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Power mode shapes for early damage detection in linear structures

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Abstract

Establishing non-modal-based damage indices for non-destructive damage detection purposes, using the statistical properties of signals is a worthwhile topic and so far little literature related to this aspect can be found. As an alternative to the conventional mode shape, a new concept of power mode shape constructed using the root mean square property of response signals is proposed in this paper. Power mode shapes possess similar shapes to conventional mode shapes but are obtained by basing them on signal power spectral densities without any modal parameter extraction. In addition, two extended parameters initially named as power mode shape curvature and power flexibility are derived using power mode shapes. Damage indices are defined by using power mode shape curvature and power flexibility as two means of locating damage in different numerical and experimental structures under random excitation with the damage being indicated by prominent peaks. Satisfactory predictions are given for both single and multiple damage situations, and minor damage inducing few changes in the structural dynamic properties can also be detected, which is meaningful and important for early damage detection. The analysis results show a potential use of the proposed method for damage detection purposes. A further experimental study on real structures is an important prerequisite before the practical application.

1. Introduction

Non-destructive damage detection has been widely studied over the past decades [1–3]. Damage detection based on modal parameters and the corresponding derived indices, such as modal curvature and flexibility, can be found in a wide range of literature [1–12]. Modal parameters can be extracted by using different techniques [13–17] and generally the spectrum estimation of signals is the first step in the entire procedure. The procedure is also expected to detect any damage based on some non-modal parameters, which avoids modal parameter extraction. The study of non-modal parameters from known power spectra or spectral densities, which may be regarded as a parallel branch to conventional modal parameter extraction in frequency domain, is worthy of investigation. This kind of non-modal parameter is based on the statistical

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properties of objective signals, such as means, variances and root mean squares (RMS), and by these means the statistical methodology is introduced into the damage detection area. Using statistical analysis is expected to reduce measurement noise influence to some extent, especially for random noise. The application potential of such research can be seen but so far little research has been carried out. Therefore this study attempts to propose damage localization indices based on the statistical properties calculated using the power spectral densities (PSDs) of vibration signals. Meanwhile, the proposed indices are also expected to locate minor damage or defects.

Related to the work developed in this paper, Das and Dey [18] studied the random responses of two numerical locally damaged beams. The response PSDs of response characteristics within a cut-off frequency range were computed for damage detection on a simply supported beam and a five-span continuous beam under clipped white noise and acoustic jet noise, respectively. It was found that the displacement PSD is sensitive to the damage especially when the damage approaches the center of the simply supported beam. Cacciola et al. [19] investigated the vibration response of a numerical nonlinear cantilever beam using the Monte Carlo method to evaluate the higher order statistical properties in time domain. It was observed that the skewness coefficient of the rotational degrees of freedom is highly sensitive to the nonlinear behavior of the structure and thus can be used for damage detection purposes. In order to locate the single damage in a simply supported beam, an index combining mode shapes and RMS values estimated in the narrow bandwidths near the frequency resonances was proposed by Liberatore and Carman [20]. It was shown that in both the numerical and the experimental analysis, distinct peaks appeared at the damage locations in the index value plot along the beam length. A minor defect could be the presence of the symmetric peaks in some scenarios in which the damage might not be uniquely identified. As an alternative method to Ref. [20], Fang et al. [21] gave a damage localization index constructed on the standard deviations of structural response PSDs. The index was verified against a numerically simulated beam and an experimental reinforced concrete beam and the results demonstrated that this index appeared to be sensitive to the damage location especially for the cases studied in which noise was not present. Another recent approach using the cross correlation function amplitude vectors (CorVs) of the measured responses was described by Yang et al. [22]. The normalized CorVs were used to construct a damage location index, which was consequently used to detect any fasteners becoming loose in an aircraft panel. The authors concluded that this non-model-based approach can be applied to structural health monitoring (SHM) with steady ambient excitations.

In this study, a new concept of power mode shape (PMS), as an alternative to the conventional mode shape concept, is proposed based on the statistical analysis of vibration signals. The PMS utilizes the RMS values calculated directly from the PSDs of objective signals and is found to have a similar shape to the conventional mode shape. Meanwhile, referring to the concepts of modal curvature and flexibility, PMS is used to derive two extended parameters power mode shape curvature (PMSC) and power flexibility (PF), which are subsequently used for establishing the desired damage localization indices. Indices based on the absolute differences in PMSC and PF between damaged and undamaged structures are used to localize the damage in different structures including a single-span beam, a mechanical system and a bridge taking account of the noise effect. On the other hand, it should be noted that this is a piece of initial research to investigate the feasibility of such a statistics-based method and thus a further validation on more engineering structures is a prerequisite before any practical application.

2. Power mode shapes

2.1. Power spectral density analysis

Provided a linear structure is excited at point r by a stationary and ergodic random force $x_r(t)$ having zero mean and the corresponding response at point i is represented as $y_i(t)$, then the autocorrelation function $R_{y_i}(\tau)$ of $y_i(t)$ agrees with its mean square value (MSV), $\psi^2_{y_i}$, when the time lag $\tau = 0$:

$$R_{y_i}(0) = \psi^2_{y_i} = \lim_{T\to\infty} \frac{1}{T} \int_0^T y_i^2(t)\,\mathrm{d}t \tag{1}$$

The last term of this equation represents the average power of the output signal, which demonstrates its relation with the MSV. Since $\psi_{y_i}^2$ and its root ψ_{y_i} vary with vibration energy changes due to the variation of structural properties, both may be used as two indicators of structural integrity status.

Furthermore, there is a direct relationship between the autocorrelation function, $R_{y_i}(\tau)$, and the PSD function, $S_{y_i}(\omega)$, such that:

$$R_{y_i}(\tau) = \int_{-\infty}^{\infty} S_{y_i}(\omega) e^{\mathrm{j}\omega\tau}\, \mathrm{d}\omega \tag{2}$$

where $S_{y_i}(\omega)$ can be defined as the Fourier transform of $R_{y_i}(\tau)$ and $\mathrm{j} = \sqrt{-1}$. Therefore, the MSV of a stationary random signal is equal to the area under its PSD curve when $\tau = 0$, i.e.

$$\psi_{y_i}^2 = \int_{-\infty}^{\infty} S_{y_i}(\omega)\, \mathrm{d}\omega \tag{3}$$

which indicates that the non-modal parameter ψ_{y_i} can be easily computed from the corresponding PSD.

Additionally, for a linear system, the output PSD under a single excitation is defined as:

$$S_{y_i}(\omega) = H_{i,r}^*(\omega) H_{i,r}(\omega) S_{x_r}(\omega) = |H_{i,r}(\omega)|^2 S_{x_r}(\omega) \tag{4}$$

where $S_{x_r}(\omega)$ is the input PSD and $H_{i,r}(\omega)$ is the frequency response function (FRF) with '*' representing the conjugate value.

