Journal of Sound and Vibration 327 (2009) 285–298

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Robust adaptive backstepping attitude and vibration control with L_2-gain performance for flexible spacecraft under angular velocity constraint

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A R T I C L E I N F O

Article history:
Received 14 September 2008
Received in revised form
30 May 2009
Accepted 3 July 2009
Handling Editor: J. Lam
Available online 29 July 2009

A B S T R A C T

This paper proposes an angular velocity bounded robust adaptive control design for attitude maneuver and vibration reduction in the presence of external disturbances and uncertainties in the inertia matrix. The control design is Lyapunov based to ensure closed-loop stability, boundedness of system states and tracking error convergence. Specifically, an adaptive controller based on backstepping technique with the assumption of bounded elastic vibrations is first designed that ensures the equilibrium points in the closed-loop system uniform ultimate bounded stability in the presence of unknown inertia matrix and bounded disturbances, incorporating constraints on individual angular velocity. The prescribed robust performance is also evaluated by L_2-gain, less than any given small level, from a torque level disturbances signal to a penalty output. Then this controller is redesigned such that this assumption is released by using an elastic vibration estimator, which supplies their estimates. The external torque disturbances attenuation along with estimate errors with respect to the performance measure are also ensured in the L_2-gain sense and the induced vibrations can be actively reduced as well. The novelty of our approach is in the strategy to construct such a Lyapunov function under bounded angular velocity recursively that ensures not only stability of a tracking error system but also an L_2-gain constraint. Compared with the conventional methods, the proposed scheme guarantees not only the stability of the closed-loop system, but also the good performance as well as the robustness. Simulation results for the spacecraft model show that the precise attitudes control and vibration suppression are successfully achieved.

1. Introduction

One of the most important problems in spacecraft design is of attitude stabilization and control. Although the missions of space vehicles and their attitude requirements vary greatly, high pointing accuracy is an important part of the overall design problem for spacecraft control system. However, the orbiting attitude slewing or maneuvering operation will introduce certain levels of vibration to flexible appendages, which will deteriorate its pointing performance. In addition, dynamics of large rotational maneuvers is time varying and nonlinear, and affected by various disturbances coming from the environment and knowledge about system parameters such as the inertia matrix and modal frequencies, which are usually not well known. A more significant challenge arises when all these issues are treated simultaneously for the designers.

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doi:10.1016/j.jsv.2009.07.002

In face of increasingly complex and highly uncertain nature of spacecraft dynamical systems requiring controls, many studies related to attitude control of flexible spacecraft have been done, and robust linear and nonlinear control systems have been designed. Control laws based on linearization and nonlinear inversion have been presented in Ref. [1]. Optimal and nonlinear control systems for the control of flexible spacecraft have been developed in Ref. [2, 3]. Variable structure control (VSC) to certain types of disturbances and uncertainties also makes it attractive for spacecraft control problems [4–7]. However, these design methods based on VSC require the information on the bounds on the uncertainties/disturbances for the computation of the control gains. Unlike these methods, nonlinear adaptive control methods do not require these bounds, instead, by including an adaptation mechanism for tuning the time-varying controller gains. A variety of adaptive spacecraft controllers have been developed [8–10]. Recently, researches have also been focused on the combination of VSC and adaptive control to develop simple and adaptive robust spacecraft controllers that work for a wide range of practical systems [11–14]. Although much progress in these works [4–14] has been made for flexible spacecraft attitude control in the presence of inertia matrix uncertainties and external disturbances, there are no means of incorporating constraints on individual angular velocity. Constraints on rigid body angular velocity might be practically required for many applications, such as rendezvous of spacecraft.

A recursive control system design methodology called adaptive backstepping [15] has received much attention in recent years. Backstepping is a nonlinear control design technique that employs Lyapunov synthesis to recursively determine controller for systems satisfying a particular cascaded structure called 'lower-triangular-feedback' form. In this approach, some system states are used as virtual control inputs for subsystems of other states, by defining a positive definite control Lyapunov function and dynamic feedback control law at each intermediate step. Based on this technique, a direct adaptive fuzzy backstepping control is presented for a class of nonlinear systems in Ref. [16]. For attitude stabilization and tracking of rigid spacecraft, the backstepping controllers with or not adaptive mechanism were presented in Refs. [17–19] and reference therein. However, in these researches, highly aggressive controllers are produced due to the unbounded inputs for the standard adaptive backstepping design. In Ref. [20], a called constrained adaptive backstepping method by using the command filters calculates the derivatives of the virtual controls such that this limitation can be removed. It should be mentioned that the main attention of these control schemes is focused on only stability analysis and evaluation for the control performance is not considered explicitly in the control system design. Even if there are some relative researches on robotic tracking control with H_∞ or L_2-gain performance to guarantee arbitrary transient performance as well as arbitrary disturbance attenuation, these control approaches for flexible robotic systems including states constraint has not been studied enough at this point. To the best of the authors' knowledge, there has been little research effort expended to study flexible spacecraft attitude control system with incorporating constraints on individual angular velocity as well as control performance evaluation in the adaptive backstepping control design paradigm.

The contributions of this paper are described as follows. First, the proposed control scheme enlarged the previous methods [4–14] by incorporating the criterion of tracking performance given by L_2-gain constraint in controller synthesis. The novelty is in the strategy to construct such a Lyapunov function recursively that ensures not only stability of the tracking error system but also satisfies the dissipation inequality ensuring L_2-gain performance. Besides, the proposed scheme was extended so as to deal with elastic vibration suppression by incorporating an elastic mode estimator to supply their estimates such that the external torque disturbances attenuation along with estimate errors with respect to the performance measure are also ensured in the L_2-gain sense and the induced vibrations can be actively reduced as well. Second, the constraints on individual angular velocity components have been considered during the whole control system design such that rigid body angular velocity operates within the given domain. Finally, both equilibrium points in the closed-loop system are proved to be stable with the choice of backstepping variables, which comprises exploiting the redundancy in the quaternion parameter representation and implies as a side effect that the shortest rotation path is always used when a given attitude change is commanded. In addition, the essential ideas and results from computer simulations are presented to illustrate the performance of the controller developed in this paper.

The paper is organized as follows. The next section states flexible spacecraft modeling and control problems. Attitude maneuver control laws based on adaptive backstepping and elastic mode estimator for vibration reduction with L_2-gain performance are derived in Section 3. Next the results of numerical simulations demonstrate various features of the proposed control law. Finally, the paper is completed with some concluding comments.

