Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 324 (2009) 892–902
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Direct and parametric excitation of a nonlinear cantilever beam of varying orientation with time-delay state feedback

Mustafa Yaman*

Department of Mechanical Engineering, Atatürk University, Erzurum, Turkey

Received 11 November 2008; received in revised form 21 January 2009; accepted 9 February 2009
Handling Editor: M.P. Cartmell
Available online 10 March 2009

Abstract

The primary and parametric resonances of a directly and parametrically excited nonlinear cantilever beam of varying orientation with time-delay in the linear state feedback are investigated. The time-delay is presented in the proportional feedback and the derivative feedback, respectively. The method of multiple scales is used to obtain the first-order approximation of response. The effect of the feedback gains and time-delay on the steady state responses of two type resonances is investigated. It is found that a proper selection of the feedback gains and time-delay can enhance the control performance.

1. Introduction

Engineering structures are subjected to vibrations caused by wind, rotating engines and cars, or other environmental disturbances. As a consequence, undesirable bifurcations and high amplitude vibrations may occur and cause failure of the structures. The task of suppressing the dangerous vibrations is very important for engineering science. Plaut and Hsieh [1] studied the effect of a damping time-delay on nonlinear structural vibrations and analyzed six resonance conditions. They presented the results in a number of figures for the steady state response amplitude versus the excitation frequency and amplitude. Hu et al. [2] studied the primary and subharmonic resonances of a harmonically forced Duffing oscillator with two identical time-delays in the state feedback. The concept of an equivalent damping was proposed, and an appropriate choice of the feedback gains and time-delay was discussed from the viewpoint of vibration control. Ji [3] investigated the saddle-node bifurcation control of a forced single degree of freedom Duffing oscillator with damping for the cases of primary and superharmonic resonances, by means of feedback control without time-delay. Ji and Leung [4] discussed the primary, subharmonic and superharmonic resonances of a Duffing system with damping under linear feedback control with two time-delays. Ji and Leung [5] demonstrated that in parametrically excited Duffing systems the stable region of the trivial solution can be broadened, a discontinuous bifurcation can be transformed into a continuous one and the jump phenomenon in the response can be removed, if an appropriate feedback control is used. Maccari [6] dealt with the principal

*Tel.: +90 442 231 4862; fax: +90 442 236 0957.
E-mail address: myaman@atauni.edu.tr

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2009.02.010

parametric resonance of a van der Pol oscillator with time-delay linear state feedback. He also investigated the vibration control for the primary resonance of a forced van der Pol oscillator using time-delay linear state feedback [7] and concluded that the suppression of quasiperiodic motion can be accomplished by appropriate choices for feedback gains and time-delay. Ji and Hansen [8] studied the effect of time-delay nonlinear state feedback on the stability of trivial equilibrium of a van der Pol-Duffing oscillator using linear stability analysis, center manifold technique, normal forms and perturbation method. Morrison and Rand [9] investigated the dynamic of the delayed nonlinear Mathieu equation in the neighborhood of 2:1 resonance.

Qian and Tang [10] studied the primary and subharmonic resonances of a nonlinear dynamic beam under a moving load with the time-delay feedback control. The perturbation method was used to obtain the bifurcation equation of the nonlinear dynamic system. They pointed out that time-delay feedback controller may work well on this system, but the selection of a proper time-delay and its coefficient may depend on the engineering condition.

In this paper, the nonlinear dynamical behavior of a directly and parametrically excited nonlinear cantilever beam of varying orientation with time-delay is analyzed under primary and parametric resonance conditions. This paper mainly focused on the effect of the time-delay and feedback gains on the steady state response of the cantilever beam of varying orientation. The dynamics of the first mode are modeled with a second-order nonlinear ordinary-differential equation. A control law based on time-delay feedback is used. In the remainder of this paper, the method of multiple scales is used to obtain an approximate solution to the differential equation. Two resonance conditions are examined.

2. Model of the system and perturbation analysis

A uniform cantilever beam carrying a mass at free end and subjected to sinusoidal base motion which is $y_g(t) = y_g \sin(\Omega t)$ is shown in Fig. 1. The beam is assumed to be initially straight, of length L, and of constant mass ρA per unit length and constant stiffness. The quantity EI, where E is Young's modulus of the material and I is the principal cross-sectional area moments of inertia, is the bending stiffness of the beam, and ψ is the orientation angle of the beam, s is used to denote arc-length along the beam. The equation of motion of the beam is given by Yaman and Sen [11]

$$\rho A\ddot{w} + c\dot{w} + EI\{w'''' + (w'(w'w'')')'\} + \{[w'\rho A(L-s)]' + m(w')'\}(\ddot{y}_{gu} + g\sin(\psi))$$
$$-\frac{1}{2}\rho A\left[w'\int_s^L \frac{\partial^2}{\partial t^2}\int_0^s w'^2 \mathrm{d}s\,\mathrm{d}s\right]' - \frac{1}{2}m\left[w'\frac{\partial^2}{\partial t^2}\int_0^L w'^2\mathrm{d}s\right]' + (\rho A + \delta[s-(L-\varepsilon)]m)\ddot{y}_{gv} = 0 \tag{1}$$

subjected to the boundary conditions:

$$w(0,t) = 0 \text{ and } w'(0,t) = 0 \tag{2}$$

$$EIw'''(L,t) + mg\,\sin(\alpha)w'(L,t) = m\ddot{w}(L,t) \text{ and } EIw''(L,t) = 0 \tag{3}$$

where $()' = \partial()/\partial s$, $(\dot{}) = \partial()/\partial t$ and y_{gu}, y_{gv} are the projection of the $y_g(t)$ on x and y axes. $w(s, t)$ denotes the component along the inertial direction (y) of the displacement of the beam. After the non-dimensionalization