From Eqs. (3) and (4), ψ_{y_i} is calculated as:

$$\psi_{y_i} = \left\{ \int_{-\infty}^{\infty} S_{y_i}(\omega)\, \mathrm{d}\omega \right\}^{1/2} = \left\{ \int_{-\infty}^{\infty} |H_{i,r}(\omega)|^2 S_{x_r}(\omega)\, \mathrm{d}\omega \right\}^{1/2} \tag{5}$$

2.2. Feasibility analysis of the PMS concept

Let us suppose a linear structure with a proportional damping $[C] = \alpha[M] + \beta[K]$, based on the modal theory, then the generalized FRF of point i under the excitation $x_r(t)$ can be written as follows:

$$H_{i,r}(\omega) = \sum_{p=1}^{n} \frac{\phi_p^i \phi_p^r}{k_p - \omega^2 m_p + \mathrm{j}\omega c_p} = \sum_{p=1}^{n} \frac{\phi_p^i \phi_p^r / m_p}{\omega_p^2 - \omega^2 + \mathrm{j}\omega(\alpha + \beta\omega_p^2)} \tag{6}$$

where k_p, m_p and $c_p = \alpha m_p + \beta k_p$ are the modal stiffness, mass and damping of pth mode, respectively; α and β are the damping coefficients for modal mass and stiffness; ϕ_p^i and ϕ_p^r are the pth mode shape vector at points i and r; ω_p and ω represent the circular frequencies of the structure and the external excitation, respectively; and n is the number of modes considered.

It can be observed from Eq. (6) that for a separate mode p, the denominator remains the same due to the identical changes of ω_p, ω and damping coefficients. Thus for different response points, the only difference in the function $H_{i,r}(\omega)$ lies in the mode vector element ϕ_p^i at point i since ϕ_p^r is defined as a reference point where the excitation locates. Then replacing Eq. (5) into (6) gives:

$$\psi_{y_i} = \left\{ \int_{-\infty}^{\infty} |H_{i,r}(\omega)|^2 S_{x_r}(\omega)\, \mathrm{d}\omega \right\}^{1/2} = \left\{ \int_{-\infty}^{\infty} \left| \sum_{p=1}^{n} \frac{\phi_p^i \phi_p^r / m_p}{\omega_p^2 - \omega^2 + \mathrm{j}\omega(\alpha + \beta\omega_p^2)} \right|^2 S_{x_r}(\omega)\, \mathrm{d}\omega \right\}^{1/2} \tag{7}$$

By considering in our analysis each mode p separately in specific bandwidths, $[\omega_{p1}, \omega_{p2}]$, Eq. (7) can be written as a bandwidth-localized RMS:

$$\psi_{y_i,p} = \left\{ (\phi_p^i)^2 \int_{\omega_{p1}}^{\omega_{p2}} \left| \frac{\phi_p^r / m_p}{\omega_p^2 - \omega^2 + \mathrm{j}\omega(\alpha + \beta\omega_p^2)} \right|^2 S_{x_r}(\omega)\, \mathrm{d}\omega \right\}^{1/2} = D\phi_p^i \tag{8}$$

where $\psi_{y_i,p}$ is the RMS value of the pth PMS localized in the frequency bandwidth $[\omega_{p1}, \omega_{p2}]$, which may be named as a fractional root mean square (FRMS); $D = \left\{ \int_{\omega_{p1}}^{\omega_{p2}} \left| \frac{\phi_p^r / m_p}{\omega_p^2 - \omega^2 + \mathrm{j}\omega(\alpha + \beta\omega_p^2)} \right|^2 S_{x_r}(\omega)\, \mathrm{d}\omega \right\}^{1/2}$ can be regarded as a coefficient and ω_{p1} and ω_{p2} are the band frequencies. In this initial study without considering the complicated modal situation, the integrated frequency bandwidth for power mode i is defined as $[(f_i + f_{i-1})/2, (f_{i+1} + f_i)/2]$ for simplicity, where f_i is the modal frequency of mode i. This definition of each individual frequency range is sure to include the core information of each mode. It should be noted that for real structures with highly coupled modes or high damping ratios, the choice of frequency bandwidths will undoubtedly be difficult and practical, appropriate bandwidths should be chosen based on further study and on specific structures.

Eq. (8) shows, from a theoretical point of view, the relation between the statistical quantity $\psi_{y_i,p}$ and the physical quantity ϕ_p^i through the coefficient D and gives the reason why the proposed concept is called 'power mode shape' by shape analogy with the conventional mode shape. However, the two concepts should not be deemed to be the same, since PMS is defined from a statistical point of view while the conventional mode shape is defined from modal analysis theory.

Eq. (8) also shows that the changes in structural properties affecting the conventional mode shapes will affect $\psi_{y_i,p}$ as well. Therefore, $\psi_{y_i,p}$ may be used as ϕ_p^i for damage detection purposes even though they come from different methodologies. And since $\psi_{y_i,p}$ is actually a statistical property of random signals, it is expected to present higher robustness against measurement noise, particularly against random noise. Hence this new concept of PMS represents an alternative to conventional mode shapes for damage detection purposes.

2.3. Construction of PMSs

In spite of Eq. (8), in the proposed method $\psi_{y_i,p}$ are not calculated from mode shapes since they are evaluated directly from the output spectrum without any modal analysis. For this purpose, if the same principle used in Eq. (8) is applied directly to the output PSD in Eq. (5), we obtain the following expression:

$$\psi_{y_i,p} = \left\{ \int_{\omega_{p1}}^{\omega_{p2}} S_{y_i}(\omega)\, \mathrm{d}\omega \right\}^{1/2} \tag{9}$$

By assembling all $\psi_{y_i,p}$ for all the measurement points considered in the structure, a pth PMS vector is generated. The same procedure would be repeated for each PMS by choosing the appropriate bandwidth affecting each resonance frequency. Since all the components of a PMS vector are always positive, a positive or negative sign should be assigned to each element of a PMS vector in order to configure a 'shape' similar to the conventional mode shapes. In fact, the PMSs also represent the vibration shapes of a structure and thus the estimation of signs for each desired position (node) can be easily implemented by experiential judgment for simple structures. If the vibration towards one side of the equilibrium position is assigned as positive, the other side is assumed to be negative. For complex structures, the main way of determining the signs is still experiential judgment but sometimes to verify the experiential estimation, an FE simulation in which the vibration shapes of an objective structure can be known can also be used if the FE model is available. However such numerical simulation does not indicate the need for an experimental modal analysis on real structures. Once the signs have been assigned, the pth PMS is finally established.

However, it should be remarked that the minimal number of measurement points for a workable PMS should guarantee an approximately smooth shape. And for complex structures with highly coupled modes, high damping or strong nonlinear behavior, the definition of the frequency bandwidth for each power mode will not be an easy task. The study performed here represents a first approximation concerning the feasibility of the proposed method in the damage identification area and, therefore, its validation is initially evaluated in more simple problems.

