Journal of Sound and Vibration 327 (2009) 440–453

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Dynamic torsional response of an end bearing pile in transversely isotropic saturated soil

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A R T I C L E I N F O

Article history:
Received 5 October 2008
Received in revised form
1 May 2009
Accepted 20 June 2009
Handling Editor: L.G. Tham
Available online 17 July 2009

A B S T R A C T

The dynamic response of an end bearing pile embedded in transversely isotropic saturated soil and subjected to a time-harmonic torsional loading is investigated. Based on the dynamic wave equations of saturated soil and the stress–strain relationships of transversely isotropic medium, the dynamic governing equations of the transversely isotropic saturated soil are derived in cylindrical coordinates and the pile is modeled using one-dimensional elastic theory. At first, the torsional response of the soil layer is solved by using the separation of variables technique. Then by utilizing the boundary and continuity conditions of the pile–soil system, the dynamic response of the pile is obtained in a closed form in the frequency domain. By virtue of inverse Fourier transform and convolution theorem, a quasi-analytical solution for the velocity response of a pile subjected to a semi-sine wave exciting torque is obtained in the time domain. Finally, a parametric study has been undertaken to investigate the influence of the soil anisotropy on the torsional vibration characteristics of the pile, and the results obtained are presented in this paper.

1. Introduction

Pile foundations of machines and structures are often subjected to dynamic loading caused by running machinery and due to impact, wind or wave forces. Generally speaking, dynamic loadings result from a combination of the vertical, horizontal and torsional loading components. In previous years, the vibration characteristics of pile subjected to dynamic vertical and horizontal loadings have received much attention from investigators and corresponding pile vibration theories have been developed to provide valuable guidance for geotechnical and foundation engineering design. For instance, Novak [1] studied the dynamic response of pile subjected to horizontal, vertical and rocking loadings and applied the corresponding theory to predict the vibration characteristic of the machine foundation. Aviles and Sanchez-Sesma [2] analyzed the problem of foundation isolation from vibrations generated in the neighborhood using piles as barriers. Yesilce and Catal [3] investigated the free vibration of the pile in soil having different modulus of subgrade reaction and provided some useful guidelines for engineering design. Wang [4] applied the pile longitudinal vibration theory to the dynamic non-destructive testing of piles by using Laplace transform techniques.

It is worth noting that torsional loading commonly occurs in typical pile foundations including those of machinery, bridges, pylons, towers and lighting posts due to eccentricity in applied lateral loads. Dynamic torsional loading in pile

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2009.06.017

foundations is therefore a technically relevant problem of some practical interest. The dynamic behavior of piles embedded in isotropic media and subjected to transient torsional loading has been investigated by a variety of methods categorized as simplified continuum, continuum, finite element and boundary element methods. Baranov [5] presented a simplified continuum method and investigated the vertical vibration of embedded foundations. In the simplified continuum method, the soil is assumed to be composed of a set of independent infinitesimally thin horizontal layers that extend to infinity radially, and the gradients of displacement and stress in the vertical direction are neglected. In subsequent studies, Novak and Howell extended the simplified continuum method to torsional mode and analyzed the torsional vibration of piles embedded in a uniform [6] and layered viscoelastic soil media [7]. In their studies, they also investigated the vibration characteristics of the machine foundation. Militano and Rajapakse [8] studied the time domain vibration characteristics of an elastic pile subjected to transient torsional and axial loading in multilayered elastic soil by utilizing the Laplace transform and inverse Laplace transform techniques. It is noted that a deficiency of the simplified continuum method is that it assumes a highly idealized mechanism of pile–soil interaction. The second method of solving the pile–soil torsional interaction problem, the continuum method, includes a more realistic pile–soil coupling. Here the pile–soil system is often decomposed into an extensive elastic half-space and a fictitious pile. The Hankel transform, integral equation formulation, Green's functions and variational schemes are then introduced to obtain the solutions of the pile response. In the subsequent studies, Rajapakse et al. [9] solved the axisymmetric torsional vibrations of an elastic pile and hemispherical foundation embedded in a homogeneous elastic half-space by means of a variational solution scheme. Using time-harmonic Green's functions, Rajapakse and Shah [10] investigated the axial, lateral, rocking, coupled and torsional impedances of an elastic pile embedded in an elastic half-space. Cai et al. [11] analyzed the dynamic torsional response of an elastic pile embedded in saturated elastic half-space by utilizing Hankel transform and Fredholm integration equation schemes. However, in most cases, the solutions derived from the continuum method are only suitable for a floating pile and the corresponding numerical calculation always requires intensive computational effort. In light of this, Wang et al. [12] developed a comprehensive analytical solution to investigate the torsional vibration of an end bearing pile embedded in a homogeneous saturated soil and subjected to a time-harmonic torsional loading by using the method of separation of variables. The third and fourth methods are the finite element method (FEM) and the boundary element method (BEM). For instance, Tham et al. [13] studied the torsional vibration of single pile embedded in a layered half-space with a coupled FEM–BEM approach. Although FEM and BEM are extensively applicable to most complicated cases, they also require considerable computational effort and lack stability in the dynamic analysis. As a consequence, relatively little has been reported about these methods for studying piles subjected to dynamic loads.