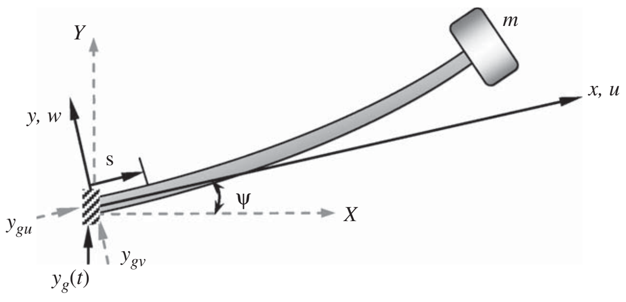

Fig. 1. A schematic of the cantilever beam under consideration.

procedure is carried out, equation of motion becomes

$$\ddot{v}+\frac{cL^2}{\sqrt{\rho AEI}}\dot{v}+v''''+(v'(v'v'')')'-\frac{1}{2}\frac{m}{\rho AL}\left[v'\frac{\partial^2}{\partial\bar{t}^2}\left(\int_0^1 v'^2\mathrm{d}x\right)\right]'-\frac{1}{2}\left[v'\int_x^1\frac{\partial^2}{\partial\bar{t}^2}\left(\int_0^x v'^2\mathrm{d}x\right)\mathrm{d}x\right]'$$
$$-\left(1+\frac{m}{\rho AL}\delta\left[x-\left(1-\frac{\varepsilon}{L}\right)\right]\right)y_0\Omega_0^2\sin(\Omega_0\bar{t})\cos(\psi)$$
$$+\left([v'(1-x)]'+\frac{m}{\rho AL}(v')'\right)(g_0-y_0\Omega_0^2\sin(\Omega_0\bar{t}))\sin(\psi)=0 \quad (4)$$

subjected to the boundary conditions:

$$v(0,\bar{t})=0 \text{ and } v'(0,\bar{t})=0 \quad (5)$$

$$v'''(1,\bar{t})+\frac{mgL^2}{EI}\sin(\psi)v'(1,\bar{t})=\frac{m}{\rho AL}\ddot{v}(1,\bar{t}) \text{ and } v'''(1,\bar{t})=0 \quad (6)$$

where dots are the derivatives with respect to the non-dimensional time $\bar{t}=\sqrt{EI/\rho AL^4}$, $v=w/L$, $y_0=y/L$, $g_0=g(\rho AL^3/EI)$, $\Omega_0=\Omega(\rho AL^4/EI)^{1/2}$ and $x=s/L$. The governing Eq. (4) is nonlinear, and does not admit a closed-form solution. Therefore, an approximate solution will be sought that satisfies both the equation and the boundary conditions. The Galerkin method is used to obtain an ordinary differential equation form of given partial differential equation for an approximate solution. In this study, it is assumed that most of the energy excites the first mode of the system, thus the first mode is dominant. Therefore, the truncated displacement function for the first mode becomes

$$v(x,\bar{t})=\phi(x)z(\bar{t}) \quad (7)$$

where $\phi(x)$ is the shape function of the linear mode, and $z(\bar{t})$ is the time modulation of the mode. The undamped linear free vibration problem under axial loading is governed by

$$\ddot{v}+v''''+\frac{\rho AgL^3}{EI}\sin(\psi)[(1-x)v']'+\frac{mgL^2}{EI}\sin(\psi)v''=0 \quad (8)$$

The governing Eq. (8) is a variable coefficient fourth-order partial differential equation, and there is no any closed-form solution. Therefore, an approximate solution has been obtained, which satisfies both the equation and the boundary conditions. The solution of this problem is obtained by applying Adomian decomposition method [12]

$$\phi(x)=0.5x^2-0.013324\sin(\psi)x^4+0.18151\times10^{-2}\sin(\psi)x^5+[0.14204\times10^{-3}\sin^2(\psi)$$
$$+0.13888\times10^{-2}\omega^2]x^6-0.41463\times10^{-4}\sin^2(\psi)x^7+\ldots C_2/C_1\{0.16x^3-0.26649\times10^{-2}\sin(\psi)x^5$$
$$+0.45379\times10^{-3}\sin(\psi)x^6+[0.19841\times10^{-3}\omega^2+0.20291\times10^{-4}\sin^2(\psi)]x^7$$
$$-0.69105\times10^{-5}\sin^2(\psi)x^8\ldots\} \quad (9)$$

where

$$C_2/C_1=-[1+0.2684\times10^{-2}\sin^2(\psi)+0.41666\times10^{-1}\omega^2-0.12359\sin(\psi)-0.45423\times10^{-4}\sin^3(\psi)$$
$$-0.88832\times10^{-3}\sin(\psi)\omega^2]/[1+0.4652\times10^{-3}\sin^2(\psi)-0.3968\times10^{-1}\sin(\psi)+0.8333\times10^{-2}\omega^2] \quad (10)$$

By substituting Eq. (7) into the partial differential Eq. (4) and orthogonalizing the error with respect to the eigenfunction, the following ordinary differential equation is obtained for the beam

$$k_1\ddot{z}+\mu k_1\dot{z}+(k_2+k_{10})z+k_5z^3-k_{11}z\frac{\partial^2}{\partial\tau^2}(z^2)-k_{12}\Omega_0^2\sin(\Omega_0\bar{t})\cos(\psi)-zk_{13}\Omega_0^2\sin(\Omega_0\bar{t})\sin(\psi)=0 \quad (11)$$

where $k_i=1,2,\ldots,13$ are constant coefficients as defined in Appendix A. To simplify, Eq. (11) is rescaled so as to make coefficients of the linear inertia and linear restoring terms equal to unity. This is achieved by defining the dimensionless parameters $t^*=\theta^2\bar{t}$, $\bar{\Omega}=\Omega_0/\theta^2$ and $u=\theta z$, where t^* is a new time scale and