2. Mathematical model of flexible spacecraft and control problems

2.1. Kinematic equation

The unit quaternion is adopted to describe the attitude of the spacecraft for global representation without singularities [21]. The unit quaternion $\bar{q}$ is defined by

$$\bar{q} = \begin{bmatrix} \cos(\Phi/2)) \\ n \sin(\Phi/2) \end{bmatrix} = \begin{bmatrix} q_0 \\ q \end{bmatrix} \tag{1}$$

where n is the Euler axis, Φ is Euler angle, q_0 and q are the scalar and vector components of the unit quaternion, respectively, but subject to the constraint: $q^T q + q_0^2 = 1$. Then the kinematic equation in terms of unit quaternion can be given by

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q^T \\ q_0 I + S(q) \end{bmatrix} \omega \tag{2}$$

where $\omega \in R^3$ is the angular velocity of a body-fixed reference frame of a spacecraft with respect to an inertial reference frame expressed in the body-fixed reference frame, $I \in R^{3\times3}$represents the identity matrix, and $S(q)$ denotes a skew-symmetric matrix which is given by

$$S(q) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{3}$$

2.2. Relative attitude error kinematics

Let $\bar{q}_e = [q_{0e} \;\; q_e^T]^T$denotes the relative attitude error from a desired reference frame to the body-fixed reference frame of the spacecraft, then one may have

$$\bar{q}_e = \bar{q} \otimes \bar{q}_d^{-1} = [q_{0e} \;\; q_e^T]^T \tag{4}$$

where $\bar{q}_d^{-1}$ denotes the inverse of the desired quaternion $\bar{q}_d$ with the definition $\bar{q}_d^{-1} = [q_{0d} \;\; -q_d^T]^T$ and $\otimes$ is the operator for quaternion multiplication, which is defined by

$$\bar{q}_a \otimes \bar{q}_b = \begin{bmatrix} q_{0a} q_{0b} - q_a^T q_b \\ q_{0a} q_b + q_{0b} q_a - S(q_a) q_b \end{bmatrix} \tag{5}$$

for any given two groups of quaternion of q_a and q_b. As a result, the relative attitude error can be obtained by

$$\begin{bmatrix} \dot{q}_{0e} \\ \dot{q}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q_e^T \\ q_{0e} I + S(q_e) \end{bmatrix} (\omega(t) - R_d \omega_d(t)) \tag{6a}$$

or

$$\begin{bmatrix} \dot{q}_{0e} \\ \dot{q}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q_e^T \\ q_{0e} I + S(q_e) \end{bmatrix} \omega(t) \quad \text{for } \omega_d = 0 \tag{6b}$$

where R_d is the rotation matrix from the desired reference frame to the body-fixed reference frame, and ω_d is the angular velocity of the desired reference frame with respected to the inertial reference frame expressed in the desired reference frame. Note that in this paper we consider the case $\omega_d = 0$, for convenient, to develop the control law. It is worth noting that the developed approach can be generalized to the case $\omega_d \neq 0$ by properly changing the designed controller form (or to say by adding the terms with relative to ω_d and $\dot{\omega}_d$, respectively, as the feed-forward control inputs of the attitude control system).

2.3. Flexible spacecraft dynamics

Under the assumption of small elastic displacements, the dynamic equations of spacecraft with flexible appendages can be found in Ref. [22] and references therein, and given by

$$J\dot{\omega} + \delta^T \ddot{\eta} = -\omega \times (J\omega + \delta^T \dot{\eta}) + u(t) + d(t) \tag{7a}$$

$$\ddot{\eta} + C\dot{\eta} + K\eta + \delta\dot{\omega} = 0 \tag{7b}$$

where J is the symmetric inertia matrix of the whole structure, δ is the coupling matrix between the elastic and rigid structure, η is the modal coordinate vector, $u(t)$ is control torque acting on the main body and generated by, such as reaction wheels, and $d(t)$ is external disturbance; $C = \text{diag}\{2\xi_i \Lambda_i^{1/2}, i = 1, 2, \ldots, N\}$ and $K = \text{diag}\{\Lambda_i, i = 1, 2, \ldots, N\}$ are the damping and stiffness matrices, respectively, in which N is the number of elastic modes considered, $\Lambda_i^{1/2}$ is the natural frequency, and ξ_i is the corresponding damping ratio.

Remark 1. The above dynamics of the spacecraft are obtained by computing the kinetic and potential energies and then applying the Lagrange equations with the assumption of small elastic displacement approximation. This simplified equation is easy to manipulate and more suitable for control law design. Of course, the exact model, time-varying and more difficult to handle, can be used instead for verifying the effectiveness of the control law derived on the basis of the simplified model, to accomplish the rational maneuver and vibration reduction for the closed-loop simulation later.

Throughout the remainder of this paper, the following two assumptions are taken:

Assumption 1. The inertia matrix J defined in Eq. (7a) is positive definite symmetric and uniformly bounded but unknown. That is, the knowledge of inertia matrix is not known for the controller design but there exist positive constants $\lambda_{\min}^J$ and $\lambda_{\max}^J$, the lower and upper bounds of inertia matrix, which satisfies $\lambda_{\min}^J I \leq J \leq \lambda_{\max}^J I$. Moreover, we assume that matrix $(J - \delta^T \delta)$ remains positive definite symmetric even if the inertia matrix J is unknown.

Assumption 2. The external disturbance $d(t)$ in the spacecraft system (3) is unknown but bounded, i.e., the external disturbance has the property of $d(t) \in L_2(0, T)$, where $L_2(0, T)$ denotes the space consisting of all functions whose 2-norm is finite.

Remark 2. For Assumption 1, it requires the knowledge of the bounds of the inertia matrix; however, this is not a strict assumption, since it can be found easily by the information of the upper bounds on parameters; for the assumption that the matrix $(J - \delta^T \delta)$ is positive definite, this can be guaranteed by properly selecting spacecraft mass center and body frame; for Assumption 2, it is feasible from the point of practical view.

2.4. Control problem statements

In this work, the objective of control design is to achieve attitude maneuver control under bounded angular velocity constraint and vibration reduction with Assumptions 1 and 2. The robust adaptive controller is designed to ensure that the angular velocity is bounded during attitude maneuver and the dissipation inequality ensuring L_2-gain performance is guaranteed from the disturbance input to the penalty output, less than a prescribed value, in the controller synthesis. More specifically, for a prescribed level of disturbance attenuation $\gamma > 0$ and the penalty parameters $\rho_1 > 0$ and $\rho_2 > 0$ of the errors, there exists a control law such that the closed-loop system (6) and (7) satisfy:

(a) equilibrium points in the closed-loop system uniform ultimate bounded stability with the angular velocity constraint: $\omega \in \Omega_\omega = \{\omega : |\omega_i| \leq \omega_{\max}, \forall t > 0, i = 1, 2, 3\}$ and here $\omega_{\max}$ is the maximum value of the required angular velocity;
(b) arbitrary disturbances/modal estimate errors attenuation with respect to both error attitude quaternion and angular velocity, the penalty signal $\bar{z} = [\rho_1 z_1^T \ \ \rho_2 z_2^T]^T$, are ensured in the L_2-gain sense;
(c) the induced elastic vibrations of flexible appendages during attitude maneuvering operations are also actively damped out, i.e., $\lim_{t\to\infty} \eta = 0$, and $\lim_{t\to\infty} \dot{\eta} = 0$.

In what follows, we shall develop such a control for attitude maneuver in flexible spacecraft.

3. Angular velocity bounded adaptive L_2-gain control of flexible spacecraft

In this section, two different controllers based on error quaternion attitude representation under the angular velocity constraints are developed to solve the problem that has just been stated above.

3.1. Basic controller design

For the above attitude maneuver control problem, under angular velocity constraint, a robust adaptive backstepping control strategy is investigated in this paper. Adaptive backstepping is a recursive Lyapunov-based scheme and the idea of it is to design a controller recursively by considering some of the state variables as 'virtual controls' and designing for them intermediate control laws. The advantage of adaptive backstepping compared with other control methods lies in its design flexibility, due to its recursive utilization of Lyapunov function such that cancellations of useful nonlinearities are avoided and often additional nonlinear terms are introduced to improve transient performance.

To carry out the controller design, let $\bar{d}$ denote the lumped perturbation of the rigid dynamics system, defined as $\bar{d} \triangleq d(t) - \delta^T \ddot{\eta} - \omega \times \delta^T \dot{\eta}$, and then Eq. (7a) can be rewritten as

$$J\dot{\omega} = -\omega \times J\omega + u(t) + \bar{d}(t) \tag{8}$$

From Assumption 2, if the elastic vibrations are assumed to be bounded during the whole attitude rotational maneuvering process, then the lumped perturbation $\bar{d}$ will be bounded. In general, this assumption is feasible and satisfied from point of the practical view; especially, when the actuators can produce the bigger enough control torque than the lumped ones. Note that in later section this assumption will be released for more general cases.

