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Nonlinear free vibration analysis of composite plates with material uncertainties: A Monte Carlo simulation approach

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Abstract

In this paper, the effects of dispersion in material properties on free vibration response of composite plates with geometric nonlinearity in von-Karman sense are investigated. The higher order shear deformation theory is employed for the study reported here. An efficient C^0 finite element formulation is developed for the analysis. Using Monte Carlo simulation, the second order statistics i.e., mean and standard deviation of the nonlinear free vibration response of the composite laminates are obtained for different thickness and amplitude ratios. The input statistics (mean and variance) of the material property are assumed to be known as a priori. The computed results demonstrate the influence of the variations in the material properties and amplitudes on the nonlinear free vibration response of the composite plates. The results are compared with those available in the literature.

1. Introduction

In the conventional structural analysis, the material properties like elastic modulus, Poisson's ratio, density etc. of the fiber reinforced composite laminates, are usually assumed to be deterministic quantities. Due to inherent uncertainties involved at different levels in fabrication and manufacturing processes exact values of these properties cannot be achieved and thus, these become random in nature. Because of the randomness in material properties, the mass and stiffness matrices of the composite plates become stochastic in nature. The uncertainties in the specification of mass and stiffness matrices may induce statistical variation in the eigenvalues and eigenvectors and consequently the dynamic response may be affected. Therefore a realistic analysis of composite laminated plates requires the uncertainties arising from the randomness in the material properties to be taken into account properly.

The studies in the area of stochastic analysis include the topics considering the influence of the spatial randomness of the material properties and geometrical parameters on the structural response variability [1–6]

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of the conventional materials. Literature available for analysis of composite structures with random material properties is limited. Spanos and Zeldin [7] presented a numerical method based on Galerkin approximation for solving stochastic mechanics problems by representing the solution using a small number of random parameters. Wang [8] investigated the effect of random initial geometric imperfections on the vibration behavior of rectangular plates where random initial geometric imperfections of plates were described by Gaussian random fields. A Monte Carlo analysis for simply supported plates is carried out in detail to illustrate the performance of the proposed approach. Shinozuka and Lenoe [9] proposed a probabilistic model that can be used for digital–analytical simulation of non-homogenous properties of materials. The developed model was compatible with the finite element method and thus extremely useful for the analysis and design of non-homogeneous structural systems. Singh et al. [10] employed a first order perturbation technique in combination with higher order shear deformation theory (HSDT) including rotary inertia effects to obtain the second order statistics of the natural frequencies of laminated composite plates with random material properties. Raj et al. [11] studied response of composite plates with random material properties using FEM and Monte Carlo simulation. They used HSDT [12] for the analysis. Onkar and Yadav [13] proposed a formulation based on the classical laminate theory and von-Karman nonlinear strain–displacement relation to obtain the second order response statistics of laminated plates by employing perturbation technique.

It is well known that for the deterministic analysis of moderately thick and very thick laminated plates, the use of higher order shear deformation theories is very important. Many researchers [14–18] studied nonlinear vibrations of laminated plates using first order and higher shear deformation theories based on the deterministic assumptions of the system properties. But studies to deal with uncertainties in the material properties using stochastic finite element method based on the HSDT are rather limited in literature. Within the framework of stochastic finite element method, the application of Monte Carlo simulation is quite popular. The MCS possesses the major advantage that accurate solutions can be obtained for any problem whose deterministic solution is known either numerically or analytically, since it statistically converges to the correct solution when a sufficiently large number of simulations are performed.

Keeping all these in mind, the present investigation aims at predicting the second order statistics of the nonlinear free vibration response of the laminated composite plates with random material properties. The proposed formulation is based on the HSDT [12] and von-Karman nonlinear strain displacement relation [19]. An efficient C^0 finite element formulation is developed. Finally, Monte Carlo simulation is carried out to obtain the second order statistics of nonlinear free vibration response of the laminated composite plate. The results are compared with those available in the literature to show the performance of the present formulation.

2. Formulation

Consider a rectangular laminated plate of length a, width b, and thickness h, which consists of N plies located in a three-dimensional Cartesian coordinate system (x, y, z), where the x–y plane passes through the middle of the plate thickness with its origin placed at the corner of the plate as shown in Fig. 1.

2.1. Displacement field

In the present study, the assumed displacement field is based on the higher order shear deformation theory [12] which requires C^1 continuous element for finite element approximation. In order to avoid the usual difficulties associated with these elements, the displacement model has been slightly modified to make it suitable for C^0 continuous element [20]. The modified displacement field may be expressed as

$$u(x,y,z,t) = u_0(x,y,t) + f_1(z)\psi_x(x,y,t) + f_2(z)\phi_x(x,y,t)$$

$$v(x,y,z,t) = v_0(x,y,t) + f_1(z)\psi_y(x,y,t) + f_2(z)\phi_y(x,y,t)$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{1}$$