$\theta^4 = (k_2 + k_{10})/k_1$. In terms of these dimensionless parameters Eq. (11) becomes the dimensionless equation:

$$\ddot{u} + u + 2\varepsilon\bar{\mu}\dot{u} + \varepsilon\alpha_1 u^3 - \varepsilon\alpha_2 u^2\ddot{u} - \varepsilon\alpha_2 u\dot{u}^2 = \varepsilon f_2\bar{\Omega}^2 \sin(\bar{\Omega}t^*)\cos(\psi) + \varepsilon u f_1\bar{\Omega}^2 \sin(\Omega t^*)\sin(\psi) \tag{12}$$

where u is the generalized coordinate, $\bar{\mu}$ is the viscous damping coefficient, α_i are the constants, f_i and $\bar{\Omega}$ are the forcing amplitude and frequency, respectively, ε is a non-dimensional bookeeping parameter. By adding a linear time-delayed state feedback to system, one can obtain the following new closed-loop system:

$$\ddot{u} + u + 2\varepsilon\bar{\mu}\dot{u} + \varepsilon\alpha_1 u^3 - \varepsilon\alpha_2 u^2\ddot{u} - \varepsilon\alpha_2 u\dot{u}^2 = \varepsilon f_2\bar{\Omega}^2 \sin(\bar{\Omega}t^*)\cos(\psi) + \varepsilon u f_1\bar{\Omega}^2 \sin(\Omega t^*)\sin(\psi) + \varepsilon g_1 u(t^* - \tau) + \varepsilon g_2\dot{u}(t^* - \tau) \tag{13}$$

To analyze the primary resonance of the system by using the method of multiple scales [13,14], one assumes an approximate solution of Eq. (13) in the form

$$u(T_0, T_1) = u_0(T_0, T_1) + u_1(T_0, T_1) + \cdots \tag{14}$$

where $T_n = \varepsilon^n t$, $n = 0, 1, 2$. The time derivatives are recast in terms of the new time scales as

$$\frac{\mathrm{d}}{\mathrm{d}t^*} = D_0 + \varepsilon D_1 + \cdots \quad \text{and} \quad \frac{\mathrm{d}^2}{\mathrm{d}t^{*2}} = D_0^2 + 2\varepsilon D_0 D_1 + \cdots \tag{15}$$

where $D_k \equiv \partial/\partial T_k$. Substituting Eqs. (14) and (15) into Eq. (13) and equating the same power of ε, we obtain a set of partial differential equations

$$D_0^2 u_0 + u_0 = 0 \tag{16}$$

$$D_0^2 u_1 + u_1 = -2D_0 D_1 u_0 - 2\mu D_0 u_0 - \alpha_1 u_0^3 + \alpha_2 u_0^2 D_0^2 u_0 + \alpha_2 u_0 (D_0 u_0)^2 + g_1 u_0(T_0 - \tau, T_1) + g_2 D_0 u_0(T_0 - \tau, T_1) + u_0 f_1\bar{\Omega}^2 \sin(\psi)\sin(\bar{\Omega}\mathrm{T}_0) + f_2\bar{\Omega}^2\cos(\psi)\sin(\bar{\Omega}T_0) \tag{17}$$

The solution of Eq. (16) is written as follows

$$u_0 = A(T_1)\mathrm{e}^{\mathrm{i}T_0} + \bar{A}(T_1)\mathrm{e}^{-\mathrm{i}T_0} \tag{18}$$

where $A(T_1)$ is a complex-valued quantity that will be determined by imposing the solvability condition at the next level of approximation. In the case of primary resonance (i.e.,$\bar{\Omega} \approx 1$), to express the nearness of $\bar{\Omega}$ to 1, a detuning parameter σ is introduced such that

$$\bar{\Omega} = 1 + \varepsilon\sigma \tag{19}$$

Substituting Eqs. (18) and (19) into (17), and eliminating secular terms lead to

$$2i(D_1 A + \bar{\mu}A - \tfrac{1}{2}g_2 A\mathrm{e}^{-\mathrm{i}\tau}) + (3\alpha_1 + 2\alpha_2)A^2\bar{A} - g_1 A\mathrm{e}^{-\mathrm{i}\tau} + \tfrac{1}{2}if_2\mathrm{e}^{\mathrm{i}\sigma T_1}\cos(\psi) = 0 \tag{20}$$

Substituting the polar form

$$A = \tfrac{1}{2}a(T_1)\mathrm{e}^{\mathrm{i}\beta(T_1)} \tag{21}$$

into Eq. (20), performing the integration and separating real and imaginary parts the followings are obtained:

$$a' = -(\bar{\mu} + \tfrac{1}{2}g_1\sin(\tau) - \tfrac{1}{2}g_2\cos(\tau))a - \tfrac{1}{2}f_2\cos(\psi)\cos(\phi) \tag{22}$$

$$a\phi' = (\sigma + \tfrac{1}{2}g_2\sin(\tau) + \tfrac{1}{2}g_1\cos(\tau))a - (\tfrac{1}{4}\alpha_2 + \tfrac{3}{8}\alpha_1)a^3 + \tfrac{1}{2}f_2\cos(\psi)\sin(\phi) \tag{23}$$

where $\phi = \sigma T_1 - \beta$ and the prime represents differentiation with respect to T_1. Steady-state solutions of Eq. (13) correspond to the fixed points of Eqs. (22) and (23), which are obtained by setting $a' = \phi' = 0$. The result is

$$-(\bar{\mu} + \tfrac{1}{2}g_1\sin(\tau) - \tfrac{1}{2}g_2\cos(\tau))a - \tfrac{1}{2}f_2\cos(\psi)\cos(\phi) = 0 \tag{24}$$

$$(\sigma + \tfrac{1}{2}g_2\sin(\tau) + \tfrac{1}{2}g_1\cos(\tau))a - (\tfrac{1}{4}\alpha_2 + \tfrac{3}{8}\alpha_1)a^3 + \tfrac{1}{2}f_2\cos(\psi)\sin(\phi) = 0 \tag{25}$$

