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Finite element model updating with damping identification

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Abstract

Most of finite element model updating techniques do not employ damping matrices and hence, cannot be used for accurate prediction of complex frequency response functions (FRFs). In this paper, damped finite element model updating procedure is proposed and tested with the objective that the damped finite element updated model is able to predict the measured FRFs accurately. The proposed damped updating procedure is a two-step procedure. In the first step, mass and stiffness matrices are updated using FRF data and in the second step, damping matrix is identified using updated mass and stiffness matrices that are obtained in the previous step. The effectiveness of the proposed procedure is demonstrated by numerical examples as well as by actual experimental data. Firstly, a study is performed using a numerical simulation based on fixed–fixed beam structure with non-proportional viscous damping model. The numerical study is followed by case involving actual measured data for the case of F-shaped test structure. The results have shown that the proposed procedure can be used for accurate prediction of the complex FRFs.

1. Introduction

The study of the dynamic behavior of a structure can be divided into two separate routes, namely theoretical route and experimental route. The most widely used theoretical route is the finite element (FE) method [1]. It is well known that FE predictions are often called into question when they are in conflict with test results [2,3]. Inaccuracies in FE model and errors in results predicted by it can arise due to use of incorrect modeling of boundary conditions, incorrect modeling of joints, and difficulties in modeling of damping. This has led to the development of model updating techniques, which aim at reducing the inaccuracies present in an analytical model in the light of measured dynamic test data. A number of model updating methods have been proposed in recent years as shown in the surveys by Imregun and Visser [4] and Mottershead and Friswell [5] and details of these can be found in the text by Friswell and Mottershead [6]. Model updating methods can be broadly classified into direct methods that are essentially non-iterative ones and the iterative methods. A significant number of methods [7–9], which were first to emerge, belonged to the direct category. Although these methods are computationally cheaper and reproduce the measured modal data exactly, they violate structural

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connectivity and updated structural matrices are difficult to interpret. On the other hand, iterative methods provide wide choices of updating parameters, structural connectivity can be easily maintained and corrections suggested in the selected parameters can be physically interpreted. Iterative methods either use eigendata or FRF data. Iterative methods are based on minimizing an objective function that is generally a nonlinear function of selected updating parameters. Quite often eigenvalues, eigenvectors or response data are used to construct the objective function. Collins et al. [10] used the eigendata sensitivity for analytical model updating in an iterative framework. Lin and Ewins [11] used measured FRF data to update an analytical model. Comparison of response function method (RFM) and inverse eigensensitivity method neglecting damping was done with an objective to study the accuracy with which they predicted the corrections required in an FE model [12]. Modak et al. [13] proposed an updating method in which updating of undamped FE model was done by imposing constraint on natural frequencies and mode shapes. Most of the updating methods neglect the damping. So these methods can be used up to the point of predicting the natural frequencies and real modes. But these cannot be used for complex frequency response functions (FRFs) and complex modes. All structures exhibit some form of damping, but despite a large literature on damping, it still remains one of the least well-understood aspects of general vibration analysis. A commonly used model originated by Rayleigh [14] assumes that instantaneous generalized velocities are the only variables. Material damping can arise from variety of micro structural mechanisms [15], but for small strains, it is often adequate to represent it through an equivalent linear model of the material. Maia et al. [16] emphasized the need for the development of identification methodologies of general damping models and indicated several difficulties that might arise. Pilkey [17] described two types of procedures, direct and iterative, for computation of the system-damping matrix. The damping matrices so identified are symmetric in nature and use either accurate mass matrix or both mass and stiffness matrices along with experimentally obtained modal data. Adhikari [18] identified viscous damping using residues. Adhikari and Woodhouse [19] identified the damping of the system as viscous damping. However, viscous damping is by no means the only damping model. Adhikari and Woodhouse [20] identified non-viscous damping model using an exponentially decaying relaxation function.

Some research efforts have also been made to update the damping matrices. Imregun et al. [21,22] conducted several studies using simulated and experimental data to gauge the effectiveness of RFM and extended the RFM to update proportional damping matrix by updating the coefficients of damping matrix. Yong and Zhenguo [23] proposed a two-step model updating procedure for lightly damped structures using neural networks. In the first step, mass and stiffness are updated using natural and antiresonance frequencies. In the second step, damping ratios are updated. Lin and Zhu [24] extended RFM to update damping coefficients in addition to mass and stiffness matrices. Arora et al. [25] proposed a complex parameter-based model updating method in which updating parameters are considered complex. Arora et al. [26] proposed a damped FE model updating procedure and studied its effectiveness for dynamic design.

In this paper, procedure for damped FE model updating is based on damping identification is proposed and tested with the objective that the damped updated model is able to predict the measured FRFs accurately. The proposed procedure is a two-step procedure. In the first step, mass and stiffness matrices are updated using FRF data and in the second step, damping is identified using updated mass and stiffness matrices that are obtained in previous step. The FRF-based FE model updating method named RFM, proposed by Lin and Ewins [11], is used for the updating of the mass and stiffness matrices of the system and damping matrix is identified using direct method, proposed by Pilkey [17]. In order to demonstrate the effectiveness of the proposed damped FE model updating procedure, first a study is performed using numerical simulation based on fixed–fixed beam structure with non-proportional viscous damping model. This is followed by a study involving actual measured data for the case of an F-shape test structure, which resembles the skeleton of a drilling machine.