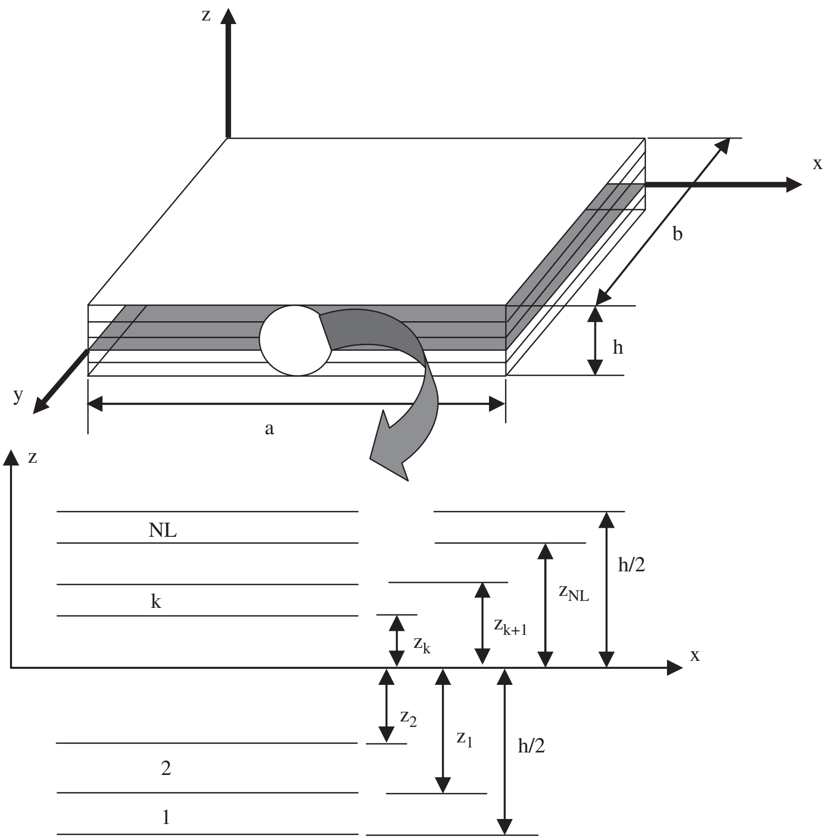

Fig. 1. Plate geometry and coordinate system.

where, u, v, and w represents the displacement of a point along the x-, y-, and z-axes respectively; u_0, v_0, and w_0 represents corresponding displacements of a point at the mid-plane, $\phi_x = \partial w_0/\partial x$ and $\phi_y = \partial w_0/\partial y$, ψ_x and ψ_y are the rotations of the normal to mid-plane about y- and x-axis, respectively; $f_1(z)$ and $f_2(z)$ may be given as $f_1(z) = C_1 z - C_2 z^3$, $f_2(z) = -C_4 z^3$, with $C_1 = 1$, $C_2 = C_4 = 4/(3h^2)$ in which h is the total thickness of the laminate.

2.2. Stress– strain relation

The stress–strain relations for a k-th lamina oriented at an arbitrary angle with respect to the reference axis for an orthotropic layer is given by

$$\{\sigma\}_k = [\overline{Q}]_k\{\varepsilon\}_k \quad \text{or} \quad \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}_k \tag{2}$$

where $[\overline{Q}]_k$, $\{\sigma\}_k$ and $\{\varepsilon\}_k$ are the transformed stiffness matrix, stress and strain vectors for the k-th lamina, respectively [21].

2.3. Strain displacement relation

The strain in the von-Karman sense is expressed as [22]

$$\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\} \tag{3}$$

where $\{\varepsilon_L\}$ and $\{\varepsilon_{NL}\}$ are linear and nonlinear strain vector, respectively.

Strain vectors corresponding to the displacement field as given in Eq. (1) may be expressed as follows:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z\begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^3\begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} + z^2\begin{Bmatrix} \gamma^2_{yz} \\ \gamma^2_{xz} \end{Bmatrix} \tag{4}$$

where

$$\begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0 \partial w_0}{\partial x \ \partial y} \end{Bmatrix}; \quad \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} = C_1\begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix}; \tag{5,6}$$

$$\begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} = -C_2\begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix} - C_4\begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}; \tag{7}$$

$$\begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} = C_1\begin{Bmatrix} \psi_y \\ \psi_x \end{Bmatrix} + \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix}; \quad \begin{Bmatrix} \gamma^2_{yz} \\ \gamma^2_{xz} \end{Bmatrix} = -3C_2\begin{Bmatrix} \psi_y \\ \psi_x \end{Bmatrix} - 3C_4\begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix}. \tag{8,9}$$

The displacement vector $\{\delta\}$ for the present model is expressed as

$$\{\delta\} = \{u_0, v_0, w_0, \phi_y, \phi_x, \psi_y, \psi_x\}^{\mathrm{T}} \tag{10}$$

2.4. Finite element modeling

A nine noded isoparametric element is employed for finite element discretization of the laminate. The displacement vector and the element geometry are represented as

$$d = \sum_{i=1}^{NN} N_i d_i, \quad x = \sum_{i=1}^{NN} N_i x_i, \quad y = \sum_{i=1}^{NN} N_i y_i \tag{11}$$

where N_i is the interpolation function for the i-th node, d_i is the vector of unknown displacements for the i-th node, NN is the number of nodes per element and x_i and y_i are Cartesian coordinate of the i-th node.

The mid-plane strain vector may expressed as

$$\{\varepsilon^0\} = \{\varepsilon^0_L\} + \{\varepsilon^0_{NL}\} = ([B_L] + [B_{NL}])\{\delta\}^e \tag{12}$$

where $\{\delta\}^e$ is the nodal displacement vector, $[B_L]$ and $[B_{NL}]$ are linear and nonlinear strain displacement matrices, respectively.