Eliminating ϕ from Eqs. (24) and (25), we obtain the bifurcation equation

$$[\mu_t^2 + (\sigma_t - \alpha_t a^2)^2]a^2 = \left[\frac{f_2 \cos(\psi)}{2}\right]^2 \tag{26}$$

where

$$\alpha_t = \tfrac{1}{4}\alpha_2 + \tfrac{3}{8}\alpha_1, \quad \mu_t = \bar{\mu} + \tfrac{1}{2}g_1 \sin(\tau) - \tfrac{1}{2}g_2 \cos(\tau), \quad \sigma_t = \sigma + \tfrac{1}{2}g_2 \sin(\tau) + \tfrac{1}{2}g_1 \cos(\tau) \tag{27}$$

The amplitude of the response is a function of the external detuning, orientation angle, feedback gains, time-delay and the amplitude of the excitation. The peak amplitude of the primary resonance response, obtained from Eq. (26), is given by

$$a_p = \frac{f_2}{2\mu_t}\cos(\psi) \tag{28}$$

The real solution a of Eq. (26) determines the primary resonance response amplitude. There can be either one or three real solutions. Three real solutions exist between two points of vertical tangents (saddle-node bifurcation), which are determined by differentiation of Eq. (26) implicitly with respect to a^2. This leads to the condition

$$\sigma_t^2 - 4\alpha a^2 \sigma_t + 3\alpha_t^2 a^4 + \mu_t^2 = 0 \tag{29}$$

The solution of Eq. (29) is written as follows

$$\sigma_{t_{12}} = 2\alpha_t a^2 \pm (\alpha_t^2 a^4 - \mu_t^2)^{1/2} \tag{30}$$

For $\alpha_t^2 a^4 > \mu_t^2$ there is an interval $\sigma_{t_1} < \sigma_t < \sigma_{t_2}$ in which three real and positive solutions amplitude a of Eq. (26) exist. In the limit $\alpha_t^2 a^4 \to \mu_t^2$, this interval becomes a point $\sigma_t = 2\alpha_t a^2$. The stability of the solutions is determined by the eigenvalues of the corresponding Jacobian matrix of Eqs. (22) and (23). The corresponding eigenvalues are the root of

$$\lambda^2 + 2\mu_t \lambda + \mu_t^2 + (\sigma_t - \alpha_t a^2)(\sigma_t - 3\alpha_t a^2) = 0 \tag{31}$$

As can be seen from Eq. (31) the sum of the two eigenvalues is $-2\mu_t$. For the uncontrolled system, the sum of the two eigenvalues is $-2\bar{\mu}$ which is negative. The addition of the feedback gains and time-delay changes the sum of the two eigenvalues. Depending on the values of the feedback gains and time-delay, three cases such as $\mu_t < 0$, $\mu_t = 0$, and $\mu_t > 0$ may occur. If the feedback gains and time-delay are selected in such a way that the sum of the two eigenvalues is positive ($\mu_t < 0$), at least one of the two eigenvalues will always have a positive real part. The system will be unstable. The selection of the feedback gains and time-delay is not possible. On the other hand, when the sum of the two eigenvalues is zero ($\mu_t = 0$) for a certain value of the feedback gains and time-delay, a pair of purely imaginary eigenvalues may occur, thus yielding a Hopf bifurcation. Therefore, the above two cases should be avoided from the viewpoint of bifurcation control. The feedback should be provided at least in such a way that $\mu_t > 0$ is satisfied. The sum of the two eigenvalues is always negative, under such feedback gains and time-delay. Accordingly, at least one of the two eigenvalues will always have a negative real part. The other eigenvalue is zero when Eq. (32) is satisfied

$$\mu_t^2 + (\sigma_t - \alpha_t a^2)(\sigma_t - 3\alpha_t a^2) = 0 \tag{32}$$

where a saddle-node bifurcation occurs. It has been shown that the feedback gains and time-delay can change the quantities of μ_t and σ_t, which govern the peak amplitude of the primary resonance response, and the stability of steady state motions. The peak amplitude of the response a_p is inversely proportional to μ_t and directly proportional to orientation angle ψ. Thus, the peak amplitude of the response a_p decreases (or increases) as μ_t and ψ increases (or decreases). On the other hand, if the resulting μ_t and σ_t maintain the inequality $\mu_t^2 + (\sigma_t - \alpha_t a^2)(\sigma_t - 3\alpha_t a^2) > 0$, there is not an unstable solution. The system will not show jump and hysteresis phenomenon. Thus, the appropriate feedback gains and time-delay can improve the control performance.

In the case of principal parametric resonance (i.e., $\bar{\Omega} \approx 2$), the excitation frequency can be expressed as $\bar{\Omega} = 2 + \varepsilon\sigma$, where the parameter σ is called the external detuning. Substituting Eqs. (18) and $\bar{\Omega} = 2 + \varepsilon\sigma$

into (17) and after the secular terms are eliminated, the solvability condition yields,

$$2i(D_1A+\bar{\mu}A-\tfrac{1}{2}g_2Ae^{-i\tau})+(3\alpha_1+2\alpha_2)A^2\bar{A}-g_1Ae^{-i\tau}+i2f_1e^{i\sigma T_1}\bar{A}\sin(\psi)=0 \tag{33}$$