2. Theory

The proposed procedure is a two-step procedure to obtain damped updated FE model. In the first step, mass and stiffness matrices are updated using FRF data, proposed in Ref. [11]. In the second step, damping matrix is identified by direct method, proposed in Ref. [17].

2.1. Response function method (RFM)

The RFM proposed by Lin and Ewins [11], is an FRF-based iterative method and uses the real part of FRF for updating of mass and stiffness matrices without requiring any modal extraction. Following identities relating dynamic stiffness matrix $[Z]$ and receptance FRF matrix $[\alpha]$ can be written for the analytical model as well as the actual structure, respectively:

$$[Z_A][\alpha_A] = [I] \tag{1}$$

$$[Z_X][\alpha_X] = [I] \tag{2}$$

where subscripts A and X denote analytical (like an FE model) and experimental model, respectively. Expressing $[Z_X]$ in Eq. (2) as $[Z_A]+[\Delta Z]$ and then subtracting Eq. (1) from it, following matrix equation is obtained:

$$[\Delta Z][\alpha_X] = [Z_A]([\alpha_A] - [\alpha_X]) \tag{3}$$

Pre-multiplying above equation by $[\alpha_A]$ and using Eq. (1) gives

$$[\alpha_A][\Delta Z][\alpha_X] = [\alpha_A] - [\alpha_X] \tag{4}$$

If only the jth column of experimentally measured FRF matrix $\{\alpha_X\}_j$, is available then the above equation is reduced to

$$[\alpha_A][\Delta Z]\{\alpha_X\}_j = \{\alpha_A\}_j - \{\alpha_X\}_j \tag{5}$$

which is the basic relationship of the RFM. With this method, physical variables based updating parameter formulation is used. Linearizing $[\Delta Z]$ with respect to $\{p\}$, $\{p\} = \{p_1, p_2, \ldots, p_{\mathrm{nu}}\}^t$ being the vector of physical variables associated with individual or group of finite elements, gives

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial [Z]}{\partial p_i} \Delta p_i \right) \tag{6}$$

where nu is the total number of updating parameters. For undamped system, $[Z]$ in Eq. (6) is replaced by $[K]-\omega^2[M]$. Dividing and multiplying Eq. (6) by p_i and then writing u_i in place of $\Delta p_i/p_i$. Then Eq. (6) becomes

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial([K] - \omega^2[M])}{\partial p_i} p_i \right) u_i \tag{7}$$

Thus $\{u\} = \{u_1, u_2, \ldots, u_{\mathrm{nu}}\}^t$ is the unknown vector of fractional correction factors to be determined during updating. The resulting equations can be framed after substituting $[\Delta Z]$ from Eq. (7) into Eq. (5) as

$$[S]\{u\} = \{\varepsilon\} \tag{8}$$

Here $[S]$ is sensitivity matrix and $\{\varepsilon\}$ is a residual vector. The parameters are updated iteratively till residual vector become sufficiently small. The performance is judged on the basis of the accuracy with which the FRFs predicted by updated FE model match with the simulated experimental FRFs or the experimental FRFs.

2.2. Damping identification

Direct method of damping identification [17] requires prior knowledge of accurate stiffness and mass matrices and also complex eigendata. The damping matrix (C) identified is both symmetric and positive definite. Direct method is based on Lancaster's formulation [27] of damping identification from the measured complex modal parameters when the complex modes are normalized in a specified way. The eigenvalue problem associated with viscously damped systems is first written in the form

$$[M\lambda_i^2 + C\lambda_i + K]\Phi_i = 0 \tag{9}$$

and damping matrix can be computed from the formula given as

$$C = -M(\Phi \Lambda^2 \Phi^{\mathrm{T}} + \overline{\Phi \Lambda^2 \Phi^{\mathrm{T}}})M \tag{10}$$

where the overall bar represents complex conjugate. Λ is the diagonal matrix of complex eigenvalues λ_i and Φ are complex eigenvectors.

This formulation requires normalized complex eigenvectors (Φ). The complex eigenvectors are normalized by direct method given by Pilkey [17] as

$$\Phi_i^{\mathrm{T}}(M\lambda_i^2 - K)\Phi_i = \lambda_i \tag{11}$$

where M and K are mass and stiffness matrices. The updated mass and stiffness obtained in the previous section are used for normalization of complex eigenvectors.

3. Case study of a fixed–fixed beam structure using simulated data

A simulated study on a fixed–fixed beam is conducted to evaluate the effectiveness of the proposed procedure. The dimensions of the beam are $910 \times 50 \times 5$ mm. The beam is modeled using 30, two-noded beam elements with nodes at the two ends being fixed as shown in Fig. 1. The displacements in y-direction and the rotation about the z-axis are taken as two dof at each node. Analytical data of the FRFs is obtained with known discrepancies in the thickness of all the FEs as presented in Table 1. The simulated complex FRF data, which is treated as experimental data, is obtained by generating a damped FE model with non-proportional viscous damping.