To give a clear idea of such controller design procedure, the following variables are defined as [18]

$$z_1 = \begin{bmatrix} 1 - |q_{0e}| \\ q_e \end{bmatrix} \tag{9a}$$

$$z_2 = \omega - \alpha(q_{0e,}q_e) \tag{9b}$$

where $\alpha(q_{0e}, q_e)$.

Remark 3. For kinematics of spacecraft described by Eq. (2), since the unit quaternion parameter set is redundant, a given physical attitude for a rigid body will have two mathematical representation, in which one of these includes a rotation of 2π about an axis relative to the other, such that it has two equilibrium points, i.e., (1,0) and (−1,0). With the choice of backstepping variables for z_1, two equilibrium points $\bar{q}_e = [\pm 1 \;\; 0]^T$ will be shown to be uniformly asymptotically stable in later proof statements when $\bar{d} = 0$, or be stable in the sense of uniform ultimate bounded stability when $\bar{d} \neq 0$ such that q_e will be regulated to the equilibrium point that comprises the shortest path of rotation as compared with only one equilibrium point being considered in the literatures.

Step 1. By considering z_2 as the virtual control variable, based on above assumption, the derivate of Eq. (9a) is defined as

$$\dot{z}_1 \triangleq \begin{bmatrix} -\text{sgn}(q_{0e})\dot{q}_{0e} \\ \dot{q}_e \end{bmatrix} = \frac{1}{2} Q(\bar{q}_e)\omega \tag{10}$$

with the definition

$$Q(\bar{q}_e) \triangleq \begin{bmatrix} \text{sgn}(q_{0e})q_e^T \\ q_{0e}I + S(q_e) \end{bmatrix}^T.$$

Remark 4. For the definition of variable$z_1$in Eq. (9a), it is not differentiable due to $|q_{0e}|$ mathematically. In order to this, here for simplicity, it is assumed that the sgn of the scalar parameter of quaternion does not change, i.e., $\text{sgn}(q_{0e}(t_0)) = \text{sgn}(q_{0e}(t))$ for all $t > t_0$. Note that this assumption is imposed for technical reasons to obtain the derivative of variables z_1. From a physical viewpoint, since both equilibriums correspond to the same orientation it is important to make a choice of the equilibrium point to be stabilized, depending on the given initial condition; logically, one aims at minimizing the path length for the desired rotation which can be ensured by choosing the equilibrium point corresponding to the sign of $q_{0e}(t_0)$. It should be noted that the definition of $\dot{z}_1$ given in Eq. (10) will be just used to state two different rotations based on the assumption that the sgn of the scalar parameter q_{0e} of quaternion does not change. This will be visualized in the numerical simulation in Section 4.

Define the following stabilizing function

$$\alpha = -GQ(\bar{q}_e)z_1 \tag{11}$$

where $G = G^T > 0$ is the designed feedback gain and the argument of the matrix $\alpha(q_{0e}, q_e)$ has been left out for readability and convenience. Note that in this step, the task is to stabilize Eq. (10) with respect to the Lyapunov function $V_1 = z_1^T z_1$ and the time derivate of V_1 can be given by

$$\dot{V}_1 = 2z_1^T\dot{z}_1 = z_1^T Q^T(\bar{q}_e)\omega = z_1^T Q^T(\bar{q}_e)(z_2 + \alpha) = z_1^T Q^T(\bar{q}_e)z_2 - z_1^T Q^T(\bar{q}_e)GQ(\bar{q}_e)z_1 \tag{12}$$

Step 2. The z_2 subsystem is considered, and the time derivate of z_2 left-multiplied by inertia matrix J, with respect to Eq. (8), can be obtained

$$J\dot{z}_2 = J\dot{\omega} - J\dot{\alpha} = -S(\omega)J\omega + u + \bar{d} - J\dot{\alpha} \tag{13}$$

From Assumption 2, even if the inertia matrix J is unknown for the system design, it can observed that the inertia parameters $J_{i,j}$ where $i,j = 1, 2, 3$, appear linearly in Eq. (13). To isolate these parameters, a linear operator $L : R^3 \to R^{3\times 6}$ acting on $b = [b_1 \;\; b_2 \;\; b_3]^T$by

$$L(b) = \begin{bmatrix} b_1 & 0 & 0 & 0 & b_3 & b_2 \\ 0 & b_2 & 0 & b_3 & 0 & b_1 \\ 0 & 0 & b_3 & b_2 & b_1 & 0 \end{bmatrix}$$

is defined as the same in Ref. [23].

Letting $\Theta \triangleq [J_{11} \;\; J_{22} \;\; J_{33} \;\; J_{23} \;\; J_{13} \;\; J_{12}]^T$, it follows that $Jb = L(b)\Theta$ and then Eq. (13) can be rewritten as

$$J\dot{z}_2 = [-S(\omega)L(\omega) - L(\dot{\alpha})]\Theta + u + \bar{d} \triangleq F(\omega, q_{0e}, q_e)\Theta + u + \bar{d} \tag{14}$$

with $F(\omega, q_{0e,} q_e) \triangleq -S(\omega)L(\omega) - L(\dot{\alpha})$.

The task in this step is to stabilize a new Lyapunov function while the constraint of angular velocity, $\omega \in \Omega_\omega = \{\omega : |\omega_i| \le \omega_{\max}, \forall t > 0, i = 1, 2, 3\}$, is considered at the same time, defined as

$$V_2 = V_1 + \frac{1}{2}z_2 J z_2^T + \frac{1}{2}\beta \ln\left(\frac{l^6}{(l^2 - z_{21}^2)(l^2 - z_{22}^2)(l^2 - z_{23}^2)}\right) + \frac{1}{2}\tilde{\Theta}^T P\tilde{\Theta} \tag{15}$$

with $|z_{2i}|<l$ is the elements of $z_2(i=1,2,3)$, $\tilde{\Theta}=\Theta-\hat{\Theta}$, $\beta>0$ is a constant number and P is a positive definite matrix. Then the following statements can be concluded. We mention that the log-term in our Lyapunov function is motivated by the Lyapunov function introduced in Ref. [24].