The amplitude a and the phase ϕ are governed by the following polar form of modulation equations

$$a'=-\mu_t a-af_1\sin(\psi)\cos(\phi) \tag{34}$$

$$a\phi'=a\sigma_t-\alpha_t a^3+2af_1\sin(\psi)\sin(\phi) \tag{35}$$

where $\phi=\sigma T_1-2\beta$

$$\alpha_t=\tfrac{1}{2}\alpha_2+\tfrac{3}{4}\alpha_1,\quad \mu_t=\bar{\mu}+\tfrac{1}{2}g_1\sin(\tau)-\tfrac{1}{2}g_2\cos(\tau),\quad \sigma_t=\sigma+g_1\cos(\tau)+g_2\sin(\tau) \tag{36}$$

Steady-state solutions of Eq. (13) correspond to the fixed points of Eqs. (34) and (35), which are obtained by setting $a'=\phi'=0$. There are two possibilities: either a trivial solution $a=0$, or non-trivial solutions

$$a^2=\frac{1}{\alpha_t}\left[\sigma_t\mp 2\sqrt{f_1^2\sin^2(\psi)-\mu_t^2}\right],\quad \phi=\arccos\left(-\frac{\mu_t}{f_1\sin(\psi)}\right) \tag{37}$$

when the right-hand side of the first formula is real and positive. Depending on the sign of the quantity $\sigma_t\alpha_t$, one or two solutions for the amplitude are available. For $\sigma_t\alpha_t>0$, non-trivial fixed points are possible for $f_1^2\sin^2(\psi)>\mu_t^2$. There are two different non-trivial solutions for $a>0$. One exists for $f_1^2\sin^2(\psi)>\mu_t^2$, which is obtained as

$$a=\left[\frac{1}{|\alpha_t|}\left(|\sigma_t|+2\sqrt{f_1^2\sin^2(\psi)-\mu_t^2}\right)\right]^{1/2} \tag{38}$$

The other exists for $\mu_t^2<f_1^2\sin^2(\psi)<\mu_t^2+\sigma_t^2/4$, which is given by

$$a=\left[\frac{1}{|\alpha_t|}\left(|\alpha_t|-2\sqrt{f_1^2\sin^2(\psi)-\mu_t^2}\right)\right]^{1/2} \tag{39}$$

For $\sigma_t\alpha_t<0$, for the existence of real solutions of amplitude a it is required that $f_1^2\sin^2(\psi)>\mu_t^2+\sigma_t^2/4$. The only single solution $a>0$ is given by

$$a=\left[\frac{1}{|\alpha_t|}\left(-|\alpha_t|+2\sqrt{f_1^2\sin^2(\psi)-\mu_t^2}\right)\right]^{1/2} \tag{40}$$

In the investigation of the stability of the trivial solutions, it is necessary to express Eqs. (34) and (35) in Cartesian form. For this purpose, A is described in the Cartesian form

$$A=\tfrac{1}{2}(p+iq)e^{i(1/2)\sigma T_1} \tag{41}$$

where p and q are real. Substituting Eq. (41) into (33) and separating real and imaginary parts, the followings are obtained:

$$p'=-[\mu_t+f_1\sin(\psi)]p+\tfrac{1}{2}\sigma_t q \tag{42}$$

$$q'=-\tfrac{1}{2}\sigma_t p-[\mu_t-f_1\sin(\psi)]q \tag{43}$$

The stability of the fixed points is determined by the eigenvalues of the corresponding Jacobian matrix of Eqs. (42) and (43). The eigenvalues for the trivial solutions are the roots of

$$\lambda^2+2\mu_t\lambda+\mu_t^2+\tfrac{1}{4}\sigma_t^2-f_1^2\sin^2(\psi)=0 \tag{44}$$

while the eigenvalues for non-trivial solutions are the roots of

$$\lambda^2+2\mu_t\lambda+\alpha_t(-\sigma_t+\alpha_t a^2)a^2=0 \tag{45}$$

As can be seen from Eq. (45) the sum of the two eigenvalues is $-2\mu_t$. For the uncontrolled system, the sum of the two egivalues becomes $-2\bar{\mu}$, which is negative. The introduction of feedback control changes the sum of the two eigenvalues. Depending on the values of the feedback gains and time-delay, three cases, such as $\mu_t<0$,

$\mu_t = 0$ and $\mu_t > 0$, may occur. From the viewpoint of bifurcation control, the feedback control should be implemented at least in such a way that $\mu_t > 0$ is satisfied. As a result, at least one of the two eigenvalues will always have a negative real part. The other eigenvalue for the trivial solutions is zero if $f_1^2 \sin^2(\psi) < \mu_t^2 + \sigma_t^2/4$, where a pitchfork bifurcation occurs. In particular, for $\sigma_t \alpha_t < 0$, a supercritical pitchfork bifurcation occurs, while for $\sigma_t \alpha_t > 0$, a subcritical pitchfork bifurcation takes place. The addition of feedback control varies the value of quantity $\mu_t^2 + \sigma_t^2/4$, and thus can enlarge the stable region of the trivial solutions. For $\sigma_t \alpha_t > 0$, the non-trivial solution amplitude a, given by Eq. (38), is stable, while the other given by Eq. (39) is unstable. A saddle-node bifurcation occurs at $f_1^2 \sin^2(\psi) = \mu_t^2$. For $\sigma_t \alpha_t < 0$, the only non-trivial solution a, given by Eq. (40), is stable.

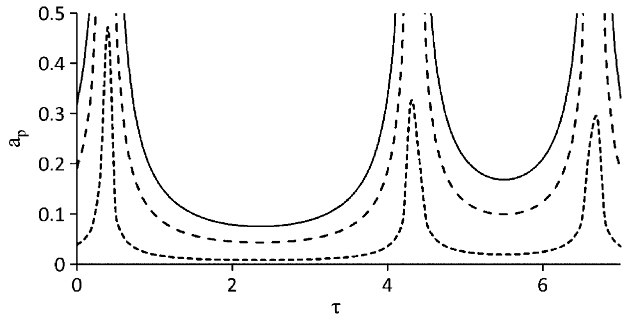

Fig. 2. The peak amplitude of the primary resonance response a as a function of the time-delay — for $\psi = 0°$, ----- for $\psi = 40°$ and ······· for $\psi = 80°$.

