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Vibration based structural damage detection in flexural members using multi-criteria approach

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Abstract

This paper uses dynamic computer simulation techniques to develop and apply a procedure using non-destructive methods for damage assessment in beams and plates, which are important flexural members in building and bridge structures. In addition to changes in natural frequencies, this multi-criteria procedure incorporates two methods, called the modal flexibility and the modal strain energy method, which are based on the vibration characteristics of the structure. Using the results from modal analysis, algorithms based on flexibility and strain energy changes before and after damage are obtained and used as the indices for the assessment of the status of the structural health. The objective is to evaluate the feasibility of the proposed multi-criteria method to identify and localise single and multiple damages in numerical models of flexural members having different boundary conditions. The application of the approach is demonstrated through two sets of numerical simulation studies on beam and plate structures with nine damage scenarios in each. Results show that the proposed multi-criteria method incorporating modal flexibility and modal strain energy method is effective in multiple damage assessment in beam and plate structures.

1. Introduction

Civil infrastructures such as bridges, buildings are normally designed to have long life spans. Changes in load characteristics, deterioration with age, environmental influences and random actions may cause local or global damage to structures. Continuous health monitoring of structures will enable the early identification of distress and allow appropriate retrofitting to prevent potential sudden structural failures. In recently times, structural health monitoring (SHM) has attracted much attention in both research and development. SHM defined by Housner et al. [1] refers to the use of in-situ, continuous or regular (routine) measurement and analyses of key structural and environmental parameters under operating conditions, for the purpose of warning impending abnormal states or accidents at an early stage to avoid casualties as well as giving maintenance and rehabilitation advice. SHM encompasses both local and global methods of damage identification [2]. In the local case, the assessment of the state of a structure is done either by direct visual

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inspection or using experimental techniques such as acoustic emission, ultrasonic, magnetic particle inspection, radiography and eddy current. A characteristic of all these techniques is that their application requires a prior localisation of the damaged zones. The limitations of the local methodologies can be overcome by using vibration-based (VB) methods, which give a global damage assessment. Health monitoring techniques based on processing vibration measurements basically handle two types of characteristics: the structural parameters (mass, stiffness, damping) and the modal parameters (modal frequencies, associated damping values and mode shapes). As the dynamic characteristics of a structure, namely natural frequencies and mode shapes are known to be functions of its stiffness and mass distribution, variations in modal frequencies and mode shapes can be an effective indication of structural deterioration. Deterioration of a structure results in a reduction of its stiffness which causes the change in its dynamics characteristics. Thus, damage state of a structure can be inferred from the changes in its vibration characteristics [3]. Usually four different levels of damage identification are discriminated [4]: damage detection (Level 1), damage localisation (Level 2), damage quantification (Level 3), and predication of the acceptable load level and of the remaining service life of the damaged structure (Level 4). The amount of literature is quite large for a single damage scenario, but limited for multiple damage cases. Also existing methods are limited in scope and may not be useful in several realistic situations. It is observed that changes in natural frequencies alone may not provide enough information for integrity monitoring. It is common to have more than one damage case giving a similar frequency-change characteristic ensemble. In case of symmetric structures, the changes in natural frequency due to damage at two symmetric locations are exactly the same. Alternatively, no changes in the mode shapes could be detected if the mode had a node point at the location of damage [5]. There is thus a need for a more comprehensive method of damage assessment in structures.

Fast computers and sophisticated finite element programs have enabled the possibility of analysing hitherto intractable problems in structural engineering while simplifying the analyses of other problems. This paper uses dynamic computer simulation techniques to develop and apply two non-destructive damage detection methods for damage assessment in beams and plates which are both important flexural members in buildings and bridges. These methods called the modal flexibility method and the modal strain energy method, which are based on the dynamic characteristics of natural frequencies and mode shapes and their variations with the state of the health of the structure. The objective is to evaluate the feasibility of the proposed methods to identify and localise single and multiple damages in numerical models of the flexural members with different boundary conditions. The application of the approach is demonstrated through two sets of numerical simulation studies on beam and plate structures with nine damage scenarios in each case. Results show the procedure incorporating the modal flexibility and modal strain energy methods is effective in the multiple damage assessment of beam and plate structures.

2. Recent studies on VB damage identification

A number of methodologies have been found recently in the literature to identify, locate and estimate the severity of damage in structures using numerical simulation. Lee et al. summarise the features applied for damage detection algorithms utilising vibration properties as shown in Table 1. It is noticed that those methods utilising mode shapes are the most developed in terms of displaying the ability to identify, locate and estimate the severity of damage [6]. The modal flexibility and modal strain energy method are chosen in this investigation as their corresponding algorithms can be applied to both beam and plate structures. The advantage of using modal flexibility method is that the flexibility matrix is most sensitive to changes in the lower-frequency modes of the structures due to the inverse relationship to the square of the natural frequencies. Therefore, a good estimate of the flexibility can be made with the inclusion of the first few frequencies and their associated mode shapes. The advantage of using modal strain energy method is that only measured mode shapes and elemental stiffness matrix are required in the damage identification without knowledge of the complete stiffness and mass matrices of the structure. Only the mode shapes of the first few modes and their corresponding derivatives are required in this proposed algorithm for accurate damage localisation. Modal assurance criterion (MAC) values are not required to determine the indication of which modes are being affected most by the damage.

Table 1
Summary of damage detection categories and methods (after [6]).

Category		Methodology
Modal parameters	Natural frequencies	Frequency changes
		Residual force optimization
	Mode shapes	Mode shape changes
		Modal strain energy
		Mode shape derivatives
Matrix methods	Stiffness-based	Optimization techniques
		Model updating
	Flexibility-based	Dynamically measured flexibility
Machine learning	Genetic algorithm	Stiffness parameter optimization
		Minimization of the objective function
	Artificial neutral network	Back propagation network training
		Time delay neural network
		Back propagation network training
		Neural network systems identification with neural network damage detection
Other techniques		Time history analysis
		Evaluation of frequency response functions (FRF)

2.1. Modal flexibility method

Pandey and Biswas [7] presented the flexibility matrix for detecting the presence and location of structural damage. All predictions of the state of damage were made from the full experimental data from modal testing. The authors treated a simply supported beam, a cantilever beam and a free–free beam to gain an insight into how the flexibility matrix is affected by the presence of damage. It was shown that the flexibility change pattern is different for different support conditions.

