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Relation of bulk to shear loss factor of solid viscoelastic materials

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Abstract

The relation between the bulk and shear loss factors of isotropic, homogeneous, linear solid viscoelastic materials is investigated in this paper by means of the complex modulus concept. It is shown that the bulk and shear loss factors can be related through the dynamic Poisson's ratio provided that the shear loss is low enough. Bounds on the ratio of the bulk to shear loss factor are derived, and the respective lower bounds are given as a function of the dynamic Poisson's ratio. The ratio of the bulk to shear loss factor is predicted to decrease with the increase of dynamic Poisson's ratio, and it is shown that the decrease may obey a simple power law if the Poisson's ratio is close to either 0 or 0.5. Experimental data on solid polymeric materials are presented which support the theoretical findings.

1. Introduction

The dynamic elastic and loss properties of linear, solid viscoelastic materials can effectively be characterized by the complex modulus concept [1–3]. An isotropic, homogeneous solid is known to have two fundamental complex moduli; namely the complex bulk and shear moduli [1–3]. From knowledge of these moduli the dynamic behaviour of the isotropic viscoelastic material can completely be described, and the velocity and attenuation of bulk and shear waves propagating in the solid can be determined. The complex bulk and shear moduli are fundamental ones, and hence are independent of each other, but can be related through a third complex modulus or the complex Poisson's ratio [3]. It is easy to show that the real parts of the complex bulk and shear moduli, i.e., the relevant dynamic moduli, can be related through the dynamic Poisson's ratio provided that the shear loss is low enough [4]. The question rightly arises; what is the relation between the bulk and shear loss properties, and whether the relevant loss factors can be related through the dynamic Poisson's ratio likewise the dynamic moduli.

The relations between the loss factors of isotropic linear viscoelastic materials have been investigated theoretically, and it has been proved that the bulk loss factor is inevitably smaller than the shear one [3,5]. In addition, several experiments have been made since about 1960s up to now to clear up the relation between

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the bulk and shear loss properties in viscoelastic solids, especially rubbers and hard plastics [2,5–10]. All experimental results support that the bulk loss factor is smaller than the shear one, but only few works investigated quantitatively the relation between these loss factors [7,8]. Furthermore, the detailed experimental data in one of the latter works suggest that the ratio of bulk to shear loss factors may be related to the dynamic Poisson's ratio [8], but neither theoretical, nor experimental works are known on studying this relation.

The idea on the relation between the bulk and shear loss factors and the dynamic Poisson's ratio arose in a recent paper by the present author [11]. The aim of this work is to present the theoretical base of this idea, to develop some relations and support them with experimental data. The essential motivation of this work is to widen our knowledge on the dynamic properties of solid linear viscoelastic materials. The new knowledge can be useful in both material characterization and research, moreover in the sound and vibration control, and wave propagation studies [1,12–16].

2. Theory

2.1. The problem

Consider the relation between the complex bulk modulus, $\overline{B}$, the complex shear modulus, $\overline{G}$, and the complex Poisson's ratio, $\overline{\nu}$, which can be given as [3]

$$\overline{B}(\mathrm{j}\omega) = \frac{2}{3}\overline{G}(\mathrm{j}\omega)\frac{1+\overline{\nu}(\mathrm{j}\omega)}{1-2\overline{\nu}(\mathrm{j}\omega)}, \tag{1}$$

where the overbar denotes the complex value, $\mathrm{j} = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$, f is the frequency in Hz, and

$$\overline{B}(\mathrm{j}\omega) = B_d(\omega) + \mathrm{j}B_l(\omega) = B_d(\omega)[1 + \mathrm{j}\eta_B(\omega)], \tag{2}$$

$$\overline{G}(\mathrm{j}\omega) = G_d(\omega) + \mathrm{j}G_l(\omega) = G_d(\omega)[1 + \mathrm{j}\eta_G(\omega)], \tag{3}$$

$$\overline{\nu}(\mathrm{j}\omega) = \nu_d(\omega) - \mathrm{j}\nu_l(\omega) = \nu_d(\omega)[1 - \mathrm{j}\eta_\nu(\omega)]. \tag{4}$$