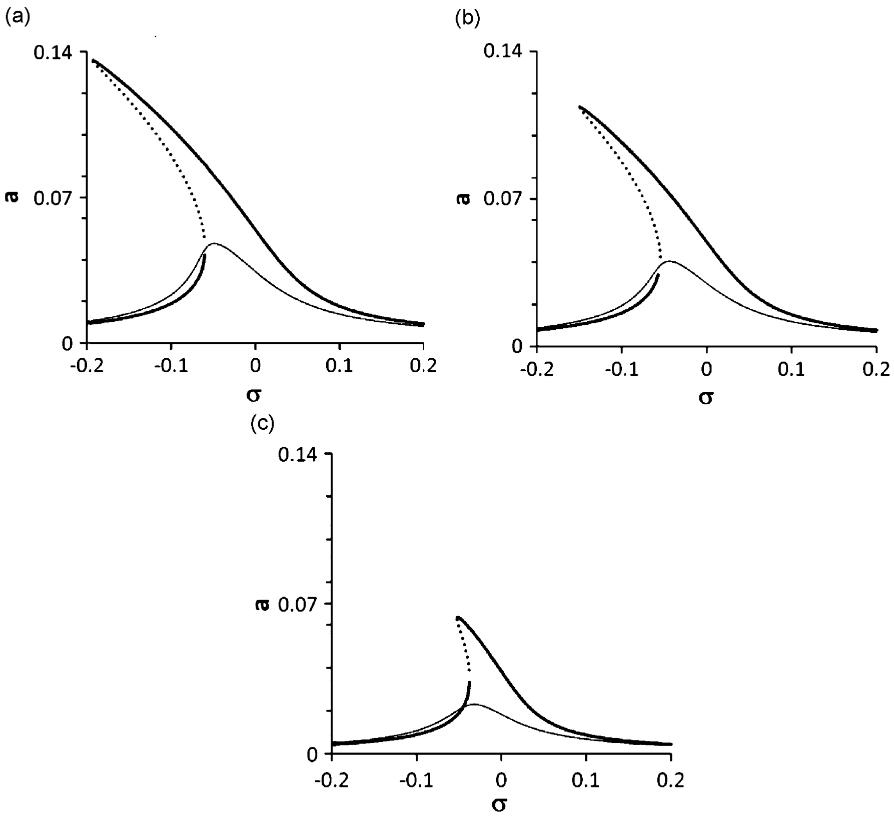

Fig. 3. Frequency-response curves for primary resonance for two sets of the time-delay, curves for $\tau = 0$ and $\pi/2$ (a) $\psi = 0°$ (b) $\psi = 30°$ and (c) $\psi = 60°$.

3. Results and discussion

This section illustrates the effect of the feedback gains and time-delay on the nonlinear dynamical behavior of the controlled system. The results of this study are presented for the system parameters: ρA = beam density = 0.3925 kg/m, m = tip mass = 0.266 kg, L = length of the beam = 0.35 m, h = thickness of the beam = 0.002 m, EI = beam flexural rigidity = 1.5158 Nm^2, c = 0.2 Ns/m.

In Fig. 2, the peak amplitude of the primary resonance response is plotted as a function of the time-delay for three different orientation angles. It is easy to see that when the orientation angle is increased the selection interval of the time-delay extends. So the proper value of τ may take into account the requirement of engineering.

Fig. 3 shows the frequency-response curves for the primary resonance response in various orientation of the directly excited beam with $g_p = 0.05$, $g_d = 0.05$ and τ (time-delay) $= \pi/2$. The thick line is corresponding to the original system and thin line corresponding controlled system with time-delay. Obviously, Fig. 3(a) shows that the saddle-node bifurcation and jump phenomenon can be eliminated by an appropriate selection of the time-delay. Figs. 3(b) and (c) show the frequency-response curves for the beam oriented 30° and 60°. As can be seen from these figures, even if the beam is oriented, both the saddle-node bifurcation and jump phenomenon can be still eliminated.

Fig. 4 shows the forcing amplitude-response diagram of parametrically excited beam when y_g is used as a control parameter for different orientation angles with $g_p = 0.2$, $g_d = 0.2$ and τ (time-delay) $= \pi/2$. In Fig. 4(a), the bifurcation diagrams for the uncontrolled system which is illustrated by thick line and controlled system which is illustrated by thick line in the figure are illustrated for $\sigma = 0.1$. In this and the subsequent figures, the solid and broken lines correspond to the stable and unstable fixed points, respectively. It is clearly seen that the region of excitation amp litude y_g for stable trivial fixed points of the controlled system is much