Patjawit and Kanok–Nukulchai [8] introduced a global flexibility index (GFI) to identify global health deteriorations of highway bridges. The index is the spectral norm of the modal flexibility matrix obtained in association with selected reference points sensitive to the deformation of the bridge structure. The modal flexibility matrix is evaluated from the dynamic responses at these reference points under forced vibration. Aging of a bridge over a period of time will be reflected by the gradual increase of GFI. The change in the GFI has been shown to be sufficiently sensitive to the global weakening of the structure and its increase in magnitude is a good indication for structural deterioration.

2.2. Modal strain energy method

Cornwell et al. [9] applied the strain energy damage detection method to plate-like structures. The method only requires the mode shapes of the structure before and after damage and the modes do not need to be mass normalised making it very advantageous when using ambient excitation. The algorithm was found to be effective in locating areas with stiffness reductions as low as 10 percent using relatively few modes. The algorithm was also demonstrated successfully using experimental data.

Hu et al. [10] applied strain energy method and modal analysis to the damage detection of a surface crack in composite laminated plates. Both experimental modal analysis (EMA) and finite element analysis (FEA) were performed to obtain the mode shapes of the laminated plates. The mode shapes were then used to calculate strain energy using differential quadrature method (DQM). The authors indicated that only a few grid points in the test plate are required for DQM to provide an accurate and rapid approach to obtain strain energy. Consequently, a damage index was established to locate the surface crack using the fractional strain energy of laminated plates before and after damage. Experimental results showed that surface crack locations in various composite laminates were successfully identified by the damage indices.

Li et al. [11] evaluated the performance of the modal strain energy based damage identification algorithm for detecting damage in a timber structure. It was shown that the method was capable of detecting single damage in timber but will experience some problems in multiple damage detection. A modified algorithm was proposed by the authors to overcome the problems associated with reliable detection of multiple damages in terms of damage location and severity.

Alvandi and Cremona [12] studied the performance of both flexibility method and strain energy method on a simply supported beam. Measured modal parameters which use only few mode shapes and modal frequencies of the structure obtained by random force excitation were used. The authors assessed the performance of these techniques by introducing different noise levels to the response signals of a simulated beam which was excited by a random force. They concluded that both methods are capable of detecting and localising damaged elements but in the case of complex and simultaneous damages, the flexibility method is less efficient. Moreover, the strain energy method demonstrates stability in the presence of noisy signals.

Based on the literature review, it is observed that the two damage localisation algorithms, each by itself, is not effective in locating multiple damages and evaluating the severity of damage. It is possible to develop a damage detection system which uses both algorithms, in addition to the change in natural frequencies, to localise multiple damages in flexural members and cross check the results. This paper proposes such a system that can guarantee the multiple damages in flexural members with different boundary conditions to be localised accurately by using two complementary damage identification algorithms.

3. Theory

3.1. Modal flexibility matrix

The modal flexibility matrix includes the influence of both the mode shapes and the natural frequencies. It is defined as the accumulation of the contributions from all available mode shapes and corresponding natural frequencies. The modal flexibility matrix associated with the referenced degrees of freedom can be established from Eq. (1) found in Huth et al. [13].

$$[F] = [\phi][1/\omega^2][\phi]^{\mathrm{T}} \tag{1}$$

where $[F]$ is the modal flexibility matrix; $[\phi]$ is the mass normalised modal vectors; and $[1/\omega^2]$ is a diagonal matrix containing the reciprocal of the square of natural frequencies in ascending order. The modal contribution to the flexibility matrix decreases as the frequency increases, i.e., the flexibility matrix converges rapidly with increasing values of frequency. From only a few of the lower frequency modes, therefore, a good estimate of the flexibility can be made. The change in the flexibility matrix $\Delta[F]$ due to structural deterioration is given by

$$\Delta[F] = [F^d] - [F^h] \tag{2}$$

where index 'h' and 'd' refer to the healthy and damaged state, respectively. Theoretically, structural deterioration reduces stiffness and increases flexibility. Increase in structural flexibility can therefore serve as a good indicator of the degree of structural deterioration.

3.2. Modal strain energy based damage index

The strain energy U of a Bernoulli–Euler beam is given as follows:

$$U = \int \frac{EI}{2}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2 \mathrm{d}x \tag{3}$$

where x is the distance measured along the length of the beam, y is the vertical deflection, EI is the flexural rigidity of the cross section and $\mathrm{d}^2y/\mathrm{d}x^2$ is the curvature of the deformed beam.

Deterioration of a structure results in a reduction of its stiffness which causes the changes in modal strain energy. The damage localisation method is based on the relative differences in modal strain energy between an undamaged and damaged structure. Information required in the identification are the measured mode shapes and elemental stiffness matrices, only without knowledge of the complete stiffness and mass matrices of the structure. The equation used to calculate the damage index β_{ji} for the jth element and i mode of a beam is given in Ref. [14].

$$\beta_{ji} = \frac{\left(\int_j \left[\phi''^*_i(x)\right]^2 \mathrm{d}x + \int_0^L \left[\phi''^*_i(x)\right]^2 \mathrm{d}x\right) \int_0^L [\phi''_i(x)]^2 \mathrm{d}x}{\left(\int_j [\phi''_i(x)]^2 \mathrm{d}x + \int_0^L [\phi''_i(x)]^2 \mathrm{d}x\right) \int_0^L \left[\phi''^*_i(x)\right]^2 \mathrm{d}x} \tag{4}$$

To account for all available modes, a single indicator for each location along the beam is given by

$$\beta_j = \frac{\sum_{i=1}^{NM} Num_{ji}}{\sum_{i=1}^{NM} Denom_{ji}} \tag{5}$$

where Num_{ji} = numerator of β_{ji} and $Denom_{ji}$ = denominator of β_{ji} in Eq. (4).

The strain energy of a plate of size $a \times b$ is given as follows:

$$U = \frac{D}{2}\int_0^b \int_0^a \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2v\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + 2(1-v)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y \tag{6}$$

where $D = Eh^3/12(1-v^2)$ is the bending stiffness of the plate, v is the Poisson's ratio, h is the plate thickness, w is the transverse displacement of the plate, $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$ are the bending curvatures, $2\partial^2 w/\partial x \partial y$ is the twisting curvature of the plate. For a particular mode shape $\phi_i(x,y)$, the strain energy U_i associated with that mode shapes is

$$U_i = \frac{D}{2}\int_0^b \int_0^a \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2}\right)^2 + 2v\left(\frac{\partial^2 \phi_i}{\partial x^2}\right)\left(\frac{\partial^2 \phi_i}{\partial y^2}\right) + 2(1-v)\left(\frac{\partial^2 \phi_i}{\partial x \partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y \tag{7}$$