2.4. PMSs vs. conventional mode shapes

Both concepts, PMSs and conventional mode shapes, should firstly require a spectral analysis of vibration signals in frequency domain. Then the modal parameter extraction procedure can be carried out on spectra, or FRFs, to obtain the mode shapes. Basic modal parameters have shown their performance in damage detection problems either directly or through parameters derived from them, such as modal curvature and flexibility. Alternatively, it might be expected that a direct derivation of spectra (spectral densities) might attenuate the influence of measurement uncertainties and using the statistical properties of random signals to establish some new concepts could be interesting. Against this background, the concept of PMSs has been proposed in this study. In this sense, PMSs and conventional mode shapes may be regarded as two different concepts using different parameter extraction methods.

3. Damage localization indices based on PMSs

By extending the concepts of modal curvature and flexibility to our case, two new damage localization indices have been defined based on PMSs.

3.1. Power mode shape curvature (PMSC) index

The PMSC at each measurement point i of pth mode is defined and formulated by using a central finite difference scheme:

$$\kappa_{i,p} = \frac{\psi_{y_{i+1},p} - 2\psi_{y_i,p} + \psi_{y_{i-1},p}}{l_i^2} \tag{10}$$

where $\kappa_{i,p}$ denotes the PMSC magnitude at point i and l_i is the distance between two measurement points $(i+1)$ and i. Replacing Eq. (8) into (10) gives:

$$\kappa_{i,p} = D\frac{\phi_p^{i+1} - 2\phi_p^i + \phi_p^{i-1}}{l_i^2} \tag{11}$$

which demonstrates that PMSC has a similar inherent ability to locate the damage as does the conventional modal curvature. However, PMSC does not require any modal parameter extraction because $\psi_{y_i,p}$ in Eq. (10) is obtained through simple spectral analysis, which is an advantage of using this method.

Then the damage locations can be detected by the largest absolute differences between the PMSCs of undamaged and damaged structures:

$$\Delta\Upsilon_p = |\Upsilon_p^u - \Upsilon_p^d| \tag{12}$$

where $\Delta\Upsilon_p$ denotes the absolute difference of two PMSC vectors; Υ_p^u and Υ_p^d denote the undamaged and damaged pth PMSC vectors comprising all $\kappa_{i,p}$, respectively.

3.2. Power flexibility (PF) index

Another damage localization index based on power flexibility is also proposed. Referring to the principle of modal flexibility, PF can be similarly defined as:

$$\check{\mathbf{F}} = \check{\mathbf{\Phi}}\Lambda^{-1}\check{\mathbf{\Phi}}^T \quad \text{with diagonal elements of } \sum_{p=1}^{n}\frac{(\psi_{y_i,p})^2}{\omega_p^2} \tag{13}$$

where $\check{\mathbf{F}}$ denotes the PF matrix; $\check{\mathbf{\Phi}}$ denotes the PMS matrix consisting of n fundamental power modes and Λ is the eigenvalue square matrix consisting of circular frequency ω in rad/s or modal frequency f in Hz. In Eq. (13), only the diagonal elements equal to the self-product of the FRMS value at each measurement point are used for simplicity.

As illustrated in subsequent case studies, PF inherits the merit of conventional flexibility in that the first few fundamental power modes are enough for a reliable prediction. But unlike conventional flexibility, which is defined by physical quantities, for its definition PF employs both statistical parameters of vibration signals and modal frequencies. It should be clarified here that in this sense, an attempt to combine both statistical and modal properties into a new parameter has been made. However, this does not invalidate the proposed PF method because the modal frequencies are relatively easy to measure from spectra and the dominant factor in PF elements (Eq. (13)) is the RMS when the frequency change before and after damage is very small.

The change of PFs between before and after damage, $\Delta\check{\mathbf{F}}$, has been used as damage localization index:

$$\Delta\check{\mathbf{F}} = \check{\mathbf{F}}_d - \check{\mathbf{F}}_u \tag{14}$$

where the subscripts d and u denote the damaged and undamaged states, respectively.

4. Case study I: a numerical single-span beam

4.1. Description

A numerical simulation on a single-span beam was firstly adopted to examine the performance of the damage indices proposed in Section 3. A schematic diagram of this beam with its geometric dimensions and material properties is shown in Fig. 1. The beam was assumed to be lightly damped with a constant damping ratio of 0.2%. For analysis purposes the beam was divided into 20 two-dimensional beam elements and fixed ends were considered.

The damage was numerically simulated by introducing stiffness reduction into the elements assumed as damaged. To excite the undamaged and damaged beam, a simulated white-noise force spectrum with constant amplitude of 1.0 was applied to node 10 adjacent to the mid-span of the beam (Fig. 1). Subsequently, the vertical acceleration response PSD of each node was analyzed to construct the PMS of the first three flexural modes. The PMSs of the beam take similar shapes to the conventional mode shapes of this beam, as compared in Fig. 2.

4.2. Results and discussions

4.2.1. Noise-free damage scenarios

To study the efficiency and reliability of the proposed damage indices, both single and multiple damage scenarios were considered with a constant stiffness reduction of 3% inducing in the first 3 frequencies an average reduction of 0.05% for single damage scenario and 0.1% for multiple damage scenario. This severity level was chosen to investigate the sensitivity of the proposed indices to minor damage. Element 10 at the mid span of the beam was simulated as damaged for the single damage scenario while elements 5, 10 and 15 were considered to be damaged in the multiple damage scenario.

Fig. 3(a) and (c) shows that both PMSC and PF methods identify the location of the damaged element suitably in the case of single damage since a clear peak appears at the location of element 10. Fig. 4(a) shows that the PMSC index performs well for the multiple damage case except the first PMSC does not give clear

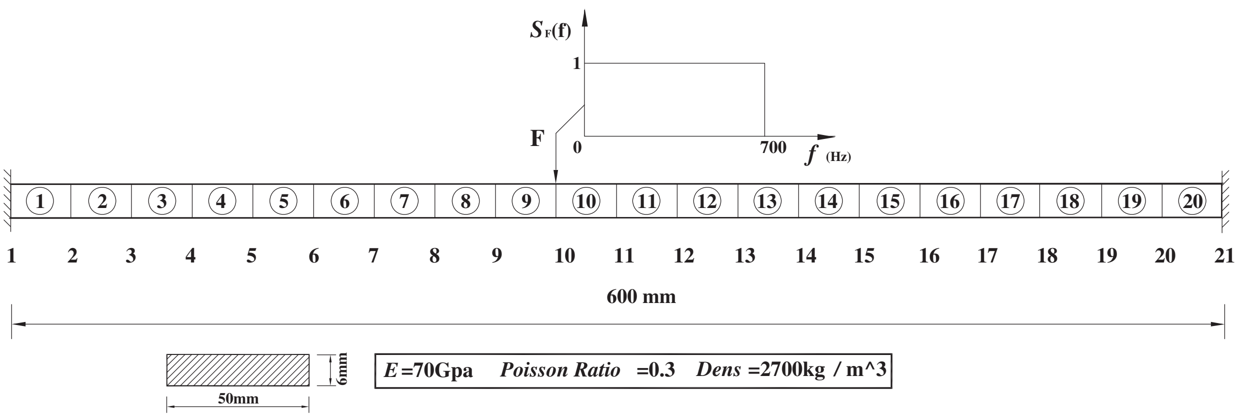

Fig. 1. Schematic diagram of the single-span beam.