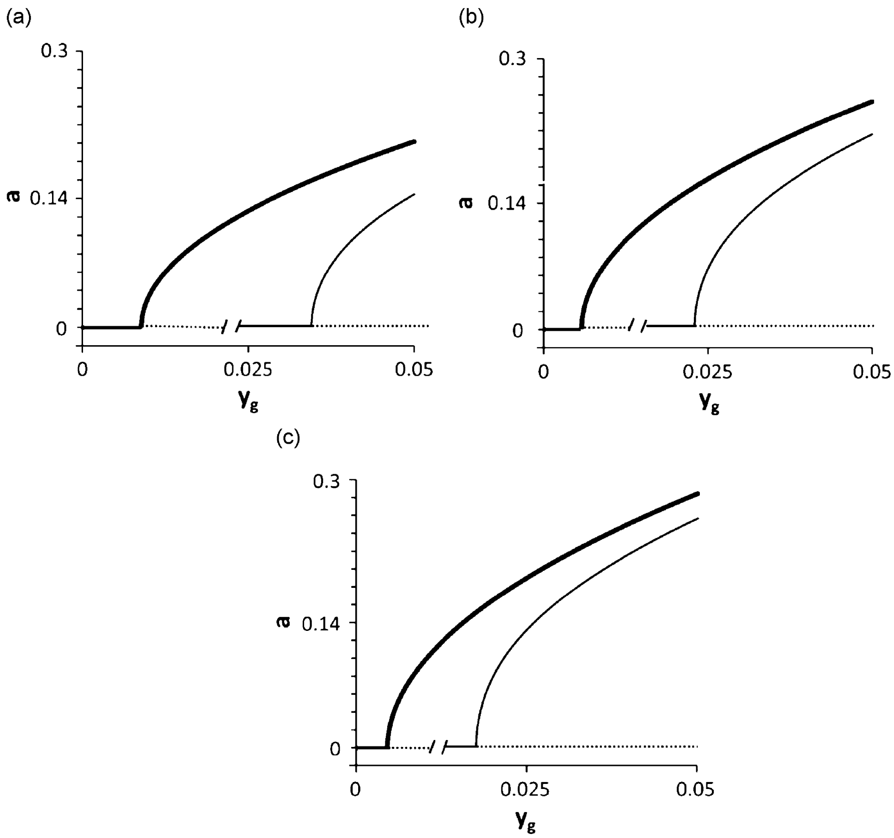

Fig. 4. Bifurcation diagram when y_g is used as a control parameter for $\sigma < 0$ (a) $\psi = 30°$ (b) $\psi = 50°$ and (c) $\psi = 90°$.

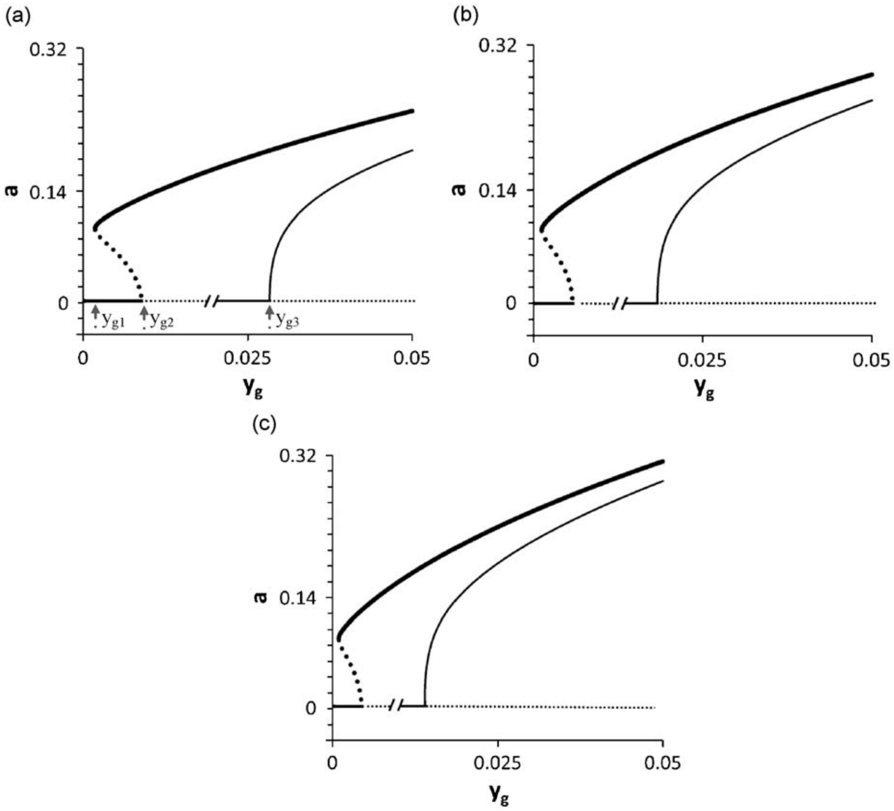

Fig. 5. Bifurcation diagram when y_g is used as a control parameter for $\sigma > 0$ (a) $\psi = 30^\circ$ (b) $\psi = 50^\circ$ and (c) $\psi = 90^\circ$.

larger than that of the uncontrolled system. This suggests that the occurrence of a supercritical pitchfork bifurcation can be delayed in the controlled system. Figs. 4(b) and (c) show bifurcation diagram curves for the beam oriented 50° and 90°. It is clearly seen from the figures that when the beam is oriented, the control system still maintains its same performance. However, when the orientation angle is increased, the region of excitation amplitude y_g for stable trivial fixed points of the controlled system decreases.

Fig. 5 shows the bifurcation diagrams for $\sigma = -0.1$. For the uncontrolled system shown in Fig. 5(a) with thick line, as the excitation amplitude y_g is gradually increased from zero, the trivial fixed point loses its stability at $y_g = y_{g2}$, where a subcritical pitchfork bifurcation originates. This discontinuous bifurcation leads to a jump from the lower branch to the upper branch. On the other hand, as y_g is decreased gradually, the non-trivial fixed point remains stable until $y_g = y_{g1}$ is reached. Here, a saddle-node bifurcation, which is another example of discontinuous bifurcation, occurs and the system suddenly jumps to the lower branch. In contrast, for the controlled system as displayed by thin line in Fig. 5(a), as the excitation amplitude y_g is gradually increased from zero, the trivial fixed point loses its stability at $y_g = y_{g3}$, where a supercritical pitchfork bifurcation occurs. For $y_g > y_{g3}$, a stable non-trivial fixed point is produced. No jump phenomenon can be observed. Due to the presence of the feedback control, the subcritical pitchfork bifurcation is transformed into a supercritical one, and the saddle-node bifurcation is excluded. Figs. 5(b) and (c) show bifurcation diagram curves for the beam oriented 50° and 90°. When the orientation angle is increased, the time-delay feedback control is still effective, but the region of excitation amplitude y_g for stable trivial fixed points of the controlled system decreases.