Non-proportional damping is obtained by reducing the distribution of the stiffness to the damping matrix is reduced by 50 percent in 10th element. It is easily verified that non-Rayleigh style damping ensues with the simple equation

$$CM^{-1}K \neq KM^{-1}C \tag{12}$$

Fig. 2 shows the overlay of analytical FRF, which is undamped, and simulated experimental FRF (noise free) obtained considering non-proportional damping in the experimental data. It can be observed that the analytical FRF ($5y5y$) and simulated experimental FRF ($5y5y$) do not match with each other. $5y5y$ represents excitation and response at node 5, both in y-direction.

In the first step, mass and stiffness matrices are updated using real part of the FRF. Thickness of each element of the beam structure is considered as updating parameters. So for the beam structure, 30 updating parameters are taken. The proposed procedure has been evaluated for the cases of incomplete and noisy data. The real life measured data is always incomplete, as it is not practical to measure all the coordinates specified in the analytical FE model and always contains some measurement noise. Incompleteness is considered by assuming that only lateral dof, at all the 29 nodes are measured. This has been referred as 50 percent incomplete data. Different levels of noise, i.e. noise-free, 1, 2 and 3 percent noise, in simulated experimental data are considered. The frequency range from 0 to 1000 Hz is considered for updating procedure. The performance of the proposed procedure is judged on the basis of accuracy with which the FRFs obtained by

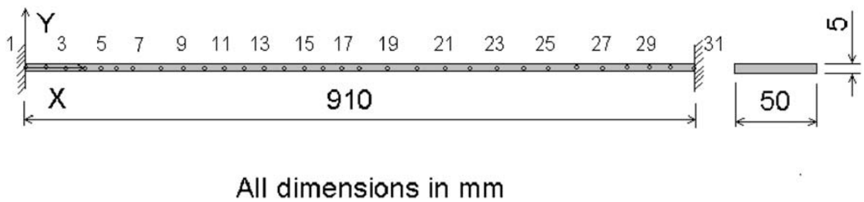

Fig. 1. Beam structure and its FE mesh.

Table 1
Discrepancies between the finite element and the simulated experimental model for the case of fixed–fixed beam structure.

Element number	3	5	11	16	25	29	All other elements
Percent deviation in thickness	+20	+40	25	+40	+30	+30	+20

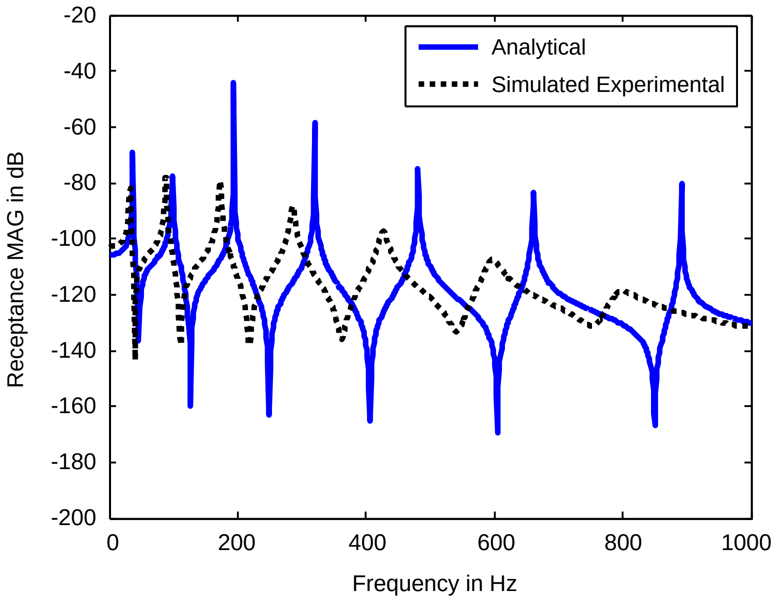

Fig. 2. Overlay of simulated experimental and analytical FRFs before updating.