Most of the previous studies on piles had regarded soil as an isotropic medium. However, in practice soil deposits, generally speaking, possess a certain degree of anisotropy due to their deposition history resulting in properties that are different in the horizontal and vertical spatial directions. The vertical and horizontal differences can be accounted for using a transversely isotropic soil model [14] which a simple isotropic model cannot. Tsai [15] investigated torsional vibration of a circular disk on a transversely isotropic half-space by using Hankel transform. It is evident from Tsai's study that the anisotropy material constants had obvious influence on the resonant amplitude and frequencies of vibration. Liu and Novak [16] studied the dynamic response of single pile embedded in transversely isotropic layered media by using FEM combined with dynamic stiffness matrices of the soil derived from Green's functions for ring loads. Notwithstanding the work above, it appears that no analytical solution corresponding to the torsional vibration characteristics of an end bearing pile embedded in transversely isotropic saturated soil has been reported in open literature until now. This paper generalizes the previous work of the authors [12] by developing analytic and quasi-analytic solutions of an embedded end bearing pile in transversely isotropic soil subjected to dynamic torsional loading. The previous work [12] sought solutions in an isotropic soil only, and is a special case of the solutions given herein. The analytic and quasi-analytic solutions are not only useful such as for providing a theoretical basis for non-destructive pile testing and for assessing the pile–soil responses under incremental dynamic torsional loading, they can also serve as benchmarks for validating the accuracy and correctness of more powerful numerical solutions. The work in this paper embodies one step in our attempt to progressively develop more complex methods and models in future work, and a conscious effort to understand the causality link of a key parameter, the anisotropy of soil through analytic solutions. Thus, using the solution developed herein, a parametric study has been undertaken to assess the influence of soil anisotropy on the vibration behavior of the pile and a discussion of the results is included in this paper.

2. Basic equations and solutions

2.1. Basic equations

The problem investigated in this paper is that of the torsional vibration of an end bearing pile embedded in a transversely saturated soil medium under small deformations and strains conditions. The present paper does not deal with a pile at failure as this is clearly beyond the scope of this study, although Zhang and Kong [17] have suggested that a pile can be loaded to failure more easily by torsional loading than by compressive loading. Solutions developed below are, however, applicable for non-destructive torsional testing of a pile and for studying pile responses under incremental

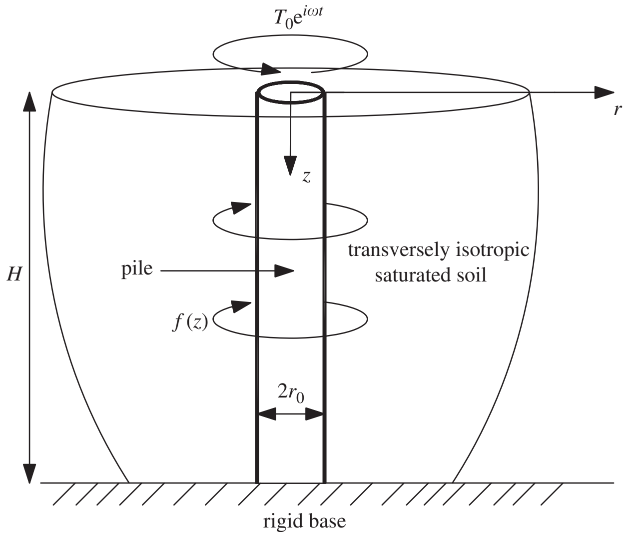

Fig. 1. Geometry of the pile and embedded poroelastic medium.

torsional loading above an initial dynamic equilibrium state. Given this, the geometric model is idealized as shown in Fig. 1 and the following assumptions are made during the analysis:

(1) The exciting torque is harmonic. The pile–soil system is subjected to small deformations and strains during the vibration. Only the circumferential displacements are considered.
(2) The pile is vertical, elastic, end bearing, circular in cross section and has a perfect contact with the surrounding soil during vibration.
(3) The soil is a linearly elastic and transversely isotropic saturated layer. The layer overlies a rigid bedrock (i.e., the rigid bedrock extends continuously in the radial direction).
(4) The free surface of the soil has no normal and shear stresses and there is no displacement occurring at the bottom of the layer. The soil is infinite in the radial direction.