If the plate is subdivided into N_x subdivisions in the x direction and N_y subdivisions in the y direction, then the energy U_{ijk} associated with sub-region jk for the ith mode is given by

$$U_{ijk} = \frac{D_{jk}}{2}\int_{b_k}^{b_{k+1}} \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2}\right)^2 + 2v\left(\frac{\partial^2 \phi_i}{\partial x^2}\right)\left(\frac{\partial^2 \phi_i}{\partial y^2}\right) + 2(1-v)\left(\frac{\partial^2 \phi_i}{\partial x \partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y \tag{8}$$

so

$$U_i = \sum_{k=1}^{N_y} \sum_{j=1}^{N_x} U_{ijk} \tag{9}$$

And the fractional energy at location jk is defined to be

$$F_{ijk} = \frac{U_{ijk}}{U_i} \tag{10}$$

and

$$\sum_{k=1}^{N_y} \sum_{j=1}^{N_x} F_{ijk} = 1 \tag{11}$$

Similar expressions can be written using the modes of the damaged structure ϕ_i^*, where the superscript * indicates damaged state. A ratio of parameters can be determined that is indicative of the change of stiffness in the structure as shown in Eqs. (12) and (13).

$$\frac{D_{jk}}{D_{jk}^*} = \frac{f_{ijk}^*}{f_{ijk}} \tag{12}$$

where

$$f_{ijk} = \frac{\int_{b_k}^{b_{k+1}} \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \phi_i}{\partial x^2}\right)\left(\frac{\partial^2 \phi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \phi_i}{\partial x \partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y}{\int_0^b \int_0^a \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \phi_i}{\partial x^2}\right)\left(\frac{\partial^2 \phi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \phi_i}{\partial x \partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y} \tag{13}$$

and an analogous term f^*_{ijk} can be defined using the damaged mode shapes. In order to account for all measured modes, the following formulation for the damage index for sub-region jk is used:

$$\beta_{jk} = \frac{\sum_{i=1}^{m} f^*_{ijk}}{\sum_{i=1}^{m} f_{ijk}} \tag{14}$$

The complete derivation of the damage indicator for beam and plate are given in Refs. [9,13,14].

4. Method

As a single damage indicator is not reliable, especially in the case of multiple damages, a damage multi-criteria approach which incorporates (1) change of frequency Δf, (2) change of flexibility matrix ΔF and (3) modal strain energy based damage index β_i is used in the damage assessment of beam and plate structures. Initial beam and plate-like structures are first defined and developed as finite element (FE) models and their modal responses are obtained using the FE software package SAP2000. After validation of these initial FE models, additional FE models of the beam and plate structures, first without damage and then with 9 different damage patterns and boundary conditions are selected for investigation. The primary modal parameters of natural frequencies and mode shapes of the first five modes of these structures, before and after damage in nine scenarios shown in Figs. 1–6, are extracted from the results of the FE analysis. These parameters are then used to determine the modal flexibility matrix and the modal strain energy based damage index and thereby assess the damage state of the test structure. The peak values of the parameter in each method indicate the location of simulated damage in the corresponding damage scenarios. The accuracy of the damage detection method is then evaluated through observations of the plots. The details of modal testing and FE modelling on beams and plates are described below.

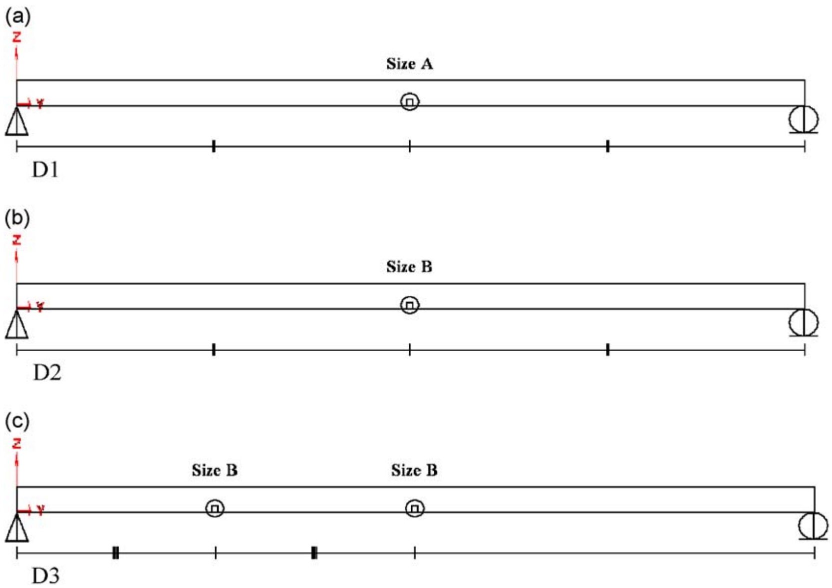

Fig. 1. Damage Case (D) for single span beam.

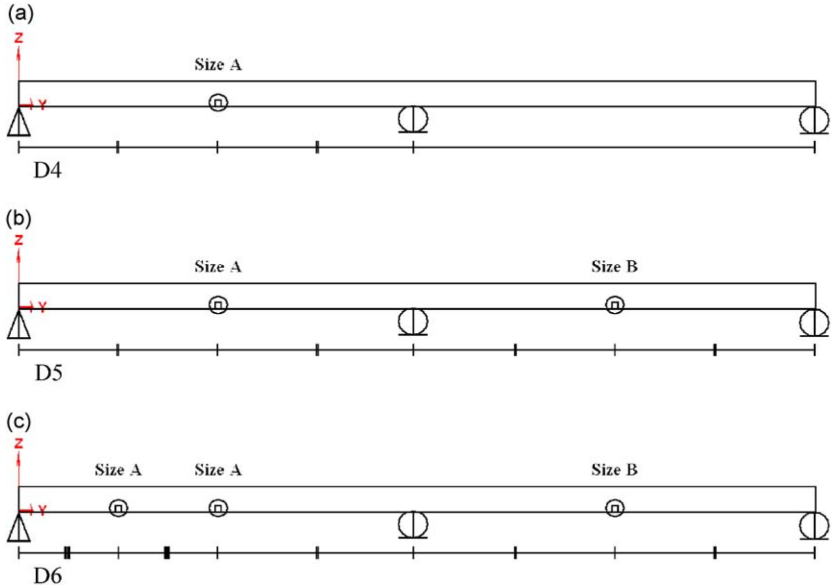

Fig. 2. Damage Case (D) for 2-span beam.