Theorem 1. *For the system* (6b) *and* (8) *under the constraint of angular velocity, suppose that Assumptions* 1 *and* 2 *are satisfied. If the lumped perturbation $\bar{d}$ is bounded, and the robust control law is designed by*

$$u=-\left[\frac{\beta\dot{z}_{21}}{(l^2-z_{21}^2)}\ \frac{\beta\dot{z}_{22}}{(l^2-z_{22}^2)}\ \frac{\beta\dot{z}_{23}}{(l^2-z_{23}^2)}\right]^T-Q(\bar{q}_e)z_1 \\ -F(\omega,q_{0e},q_e)\hat{\Theta}-Hz_2 \tag{16}$$

with the parameters updating law

$$\dot{\hat{\Theta}}=P^{-1}F(\omega,q_{0e},q_e)z_2 \tag{17}$$

in the presence of parameter constraints

$$0<\rho_1^2 I<\underline{\lambda}_1 I\le Q^T(\bar{q}_e)GQ(\bar{q}_e)\le\bar{\lambda}_1 I \quad \text{and} \quad G=G^T>0 \tag{18a}$$

$$0<\left(\frac{1}{4\gamma^2}+\rho_2^2\right)I<\underline{\lambda}_2 I\le H\le\bar{\lambda}_2 I \quad \text{and} \quad H=H^T>0 \tag{18b}$$

$$0<l+\bar{\lambda}_1<\omega_{\max} \tag{18c}$$

where $\underline{\lambda}_i$ and $\bar{\lambda}_i$ $(i=1,2)$ denote the minimum and maximum eignvalues of the matrices G and H, respectively, then for the zero initial conditions, the following control objectives can be guaranteed: (*i*) *the equilibrium points $\bar{q}_e$ and ω can be made uniformly ultimately bounded;* (*ii*) *the L_2-gain control performance from the perturbation to $\bar{z}$ is achieved;* (*iii*) *if $\bar{d}=0$, the equilibrium points $\bar{q}_e$ and ω can be made uniformly asymptotically stable.*

Proof. Using Eq. (12) the time derivate V_2 along Eq. (9) is given by

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+z_2J\dot{z}_2^T+z_2^T\left[\frac{\beta\dot{z}_{21}}{(l^2-z_{21}^2)}\ \frac{\beta\dot{z}_{22}}{(l^2-z_{22}^2)}\ \frac{\beta\dot{z}_{23}}{(l^2-z_{23}^2)}\right]^T+\tilde{\Theta}^TP\dot{\tilde{\Theta}}\\&=z_1^TQ^T(\bar{q}_e)z_2-z_1^TQ^T(\bar{q}_e)GQ(\bar{q}_e)z_1+z_2^T(F(\omega,q_{0e},q_e)\Theta+u+\bar{d})+\tilde{\Theta}^TP\dot{\tilde{\Theta}}\\&\quad+z_2^T\left[\frac{\beta\dot{z}_{21}}{(l^2-z_{21}^2)}\ \frac{\beta\dot{z}_{22}}{(l^2-z_{22}^2)}\ \frac{\beta\dot{z}_{23}}{(l^2-z_{23}^2)}\right]^T\\&=z_2^T\left\{(F(\omega,q_{0e},q_e)\Theta+u+d)+Q(\bar{q}_e)z_1+\left[\frac{\beta\dot{z}_{21}}{(l^2-z_{21}^2)}\ \frac{\beta\dot{z}_{22}}{(l^2-z_{22}^2)}\ \frac{\beta\dot{z}_{23}}{(l^2-z_{23}^2)}\right]^T\right\}\\&\quad-z_1^TQ^T(\bar{q}_e)GQ(\bar{q}_e)z_1+\tilde{\Theta}^TF(\omega,q_{0e},q_e)z_2\\&=-z_1^TQ^T(\bar{q}_e)GQ(\bar{q}_e)z_1-z_2^THz_2+z_2^T\bar{d}\end{aligned} \tag{19}$$

By adding and subtracting the term $(\gamma^2\|\bar{d}\|^2-\|\bar{z}\|^2)$ on the right side of Eq. (19), it can be rewritten as

$$\begin{aligned}\dot{V}_2&=-z_1^TQ^T(\bar{q}_e)GQ(\bar{q}_e)z_1-z_2^TKz_2+z_2^T\bar{d}-\|\bar{z}\|^2+(\rho_1^2\|z_1\|^2+\rho_2^2\|z_2\|^2)-\gamma^2\|\bar{d}\|^2+\gamma^2\|\bar{d}\|^2\\&\le-z_1^T(Q^T(\bar{q}_e)GQ(\bar{q}_e)-\rho_1^2)z_1-z_2^T\left[H-\left(\frac{1}{4\gamma^2}+\rho_2^2\right)I\right]z_2-\|\bar{z}\|^2+\gamma^2\|\bar{d}\|^2\\&\le-\kappa\|z\|^2-\|\bar{z}\|^2+\gamma^2\|\bar{d}\|^2\end{aligned} \tag{20}$$

where $\bar{z}=[\rho_1z_1^T\ \ \rho_2z_2^T]^T$ is used and

$$\kappa=\min\left\{(\underline{\lambda}_1-\rho_1^2),\left(\underline{\lambda}_2-\left(\frac{1}{4\gamma^2}+\rho_2^2\right)\right)\right\}.$$

Take $\bar{d}=0$, from Eq. (20), one can have

$$\dot{V}_2\le-\kappa\|z\|^2-\|\bar{z}\|^2\le0 \tag{21}$$

According to the LaSalle–Yoshizawa theorem, this establishes that both of the equilibrium points $\bar{q}_e=[\pm1\ \ 0]^T$ are uniformly asymptotically stable, which implies that $q_{0e}\to\pm1$, $q_e\to0$ and $\omega\to0$ as $t\to\infty$; moreover, in the whole process, the angular velocity has to lie in within the hyper-rectangle with sides $2l$ by the constraint $|\omega_i|<l<\omega_{\max}$. The proof for case (iii) is completed. □

For the case $\bar{d} \neq 0$, integrating the above inequality from $t=0$ to any $T \geq 0$ with zero initial conditions yields

$$V_2(t) - V_2(0) \leq \gamma^2 \int_0^t \|\bar{d}\|^2 dt - \int_0^t \|\bar{z}\|^2 \, dt \tag{23}$$

Using the results of Ref. [25] we can conclude that the closed-loop system is globally stable in the sense of uniform ultimate bounded stability, and the L_2-gain attenuation level of γ is also guaranteed for the physically realizable initial conditions. The proof for cases (i) and (ii) are completed.

Remark 5. It is seen that control scheme Eq. (16) with proper designed parameters not only achieve the control objectives (i–iii) even under the constraint of angular velocity $\omega \in \Omega_\omega = \{\omega : |\omega_i| \leq \omega_{\max}, \forall t > 0, i = 1, 2, 3\}$, but also both equilibrium points in the closed-loop system are proved to be stable with the choice of backstepping variables, which comprises exploiting the redundancy in the quaternion parameter representation and implies as a side effect that the shortest rotation path is always used when a given attitude change is commanded.

To guarantee the boundedness of $\hat{\Theta}$, a projection operator [26] is adopted in the parameter update law and then the modified one is given by

$$\dot{\hat{\Theta}} = \mathrm{Proj}(\hat{\Theta}, P^{-1}F(\omega, q_{0e}, q_e)z_2) \tag{24}$$

where the projection operator is defined as

$$\mathrm{Proj}(\hat{\Theta}, P^{-1}F(\omega, q_{0e}, q_e)z_2) = \begin{cases} 0 & \text{if } \hat{\Theta}_i = \Theta_{i\max} \text{ and } (P^{-1})_{ii}[F(\omega, q_{0e}, q_e)z_2]_i < 0 \\ (P^{-1})_{ii}[F(\omega, q_{e0,} q_e)z_2]_i & \begin{cases} \text{if } \Theta_{i\min} < \hat{\Theta}_i < \Theta_{i\max} \\ \text{or } \hat{\Theta}_i = \Theta_{i\min} \text{ and } (P^{-1})_{ii}[F(\omega, q_{0e}, q_e)z_2]_i \geq 0 \\ \text{or } \hat{\Theta}_i = \Theta_{i\min} \text{ and } (P^{-1})_{ii}[F(\omega, q_{0e}, q_e)z_2]_i \leq 0 \end{cases} \\ 0 & \text{if } \hat{\Theta}_i = \Theta_{i\min} \text{ and } (P^{-1})_{ii}[F(\omega, q_{0e}, q_e)z_2]_i > 0 \end{cases} \tag{25}$$

$\Theta_{i\max}$ and $\Theta_{i\min}$ are real numbers denoting the upper and lower bounds of the ith element $\hat{\Theta}$, respectively, $(P^{-1})_{ii}$ is the (i, i) element of P^{-1}, and $[F(\omega, q_{0e}, q_e)z_2]_i$ is the ith element of $F(\omega, q_{0e}, q_e)z_2$.