2.5. Potential energy and kinetic energy

The elemental potential energy of the plate with large deflection can be written as

$$U^e = \frac{1}{2}\int\int\left[\sum_{i=1}^{N}\int_{z_{k-1}}^{z_k}\{\varepsilon\}^{\mathrm{T}}[\overline{Q}_k]\{\varepsilon\}dz\right]dx\,dy \tag{13}$$

With the help of Eqs. (12) and (13), above equation may be expressed as

$$U^e = \frac{1}{2}\int\int\left[\sum_{i=1}^{N}\int_{z_{k-1}}^{z_k}\{\delta\}^{e\mathrm{T}}[\overline{B}]^{\mathrm{T}}[D][\overline{B}]\{\delta\}^e\,dz\right]dx\,dy \tag{14}$$

where $[\overline{B}] = [B_L] + [B_{NL}]$ is strain displacement matrix for large deformation of plates and $[D]$ is defined as

$$[D] = \begin{bmatrix} [A] & [B] & [E] & 0 & 0 \\ [B] & [D_1] & [F_1] & 0 & 0 \\ [E] & [F_1] & [H] & 0 & 0 \\ 0 & 0 & 0 & [A_2] & [D_2] \\ 0 & 0 & 0 & [D_2] & [F_2] \end{bmatrix} \tag{15}$$

with the elements of plate stiffness matrices are defined as

$$(A_{1ij}, B_{ij}, D_{1ij}, E_{1ij}, F_{1ij}, H_{ij}) = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}[\overline{Q}_{ij}]_k(1, z, z^2, z^3, z^4, z^6)\,dz, \quad \text{for } i,j = 1, 2 \text{ and } 6,$$

$$(A_{2ij}, D_{2ij}, F_{2ij}) = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}[\overline{Q}_{ij}]_k(1, z^2, z^4)\,dz, \quad \text{for } i,j = 4 \text{ and } 5. \tag{16}$$

The elemental kinetic energy (T^e) of the vibrating laminated plate can be expressed as

$$T^e = \frac{1}{2}\int\int\left[\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\rho_k\{\dot{f}\}_k^T\{\dot{f}\}_k\,dz\right]dx\,dy \tag{17}$$

where ρ_k and $\{\dot{f}\}_k = \{\dot{u}^k \;\; \dot{v}^k \;\; \dot{w}^k\}^{\mathrm{T}}$ are the mass density and the velocity vector of the k-th layer of the plate, respectively. The dot represents differentiation of displacement field with respect to time 't'. The above equation may further be expressed using Eq. (11) as

$$T^e = \frac{1}{2}\int\int\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}[[N]\{\dot{\delta}\}^e]^{\mathrm{T}}\rho_k[[N]\{\dot{\delta}\}^e]\,dx\,dy\,dz = \frac{1}{2}\int\int\{\dot{\delta}\}e\mathrm{T}[m]^e\{\dot{\delta}\}edx\,dy \tag{18}$$

where $[m]^e$ is called inertia matrix of element, $[N]$ is shape function matrix.

After evaluating the elemental potential energy (U_e) and kinetic energy (T_e) of all elements in the finite element mesh, they are assembled to get the total potential energy (U) and kinetic energy (T) of the whole laminate.

2.6. Equation of motion

The governing equation is derived using principle of virtual displacements, dynamic version of the principle of virtual work

$$0 = \int_0^t(\delta U - \delta T)\,dt \tag{19}$$

Putting the expressions of potential and kinetic energy in Eq. (19), the equation of motion of the plate is obtained as follows:

$$[M]\{\ddot{\delta}\} + [K_T]\{\delta\} = 0 \tag{20}$$

where $[M]$ and $[K_T]$ are consistent mass matrix and total tangential stiffness matrix, respectively. It is noted that the tangential stiffness matrix $[K_T]$ depends on the displacements. The expressions of $[M]$ and $[K_T]$ are given as

$$[M] = \int\int [N]^{\mathrm{T}}[m]^e[N]\,dx\,dy; \quad [K_T] = [K_L] + [K_{NL}] + [K_\sigma] \tag{21}$$

where $[K_L]$ and $[K_{NL}]$ are linear and nonlinear stiffness matrices, respectively, and $[K_\sigma]$ is the geometric stiffness matrix.

Eq. (20) may further be written assuming $\{\delta\} = \{\delta_0\}\mathrm{e}^{-\mathrm{i}\omega t}$ as

$$[K_T] - \omega^2[M] = 0. \tag{22}$$

This is the standard eigenvalue problem for solving free vibration problem.

2.7. Solution technique for solving nonlinear eigenvalue problem

The finite element model is a system of algebraic equations among the nodal values of primary variables (generalized displacements) and secondary variables (generalized forces). The coefficients of these algebraic equations contain integrals of the physical parameters (e.g., material properties) and functions used for the approximation of the primary variables. The integral expression is, in general, complicated algebraically due to spatial variation of the parameters or coordinate transformations. Therefore, a numerical integration method, the Gauss quadrature, is used to evaluate them. The nonlinear eigenvalue problem is solved employing an iterative procedure as highlighted below.

The iteration starts from a corresponding normalized initial mode shape obtained from linear analysis, putting $[K_{NL}] = 0$ and $[K_\sigma] = 0$, with amplitude scaled up by the desired value. This initial mode shape is used for evaluating the nonlinear stiffness matrix $[K_{NL}]$ and geometric stiffness matrix $[K_\sigma]$. Then an eigenvalue and associated eigenvector are obtained using standard eigenvalue extraction algorithm. This eigenvalue is again normalized and scaled up for updating the nonlinear stiffness matrix and iteration continues until the frequency evaluated from the subsequent two iterations is within the tolerance limit of 0.001%.