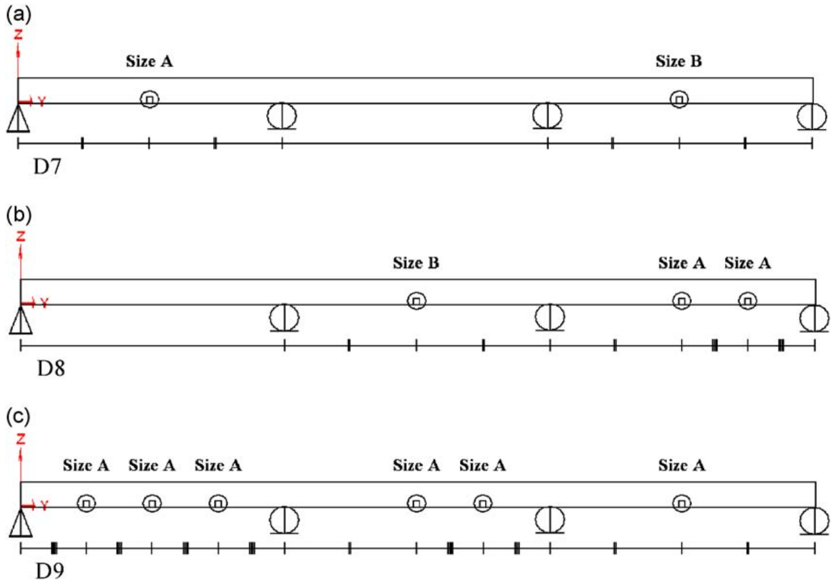

Fig. 3. Damage Case (D) for 3-span beam.

4.1. Finite element modelling and analysis of beams

Experimental modal testing is conducted on the simply supported steel beam to validate the initial FE beam model. The undamaged beam is first excited by an impact hammer and the dynamic responses are measured by an accelerometer fixed at the mid-span of the beam. A software known as, SignalCalc ACE Dynamic Signal Analyzer is used to extract the pre-damage modal parameters. Then the test beam is cut to cause a small flaw at mid-span and the testing is repeated to extract the post-damage modal parameters. Finite element models (FEM) of the undamaged and damaged simply supported beams, tested previously, having a span length of 2.8 m are generated using the FE program SAP2000. Plane elements are used for the FEM. The details of the

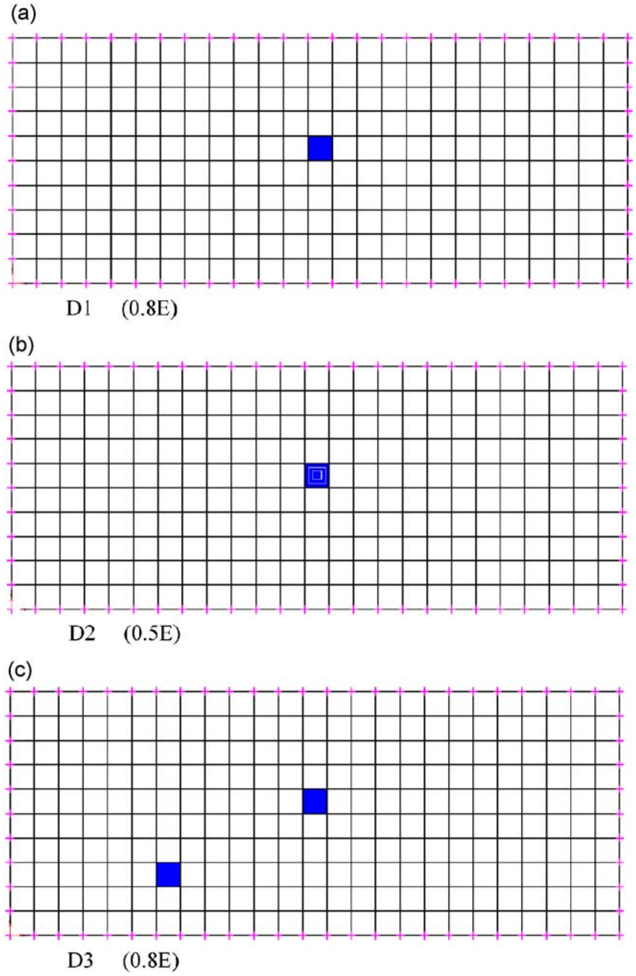

Fig. 4. Damage Case (D) for plate with all edges clamped (values in brackets show damages with percentage remaining in E).

beam are given in Table 2. The flexural rigidity EI is assumed constant over the beam span and damping effect is not taking into account. Modal analysis is performed to obtain the natural frequencies and the associated mode shapes of the beam. As a low frequency accelerometer is chosen in the measurement of dynamic responses, only the two lowest natural frequencies are captured from the experiments. The obtained experimental results are compared with those from FE analysis to validate the FEM. It is noted that the experimental and FE results showed good agreement as seen in Table 3. This provides adequate confidence with FE modelling and analysis of other beam models. Further FE analysis is performed to extract the modal parameters of 2-span and 3-span continuous beams. All continuous beams have the same span length of 2.8 m and are simply supported at their ends. To simulate damage, the selected damaged elements are removed from the bottom of the beams in the FE models. Nine such damage cases are investigated with two different sizes of flaws listed in Table 4. Fig. 1 shows three damage scenarios in a single span simply supported beam in which size B flaw represents larger size damage than size "A" flaw. Figs. 2 and 3 show the other six damage scenarios in the 2-span and 3-span beams, respectively. Fig. 7 shows the details of flaw size "A" simulated in the FE models. The damage simulation technique used in this paper has been adopted from established work in the literature, such as [9], where structural damage or loss of stiffness is simulated by reducing the flexural rigidity (e.g. E elastic modulus or I second moment of area) of the structure.