As shown in Fig. 1, the dynamic equilibrium equation for the saturated soil undergoing torsional axisymmetric deformations about the z-axis of a cylindrical polar coordinate system can be expressed as follows [18]:

$$\frac{\partial \sigma_{r\theta}(r,z,t)}{\partial r} + \frac{\partial \sigma_{\theta z}(r,z,t)}{\partial z} + \frac{2\sigma_{r\theta}(r,z,t)}{r} = \rho \frac{\partial^2 u_\theta(r,z,t)}{\partial t^2} + \rho_f \frac{\partial^2 w_\theta(r,z,t)}{\partial t^2} \tag{1}$$

where $\sigma_{r\theta}(r,z,t)$ and $\sigma_{\theta z}(r,z,t)$ denote the incremental shear stresses of the soil, $u_\theta(r,z,t)$ and $w_\theta(r,z,t)$ are, respectively, the circumferential displacement of the solid phase and that of the fluid phase relative to the solid phase but measured in terms of the volume per unit area of the bulk material; n is the porosity; ρ_s and ρ_f are the mass densities of soil particle and fluid, respectively; and $\rho = (1-n)\rho_s + n\rho_f$, the mass density of soil.

The stress–strain relationship in cylindrical coordinates for a transversely isotropic medium can be written in the following form [14]:

$$\sigma_{\theta z} = c_{44}\gamma_{\theta z}$$

$$\sigma_{r\theta} = (c_{11} - c_{22})\gamma_{r\theta}/2 \tag{2}$$

where c_{11}, c_{22} and c_{44} are the material constants. In the soil mechanics, $c_{44} = G_{\mathrm{sv}}$ and $(c_{11}-c_{22})/2 = G_{\mathrm{sH}}$, where G_{sv} and G_{sH} are the vertical and horizontal shear modulus of the soil, respectively.

Due to the symmetry of the problem, the motion is independent of angle θ. Then, the strain–displacement relations written in cylindrical coordinates can be simplified as [15]

$$\gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta} = \frac{\partial u_\theta}{\partial z}$$

$$\gamma_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), the equilibrium equation of transversely isotropic saturated soil can be expressed as

$$G_{sH}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)u_\theta(r,z,t)+G_{sv}\frac{\partial^2 u_\theta(r,z,t)}{\partial z^2}=\rho\frac{\partial^2 u_\theta(r,z,t)}{\partial t^2}+\rho_f\frac{\partial^2 w_\theta(r,z,t)}{\partial t^2} \tag{4}$$

Considering the symmetry of the problem, the equilibrium equation of the pore fluid can be written as

$$\frac{\rho_f g}{k_h}\frac{\partial w_\theta(r,z,t)}{\partial t}+\rho_f\frac{\partial^2 u_\theta(r,z,t)}{\partial t^2}+\frac{\rho_f}{n}\frac{\partial^2 w_\theta(r,z,t)}{\partial t^2}=-\frac{1}{r}\frac{\partial P_w}{\partial \theta}=0 \tag{5}$$

where k_h denotes the horizontal dynamic permeability coefficient containing the viscosity of the liquid; g is the gravitational acceleration; and P_w is excess pore water pressure.