In Eqs. (2) and (3) the subscripts d and l refer to the dynamic and loss moduli, respectively, η_B is the bulk loss factor and η_G is the shear loss factor. In Eq. (4) ν_d is the dynamic Poisson's ratio, ν_l is the relevant loss part, and η_ν is referred to as Poisson's loss factor [11]. The loss factors are defined as

$$\eta_B = B_l/B_d, \tag{5}$$

$$\eta_G = G_l/G_d, \tag{6}$$

$$\eta_\nu = \nu_l/\nu_d. \tag{7}$$

Starting out of Eq. (1), formally the same equation can be derived for the relation between the dynamic moduli and Poisson's ratio under the assumption that the shear loss factor is low enough, namely $\eta_G < 0.3$, and bearing in mind that η_G is the highest among the material loss factors [3,4]. Consequently, the ratio of the dynamic bulk modulus to the dynamic shear modulus can be written as

$$\frac{B_d}{G_d} \approx \frac{2}{3}\frac{1+\nu_d}{1-2\nu_d}. \tag{8}$$

This equation shows that the dynamic bulk and shear moduli are related through the dynamic Poisson's ratio. In addition, Eq. (8) offers a possibility to investigate the relation between B_d and G_d, since the physically possible values of dynamic Poisson's ratio are known from the theory of elasticity, namely: $-1 < \nu_d \leqslant 0.5$ for isotropic, homogeneous solids. The common viscoelastic solids (rubbers, hard plastics) have positive Poisson's ratio, i.e., $0 \leqslant \nu_d \leqslant 0.5$, and only these materials are in focus of this work. It can be read from Eq. (8) that the larger ν_d, the larger B_d/G_d, moreover $B_d/G_d \to \infty$ as $\nu_d \to 0.5$.

Seeing the simple equation between B_d/G_d and v_d, the question arises; whether the bulk and shear loss factors can be related through the dynamic Poisson's ratio likewise the dynamic moduli. By studying the magnitudes of η_B and η_G at $v_d = 0$, in the vicinity of $v_d = 0$ and 0.5, respectively, it is easy to show that the relation between the bulk and shear loss factors and the dynamic Poisson's ratio may exist. Consider firstly the case if $\overline{v} = v_d = 0$, then it is clear from Eq. (1) that $\overline{B} = 2\overline{G}/3$, from which: $B_d = 2G_d/3$ and $B_l = 2G_l/3$, and therefore: $\eta_B = \eta_G$. Bearing in mind that $\eta_B < \eta_G$ for $v_d > 0$ [3,5], it can be concluded that η_B/η_G should start to decrease from unity with increasing v_d. Furthermore, from knowledge of $\eta_B/\eta_G = (B_l/G_l)/(B_d/G_d)$, and $B_d/G_d \to \infty$ as $v_d \to 0.5$, one can predict that, in principle, $\eta_B/\eta_G \to 0$ as $v_d \to 0.5$, since the loss moduli, which are proportional to the dissipated energy, should remain finite in a real solid regardless of the magnitude of v_d. It follows that the bulk loss factor can be much lower than the shear one if v_d is close to 0.5, which agrees with the experimental observations on rubbery materials [2,6,11]. On the basis of these one can conclude that the ratio of bulk to shear loss factor is related to the dynamic Poisson's ratio, and η_B/η_G is expected to decrease with increasing v_d.