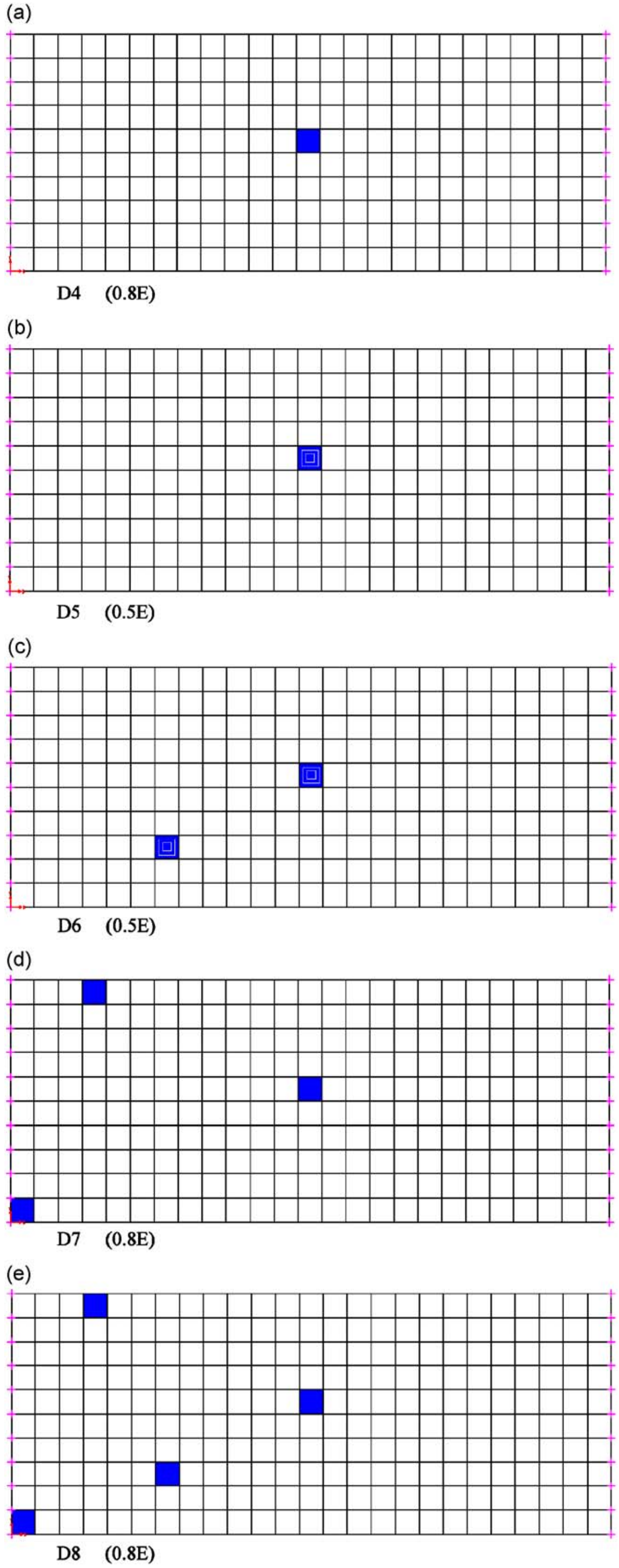

Fig. 5. Damage Case (D) for simply supported plate.

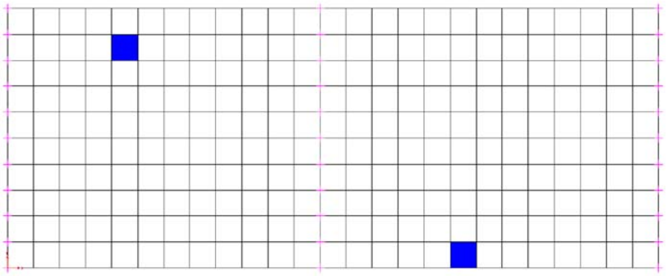

Fig. 6. Damage Case (D) for 2-span plate.

Table 2
Geometric and material properties of beam and plate.

Flexural member	Beam (2D)	Plate (2D)
Element type	Plane stress	Plate
Material	Steel	Steel
Length	2.8 m	2.5 m
Width	40 mm	1 m
Depth	20 mm	2 mm
Poisson's ratio	0.3	0.3
Mass density	7850 kg/m^3	7800 kg/m^3
Modulus of elasticity	200 GPa	210 GPa

Table 3
Validation of FEM for simply supported beam.

State	Frequency mode	Experiment (Hz)	FEM (Hz)	% Diff.
Undamaged	f_1	5.94	5.84	1.7
	f_2	24.38	23.33	4.3
Damaged at mid-span	f_1^*	5.63	5.65	0.4
	f_2^*	23.13	23.33	0.9

Table 4
Dimension of flaws in beam.

Size	Length (mm)	Depth (mm)	Width (mm)
A	10	5	40
B	20	5	40

4.2. Finite element modelling and analysis of plates

FE techniques are used to carry out modal analysis of the plate-like structures. Initially, FE model of a steel plate clamped along all four edges is analysed and the first three natural frequencies and the associated mode shapes of the plate provided by Ulz and Semercigil are compared with the results obtained from modal analysis [15]. The two sets of results compare well as seen in Table 5, providing adequate confidence in the present FE modelling and analysis of plate structures. Additional FE models of single and two-span plate structures are developed and their modal analysis is carried out before and after damage. The chosen plate for

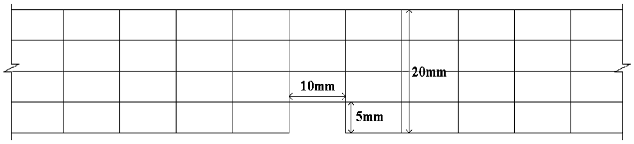

Fig. 7. Flaw size 'A' simulated in FEM.

Table 5
Validation of FEM for plate with clamped boundaries.

State	Frequency f_i	Ref. [15] (Hz)	SAP2000 (Hz)	% Diff.
Undamaged	f_1	11.81	11.78	0.3
	f_2	13.89	13.77	0.8
	f_3	17.68	17.44	1.4

Table 6
Natural frequencies of undamaged beam and plate from FEM.

Member type	Boundary condition	Mode 1 f_1 (Hz)	Mode 2 f_2 (Hz)	Mode 3 f_3 (Hz)	Mode 4 f_4 (Hz)	Mode 5 f_5 (Hz)
Beam	SS (1-span)	5.84	23.33	52.45	93.10	145.16
	SS (2-span)	5.84	9.12	23.33	29.52	52.45
	SS (3-span)	5.84	7.48	10.92	23.33	26.58
Plate	Edges clamped (1-span)	11.78	13.77	17.44	22.91	30.15
	SS (1-span)	0.76	2.66	3.07	5.95	6.95
	SS (2-span)	3.04	4.90	5.88	7.36	12.19

Note: SS means simply supported.

numerical analyses is a steel plate divided into 250 plate elements. The geometry is rectangular, 2.5 m in length, 1 m in width and 2 mm in thickness. The material properties of the steel plate are shown in Table 2. Damage is simulated by reducing the elastic modulus (E) to 80 percent and 50 percent of selected elements as shown in Figs. 4–6. An assumption is made that the mass of the plate does not change appreciably as a result of the damage. No structural damping is used in the FE analysis. Nine damage cases are investigated in this study. Fig. 4 shows three damage scenarios for the plate with all edges clamped. Figs. 5 and 6 show the other six damage scenarios for simply supported plates with single and 2 spans, respectively.