2.8. Solution approach for nonlinear free vibration: Monte Carlo simulation technique

The tangential stiffness matrices $[K_T]$ involves the material properties E_L, E_T, G_{LT}, G_{TZ}, ν_{LT}, ν_{TZ} out of which four material properties namely E_L, E_T, G_{LT} and ν_{LT} are treated as independent random variables. The second order statistics of the non-dimensional free vibration fundamental frequency is obtained by Monte Carlo simulation approach. A set of input sample of random numbers is generated by MATLAB software having given size, mean, and standard deviation assuming Gaussian distributions of the properties. The samples are checked for shift from the target mean value and standard deviation.

The formula for Mean and standard deviation (SD) of property x to be varied are as

$$\text{Mean}: \quad \mu = \frac{\sum_{i=1}^{n} x_i}{n} \tag{23}$$

$$\text{Standard deviation (SD)}: \quad \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{(n-1)}} \tag{24}$$

where $i = 1,2,3,\ldots, n$, with $n =$ total no. of random numbers.

3. Results and discussion

The finite element formulation developed in previous section based on Monte Carlo simulation is used to obtain the mean and standard deviation of the fundamental frequency for the composite plate with random material properties. The effect of dispersion in material property along with variation in aspect ratio and oscillation amplitude on the frequency statistics is studied. The approach is validated with those results available in literature. The second order statistics of the nonlinear fundamental frequency has been obtained for a rectangular square laminated plate with all edges simply supported cross-ply and angle ply laminates. The examples considered here assume the plate to be undergoing large amplitude vibrations. A computer program is developed in MATLAB to compute nonlinear fundamental frequency for validation of results and variation in fundamental frequency of free vibration due to variation in material properties. This program estimates the expected value and standard deviation of the nonlinear fundamental frequency by Monte Carlo simulation technique.

Boundary condition: Boundary condition considered is of simply supported (SS-2) type, *i.e*. all edges movable in their normal direction.

$$\text{at} \quad x = 0 \text{ and } a; \quad v = 0, \quad w = 0, \quad \phi_y = 0, \quad \psi_x = 0,$$

$$\text{at} \quad y = 0 \text{ and } b; \quad u = 0, \quad w = 0, \quad \phi_x = 0, \quad \psi_y = 0.$$

The fundamental frequency is non-dimensionalized as, $\varpi = \omega(a^2/h)\sqrt{\rho/E_T}$, while the standard deviation is normalized with the corresponding mean value.

3.1. Convergence and validation study: mean and standard deviation

Problem of an antisymmetric angle-ply [30°/−30°/30°/−30°/30°/−30°] square plate with all four edges simply supported is considered. All of the laminae are assumed to be of the same thickness and made of the same orthotropic material. The mean values of the elastic properties are, $E_L/E_T = 40$, $G_{LT}/E_T = G_{TZ}/E_T = 0.6$, $v_{LT} = v_{TZ} = 0.25$, $E_T = 10.23\,\text{GN}\,\text{m}^{-2}$, $\rho = 1630\,\text{kg}\,\text{m}^{-3}$. The non-dimensional linear fundamental frequencies of the plate with various mesh sizes have been obtained for the laminated composite plate with $a/h = 100$ and are presented in Table 1 which shows that there is very small change in non-dimensional fundamental frequency ϖ for increasing mesh from 7×7 to 8×8. Hence, it may be concluded that for mesh 7×7 the value of normalized fundamental frequency ϖ becomes stable. In Table 1, the linear frequency is also compared with those of Reddy [22]. The agreement between the two results is excellent.

Now, a symmetric square cross-ply [0°/90°/90°/0°] laminated plate with simply supported edges is studied. The mean values of elastic properties of lamina are $E_L/E_T = 40.0$, $G_{LT}/E_T = G_{LZ}/E_T = 0.6$, $G_{TZ}/E_T = 0.5$, $v_{LT} = v_{TZ} = 0.25$, $E_T = 10.3\,\text{GN}\,\text{m}^{-2}$, $\rho = 1522\,\text{kg}\,\text{m}^{-3}$. For normalized nonlinear fundamental frequency ratio the 7×7 mesh size is found to be adequate in modeling the full laminated plate, based on the progressive mesh refinement. The ratio of SD and mean of the natural frequency ratio, ω_{NL}/ω_L is obtained by allowing composite material properties (E_L, E_T, G_{LT}, and v_{LT}) to vary simultaneously with the SD/mean = 5%. The elastic properties of composite lamina are modeled as the basic random variables. Convergence for random numbers to be generated for various elastic composite material properties has been shown with the help of plot SD/mean of the nonlinear frequency ratio (ω_{NL}/ω_L) versus random numbers in Fig. 2. It can be observed from the figure that the curve of the SD/mean of ω_{NL}/ω_L versus number of random numbers generated becomes nearly straight line at value 9000 along x-axis. So, it is understood that 9000 random values of the

Table 1
Non-dimensional fundamental frequency $\varpi = (\omega a^2/h)\sqrt{\rho/E_T}$.

Stacking sequence [30/−30/30/−30/30/−30] antisymmetric	Mesh size	Non-dimensional fundamental frequency, $\varpi = (\omega a^2/h)\sqrt{\rho/E_T}$	
		Present study	Reddy [22], exact
$a/h = 100$	4×4	18.2541	18.1567
	5×5	18.1743	
	6×6	18.1603	
	7×7	18.1598	
	8×8	18.1591	

elastic material properties are to be generated for the satisfactory convergence of the nonlinear response of the composite plate.