4. Conclusions

The nonlinear response of a directly and parametrically excited nonlinear cantilever beam of varying orientation with time-delay is investigated under primary and parametric resonances. The effect of the feedback gains and time-delay on the nonlinear response of the system is discussed. It is found that an appropriate feedback can enhance the control performance. A suitable choice of the feedback gains and time-delay can eliminate saddle-node bifurcation, thereby eliminating jump and hysteresis phenomena taking place in the corresponding uncontrolled system for primary resonance.

The steady-state response of a parametrically excited system can exhibit a jump and hysteresis phenomenon under the principal parametric resonance. It is found that an appropriate linear time-delayed feedback control is effective in delaying the occurrence of the pitchfork bifurcations, stabilizing the subcritical pitchfork bifurcations, and eliminating the saddle-node bifurcations.

Appendix A

$$k_1 = \int_0^1 \phi^2 \mathrm{d}x, \quad k_2 = \int_0^1 \phi'''' \phi \, \mathrm{d}x, \quad k_3 = \int_0^1 \phi[\phi'\{1-x\}]' \mathrm{d}x$$

$$k_4 = \int_0^1 \phi[\phi']' \mathrm{d}x, \quad k_5 = \int_0^1 \phi\left[\phi'(\phi'\phi'')'\right]' \mathrm{d}x, \quad k_6 = \int_0^1 \phi\left[\phi' \int_x^1 \int_0^x \phi'^2 \mathrm{d}x \, \mathrm{d}x\right]' \mathrm{d}x$$

$$k_7 = \int_0^1 \phi\left[\phi' \int_0^1 \phi'^2 \mathrm{d}x\right]' \mathrm{d}x, \quad k_8 = \int_0^1 \phi\delta(x-(1-\varepsilon/L))\mathrm{d}x, \quad k_9 = \int_0^1 \phi \, \mathrm{d}x,$$

$$k_{10} = \left(k_3 + \frac{m}{\rho AL}k_4\right) g_0 \sin(\alpha), \quad k_{11} = \tfrac{1}{2}\left(k_6 + \frac{m}{\rho AL}k_7\right), \quad k_{12} = \left(k_9 + \frac{m}{\rho AL}k_8\right) y_o,$$

$$k_{13} = \left(k_3 + \frac{m}{\rho AL}k_4\right) y_o, \quad \mu = cL^2/\sqrt{\rho AEI}, \quad 2\varepsilon\bar{\mu} = \mu/\theta^2, \quad g_0 = g\frac{\rho AL^3}{EI},$$

$$\varepsilon\alpha_1 = \frac{k_5}{k_1\theta^6}, \quad \varepsilon\alpha_2 = 2\frac{k_{11}}{k_1\theta^2}, \quad \varepsilon f_1 = \frac{k_{13}}{k_1}, \quad \varepsilon f_2 = \frac{k_{12}}{k_1}\theta, \quad \Omega_0 = \Omega\sqrt{\frac{\rho AL^4}{EI}}.$$

References

[1] R.H. Plaut, J.C. Hsieh, Nonlinear structural vibrations involving a time-delay in damping, *Journal of Sound and Vibration* 117 (1987) 497–510.

[2] H. Hu, E.H. Dowell, L.N. Virgin, Resonances of a harmonically forced Duffing oscillator with time delay state feedback, *Nonlinear Dynamics* 15 (1998) 311–327.

[3] J.C. Ji, Local bifurcation control of forced single-degree-of-freedom nonlinear system: saddle-node bifurcation, *Nonlinear Dynamics* 25 (2001) 369–382.

[4] J.C. Ji, A.Y.T. Leung, Resonances of a nonlinear SDOF system with two time-delays on linear feedback control, *Journal of Sound and Vibration* 253 (2002) 985–1000.

[5] J.C. Ji, A.Y.T. Leung, Bifurcation control of parametrically excited Duffing system, *Nonlinear Dynamics* 27 (2002) 411–417.

[6] A. Maccari, The resonances of a parametrically excited van der Pol oscillator to a time delay state feedback, *Nonlinear Dynamics* 26 (2001) 105–119.

[7] A. Maccari, Vibration control for the primary resonance of the van der Pol oscillator by a time delay state feedback, *International Journal of Nonlinear Mechanics* 38 (2003) 123–131.

[8] J.C. Ji, C.H. Hansen, Stability and dynamics of a controlled van der Pol-Duffing oscillator, *Chaos, Solutions Fractals* 28 (2006) 555–570.

[9] T.M. Morrison, R.H. Rand, 2: 1 Resonance in the delayed nonlinear Mathieu equation, *Nonlinear Dynamics* 50 (2007) 341–352.

[10] C.Z. Qian, J.S. Tang, A time delay control for a nonlinear dynamic beam under moving load, *Journal of Sound and Vibration* 309 (2008) 1–8.

[11] M. Yaman, S. Sen, The analysis of the orientation effect of non-linear flexible systems on performance of the pendulum absorber, *International Journal of Non-Linear Mechanics* 39 (5) (2004) 741–752.

[12] M. Yaman, Adomian Decomposition Method for Solving a Cantilever Beam of Varying Orientation with Tip Mass, *Journal of Computational and Nonlinear Dynamics* 2 (2007) 52–57.
[13] A.H. Nayfeh, *Perturbation Methods*, Wiley, New York, 1973.
[14] A.H. Nayfeh, D.T. Mook, *Nonlinear Oscillations*, Wiley, New York, 1979.