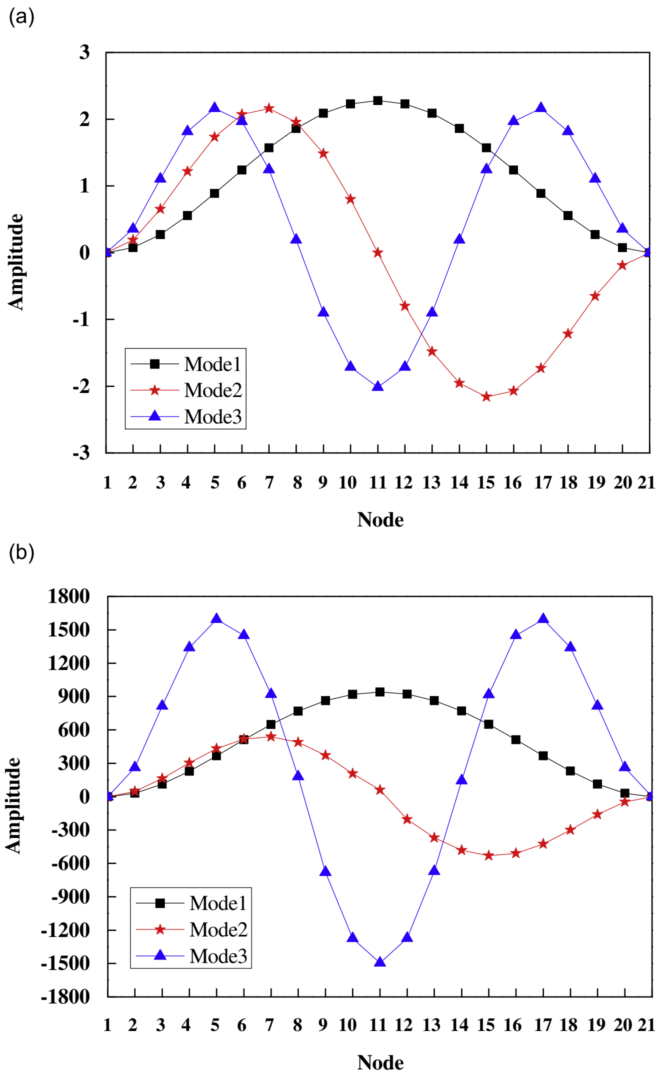

Fig. 2. Comparison of (a) conventional and (b) power mode shapes of the single-span beam.

peaks for damaged elements of 5 and 15. Fig. 4(c) presents a good prediction using the PF index for a noise-free situation and it should be mentioned that only the second and third PMSs were used to calculate the PFs here due to the unsatisfactory performance of the first PMS in PMSC prediction. This means that the PMSs for PF calculation may be selected according to their performance in the PMSC index prediction.

4.2.2. Noisy damage scenarios

The proposed indices satisfactorily predict damage distribution when noise is not present. However, for experimental modal testing, recorded data that are contaminated with measurement noise are usual. For this reason, the consideration of simulated measurements with artificial noise is very important for testing the stability and robustness of the proposed algorithm. To perform this, some random noise was added to the theoretically calculated spectral densities. A noise level of 15% normally distributed with mean zero and a standard deviation of one was considered.

It can be observed in Fig. 3(b) and (c) that for the single damage case, the proposed two indices predict the damage location satisfactorily except for a false detection by the third PMSC. However for the multiple damage scenario, the prediction becomes worse, as shown in Fig. 4(b) and (c). The simultaneous presence

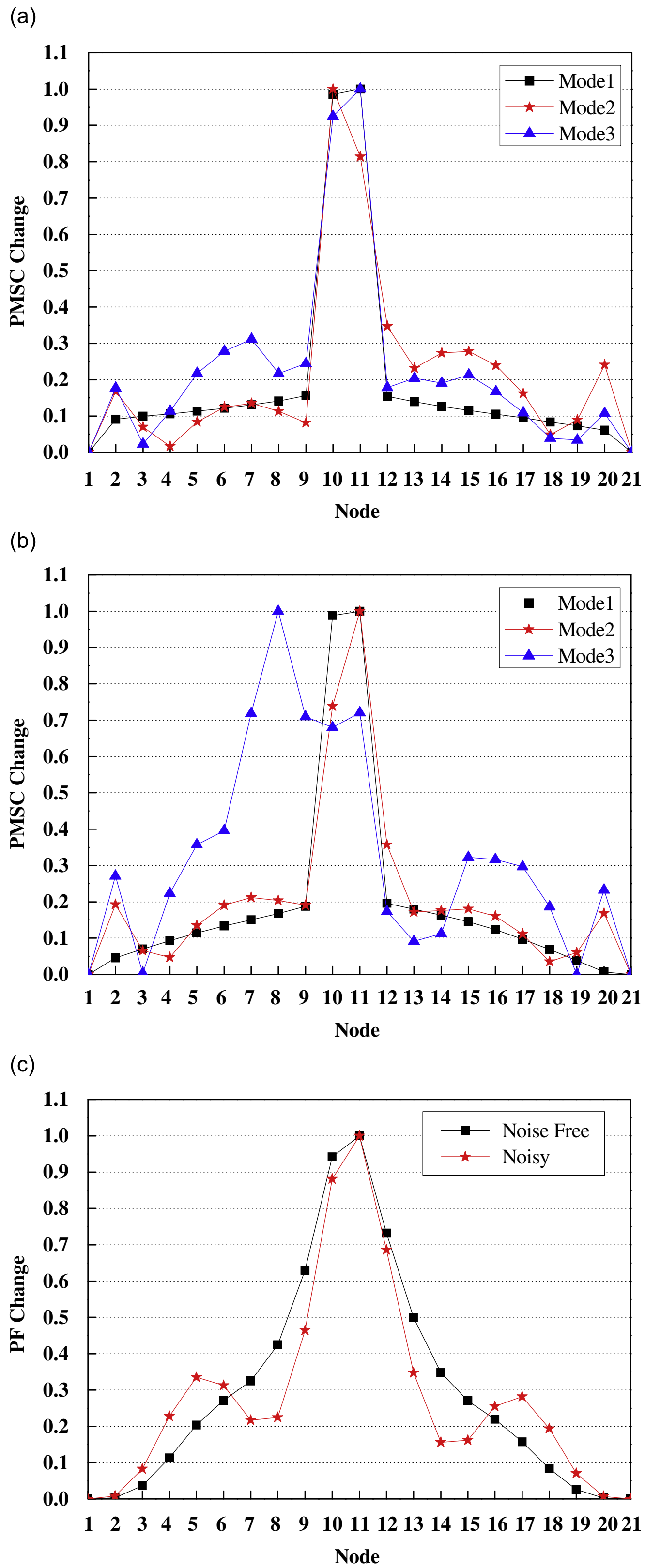

Fig. 3. Prediction of (a) noise free PMSC, (b) noisy PMSC and (c) PF indices with damaged element 10.