The results from present formulation are now compared with those from Singh et al. [10] and Onkar and Yadav [13]. The variation in frequencies for linear strain displacement relation is obtained by putting $[K_{NL}] = 0$, in the present nonlinear formulation for simultaneous change in all basic material properties. These linear results for thickness ratio $b/h = 10$, are presented with those of Singh et al. [10] and Onkar and Yadav [13] in Fig. 3. The figure shows good agreement between the results. On the other hand, for nonlinear free vibration analysis with random material properties the results are compared only with the results given by Onkar and Yadav [13], who obtained the results based on exact analysis for nonlinear free vibration using CLT and ignoring in-plane inertias. Table 2 presents a comparison of the non-dimensional mean frequency with results given by Onkar and Yadav [13] for the plate with $a/b = 2$. The present results obtained using MCS with nonlinear formulation are placed in the table for comparison. A reasonable good agreement between the two is observed. The effect of nonlinearity is apparent with different values of the amplitudes. The difference in the results demonstrates the importance of MCS technique in the probabilistic analysis. The difference in the results is due to use of CLT by Onkar and Yadav [13] and HSDT by the present formulation besides the different solution approaches closed form and MCS.

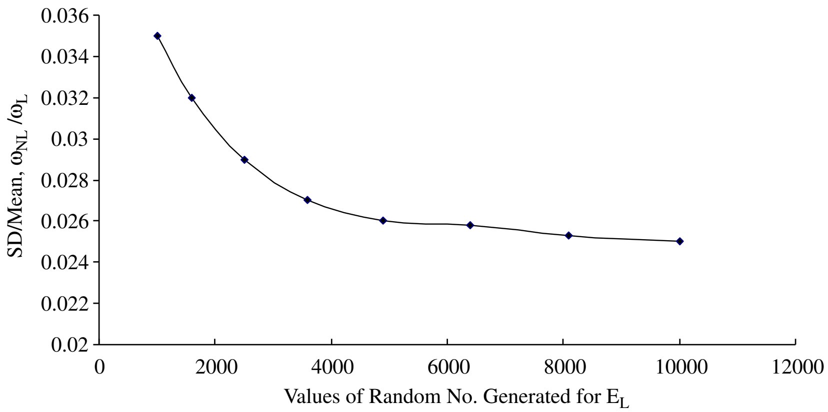

Fig. 2. Convergence of SD/mean, ω_{NL}/ω_L for simultaneous variation in E_L, E_T, G_{LT}, and v_{LT}.

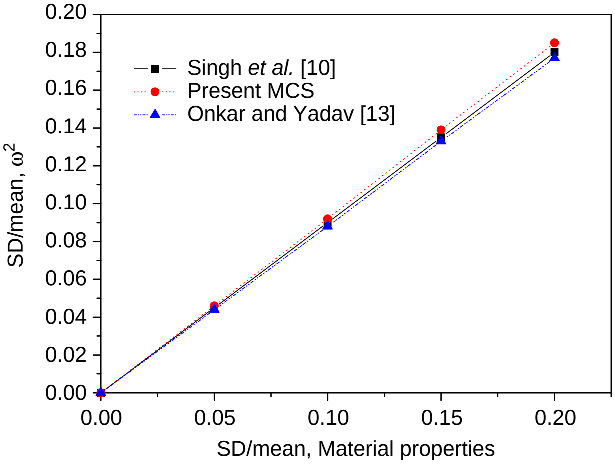

Fig. 3. Influence of scattering in all material properties simultaneously on the linear frequency with $b/h = 10$ and $a/b = 1$.

Table 2
Non-dimensional fundamental frequency ratio, ω_{NL}/ω_L for simultaneous variation in all properties.

SD/mean, material properties	References	SD/mean, ω_{NL}/ω_L		
		$w/h=0.3$	$w/h=0.6$	$w/h=0.9$
0.050	Present	0.050	0.057	0.059
	Onkar and Yadav [13]	0.048	0.054	0.058
0.100	Present	0.101	0.109	0.117
	Onkar and Yadav [13]	0.097	0.107	0.115
0.150	Present	0.148	0.163	0.176
	Onkar and Yadav [13]	0.146	0.160	0.172
0.200	Present	0.198	0.216	0.231
	Onkar and Yadav [13]	0.194	0.213	0.229

Table 3
Non-dimensional fundamental frequency ratio, ω_{NL}/ω_L for different b/h ratios and amplitude ratios (w/h).

b/h	w/h	Non-dimensional fundamental frequency ratio, ω_{NL}/ω_L	ω_L
50	0.2	1.0021	22.179
	0.4	1.0098	
	0.6	1.0189	
	0.8	1.0923	
	1.0	1.1201	
100	0.2	1.0056	22.739
	0.4	1.0278	
	0.6	1.0639	
	0.8	1.1185	
	1.0	1.1700	

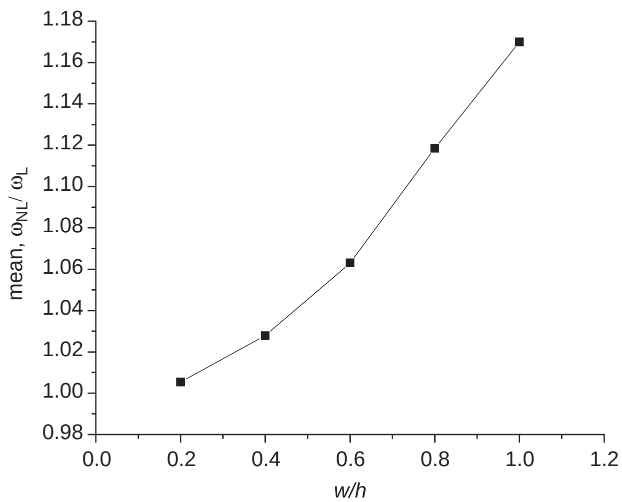

Fig. 4. Effect of the amplitude ratio (w/h) on non-dimensional frequency ratio, ω_{NL}/ω_L.