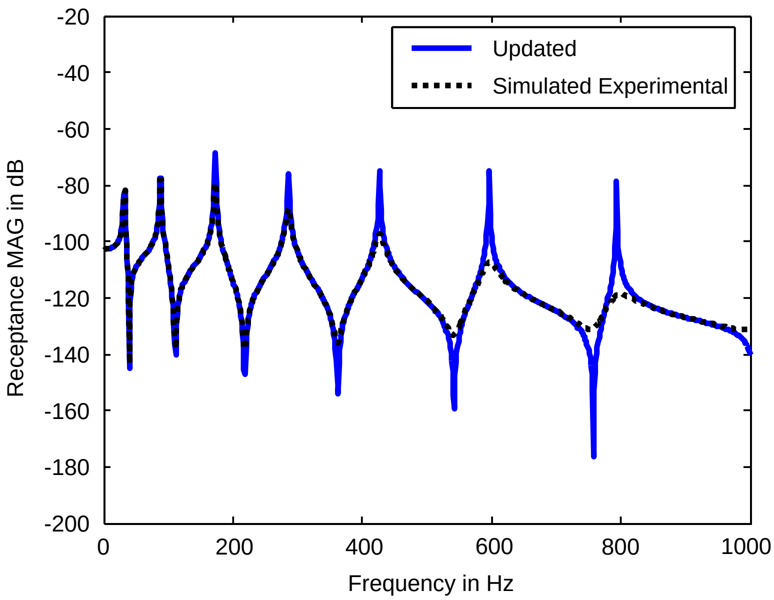

Fig. 3. Overlay of simulated experimental and undamped updated FRFs.

updated model match with the simulated experimental FRFs. After updating, simulated experimental FRF and updated FRF match as shown in Fig. 3. It can be observed from Fig. 3 that simulated experimental and updated FRFs matches with each other except at the resonance and antiresonance frequencies. The final value of the thickness of each element is close to 5 mm (the original thickness).

In the second step, damping is identified using complex eigendata. Direct method of damping identification is applied. As discussed above, only lateral dof, at all the 29 nodes are measured. For direct method of damping identification, updated mass and stiffness matrices are reduced to the dof measured using iterated

IRS method [28] considering the measurement points. In real life all the modes can not be measured. Three typical cases are presented here:

(a) All modes are measured.
(b) First five modes are measured.
(c) First three modes are measured.

For the case of all measured modes, when all the modes are measured. It can be observed from the Fig. 4 that damped updated and simulated experimental FRFs matches completely and the error in simulated damping

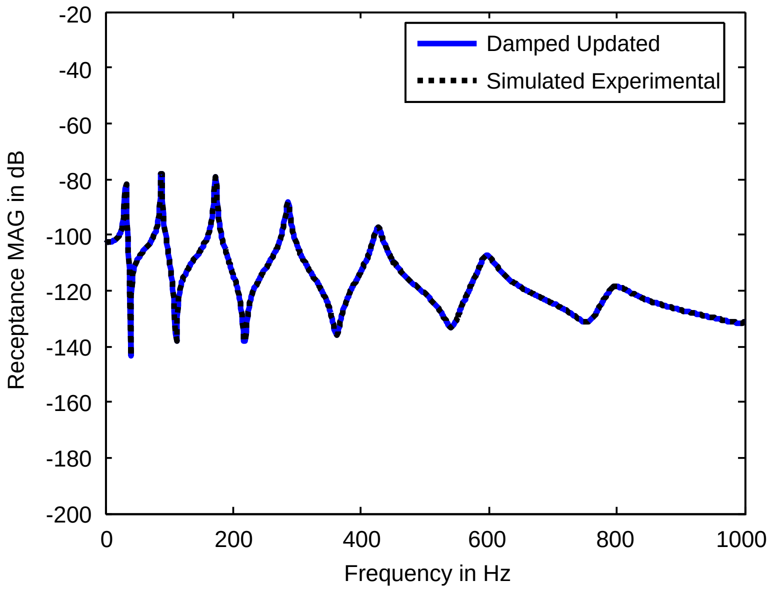

Fig. 4. Overlay of simulated experimental and damped updated FRFs when all modes are measured.

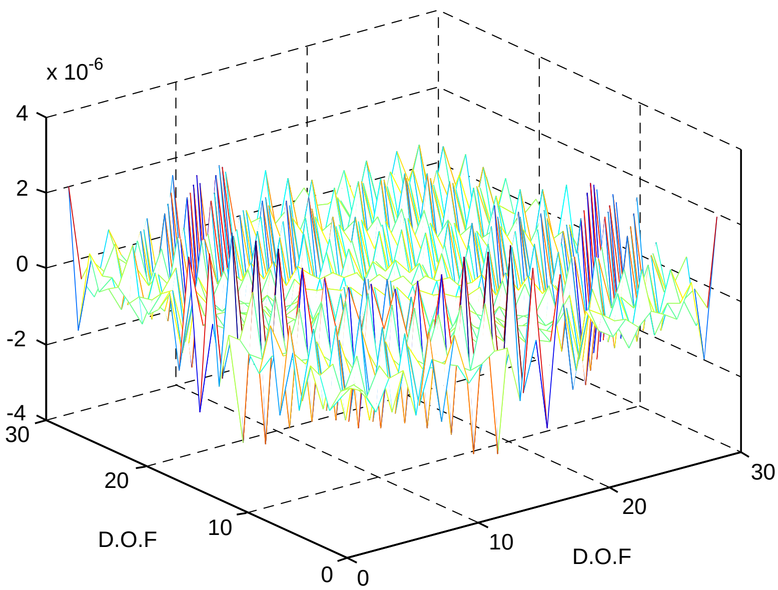

Fig. 5. Error in identified and simulated damping.

and identified damping is very low as shown in Fig. 5. Similarly, overlay of the damped updated and simulated experimental FRFs for the cases of first five and three modes measured are shown in Figs. 6 and 7. It can be observed from Figs. 6 and 7 that the updated and simulated FRFs matches up to the mode considered for damping identification beyond that the matching is very poor. So from the simulated study, it can be concluded that the proposed procedure is able to predict FRFs accurately for the range covering the number of modes considered. However, beyond the modes considered, the FRFs predicted do not match with simulated experimental FRFs.