5. Results and discussions

The natural frequencies of the first five modes of the beams and plates before and after damage in nine scenarios obtained from the results of the FE analysis are shown in Tables 6–8. Percentage changes in the natural frequencies between the undamaged and damages conditions are listed within brackets in Tables 7 and 8. It can be observed that in general the presence of damage in flexural members causes a small decrease in the natural frequencies in all damage cases, with very few exceptions. If the damage cases 1 and 2 for the single span beam are considered, change (i.e. decrease) in the frequency Δf is evident for the 1st, 3rd and 5th modes, while there is no change for the 2nd and 4th modes. This is because the damage elements are located at the nodes of these anti-symmetric modes of vibration and hence have no influence on the corresponding natural frequencies. It may be concluded that by observing the changes in the natural frequencies, it is more possible to achieve Level 1 of identification of macro-damage in flexural members, rather than micro-damage or small damage. The detection of small damage can be supplemented by advanced techniques such as acoustic emission monitoring.

Table 7
Natural frequencies from FEM for damaged beam (percentage changes wrt the undamaged conditions are listed within brackets).

Damage case	Mode 1 f_1 (Hz)	Mode 2 f_2 (Hz)	Mode 3 f_3 (Hz)	Mode 4 f_4 (Hz)	Mode 5 f_5 (Hz)
1	5.80 (0.68)	23.33 (0.00)	52.08 (0.71)	93.10 (0.00)	144.13 (0.71)
2	5.77 (1.20)	23.33 (0.00)	51.90 (1.05)	93.10 (0.00)	143.65 (1.04)
3	5.74 (1.71)	23.08 (1.07)	51.62 (1.58)	93.10 (0.00)	142.95 (1.52)
4	5.82 (0.34)	9.10 (0.22)	23.33 (0.00)	29.51 (0.03)	52.26 (0.36)
5	5.78 (1.03)	9.07 (0.55)	23.33 (0.00)	29.49 (0.10)	51.99 (0.88)
6	5.77 (1.20)	9.05 (0.77)	23.25 (0.34)	29.39 (0.44)	51.90 (1.05)
7	5.80 (0.68)	7.43 (0.67)	10.90 (0.18)	23.33 (0.00)	26.57 (0.04)
8	5.79 (0.86)	7.45 (0.40)	10.86 (0.55)	23.28 (0.21)	26.48 (0.38)
9	5.77 (1.20)	7.42 (0.80)	10.86 (0.55)	23.16 (0.73)	26.42 (0.60)

Table 8
Natural frequencies from FEM for damaged plate (percentage changes wrt the undamaged conditions are listed within brackets).

Damage case	Mode 1 f_1 (Hz)	Mode 2 f_2 (Hz)	Mode 3 f_3 (Hz)	Mode 4 f_4 (Hz)	Mode 5 f_5 (Hz)
1	11.77 (0.14)	13.77 (0.01)	17.42 (0.12)	22.91 (0.00)	30.11 (0.15)
2	11.74 (0.39)	13.77 (0.01)	17.38 (0.38)	22.91 (0.02)	30.02 (0.43)
3	11.77 (0.14)	13.77 (0.02)	17.41 (0.15)	22.90 (0.04)	30.10 (0.18)
4	0.76 (0.08)	2.66 (0.00)	3.07 (0.00)	5.94 (0.06)	6.95 (0.08)
5	0.76 (0.24)	2.66 (0.00)	3.07 (0.00)	5.94 (0.17)	6.94 (0.23)
6	0.76 (0.13)	2.66 (0.04)	3.06 (0.08)	5.94 (0.09)	6.95 (0.11)
7	0.76 (0.11)	2.66 (0.15)	3.07 (0.07)	5.93 (0.19)	6.94 (0.18)
8	0.76 (0.13)	2.66 (0.04)	3.06 (0.08)	5.94 (0.09)	6.95 (0.11)
9	3.03 (0.16)	4.90 (0.12)	5.87 (0.10)	7.36 (0.10)	12.18 (0.08)

5.1. Modal flexibility change (MFC)

The first five natural frequencies and associated mode shapes obtained from the results of the FE analysis are used to calculate the MFC by using Eqs. (1) and (2). The plot of MFC as a percentage along the beam for some representative cases are shown in Figs. 8(a)–(d). In all cases, the peak values indicate the location of damage in the beams. Figs. 8(a) and (b) show the results for single damage cases and it is evident that the peak for the more severe damage, case 2, is higher than that for case 1. In Fig. 8(c) there are two unequal peaks corresponding to the 2 different damages in this beam and once again it is seen that greater damage in the beam attracts a greater peak in the MFC. Finally, Fig. 8(d) clearly shows that this damage case with triple damages has three distinct peaks in the MFC. The results for the other damage cases are not shown for want of space. From the above observations, it is evident that the damage detection algorithm for MFC is able to locate the damage in the beam structure correctly in all damage cases and also give an indication of its severity in single damage cases. This confirms that the modal flexibility method is sufficiently sensitive to the damages in the beams.

To optimise the damage detection results for the plate structure, plots of MFC for damage cases 1, 2 and 9 are shown in Figs. 9(a), (b) and (d), respectively, and the plot of MFC for damage case 8 expressed as a percentage with respect to undamaged modal flexibility matrix is shown in Fig. 9(c). Results for the other damage cases are not shown for want of space. The peak values indicate the location of damage in the plate. Comparison of Figs. 9(a) and (b) pertaining to detection of a single damage in a plate shows that as the severity of the single damage at mid-span increases, the corresponding MFC also increases, as demonstrated by the higher peak. For the case of multiple damage detection in damage case 9, the modal flexibility method is able to correctly locate the damage. But for multiple damage case 8, results in Fig. 9(c) does not clearly indicate the damage locations and the damage indicator has partly missed the damage at the mid-span of the plate. Overall, the results showed that the modal flexibility method is able to correctly locate the damage in