3.2. Numerical results: mean and standard deviation

The vibration of an angle-ply [45°/–45°/45°/–45°/45°] laminated plate is taken up for consideration here. The mean values of the elastic properties of lamina are, $E_L/E_T = 15.4$, $G_{LT}/E_T = G_{TZ}/E_T = 0.79$, $\nu_{LT} = \nu_{TZ} = 0.30$, $E_T = 5.171\,\mathrm{GN\,m^{-2}}$, $\rho = 1630\,\mathrm{kg\,m^{-3}}$. The non-dimensional mean fundamental nonlinear frequency ratio (ω_{NL}/ω_L) for plate considered with all edges simply supported (SS-2, type) are presented in Table 3 for $b/h = 100$, and 50. The four material properties, which are considered as random variables for the present analysis are E_T, E_L, G_{LT}, and ν_{LT}. Fig. 4 shows influence of amplitude ratio (w/h) on non-dimensional mean fundamental frequency ratio (ω_{NL}/ω_L) assuming SD/mean of material properties to be 5%. It can be observed from the figure that the mean frequency ratio increases with the increase in amplitude of vibration.

Influence of scattering in the material properties on the mean frequency ratio is obtained by allowing the ratio of standard deviation to mean to vary from 0% to 20% [23] with a sample size of 9000. In the first set of studies, dispersion of one random variable is assessed, while all other random variables are kept constant at

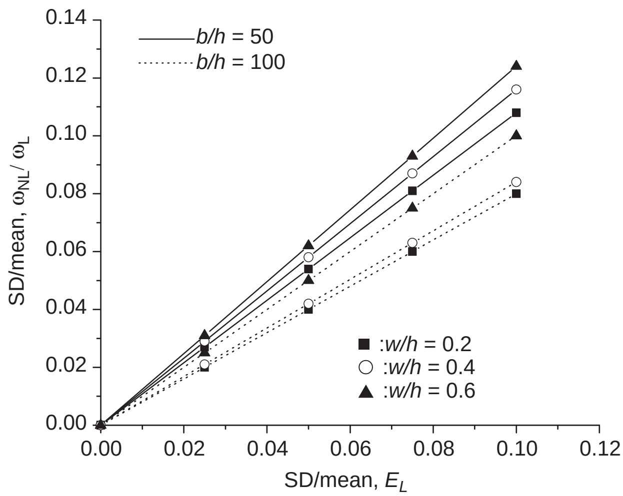

Fig. 5. Influence of the variation in E_L on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) with $a/b = 1$ for $b/h = 50$ and $b/h = 100$.

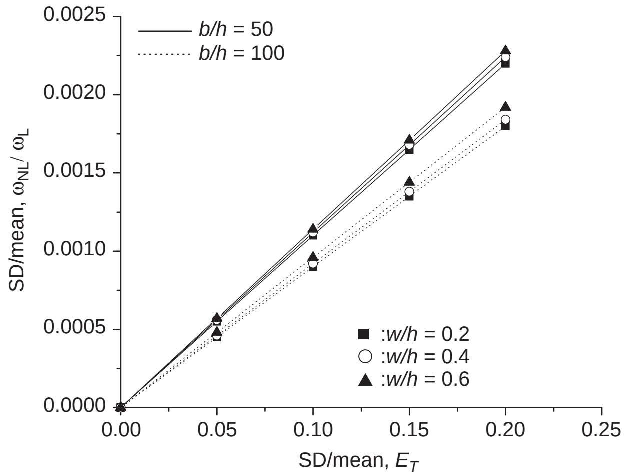

Fig. 6. Influence of the variation in E_T on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) with $a/b = 1$ for $b/h = 50$ and $b/h = 100$.

their mean values. The geometric parameter thickness ratio (b/h) is varied. The variation of non-dimensional frequency with dispersion in all basic material properties changing simultaneously are presented subsequently for square symmetric five-layered angle ply laminate for $b/h = 50$ and 100.