3.2. Modified controller design

In subsection 3.1, we have shown how to design a stable system by adaptive backstepping control for the flexible spacecraft system with unknown inertia matrix and disturbances. However, the assumption for boundedness of the lumped perturbation must be satisfied in advance, this is to say, the elastic vibrations should be bounded during the whole attitude rotational maneuvers. Moreover, active attenuation of the flexible oscillations induced by spacecraft maneuvers is not explicitly considered for improving the precision pointing. To overcome this problem and relax the assumption, a modified adaptive backstepping controller is proposed hereinafter, which makes use of spacecraft position and angular velocity measures to estimate what we need are information are regarding the modal variable η and velocity $\dot{\eta}$. The estimate of the modal variables from measurements of $\bar{q}_e$ and ω is possible since the rigid dynamics are influenced by the flexible ones through the coupling matrix δ. The observer is given by

$$\hat{\eta}_i(s) = -\frac{\sum_{j=1}^{3} \delta_{ij}\omega_j(s)s}{s^2 + 2\xi_i\Lambda_i^{1/2}s + \Lambda_i} \tag{26}$$

where δ_{ij} denotes the (i, j) element of δ.

Let $\tilde{\eta} = \eta - \hat{\eta}$, then we have

$$\ddot{\tilde{\eta}} + C\dot{\tilde{\eta}} + K\tilde{\eta} = 0 \tag{27}$$

From Eq. (27), it can be easy to obtain the time response of $\tilde{\eta}$, which can be algebraically rearranged as

$$\tilde{\eta}_i(t) = e^{-\xi_i\Lambda_i^{1/2}t}\left(\tilde{\eta}_i(0)\sin(\sqrt{1-\xi_i^2}\Lambda_i^{1/2}t) + \frac{\Lambda_i^{1/2}\dot{\tilde{\eta}}_i(0) + \xi_i\tilde{\eta}_i(0)}{\sqrt{1-\xi_i^2}}\cos(\sqrt{1-\xi_i^2}\Lambda_i^{1/2}t)\right) \tag{28}$$

It can be seen that $\tilde{\eta}_i(t)$ tends to zero as $t \to \infty$. Then we have the following statements.

Theorem 2. *Consider the system* (6b) *and* (8) *under Assumptions* 1 *and* 2 *using the following control law*

$$u = -\left[\frac{\beta\dot{z}_{21}}{(l^2 - z_{21}^2)} \ \frac{\beta\dot{z}_{22}}{(l^2 - z_{22}^2)} \ \frac{\beta\dot{z}_{23}}{(l^2 - z_{23}^2)}\right]^T - Q(\bar{q}_e)z_1 - F(\omega, q_{0e,}q_e)\hat{\Theta} - Hz_2 + [S(\omega)\delta^T - \delta^T C]\dot{\hat{\eta}} - \delta^T K\hat{\eta} \tag{29}$$

with the parameters updating law

$$\dot{\hat{\Theta}} = P^{-1}F(\omega, q_{0e}, q_e)z_2 \tag{30}$$

under the parameters constraints

$$0<\rho_1^2 I<\underline{\lambda}_1 I \le Q^T(\bar{q}_e)GQ(\bar{q}_e) \le \bar{\lambda}_1 I \quad \text{and} \quad G=G^T>0 \tag{31a}$$

$$0<\left(\frac{1+\bar{\lambda}_3^2}{4\gamma^2}+\rho_2^2\right)I<\underline{\lambda}_2 I \le H \le \bar{\lambda}_2 I \quad \text{and} \quad H=H^T>0 \tag{31b}$$

$$0<\underline{\lambda}_3 \le \|[(\delta^T C - S(\omega)\delta^T) \;\; \delta^T K)]\| \le \bar{\lambda}_3 \tag{31c}$$

$$0<l+\bar{\lambda}_1<\omega_{\max} \tag{31d}$$

Then the closed-loop system satisfies: (*i*) *the equilibrium points $\bar{q}_e$ and ω can be made uniformly ultimately bounded;* (*ii*) *the L_2-gain control performance from the perturbation to $\bar{z}$ is achieved.*

Proof. Define the following Lyapunov function candidate

$$V_3 = V_1 + \frac{1}{2}z_2(J-\delta^T\delta)z_2^T + \frac{1}{2}\beta\ln\left(\frac{l^6}{(l^2-z_{21}^2)(l^2-z_{22}^2)(l^2-z_{23}^2)}\right) + \frac{1}{2}\tilde{\Theta}^T P\tilde{\Theta} \tag{32}$$

When Eqs. (29) and (30) are applied, the Lyapunov derivative in Eq. (32) can be algebraically rearranged in steps identical to those employed in deriving Eq. (19), namely,

$$\begin{aligned}
\dot{V}_3 = & -z_1^T Q^T(\bar{q}_e)GQ(\bar{q}_e)z_1 - z_2^T H z_2 + z_2^T d + z_2^T[(\delta^T C - S(\omega)\delta^T) \;\; \delta^T K)][\dot{\tilde{\eta}}^T \;\; \tilde{\eta}^T]^T \\
& - \|\bar{z}\|^2 + (\rho_1^2\|z_1\|^2 + \rho_2^2\|z_2\|^2) - \gamma^2\|d\|^2 + \gamma^2\|d\|^2 \\
\le & -z_1^T Q^T(\bar{q}_e)GQ(\bar{q}_e)z_1 - z_2^T H z_2 - \left\|\frac{1}{2\gamma}z_2 - \gamma d\right\|^2 \\
& - \|\bar{z}\|^2 + \frac{1}{4\gamma^2}\|z_2\|^2 + (\rho_1^2\|z_1\|^2 + \rho_2^2\|z_2\|^2) + \gamma^2\|d\|^2 \\
& - \left\|\frac{\bar{\lambda}_3}{2\gamma}\|z_2\| - \gamma\|[\dot{\tilde{\eta}}^T \;\; \tilde{\eta}^T]\|\right\|^2 + \frac{\bar{\lambda}_3^2}{4\gamma^2}\|\bar{z}\|^2 + \gamma^2\|[\dot{\tilde{\eta}}^T \;\; \tilde{\eta}^T]\|^2 \\
\le & -z_1^T(\underline{\lambda}_1 - \rho_1^2)z_1 - z_2^T\left[\underline{\lambda}_2 - \left(\frac{1+\bar{\lambda}_3^2}{4\gamma^2}+\rho_2^2\right)I\right]z_2 - \|\bar{z}\|^2 + \gamma^2(\|d\|^2 + \|[\dot{\tilde{\eta}}^T \;\; \tilde{\eta}^T]\|^2) \\
\le & -\kappa\|z\|^2 - \|\bar{z}\|^2 + \gamma^2(\|d\|^2 + \|[\dot{\tilde{\eta}}^T \;\; \tilde{\eta}^T]\|^2)
\end{aligned} \tag{33}$$

where

$$\kappa = \min\left\{(\underline{\lambda}_1 - \rho_1^2), \left(\underline{\lambda}_2 - \left(\frac{1+\bar{\lambda}_3^2}{4\gamma^2}+\rho_2^2\right)\right)\right\}.$$

The validation of the specifications on cases (i) and (ii) follows the same argument developed in subsection 3.1. This completes the proof. □

Remark 6. The effect of the elastic mode estimate errors $\tilde{\eta}$ and $\dot{\tilde{\eta}}$ are also considered into robustness design, since a poor state estimation will lead to an unexpected transient response. The arbitrary attenuation of both disturbances and estimate errors is achieved for a prescribed level of attenuation γ in the sense of L_2-gain. In addition, to guarantee the boundedness of the estimated parameter $\hat{\Theta}$, the modified parameter updating law in Eq. (25) can be adopted for this new adaptive backstepping control law.