2.2. Bounds on η_B/η_G

The knowledge of bounds on η_B/η_G, if exists any, can be a useful tool to investigate the adequacy of a relation found between the bulk and shear loss factors. The existence of upper bound is plausible from the theoretical finding that the bulk loss factor is inevitably lower than the shear one [3,5]. Moreover, $\eta_B = \eta_G$ at $v_d = 0$ as mentioned above, consequently one can write: $\eta_B/\eta_G \leqslant 1$ for $v_d \geqslant 0$. To find the lower bounds, the complex form of the first Lame's constant, λ, is considered, which is [3]

$$\overline{\lambda}(\mathrm{j}\omega) = \lambda_d + \mathrm{j}\lambda_l = \overline{B} - \frac{2}{3}\overline{G}. \tag{9}$$

It follows from Eq. (9) that

$$\lambda_l = B_l - \frac{2}{3}G_l. \tag{10}$$

Bearing in mind that $\lambda_l > 0$ for $v_d > 0$ [5], it can be concluded from Eq. (10) that

$$\frac{B_l}{G_l} > \frac{2}{3}, \tag{11}$$

which can be written as

$$\frac{\eta_B}{\eta_G}\frac{B_d}{G_d} > \frac{2}{3}. \tag{12}$$

Combining Eq. (12) with Eq. (8) yields:

$$\frac{\eta_B}{\eta_G} > \frac{1 - 2v_d}{1 + v_d} \tag{13}$$

for $0 < v_d < 0.5$. Eq. (13) defines the lower bounds for the ratio of the bulk to shear loss factors, and these bounds are functions of the dynamic Poisson's ratio. The upper and lower bounds on η_B/η_G are shown in Fig. 1 as a function of v_d.

2.3. Relation between η_B/η_G and dynamic Poisson's ratio

The upper and lower bounds given in Fig. 1 indicate the area where the magnitudes of η_B/η_G may occur. In order to find the relation between η_B, η_G and v_d, and to predict the variation of η_B/η_G with v_d within the bounds, the bulk loss factor is derived from Eq. (1). The separation of Eq. (1) into real and imaginary parts and some transformations result in

$$\eta_B = \frac{[(1 + v_d)(1 - 2v_d) - 2(v_d\eta_v)^2]\eta_G - 3v_d\eta_v}{(1 + v_d)(1 - 2v_d) - 2(v_d\eta_v)^2 + 3v_d\eta_v\eta_G}. \tag{14}$$

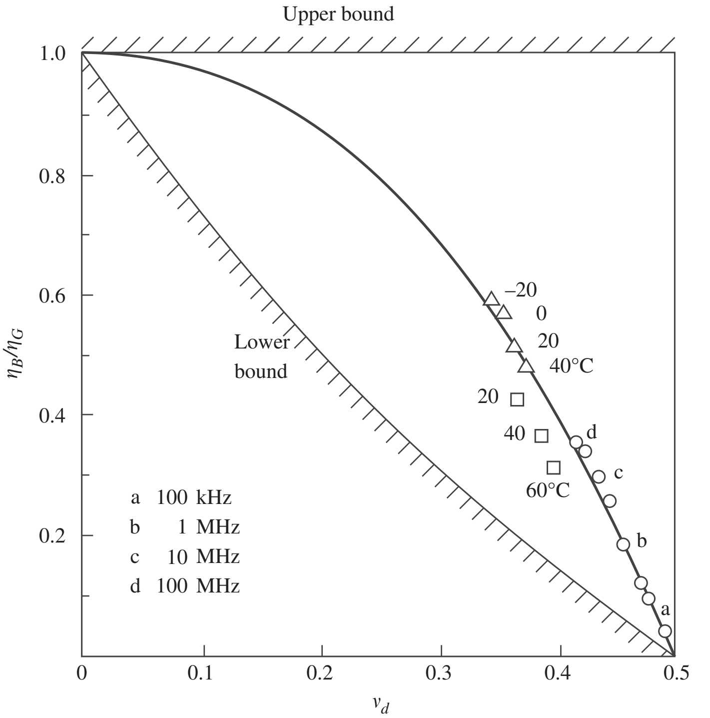

Fig. 1. The ratio of bulk to shear loss factor plotted against the dynamic Poisson's ratio: ○: styrene-butadiene rubber, $f = 100$ k, 300 k, 500 k, 1 M, 3 M, 10 M, 30 M, 100 MHz, 20 °C [6,11]; Δ: poly(methyl methacrylate), $f = 1$ Hz [9]; □: poly(4-thiacyclohexyl methacrylate), $f = 800$ kHz [17]; and –: fitting the data on the styrene-butadiene rubber by Eq. (19), $n = 2.3$.