Figs. 5–8 show effect of individual variation in the material properties E_L, E_T, G_{LT} and v_{LT} on the non-dimensional fundamental frequency ratio for different b/h ratios. In addition, the effect of simultaneous variations in all the properties, i.e., E_L, E_T, G_{LT}, and v_{LT} on non-dimensional fundamental frequency ratio are demonstrated in Fig. 9 in which the plot of SD/mean ω_{NL}/ω_L versus SD/mean of material properties are presented for different b/h ratios and amplitude ratios (w/h). The plate shows linear variation in nonlinear fundamental frequency to linear frequency ratio within studied range, as a result of individual as well as simultaneous variations in E_L, E_T, G_{LT} and v_{LT}, for all amplitude ratios (w/h). The influence of SD of frequency shows different sensitivity at different amplitudes, at higher amplitude the variation is more than at lower amplitude, i.e, at higher amplitude the plate shows higher sensitivity to change in the material properties. The SD/mean of frequency is more for $b/h = 50$, than for $b/h = 100$, for the same plate. The effect of the individual variation in E_L is the highest on the non-dimensional fundamental frequency ratio of the

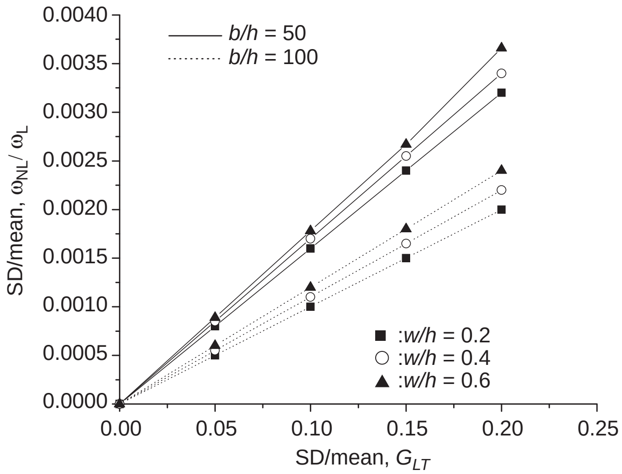

Fig. 7. Influence of the variation in G_{LT} on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) with $a/b = 1$ for $b/h = 50$ and $b/h = 100$.

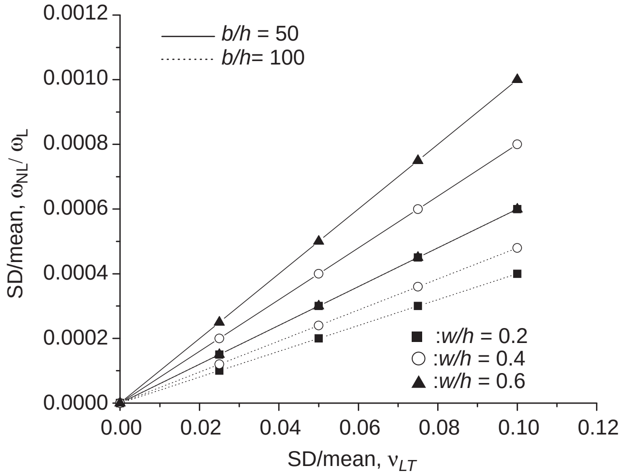

Fig. 8. Influence of the variation in υ_{LT} on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) with $a/b = 1$ for $b/h = 50$ and $b/h = 100$.

plate for both the b/h ratios. It is noted that the plate is less sensitive to the variation in E_T, in comparison to variation in E_L and it is less sensitive to the variation in G_{LT}, in comparison to variations in E_L and E_T. It can also be noted that the plate is least sensitive to the variation in v_{LT}, in comparison to the variations in E_L, E_T, and G_{LT}.

Finally, the influence of amplitude on variation in frequency has been investigated for $b/h = 100$ when all material properties assumed to vary simultaneously. The same has been presented in Fig. 10. It is found that the increase in the frequency scatter with increase in the amplitude is slightly nonlinear for the range considered for the study. It shows a rising tendency of the scatter in ω_{NL}/ω_L with the variations in the material properties.

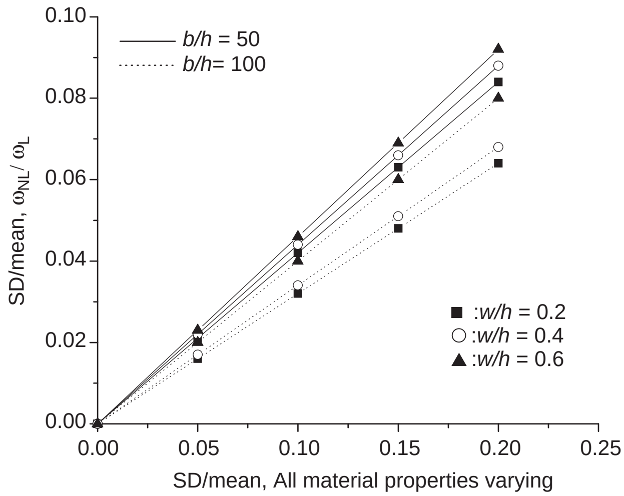

Fig. 9. Influence of scattering in all material properties varying simultaneously on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) with $a/b = 1$ for $b/h = 50$ and $b/h = 100$.

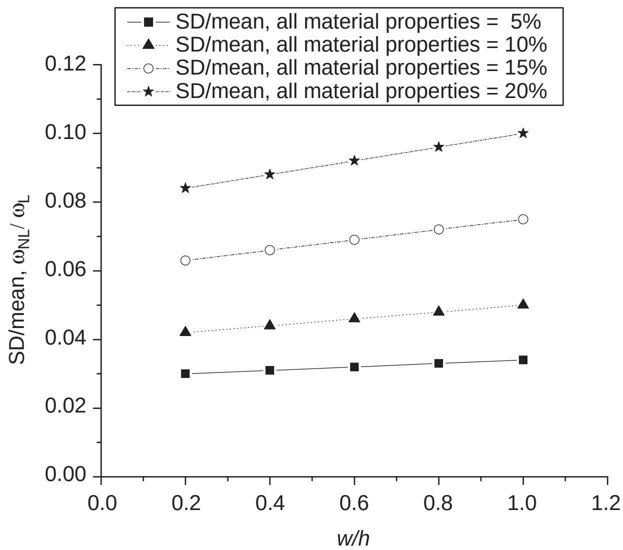

Fig. 10. Influence of amplitude ratio (w/h) on the non-dimensional frequency ratio of an angle-ply laminate (45°/–45°/45°/–45°/45°) for $a/b = 1$, $b/h = 100$, and all material properties changing simultaneously.

