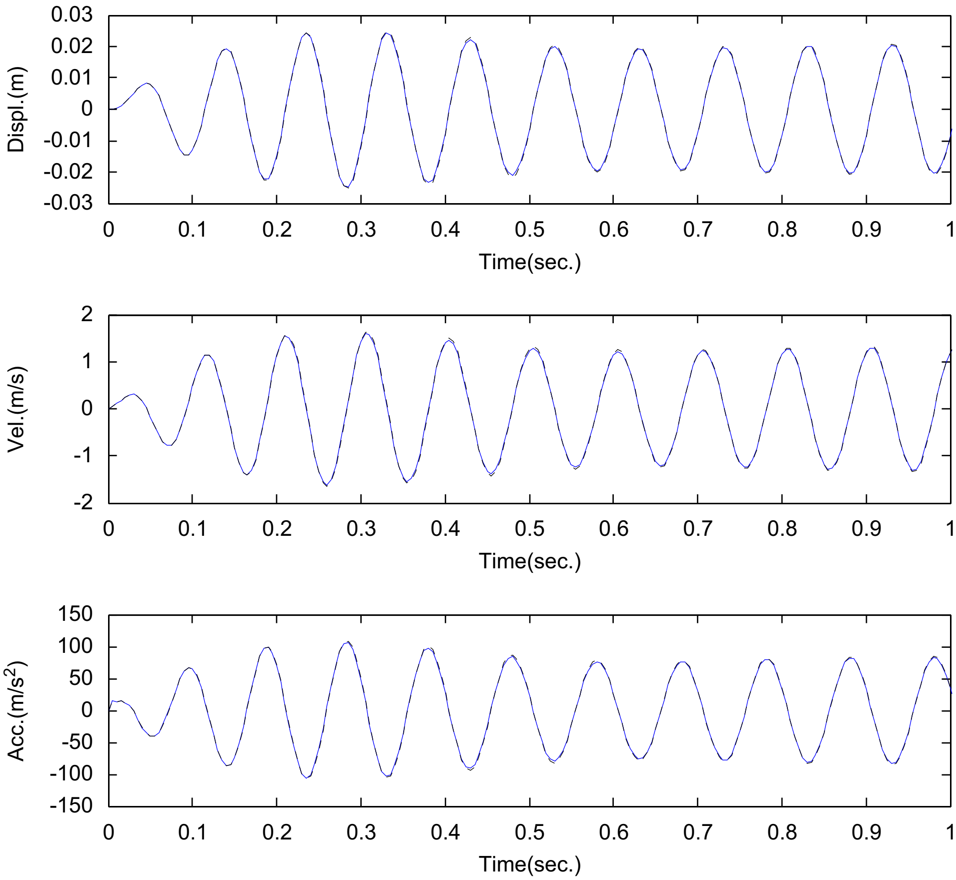

Fig. 10. Forced vibration dynamic response comparison between the CEM and FEM (a close view; - solid: CEM, - - dashed: FEM).

4. Conclusions

The composite element method is proposed for both free and forced vibration analyses of beams with multiple steps. As the composite beam element is of a one-element-one-member configuration, modeling with this type of element would not need to take into account the discontinuity between different parts of the beam. The accuracy and convergence of this new composite element has been compared satisfactorily with existing theoretical and experimental results. One advantage of the method proposed is that it can be extended easily to deal with beams consisting of an arbitrary number of nonuniform segments.

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