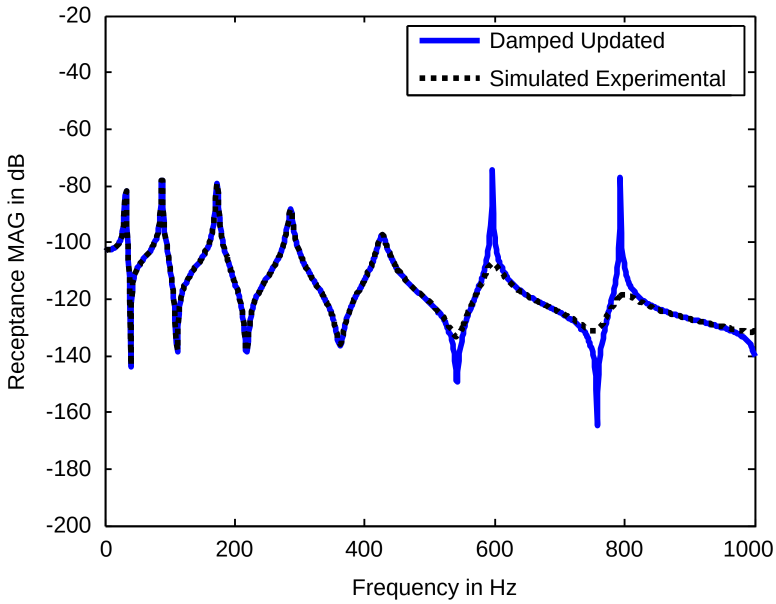

Fig. 6. Overlay of simulated experimental and damped updated FRFs when first five modes are measured.

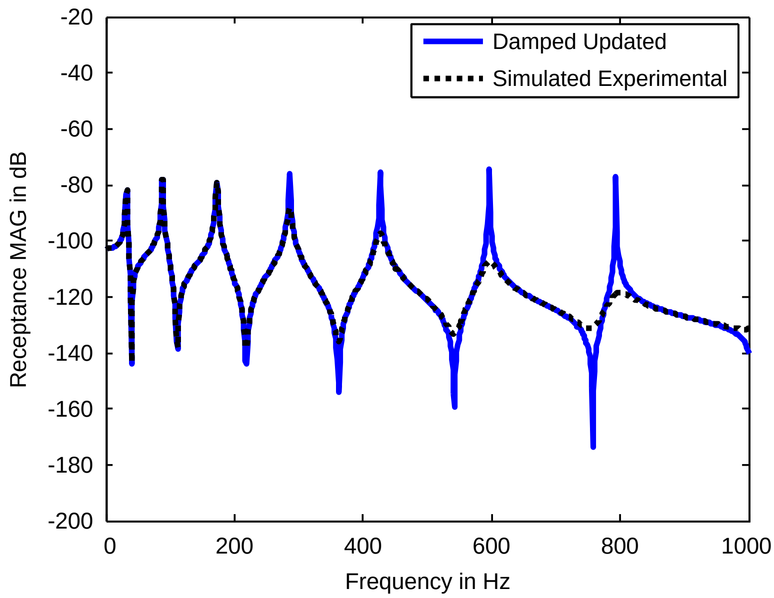

Fig. 7. Overlay of simulated experimental and damped updated FRFs when first three modes are measured.

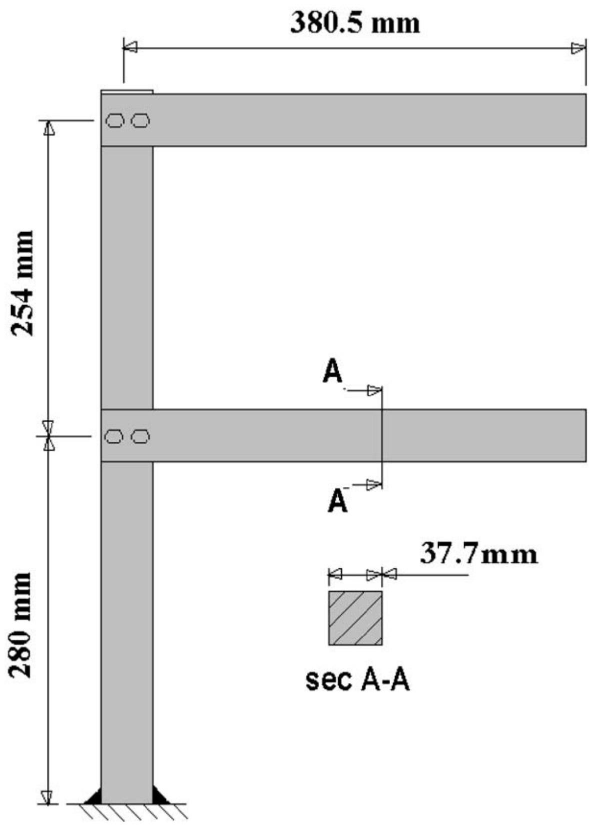

Fig. 8. F-shaped structure.