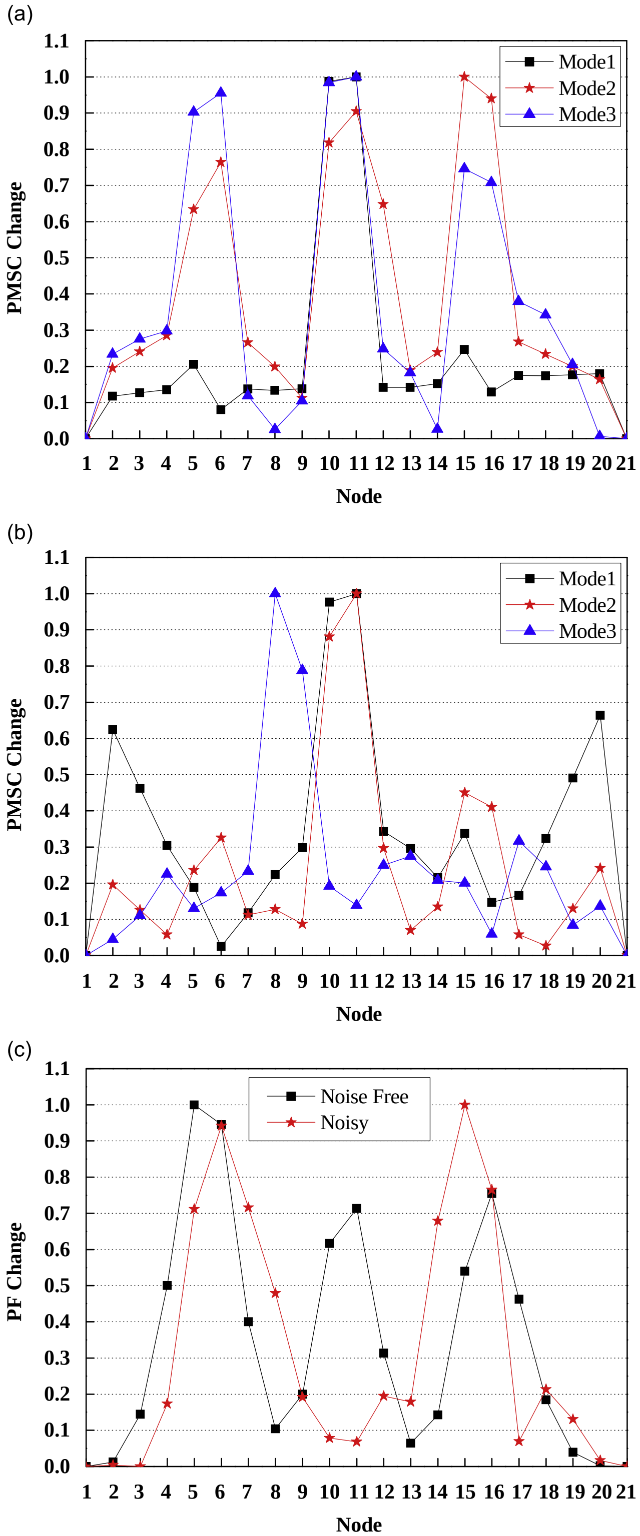

Fig. 4. Prediction of (a) noise free PMSC, (b) noisy PMSC and (c) PF indices with damaged elements 5, 10 and 15.

of 3 damaged elements makes the damage detection more difficult. Meanwhile, minor damage inducing almost no changes to the structural properties plus a noise level of 15% also increases the difficulty in localization. Fortunately, the PMSC index can approximately point out the possible damaged elements and the PF index predicts two of the three damaged elements.

In this case study, the proposed PMSC and PF indices are numerically validated in a beam with minor damage and the successful predictions for both noise-free and noisy scenarios indicate the potential use of these two indices for early damage detection.

5. Case study II: eight degree-of-freedom (8-dof) mass–spring system

5.1. Problem description

An experimentally tested 8-dof mechanical system [23] (Fig. 5) was also used for the validation of the proposed damage indices. This system consists of eight translating masses connected by seven coil springs where the masses are able to slide along a highly polished steel rod. In the undamaged state, all the masses are identical with the same geometric dimensions and weight of 419.4 g except the 559.3 g weight attached to the force transducer. All the springs share the same material properties with a linear spring constant of 56.7 kN/m. The linear experiment with single damage under random excitation was adopted for this study. The linear damage was implemented by replacing the original spring of stiffness k_5 with another resulting from applying a stiffness reduction of 14%.

A peak force electro-dynamic shaker was used to apply a random excitation to the leftmost mass of 559.3 g and one force transducer and eight accelerometers were used to record the excitation and the corresponding responses of the eight masses. The record duration was 8 seconds with a sampling rate of 512 Hz, and spectra with a frequency up to around 75 Hz were used. The first 3 modes were analyzed with the corresponding frequencies of 22.57, 44.82 and 65.08 Hz for the undamaged system and 19.89, 41.66 and 64.38 Hz for the damaged system. It should be mentioned that this system has complex vibration modes and is damped mainly by Coulomb friction. A conventional modal analysis on this system might be difficult to implement and, therefore, an analysis using PMS-based methods was performed.

5.2. Results and discussion

In order to examine the detection ability of the proposed indices in the case of noisy signals, the eight experimental output PSDs with and without filtering were used to construct the PMSs. The comparison of different predictions using PMSC and PF indices are shown in Figs. 6 and 7, respectively. It should be mentioned that because the damage was placed on the spring k_5 affecting the measurements of two accelerometers located at the masses 5 and 6, the appearance of peaks at dofs 5 and 6 in the PMSC- and PF-Change plots was acceptable. For the PMSC index, Fig. 6 shows that the damaged spring k_5 can be located in an approximate way for both filtered and noisy scenarios using the first 3 modes. In addition, using the filtered PSDs, the predictions seem better particularly for the second and third PMSCs. Then for the PF index,

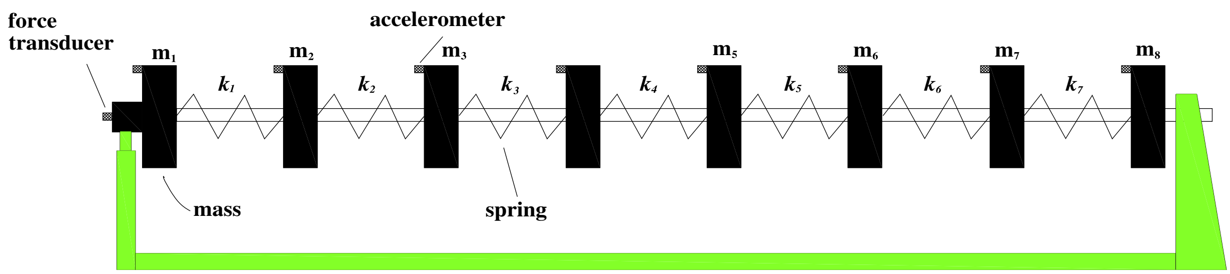

Fig. 5. Schematic diagram of the 8-dof mass–spring system.

the first 3 power modes were used to compute the damage index and the damage was successfully detected for both filtered and noisy scenarios, as shown in Fig. 7. It was found that the PF index performs well in this case study with single damage even when the experimental PSDs are corrupted by noise.