This equation can be simplified if the shear loss factor is as low as $\eta_G < 0.3$, and by considering that $\eta_\nu < \eta_G$ [5]. Some transformations of Eq. (14) and the neglecting of the small quantities yield:

$$\eta_B \approx \eta_G - \eta_\nu \frac{3\nu_d}{(1+\nu_d)(1-2\nu_d)}. \tag{15}$$

It is emphasized that the relatively low shear loss was the basic assumption required to derive Eq. (15). The low shear loss, namely $\eta_G < 0.3$ is characteristic of the majority if not all stiff structural materials (metals, ceramics, etc.), hard plastics at room temperature and rubbers in the rubbery and glassy ranges. Moreover, it has been pointed out in Ref. [11], where the rearranged form of this equation has been derived in another way, that Eq. (15) may be accurate enough even if the shear loss is high, e.g., $\eta_G \approx 1.0$, provided that the dynamic Poisson's ratio is close to 0.5, say $\nu_d > 0.45$ (rubbers in transition range). It follows that Eq. (15) may hold true for wide class of materials.

Eq. (15) demonstrates the relation between the bulk and shear loss factors and the dynamic Poisson's ratio, but involves the Poisson's loss factor too. The latter is a peculiar loss factor, which is known to be a complicated function of the dynamic Poisson's ratio itself, and depends on the material loss properties characterized by the modulus loss factors [4,11]. The dependence of η_ν on ν_d is a further support of the assumption that relation is between the bulk and shear loss factors and the dynamic Poisson's ratio, but this relation cannot be expressed explicitly from Eq. (15), unfortunately. Notwithstanding, Eq. (15) enables one to predict some relations between η_B/η_G and ν_d for the cases if ν_d is close to either 0 (cork and some polymeric foams) or 0.5 (rubbers). To find a relation for the first case, the Poisson's loss factor is expressed from Eq. (15), which can be written as

$$\eta_\nu \approx \eta_G \frac{1-\eta_B/\eta_G}{3\nu_d}, \tag{16}$$

if $v_d \to 0$. It is known that $\eta_v \to 0$ as $v_d \to 0$ [11], and this behaviour can be satisfied if the numerator in Eq. (16) obeys a power function of v_d, i.e.,

$$\frac{\eta_B}{\eta_G} = 1 - a v_d^n, \tag{17}$$

where $a > 0$ and $n > 1$ should stand. It is easy to show that this power law also may hold true if v_d is close to 0.5. To see it, consider again the Poisson's loss factor, which can be written from Eq. (15) as

$$\frac{\eta_v}{\eta_G} \approx 1 - 2v_d, \tag{18}$$

seeing that $\eta_B \ll \eta_G$ if $v_d \approx 0.5$. Moreover, it is known that $\eta_v \leqslant \eta_B$ [5], and this inequality is satisfied if η_B/η_G obeys the equation

$$\frac{\eta_B}{\eta_G} = 1 - (2v_d)^n, \tag{19}$$

where $n > 1$. Both Eqs. (17) and (19) predict the decrease of η_B/η_G with increasing the dynamic Poisson's ratio in agreement with the prediction made in Section 2.1. In addition, both equations satisfy the requirement on the lower bound defined by Eq. (13).

To sum up, the loss factor ratio, η_B/η_G, can be predicted to decrease with the increase of dynamic Poisson's ratio and the decrease may obey a simple power law if v_d is close to either 0 or 0.5. It is a reasonable assumption that a power law may be adequate to describe the variation of the loss factor ratio in other ranges of dynamic Poisson's ratio as well. Nevertheless, it should be emphasized that it is not thought at all that a power law with specified values of *a* and *n* would universally be valid for all viscoelastic materials over the whole range of v_d. In contrast, the assumption is that a power law may relate the ratio of bulk to shear loss factors to the dynamic Poisson's ratio over some limited intervals, and the parameters, *a* and *n,* certainly vary from material to material.