4. Conclusions

An approach is presented for the nonlinear free vibration analysis of composite plate with random material properties in the framework of the HSDT with nonlinear strain–displacement relations (von-Karman type), using finite element method. The following conclusions are drawn from the presented results.

1. The standard deviation in frequency shows different sensitivity to different material properties.
2. The sensitivity changes with the change in thickness of laminate and oscillation amplitude.
3. The standard deviation in frequency shows linear variation with change in standard deviation in the material properties.
4. Variation in E_L has dominant effect on the scattering of frequency as compared to other properties.
5. Plate with lower b/h ratio shows higher scattering in the frequency response.
6. At higher amplitudes the plate shows higher sensitivity than at lower amplitude.

References

[1] C.K. Choi, H.C. Noh, Weighted integral SFEM including higher order terms, *Journal of Engineering Mechanics* 126 (8) (2000) 859–866.
[2] G. Deodatis, M. Shinozuka, Bounds on response variability of stochastic systems, *Journal of Engineering Mechanics* 115 (11) (1989) 2543–2563.
[3] G. Falsone, N. Impollonia, A new approach for the stochastic analysis of finite element modeled structures with uncertain parameters, *Computer Methods in Applied Mechanics and Engineering* 191 (2002) 5067–5085.
[4] L.L. Graham, G. Deodatis, Response and eigenvalue analysis of stochastic finite element systems with multiple correlated material and geometric properties, *Probabilistic Engineering Mechanics* 16 (2001) 11–29.
[5] M. Shinozuka, G. Deodatis, Response variability of stochastic finite element systems, *Journal of Engineering Mechanics* 114 (3) (1988) 499–519.
[6] G. Stefanou, M. Papadrakakis, Stochastic finite element analysis of shells with combined random material and geometric properties, *Computer Methods in Applied Mechanics and Engineering* 193 (1–2) (2004) 139–160.
[7] P.D. Spanos, B.A. Zeldin, Galerkin sampling method for stochastic mechanics problems, *Journal of Engineering Mechanics* 120 (5) (1994) 1091–1106.
[8] F.Y. Wang, Monte Carlo analysis of nonlinear vibration of rectangular plates with random geometric imperfections, *International Journal of Solids and Structures* 26 (1) (1990) 99–109.
[9] M. Shinozuka, E. Lenoe, A probabilistic model for spatial distribution of material properties, *Engineering Fracture Mechanics* 8 (1976) 217–227.
[10] B.N. Singh, D. Yadav, N.G.R. Iyengar, Natural frequency of composite plates with random material properties using higher-order shear deformation theory, *International Journal of Mechanical Sciences* 43 (2001) 2193–2214.
[11] B.N. Raj, N.G.R. Iyengar, D. Yadav, Response of laminated composite plates with random material properties using FEM and Monte Carlo simulation, *Advanced Composite Materials* 7 (3) (1998) 219–237.
[12] J.N. Reddy, A simple higher order theory for laminated composite plates, *Journal of Applied Mechanics* 51 (1984) 745–751.
[13] A.K. Onkar, D. Yadav, Nonlinear free vibration of laminated composite plate with random material properties, *Journal of Sound and Vibration* 272 (2004) 627–641.
[14] J.N. Reddy, W.C. Chao, Large-deflection and large-amplitude free vibrations of laminated composite-material plates, *Computers and Structures* 13 (1981) 341–347.
[15] A. Bhimaraddi, Large amplitude vibrations of imperfect antisymmetric angle-ply laminated plates, *Journal of Sound and Vibration* 162 (1993) 457–470.
[16] S.R. Rao, A.H. Sheikh, M. Mukhopadhyay, Large-amplitude finite element flexural vibration of plates/stiffened plates, *Journal of the Acoustical Society of America* 93 (1993) 3250–3257.
[17] X.L. Huang, J.J. Zheng, Nonlinear vibration and dynamic response of simply supported shear deformable laminated plates on elastic foundations, *Engineering Structures* 25 (2003) 1107–1119.
[18] J.N. Reddy, A refined nonlinear theory of plates with transverse shear deformation, *International Journal of Solids and Structures* 20 (1984) 881–896.
[19] C.Y. Chia, *Nonlinear Analysis of Plates*, McGraw-Hill, New York, 1980.
[20] C.A. Shankara, N.G.R. Iyengar, A C^0 element for the free vibration analysis of laminated composite plates, *Journal of Sound and Vibration* 191 (5) (1996) 721–738.
[21] R.M. Jones, *Mechanics of Composite Materials*, 2nd ed., McGraw-Hill, Taylor & Francis Group, New York, 1998.
[22] J.N. Reddy, *Mechanics of Laminated Composite Plates Theory and Analysis*, CRC Press, Florida, 1996.
[23] W.K. Liu, T. Belytschko, A. Mani, Random field finite elements, *International Journal for Numerical Methods in Engineering* 23 (10) (1986) 1831–1845.