It can be seen that the proposed indices work well in this experimental case, even in the noisy environment. This proves the possible usage of these indices in real structures although more general conclusions would require further studies.

6. Case study III: numerical I-40 bridge

6.1. Problem description

The I-40 bridge over the Rio Grande in New Mexico (USA) was dynamically tested by Farrar et al. [24] and its dataset was analyzed in the past by many researchers using different damage detection methods. With the

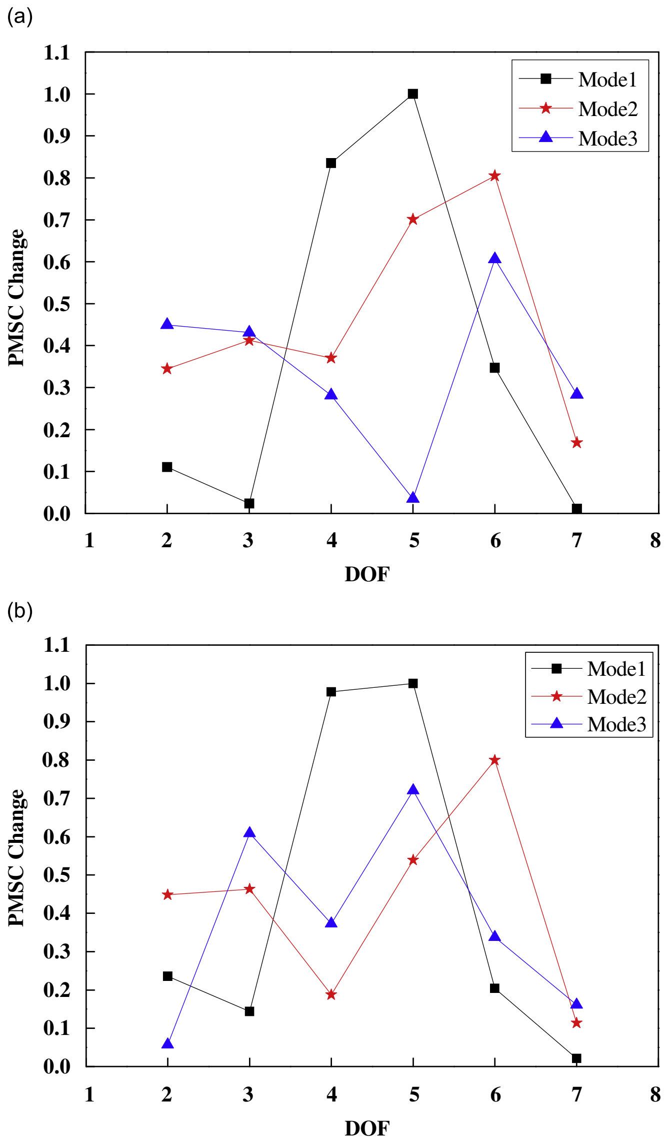

Fig. 6. Prediction of the PMSC index using (a) filtered and (b) noisy spectral densities.

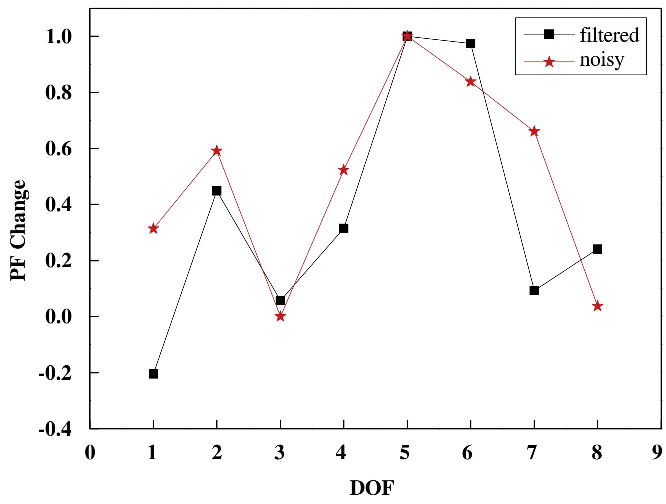

Fig. 7. Prediction of the PF index using filtered and noisy spectral densities.

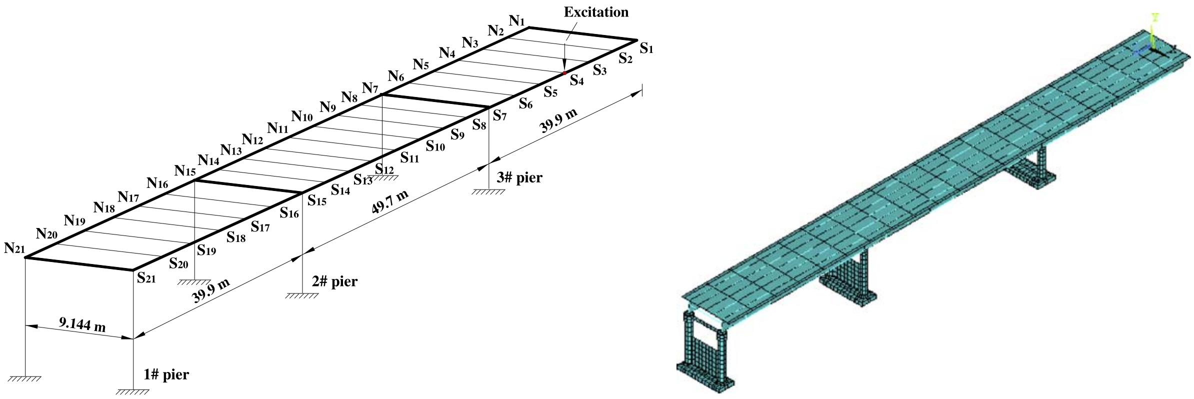

Fig. 8. Schematic diagram of the I-40 bridge and its benchmarked FE model.

same purpose, an FE benchmarked model of the bridge has been used in this work to evaluate the feasibility and reliability of the proposed method under different damage scenarios.

The I-40 bridge consists of twin spans made up of a concrete deck supported by two welded-steel plate girders and three steel stringers [24]. Loads from the stringers are transferred to the plate girders by floor beams. One portion of one of the bridge's twin spans with 3 continuous spans (39.9 m + 49.7 m + 39.9 m) has been considered in this study. The benchmarked FE model named BR3W developed by Farrar et al. [25] has been adopted in this study (Fig. 8). In the FE model, shell elements were used to model the girder webs and the concrete decks. Beam elements were used to model the girder flanges, floor beams and stringers and solid elements were used to model the concrete piers. The corresponding geometric and material properties for each element type were defined according to Ref. [25] and the numerical bridge was assumed to have a constant damping ratio of 1% for each mode. The calculation of desired PSDs was implemented with the ANSYS® finite element code [26].