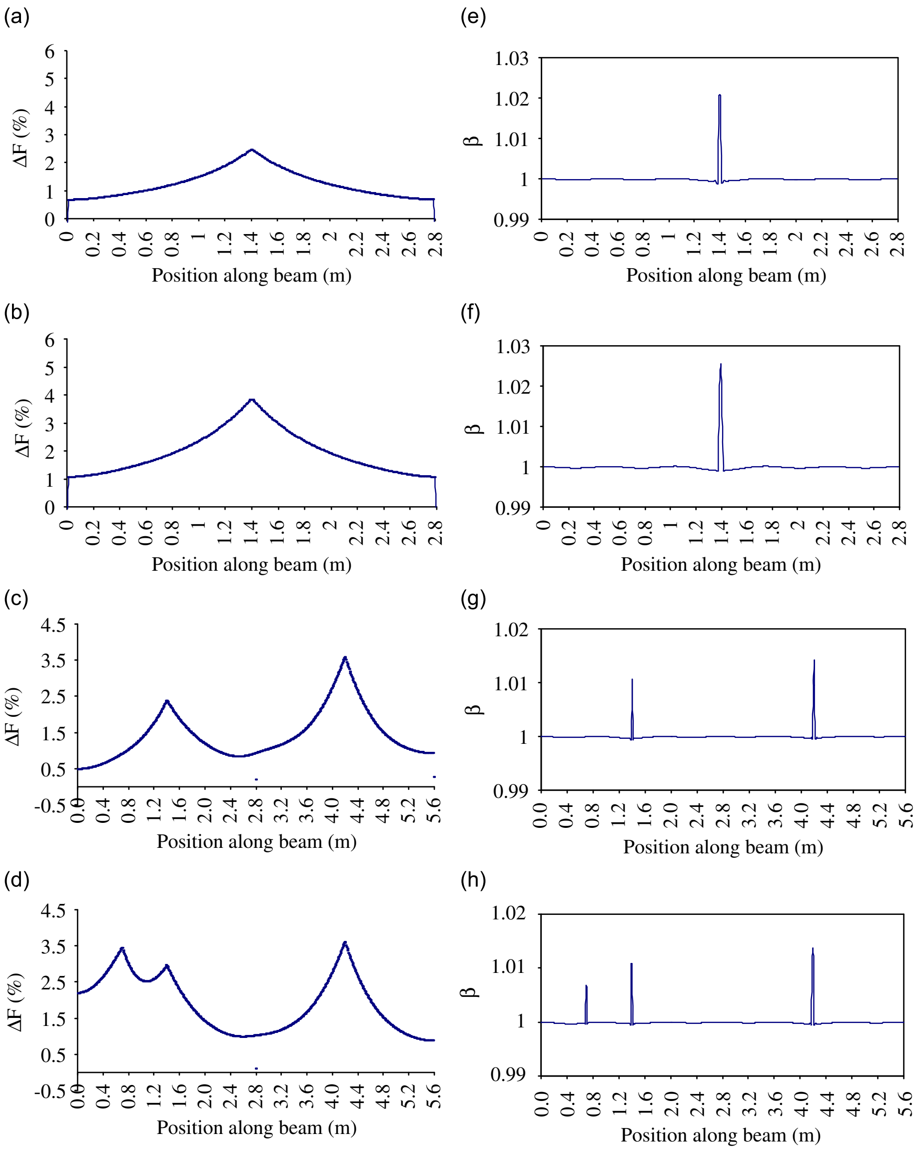

Fig. 8. Modal flexibility change (Left) and Modal strain energy based damage index (Right) on beam: (a) MFC (percent) in D1, (b) MFC (percent) in D2, (c) MFC (percent) in D5, (d) MFC (percent) in D6, (e) damage index in D1, (f) damage index in D2, (g) damage index in D5, and (h) damage index in D6.

most multiple damage cases, except in cases 7 and 8 where the damage indicator seems to have partly missed the damage at the mid-span of the plate. Similar to damage at nodes of vibrating modes not influencing the corresponding natural frequencies, this feature further demonstrates the need for multi-criteria damage assessment.

5.2. Modal strain energy change (MSEC)

The first five mode shapes and their corresponding mode shape curvatures obtained from the results of FE analysis are used to calculate the modal strain energy based damage index on beams by using Eqs. (4) and (5). The plot of damage indices along the beam for damage case 1, 2, 5 and 6 are shown in Figs. 8(e)–(h). The spikes with magnitudes greater than 1 indicate the location of damaged elements. Comparison of Fig. 8(e) and

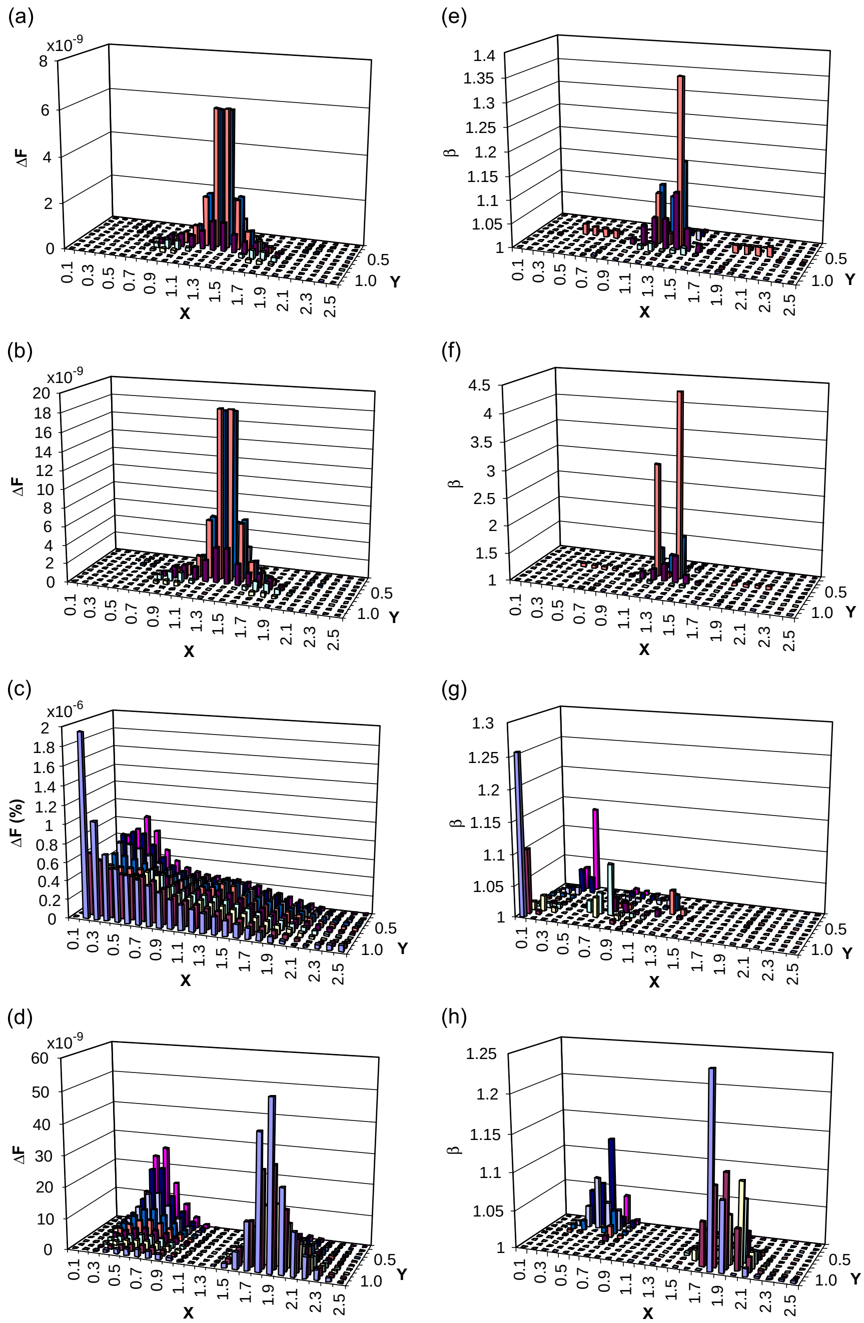

Fig. 9. Modal flexibility change (Left) and Modal strain energy based damage index (Right) on plate: (a) MFC in D1, (b) MFC in D2, (c) MFC (percent) in D8, (d) MFC in D9, (e) damage index in D1, (f) damage index in D2, (g) damage index in D8, and (h) damage index in D9.