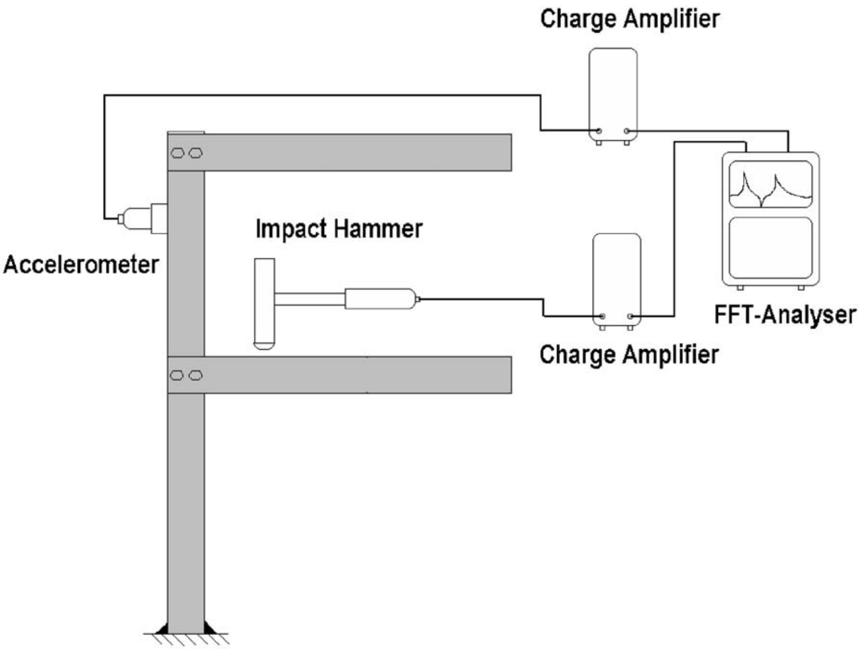

Fig. 9. Instrumentation set-up for modal test using impact excitation.

4. Case study of F-shape structure using experimental data

The proposed procedure is also evaluated using experimental data of F-shape structure. For updating of mass and stiffness matrices, the real part of experimental FRFs is taken directly from the experiments.

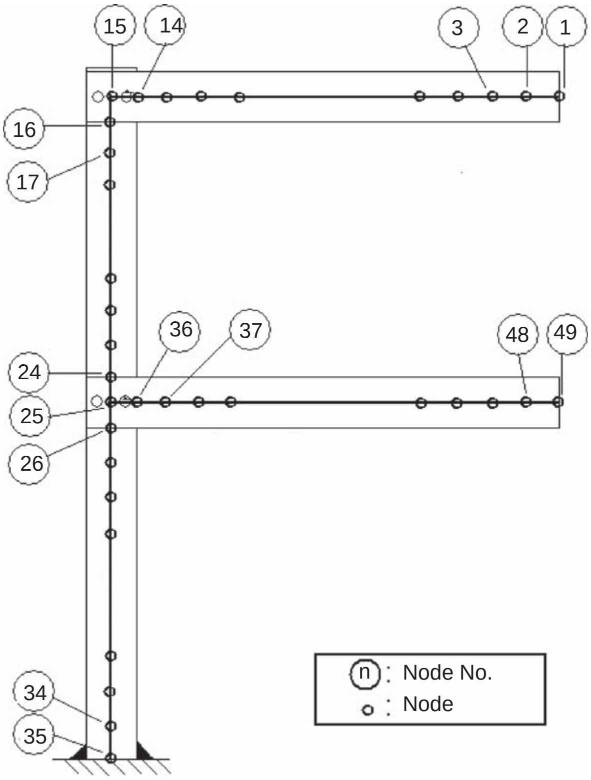

Fig. 10. Initial FE model of F-shaped structure.