Remark 7. Notice that the computation of the designed control $u(t)$ requires the use of $\dot{z}_{2i}$ $(i=1,2,3)$, which can hardly be used in practical, even if Eq. (9b) is employed. The following difference equation is used in this work

$$\dot{z}_{2i}(t-T_c) = \frac{z_{2i}(t-T_c) - z_{2i}(t-2T_c)}{T_c} = \frac{[\omega_i(t-T_c) - \omega_i(t-2T_c)] - [\alpha_i(t-T_c) - \alpha(t-2T_c)]}{T_c} \tag{34}$$

for this computation, instead of $\dot{z}_{2i}$ for computation control voltage $u(t)$ and here T_c is selected as the control update period.

Remark 8. In Theorem 1 and 2, there are many parameters to be determined by the designers, such as γ, ρ_1, ρ_2, G and H. Note that here parameters ρ_1 and ρ_2 are the weighting coefficients of penalty signal $\bar{z}$ and usually selected as 1, respectively; while the high robustness to external disturbance/elastic vibrations is guaranteed by a prescribed level γ. Theoretically we can select infinitely smaller γ for the given G and H, which satisfied the inequalities (18) or (31); however, such a small choice of γ allows excessive large control input. Since saturation for actuators is inevitable in practical problems, trade-off is required between choice of smaller γ and practical tracking performance. The proper value of γ, G and H will only be found by trial-and error through the simulations. While for parameters $\bar{\lambda}_3$ and $\underline{\lambda}_3$, they can also be easily determined according to the plant parameters and the boundedness of the angular velocity.

4. Simulation and comparison results

The numerical application of the proposed control scheme to the attitude control of flexible spacecraft is presented using MATLAB/SIMULINK software. The spacecraft is characterized by a nominal main body inertia matrix [22]

$$J = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} \mathrm{kg\,m^2}$$

and the coupling matrices

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix} \mathrm{kg^{1/2}\,m/s^2}$$

respectively; the first four elastic modes have been taken into account in the model used for simulating the spacecraft at $\omega_{n1} = 0.7681\,\mathrm{rad/s}$, $\omega_{n2} = 1.1038\,\mathrm{rad/s}$, $\omega_{n3} = 1.8733\,\mathrm{rad/s}$, $\omega_{n4} = 2.5496\,\mathrm{rad/s}$ with damping $\xi_1 = 0.0056$, $\xi_2 = 0.0086$, $\xi_3 = 0.013$, $\xi_4 = 0.025$, while for designing the controller only first three mode have been involved.

Here the rest-to-rest maneuver is considered in the simulation, and the initial conditions have been set at $q_0 = 0.173648$, $q_1 = -0.263201$, $q_2 = 0.789603$ and $q_3 = -0.526402$, i.e., a rotation of 160° is to be considered in the attitude maneuvering. Note that, due to the commanded change in reference, the controller regulates the quaternion to the equilibrium point [1,0,0,0], since this is the closest equilibrium point in terms of rotation path. In addition, the initial modal variables and its time derivative $\eta_i(0)$ and $\dot{\eta}_i(0)\,(i = 1,2,3,4)$ are supposed given by $\eta_i(0) = \dot{\eta}_i(0) = 0$, i.e., the flexible appendages are un-deformed. To examine the robustness to external disturbance, simulation was done corresponding to the periodic disturbance torque

$$T_d(t) = [0.3\cos(0.01t) + 0.1 \;\; 0.15\sin(0.02t) + 0.3\cos(0.025t) \;\; 0.3\sin(0.01t) + 0.1]^T \tag{35}$$

was considered. The angular velocity bounds are chosen to be: $\omega^x_{\max} = 6\,\mathrm{deg/sec}$, $\omega^y_{\max} = 15\,\mathrm{deg/sec}$ and $\omega^z_{\max} = 10\,\mathrm{deg/sec}$.

For the purpose of comparison, four different sets of simulation are conducted to demonstrate the effective of the proposed approach:

A. Attitude control using the proposed adaptive backstepping control law in Eq. (16).
B. Attitude control with using the modified adaptive backstepping control law in Eq. (29) with elastic mode estimator.
C. Attitude control using traditional proportional-integral-derivative (PID) control scheme.
D. Attitude control using backstepping control scheme given in Ref. [18].

In the following simulations, the control and adaptation gains were selected by trial-and-error until a good performance was obtained for above cases. The controller parameters of the different methods: proposed adaptive backstepping, PID and OVSC were determined so that all the settling time was almost the same in all the schemes; they are tabulated in Table 1. In addition, simulations have been rendered more realistic by considering saturation on the inputs. The maximum value of control torque of actuators (reaction wheel) is assumed to be 20.0 Nm. All computations and plots shown in the paper were performed using MATLAB/SIMULINK software package.

4.1. Attitude maneuver control using the proposed adaptive backstepping

In this case, firstly, to show the effect of the proposed adaptive backstepping controller in Eq. (16), simulation was done under the initial condition and velocity bounded requirements. The responses of quaternion, velocity of the spacecraft, modal displacement and the required control torque are shown in Fig. 1 (a–d, solid line). It is noted that an acceptable desirable orientation response is achieved, and the spacecraft reached the demanded angle with a settling time less than 20 s. Furthermore, Fig. 1(e) shows the plots of estimated inertia parameters corresponding to update law of Eq. (25). From these plots, it is clear that although rigid body angular velocity is well with in the prescribed bounds, there exist serve oscillations for the flexible appendages because no vibration mode information is used in the feedback loop to reduce the oscillations.

For comparison, to reduce the flexible oscillations, the system is also controlled by using the proposed adaptive backstepping law with elastic modal estimator in Eq. (29). The same simulation case is repeated with the control law, and the results are shown in Fig. 1 (a–e, dotted line). For this case, it can be observed that not only desired attitude rotational maneuver can be achieved but also the oscillations are actively suppressed during maneuvering. The estimated modal displacement was shown in Fig. 1 (d, dashed line), in which the modal displacement can be well estimated by this estimator. Moreover, it is clear that angular velocity is well with in the prescribed bounds and the regulation performance of the controllers is marginally better with the elastic modes estimator than the first case. These results completely support the theoretical result that performance of the controller can be improved with the estimated elastic modes even if the inertia matrix is unknown.

4.2. *Attitude maneuver control using the OVSC and PID*

For the purpose of comparison, the system is also controlled by using the traditional PID control. The same simulation cases are repeated with PID under the angular velocity bounds. The results of simulation are shown in Fig. 2 (a–d, dotted line). For this case, it can be observed that no desired attitude rotational maneuver can be achieved, and serve oscillations are also excited during maneuvering as shown in Fig. 2 (d, dotted line). Moreover, there exists the saturation value of angular velocity of y-axis. Despite the fact that there still exists some room for improvement with different design parameter sets, there is not much improvement in the attitude and velocity responses.

Table 1
Design parameters for the different controllers.