(f) show that the peak in the damage index increases with the severity of damage. The peaks in Figs. 8(g) and (h) clearly indicate the multiple damages in the beam. From the results for all cases, it is evident that the damage index on the modal strain energy method is able to correctly locate the damage in beams in all damage cases.

Eqs. (13) and (14) are used to calculate the modal strain energy based damage index for plates. The plot of damage indices of the plates for damage case 1, 2, 8 and 9 are shown in Figs. 9(e)–(h), respectively. The spikes with the magnitudes greater than 1 indicates the location of damaged elements. The peak in Fig. 9(f) corresponding to a more severe damage case is higher than the peak in Fig. 9(e). Again, multi-peaks in the figures indicate multiple damages in the plate. Overall, the results indicate that the strain energy method is capable of detecting multiple damages in plates for all damage cases (Table 9).

A flow chart for the proposed multi-criteria damage detection system on flexural members is shown in Fig. 10. Firstly, an initial FEM is generated using FE software package. Then the experimental dynamic testing is carried out to capture the primary modal parameters. A sensitivity analysis techniques could be used for model updating and calibrate the FEM. After that, the primary modal parameters are obtained from the validated baseline model using modal analysis. Finally Level 1 damage alarm is achieved by observing the change of natural frequencies of flexural members. Level 2 damage localisation is achieved accurately by using

Table 9
Performance of damage detection algorithms.

Damage case	Beam		Plate	
	MFC	MSEC	MFC	MSEC
1	✓	✓	✓	✓
2	✓	✓	✓	✓
3	✓	✓	✓	✓
4	✓	✓	✓	✓
5	✓	✓	✓	✓
6	✓	✓	✓	✓
7	✓	✓	✓*	✓
8	✓	✓	✓*	✓
9	✓	✓	✓	✓

Note: ✓ means accurate damage localization.
*Means unclear damage indication at mid-span.

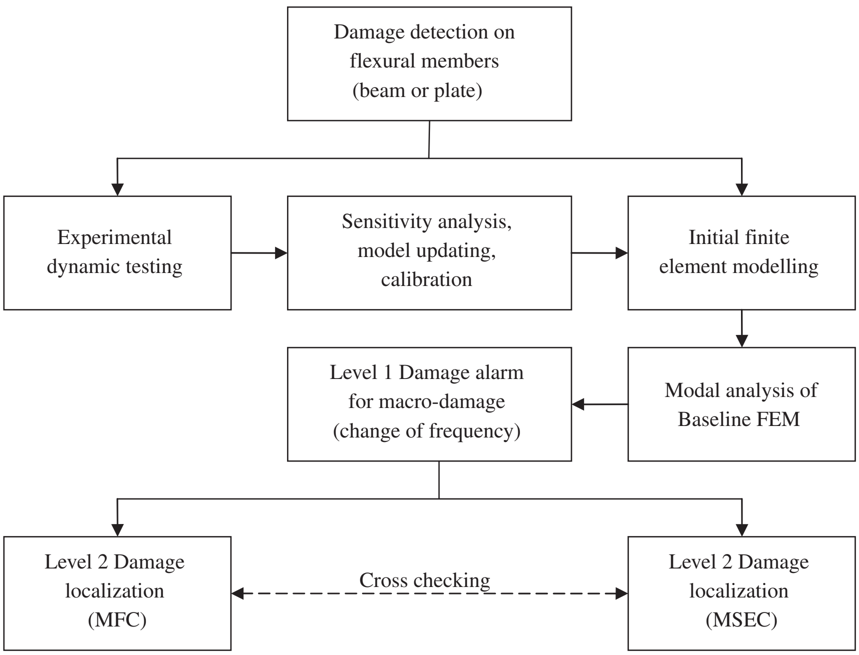

Fig. 10. Flowchart of damage detection on flexural members.

two complementary damage identification method, (1) modal flexibility method and (2) modal strain energy method.

6. Conclusions

This paper uses dynamic computer simulation techniques to develop and apply a multi-criteria based non-destructive damage detection methodology for beam and plate structures which are important flexural members. The proposed procedure involves two damage detection methods (1) modal flexibility matrix and (2) modal strain energy based damage index, in addition to change in natural frequencies, all of which are evaluated from the results of free vibration analysis of the damaged and healthy structural models. As a starting point, changes in natural frequencies can be used to detect the presence of a state of damage, since this can be done from a single point measurement. Once the presence of damage is detected, modal flexibility method and modal strain energy method can be used to locate the damage in flexural members. The changes in modal flexibility matrix and modal strain energy between the undamaged structure and the damaged structure provide a basis for identification of localised damage. For cases involving detection of a single damage in flexural members, the modal flexibility method and modal strain energy method produce similar results with no localisation error. However, for multiple damage scenarios, MSEC enhanced the accuracy of the damage localisation in plate. It is also evident that the peaks (or maxima) in the plots of MFC and damage index represent the severity of damage in single damage scenarios. In general, based on some sensitivity studies carried out, it may be concluded that in the case of a single damage, the magnitude of the spikes (or peaks) in the MFC and MSEC diagrams depend on the damage intensity. Moreover, damage at mid-spans causes a larger spike compared to damage elsewhere. However, the situation with multiple damages seems to be more complex and needs further investigation. The study also found that the damaged elements of the flexural members located near the supports can also be localised correctly provided that appropriate damage detection algorithms are used. It is concluded that applying modal flexibility method and modal strain energy method to beam and plate members provides sensitive, reliable and accuracy on multiple damage localisation. As there are some discrepancies in both the (individual) damage assessment methods, a multi-criteria procedure incorporating the changes in natural frequency, modal flexibility matrix and modal strain energy based damage index is required for accurate damage assessment, as evidenced through the examples treated in this paper.

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