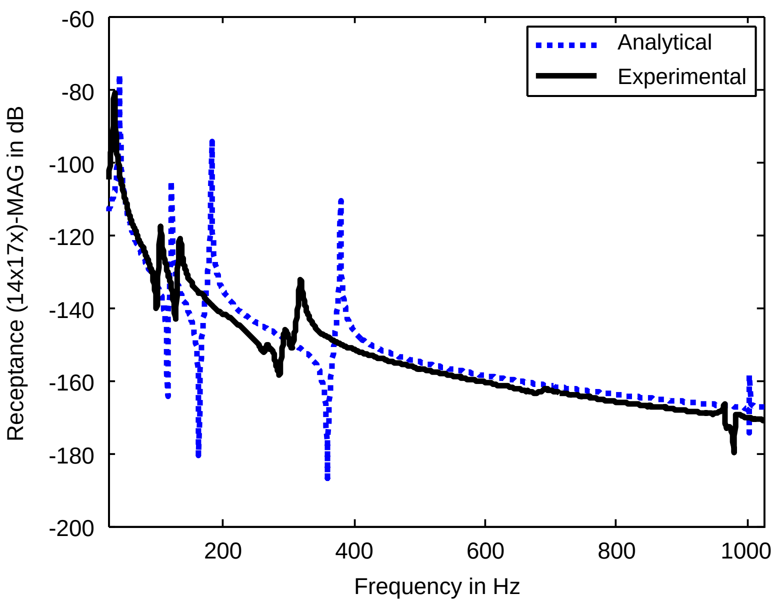

Fig. 11. Overlay of the measured FRF (____) and the corresponding FE model FRF (- - - -) before updating.

For damping identification, complex eigendata, which is obtained using curve fitting technique available in ICATS [29], is used.

An F-shaped test structure, resembling the skeleton of a drilling machine tool, as shown in Fig. 8, is considered for proposed procedure of damped model updating case study using experimental data. F-shaped structure is assembled from square cross section beam members connected by one welded and two bolted. The structure is welded at the bottom to a base plate. The structure is 13.8 kg in weight. The instrumentation set-up used for performing the modal test on the F-shape structure using impact excitation is shown in Fig. 9. The response accelerometer (B&K model 4368) is fixed at one of the locations while the structure is excited with an impact hammer (B&K model 8202). The response and force signals are fed to an FFT analyzer (DI model PL 202) via charge amplifiers (B&K model 4368).

FE model of the F-structure is built, as shown in Fig. 10, using 48 two dimensional frame elements (Two translational dof in x and y direction and one rotational dof) to model in-plane dynamics. In the F-shaped structure there are three joints and the joints are modeled by taking coincident nodes at each of them. Thus now, two nodes that are geometrically coincident are taken as joint instead of one node. A horizontal, a vertical and a torsional spring couples two nodes at each of such coincident pair of nodes and the stiffness of these springs is K_x, K_y and K_t, respectively. The FRFs of the F-structure are obtained by impacting the structure one by one at 16 locations.

Fig. 11 shows an overlay of the analytical and experimental FRF of the F-shaped structure before updating. It is clear that the mass and stiffness matrices are not accurate, since even the natural frequencies of the FRF do not match. For direct method of damping identification the prior knowledge of the correct mass and stiffness matrices are required. So before damping identification, the mass and stiffness matrices are updated using RFM. In this work FRF is matched in two steps:

Step 1: Updating of mass and stiffness matrices.
Step 2: Identification of damping matrix.

Step 1: *Updating of mass and stiffness matrices*. Choice of updating parameters on the basis of engineering judgment about the possible locations of modeling errors in a structure is one of the strategies to ensure that only physical meaningful corrections are made. In case of F-structure, due to presence of three joints and modeling of stiffness at these places are expected to be the dominant source of inaccuracy in the FE model. The stiffness of horizontal, vertical and torsional spring of each joint are potential updating parameters allowing to account for the deviation in the stiffness of the regions covered by each joint. A sensitivity analysis was performed to reduce the number of unknowns. It is noticed from sensitivity analysis that FRFs are far more sensitive to the torsional stiffness at three joints than to other spring-stiffness parameters. In light of these observations, the three torsional stiffness parameters are chosen as updating variables. The other stiffness is taken very large to represent rigid coupling of those dof. Since the undamped FE model is being updated, the FRF corresponding to the FE model has only real part so only the real part of the measured FRFs is considered.

The initial and final values of the torsional spring of each joint are given in Table 2. It is observed that the values of stiffness of the torsional spring corresponding to three joints are reduced and also values of three springs are not very different from each other. A comparison of the correlation between the measured and the updated model is given in Table 3. It is observed from Table 3 that for the updated model there is a significant reduction in the error in natural frequencies in the frequency range of 0–1000 Hz. Fig. 12 shows the overlay of

Table 2
Values of torsional springs of each joint after updating.

Updating variable	Initial value (N m rad^{-1})	Updated values (N m rad^{-1})
K_{t1}	3.28E+06	2.57E+05
K_{t2}	3.28E+06	2.83E+05
K_{t3}	3.28E+06	3.1E+05

Table 3
Comparison of correlation between measured, FE model and updated model.