3. Experimental evidences

Some experimental data on η_B/η_G as functions of v_d are given in Fig. 1, where the lower and upper bounds are shown too. The data are from literature, and concern a styrene-butadiene rubber (SBR) [6] and two hard plastics (poly(methyl methacrylate) (PMMA) [9] and poly(4-thiacyclohexyl methacrylate) (P4OCHMA) [17]). All data have been selected carefully, and are very reliable due to the accurate experimental procedures and preparation of the data described in detail in the relevant works.

The data on the SBR fall into the main transition range of viscoelastic behaviour, which was studied over wide frequency range. The real and imaginary parts of the complex bulk, longitudinal and shear moduli of SBR are available from 10 kHz to 1 GHz at 20 °C [6], and η_B, η_G and v_d were calculated from the published data at some frequencies as described in Ref. [11]. It should be noted that the frequency–temperature equivalence principle was applied in the work [6] to create the SBR data covering wide frequency range. While the frequency–temperature principle has been used intensively for a long time by a number of authors, today it is known that this principle may lead to inaccurate data for some viscoelastic materials. The present author studied carefully the experimental procedure and preparation of the SBR data given in Ref. [6], and had not found any reason to query the accuracy of these data. In contrast to SBR, the data on both hard plastics fall into the secondary transition of viscoelastic behaviour, which was investigated as a function of temperature, and the frequency–temperature principle was not used. In case of PMMA, both η_G, η_B and v_d were determined between −40 and 100 °C at 1 Hz, the data are directly available in Ref. [9] (Figs. 4 and 7). The data at −20, 0, 20 and 40 °C were selected to verify the theoretical predictions. Similarly, in case of P4OCHMA, the magnitudes of both η_G, v_d and v_l measured as a function of temperature between 20 and 120 °C at 800 kHz are available in Ref. [17] (Figs. 2 and 8). From these data the present author calculated the loss factors η_v and η_B at 20, 40 and 60 °C by means of Eqs. (7) and (15), respectively.

The experimental data seen in Fig. 1 convincingly demonstrate that relation exists between the ratio of bulk to shear loss factor and the dynamic Poisson's ratio. It is clear that the larger v_d, the smaller η_B/η_G is. The power law defined by Eq. (19) was fitted to the data on SBR; the result is shown in Fig. 1. The fitting with exponent $n = 2.3$ is rather good not only for this rubber, but also for PMMA. The experimental data definitely support that the simple power law may be adequate to describe the relation in question over some range of dynamic Poisson's ratio. Notwithstanding, it should be emphasized again, that with other materials (e.g., P4OCHMA), and in different ranges of Poisson's ratio, other parameters in the power law may be required to describe the relation between η_B/η_G and ν_d.

4. Conclusions

It has been shown in this paper that the bulk and shear loss factors of isotropic, homogeneous, linear solid viscoelastic materials can be related through the dynamic Poisson's ratio by assuming that the shear loss factor is lower than 0.3. The ratio of the bulk to shear loss factor has been found to be a bounded quantity, and the respective lower bounds have been given as a function of dynamic Poisson's ratio. It has been proved that the bulk and shear loss factors are identical if the dynamic Poisson's ratio is 0, while the bulk loss factor can be much smaller than the shear one if the Poisson's ratio is close to 0.5. The ratio of bulk to shear loss factor has been predicted to decrease by the increase of dynamic Poisson's ratio, and it has been shown that the decrease may obey a simple power law if the Poisson's ratio is close to either 0 or 0.5. The experimental data on a rubber and two hard plastics convincingly support the theoretical predictions. It is hoped that this work will inspire further research to investigate the idea outlined in the paper for viscoelastic materials of different kind, and in other ranges of dynamic Poisson's ratio.

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