6.2. Damage detection of the numerical model of the I-40 bridge

In order to study the performance of the proposed damage indices on this large-scale bridge, both single and multiple damage scenarios were investigated on the FE numerical model considering both noise-free and noisy cases. The damage was introduced through a stiffness reduction of the north plate girder. Two different single

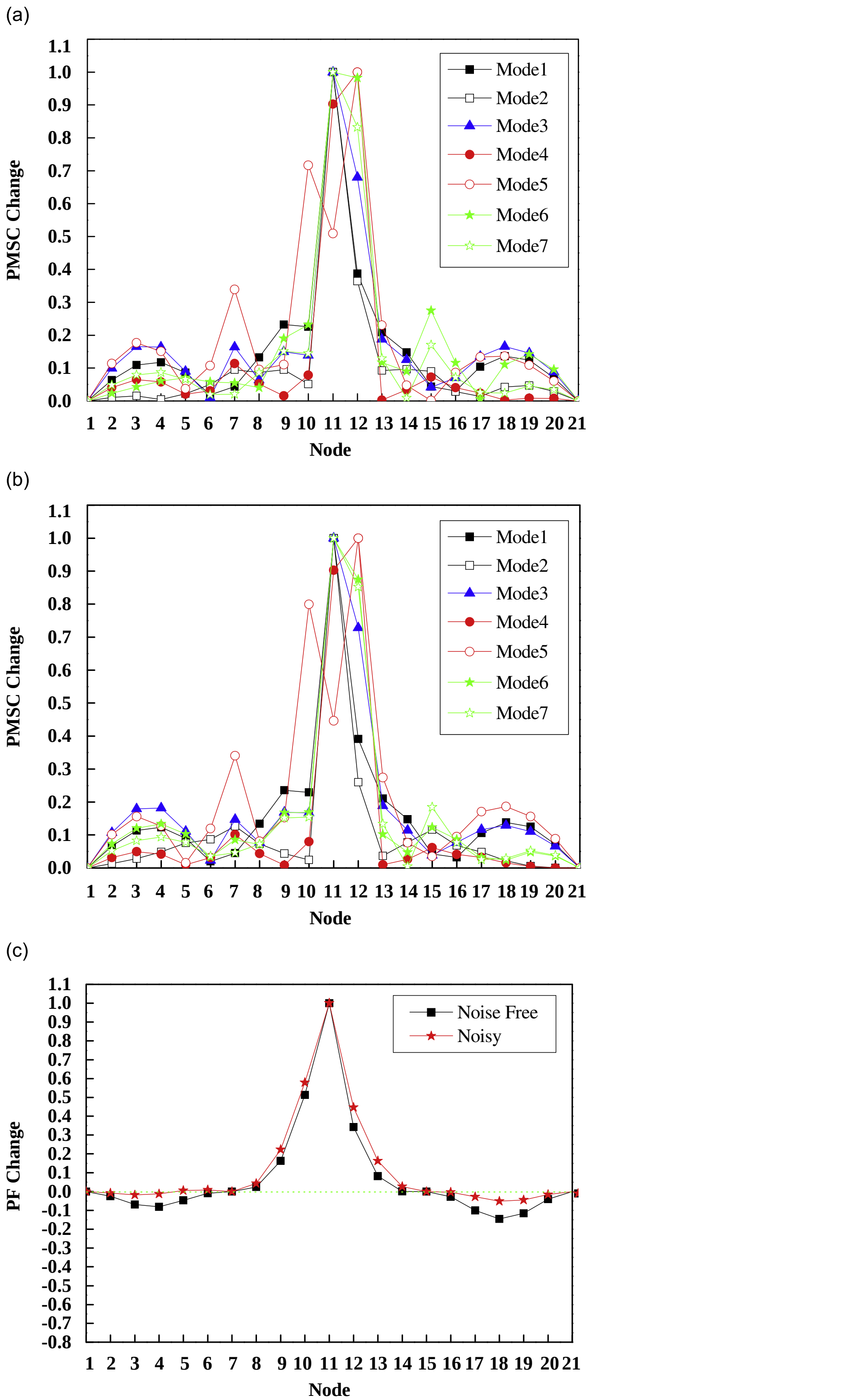

Fig. 9. Prediction of (a) noise free PMSC, (b) noisy PMSC and (c) PF indices with single damage between measurement nodes 11 and 12.

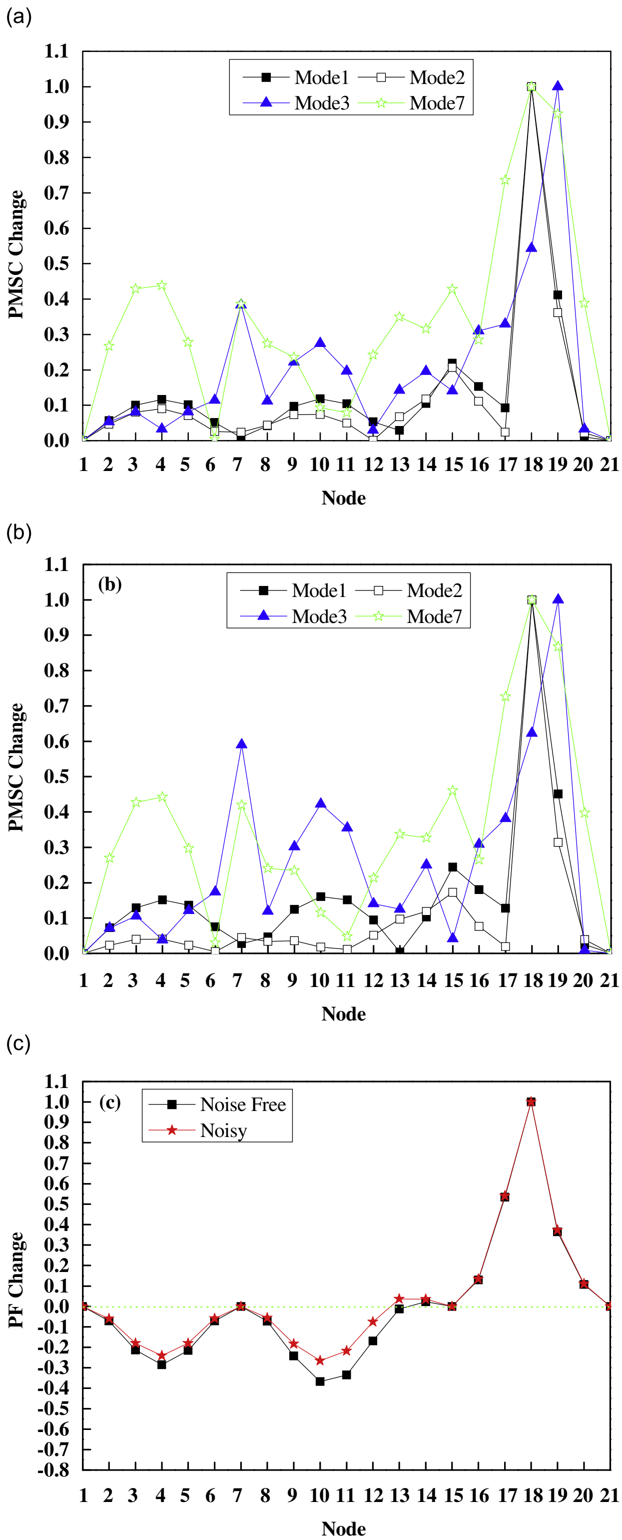

Fig. 10. Prediction of (a) noise free PMSC, (b) noisy PMSC and (c) PF indices with single damage between measurement nodes 18 and 19.