Mode no.	Measured frequency in Hz	FE-model predictions			Updated model predictions		
		Frequency in Hz	Percent error	MAC-value	Frequency in Hz	Percent error	MAC-value
1	34.95	43.05	23.17	0.9650	34.25	−2.0	0.9923
2	104.02	123.67	18.89	0.9364	100.27	−3.60	0.9693
3	133.96	185.21	38.26	0.9311	134.42	0.34	0.9675
4	317.52	385.17	21.30	0.9141	313.73	−1.19	0.9423
5	980.16	1020.06	4.07	0.6908	973.44	−0.68	0.4370

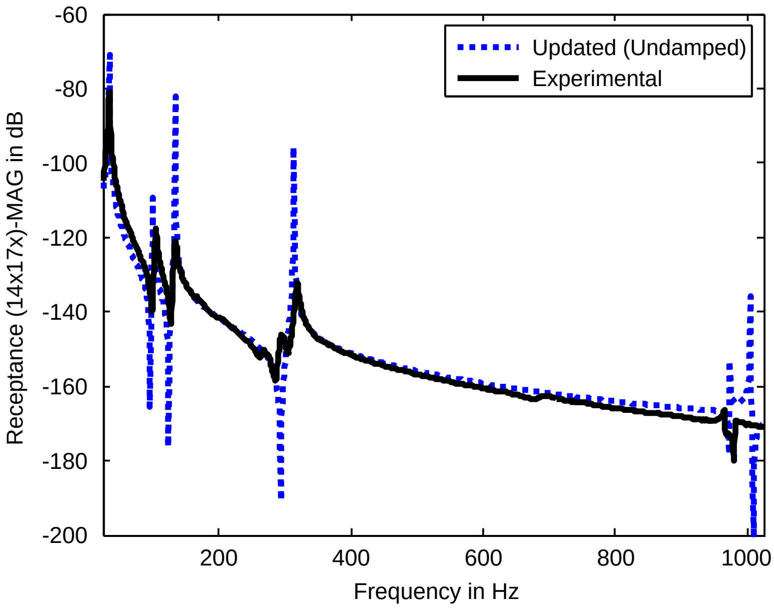

Fig. 12. Overlay of the measured FRF and the corresponding undamped FE model FRF (----) after updating.

measured and undamped updated FRF. It can be observed that the shape of the updated FRFs is same as that of measured FRFs. But, near the resonance and antiresonance frequency points, the FRFs do not match since updating is done by neglecting damping and the effect of damping is maximum near the resonance and antiresonance regions. In the second step, damping is identified so that the resonant and antiresonance frequency points of measured and updated FRFs also match with each other.

Step 2: *Identification of damping matrix*. In this step, damping is identified using direct method of damping identification. The direct method of damping identification requires prior knowledge of accurate mass and stiffness matrices. For direct method updated mass and stiffness matrices are reduced to the dof measured using iterated IRS method [28] considering the measurement points. For the case of F-shaped structure, total dof are 153 and number of measured points is 16. So, the updated mass and stiffness matrices are reduced from 153 dof to only 16 measured dof. At this point, it can be noted that only five modes have been captured experimentally and first four modes are used for identification of damping matrix. For the direct method the damping matrix is found out in one step and the FRFs of damping updated and measured FRFs are shown in Fig. 13. It can be observed that the proposed procedure able to predict damping in the system with reasonable good accuracy and there is good FRF matching between updated FRFs and the experimental FRFs for the

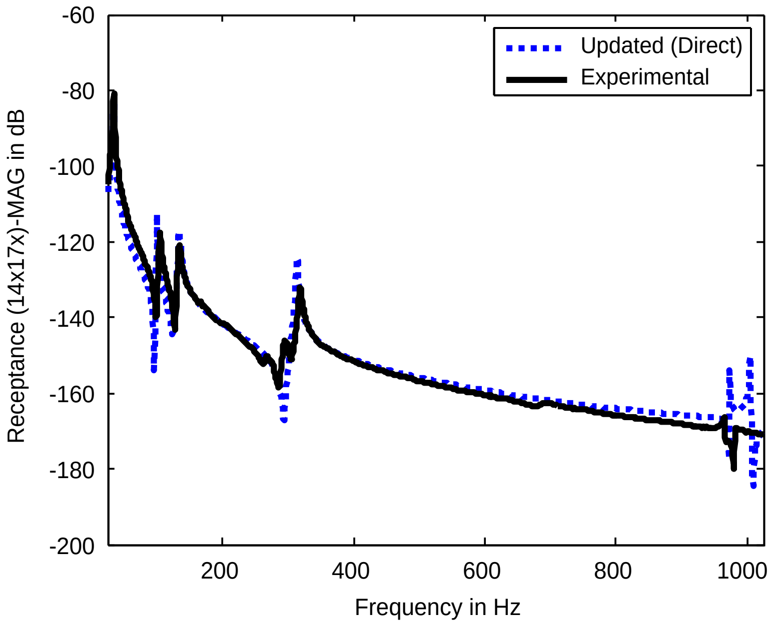

Fig. 13. Overlay of the measured FRFs and the corresponding damped FE model FRF with damping identification using direct method.

range covering the number of modes considered. However, beyond the modes considered, the FRFs predicted do not match with experimental FRFs.

5. Conclusions

In this paper, a new procedure for damped FE model updating has been proposed for better FRF matching based on application of damping identification procedure. This is done in two steps. In the first step, mass and stiffness matrices are updated using RFM, which is an iterative method and in the second step, damping matrix is identified using updated mass and stiffness matrices. The damped FE model updating procedure predicts accurately the measured FRFs. The effectiveness of the proposed procedure is demonstrated by numerical examples as well as by actual experimental data.

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