Parameter and value	
Proposed controller in Eq. (16)	$\gamma = 0.01$, $\beta = 1$, $\rho_1 = \rho_2 = 1$, $K_1 = 10I_3$, $l = 5$, $H = 10I_3$, $P = diag\{10, 5, 5\}$
Proposed controller in Eq. (29)	$\gamma = 0.01$, $\beta = 1$, $\rho_1 = \rho_2 = 1$, $K_1 = 10I_3$, $l = 5$, $H = 10I_3$, $P = diag\{10, 5, 5\}$
Backstepping controller	$K_1 = 10I_3$, $K_2 = 500I_3$
PID controller	$K_p = 800I_3$, $K_i = 0.001I_3$, $K_d = 2800I_3$

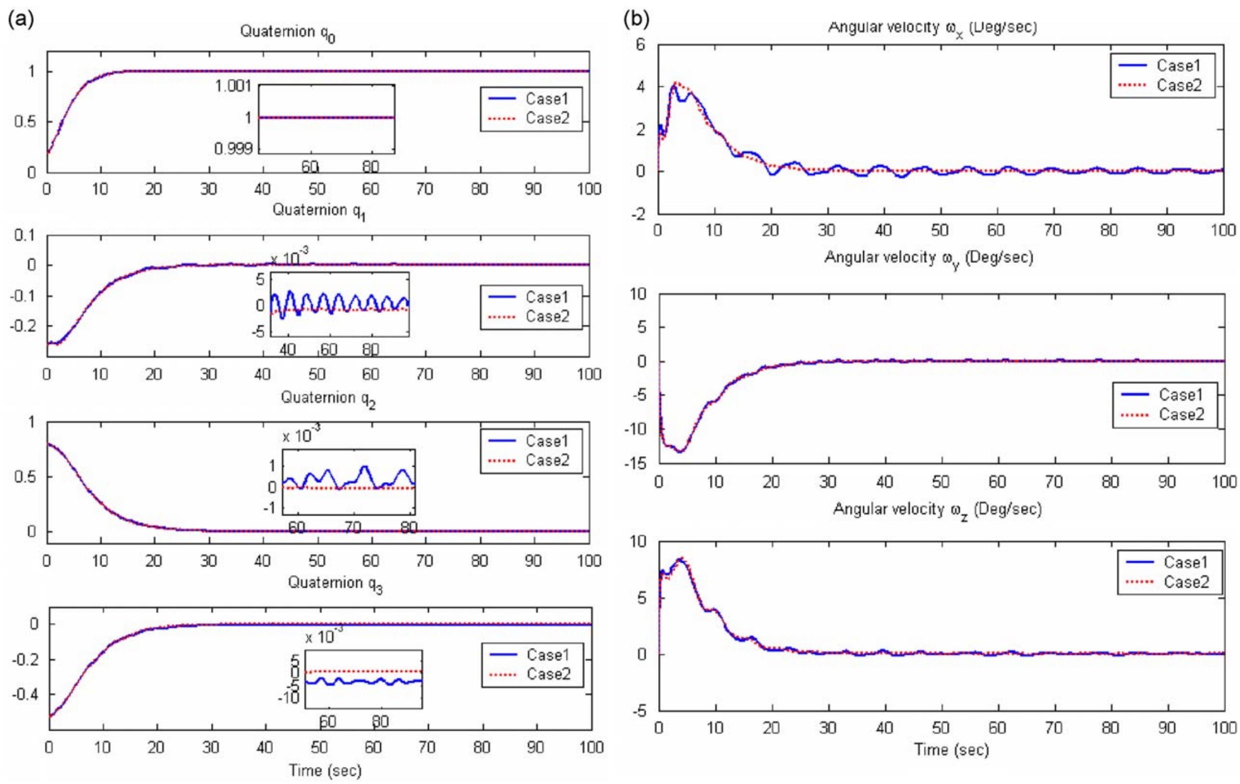

Fig. 1. Attitude maneuvers control using the proposed methods. Case1: Proposed adaptive backstepping controller (solid line); Case2: Proposed adaptive backstepping controller with elastic mode estimator (dotted line); Case3: The estimated modal displacements (dashed line). (a) Time response of quaternion, (b) time response of angular velocity, (c) time response of control torque, (d) time response of vibration displacements and (e) time responses of estimated parameters.

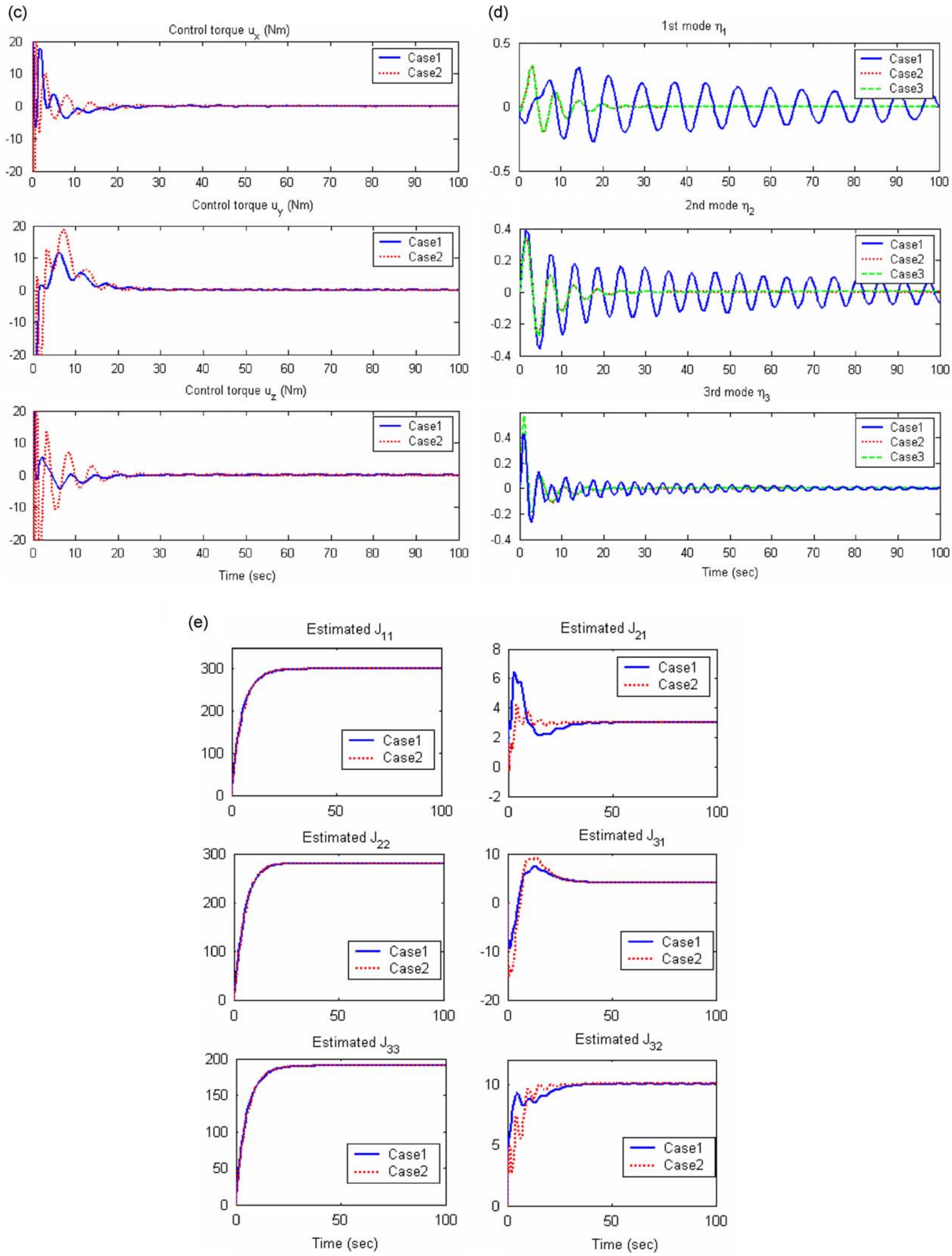

Fig. 1. (*Continued*)