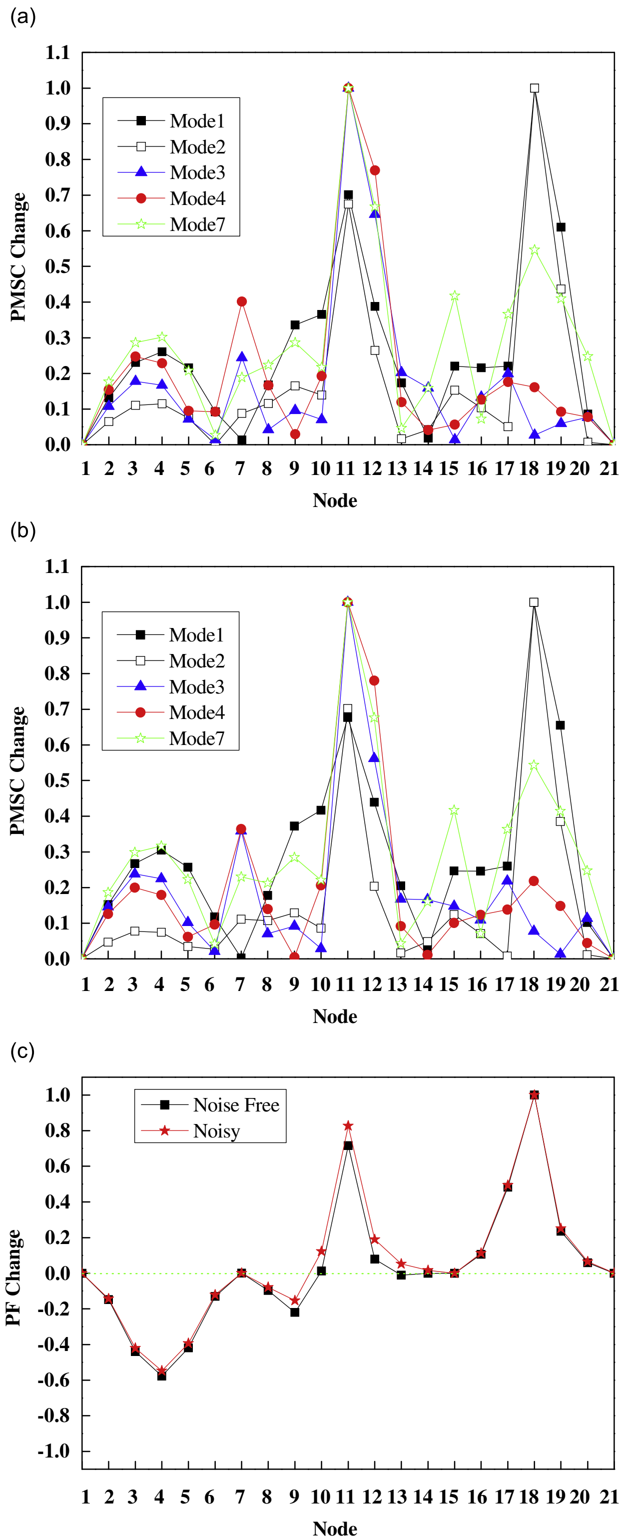

Fig. 11. Prediction of (a) noise free PMSC, (b) noisy PMSC and (c) PF indices with double damage between measurement nodes 11 and 12, and 18 and 19.

damage scenarios were considered by introducing damage into the girder segment between measurement nodes 11 and 12 (Fig. 8) or 18 and 19. Meanwhile, the multiple damage scenario is a combination of both locations. The simulated damage may be regarded as minor since it results in reductions in the average global frequencies (first 7 frequencies considered) of 0.33% and 0.34% for single damage scenarios, and 0.67% for multiple damage scenario, respectively.

A simulated white-noise force excitation with a constant spectrum amplitude of 10 000 N/Hz^2 and within a range of [2.2,6.0] Hz was applied to the location of S_4 (Fig. 8). A total of 21 nodes on the plate girder were used to obtain the numerical acceleration responses. In addition to a noise-free situation, a 15% level of random noise was introduced to the response PSDs for single and multiple damage scenarios. And to retain the consistent implementation of the PF index method, only the first 3 power modes were used to calculate the PF indices for all three damage scenarios.

It can be observed from Fig. 9 that both PMSC and PF indices give satisfactory predictions for the first single damage scenario. However, for the end-span damage scenario case using the PMSC index (Fig. 10), only the first 3 and the 7th modes can accurately locate the damage. The other three modes give poor predictions of damage location and are thus not presented in order to have a clear figure. Fortunately, the PF index accurately locates the damage in this scenario. At the same time it is interesting to remark that even for noisy cases, the proposed damage indices present a similar prediction to that found in the noise-free cases. This observation indicates the possible anti-noise ability of the proposed indices, especially to random noise. Meanwhile, Fig. 11 shows that for the multiple damage case, the two damage locations are detected correctly and again the first 3 power modes were enough to calculate the PF indices. Furthermore, apparently, the noise has little influence on the performance of the proposed damage indices. From the above damage scenarios, it seems that the fundamental PMSCs (e.g. first 3 power modes) are reliable in damage prediction and the PF index might work better than the PMSC index.

Although the formulation of reliable and effective damage indices to be applied in large-scale engineering structures is a difficult task, in the preliminary studies performed here the proposed indices have shown a good performance which might constitute a starting point for future studies. Furthermore, it is interesting to remark that even for minor damage causing minor changes in the dynamic properties of structures, the proposed damage indices can accurately locate the damage taking account of the noise influence. This observation is highly significant in practice, since it means the damage can be detected in its early state resulting in lower maintenance and property costs.

7. Conclusions

Two damage localization indices based on a novel concept of power mode shapes are proposed in this paper. Instead of the conventional modal parameters, the statistical properties of random signals and bandwidth-localized energy concept are used for their formulation. The proposed indices are designed for linear and lightly damped structures and were found to be capable of locating minor damage in different numerical and experimental cases. The ability to detect minor damage qualifies the proposed indices for a possible application in the early damage detection of real structures under random excitation.

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