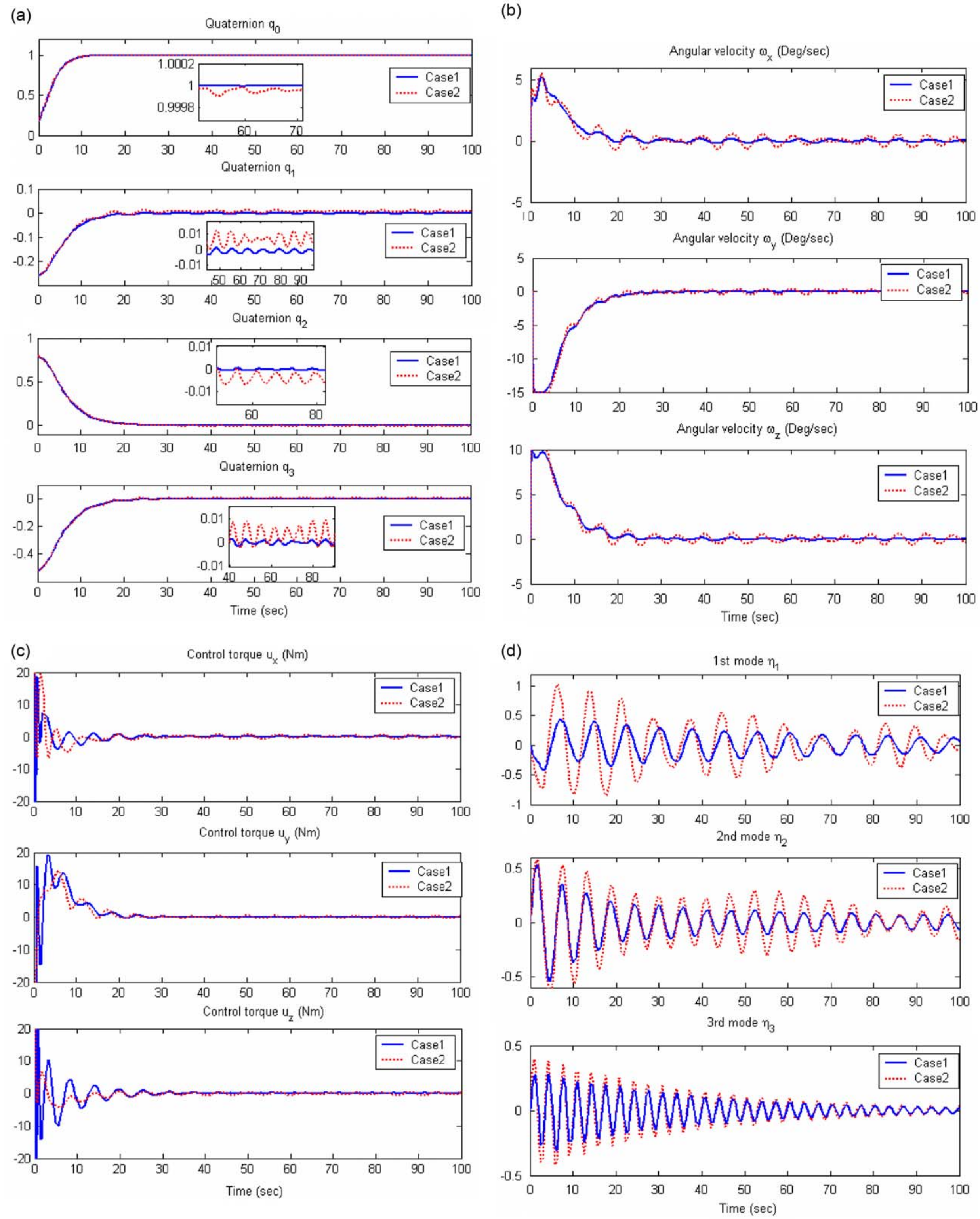

Fig. 2. Attitude maneuvers control using the conventional methods. Case1: Backstepping controller (solid line); Case2: PID controller (dotted line). (a) Time response of quaternion, (b) time response of angular velocity, (c) time response of control torque and (d) time response of vibration displacements.

For the purpose of further comparison, optimal variable structure controller designed in Ref. [18] is also employed for the system. The same simulation case is repeated with this variable structure controller and the results of simulation are shown in Fig. 2 (solid line). As one can see in Fig. 2 (solid line), even if the responses of attitude and velocity can be improved a lot, but severe vibration are the results of the control law as compared with the proposed methods. From the comparison between Figs. 1 and 2, the performance of the proposed two designs is better than the last two even if these designs will adapt the system parameters under the external disturbances.

Extensive simulations were also done using different control parameters, and disturbance inputs. These results show that in the closed-loop system attitude control and vibration stabilization are accomplished in spite of disturbances in the system. Moreover, the flexibility in the choice of control parameters can be utilized to obtain desirable performance while meeting the constraints on the control magnitude and elastic deflection.

From the comparison of above cases, it is shown that the proposed approach cannot only accomplish the quick attitude rotational maneuver with least control chattering, but also simultaneously suppress the undesired vibrations of the flexible appendages even though uncertainties and disturbances are explicitly considered, so as to obtain the precise attitude control of flexible spacecraft. Furthermore, the information of upper bound of the perturbations and uncertain is not required beforehand when the adaptive law of the developed control is adopted. This control approach provides the theoretical basis for the practical application of the advanced control theory to flexible spacecraft attitude control system.

5. Conclusions

In this paper, an adaptive backstepping controller has been designed for flexible spacecraft attitude maneuver and elastic vibration control which explicitly takes into consideration the bounds on angular velocity. The adaptive control formulation in this paper is based upon Lyapunov's direct stability theorem by incorporating the performance criterion given by L_2-gain constraint in controller synthesis. The uniform ultimate bounded stability of the system is ensured and the robustness to both disturbance and elastic mode estimation error is also guaranteed with the L_2-gain less than any given small level. The control designs are evaluated using numerical simulation comparisons between the developed approach and other referred schemes have been made, where the expected performances have been shown. In addition, the proposed control law is shown to work well in the presence of bounded angular velocity constraints fully consistent with the stability analysis presented. While the simulation results presented in this paper merely illustrate formulations for a particular attitude maneuver, further testing would be required to reach any conclusions about the efficacy of the control and adaptation laws for tracking arbitrary maneuvers.

Acknowledgments

This present work was supported by the Scientific Research Foundation for the Returned Overseas Chinese Scholars of State Education Ministry, Scientific Research Foundation for the Returned Overseas Chinese Scholars of Heilongjiang Province (LC08C01), the Development Program for Outstanding Young Teachers in Harbin Institute of Technology (HITQNJS.2007.001), National Natural Science Foundation of China (Project Number: 60774062), and Research Fund for the Doctoral Program of Higher Education of China (Project Number: 20070213061). The author fully appreciates the financial supports of National 973 Program of China (2009CB320604). The author would also like to thank the reviewers and the editor for many suggestions that helped improve the paper.

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