For the harmonic motion, $u_\theta(r,z,t)=u_\theta(r,z)e^{i\omega t}$ and $w_\theta(r,z,t)=w_\theta(r,z)e^{i\omega t}$, where $i=\sqrt{-1}$, ω is circular frequency of excitation and t is usual time variable. Then, Eqs. (4) and (5) can be expressed as

$$G_{sH}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)u_\theta(r,z)+G_{sv}\frac{\partial^2 u_\theta(r,z)}{\partial z^2}=-\rho\omega^2 u_\theta(r,z)-\rho_f\omega^2 w_\theta(r,z) \tag{6}$$

$$\frac{i\rho_f g}{k_d}w_\theta(r,z)-\rho_f\omega u_\theta(r,z)-\frac{\rho_f\omega w_\theta(r,z)}{n}=0 \tag{7}$$

Substituting Eq. (7) into Eq. (6), the governing equation of saturated soil can be written as

$$\delta\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)u_\theta(r,z)+\frac{\partial^2 u_\theta(r,z)}{\partial z^2}=-\frac{\omega^2}{G_{sv}}\left(\rho+\frac{n\rho_f\omega}{ib/\rho_f-\omega}\right)u_\theta(r,z) \tag{8}$$

where $b=n\rho_f g/k_h$; $\delta=G_{sH}/G_{sv}$ is the material parameter of transversely isotropic soil and denotes the ratio of the soil horizontal shear modulus to the soil vertical shear modulus, which describes the degree of anisotropy of the soil.

For an elastic pile subjected to harmonic torsional loading (see Fig. 1), the twist angle, $\phi(z,t)=\phi(z)e^{i\omega t}$, is governed by the following one-dimensional equation of motion [8]:

$$G_p\frac{\partial^2[\phi(z)e^{i\omega t}]}{\partial z^2}+4\frac{f(z)e^{i\omega t}}{r_0^2}=\rho_p\frac{\partial^2[\phi(z)e^{i\omega t}]}{\partial t^2} \tag{9}$$

where G_p, ρ_p, r_0 and $\phi(z)$ are the shear modulus, mass density, radius and the twist angle amplitude at depth z of the pile, respectively; $f(z)$ denotes amplitude of the contact traction along pile–soil interface.

The boundary conditions of the soil layer can be written as

$$\sigma_{\theta z}(r,z)|_{z=0}=0 \tag{10}$$

$$u_\theta(r,z)|_{z=H}=0 \tag{11}$$

The boundary conditions of the pile can be expressed as

$$\left.\frac{d\phi(z)}{dz}\right|_{z=0}=-\frac{T_0}{G_p I_p} \tag{12}$$

where $I_p=\pi r_0^4/2$ denotes the polar moment inertia of the pile and T_0 is the amplitude of the harmonic excitation torque acting at the pile top.

$$\phi(z)|_{z=H}=0 \tag{13}$$

The continuity conditions of interface of the pile and soil can be written as

$$u_\theta(r,z)|_{r=r_0}=\phi(z)r_0 \tag{14}$$

$$f(z)=\sigma_{r\theta}(r,z)|_{r=r_0}=G_{sH}\left.\left\{\frac{\partial u_\theta(r,z)}{\partial r}-\frac{u_\theta(r,z)}{r}\right\}\right|_{r=r_0} \tag{15}$$

2.2. Dynamic response of a transversely isotropic saturated soil

To solve Eq. (8), the following single-variable function $u_\theta(r,z)$ is introduced:

$$u_\theta(r,z)=R(r)Z(z) \tag{16}$$

The substitution for $u_\theta(r,z)$ from Eq. (16) into Eq. (8) yields

$$\delta\left[\frac{\mathrm{d}^2R(r)}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}R(r)}{\mathrm{d}r}-\frac{1}{r^2}R(r)\right]Z(z)+R(r)\frac{\mathrm{d}^2Z(z)}{\mathrm{d}z^2}=-\frac{\omega^2}{G_{\mathrm{sv}}}\left(\rho+\frac{n\rho_f\omega}{\mathrm{i}b/\rho_f-\omega}\right)R(r)Z(z) \tag{17}$$

Then, Eq. (17) can be split into two differential equations:

$$\frac{\mathrm{d}^2Z(z)}{\mathrm{d}z^2}+h^2Z(z)=0 \tag{18}$$

$$r^2\frac{\mathrm{d}^2R(r)}{\mathrm{d}r^2}+r\frac{\mathrm{d}R(r)}{\mathrm{d}r}-(1+q^2r^2)R(r)=0 \tag{19}$$

where constants h and q satisfy the following relationship:

$$q^2=\frac{h^2}{\delta}-\frac{\omega^2}{G_{\mathrm{sv}}\delta}\left(\rho+\frac{n\rho_f\omega}{\mathrm{i}b/\rho_f-\omega}\right) \tag{20}$$

The general solutions of Eqs. (19) and (20) can be expressed as

$$Z(z)=C\,\sin(hz)+D\,\cos(hz) \tag{21}$$

$$R(r)=AK_1(qr)+BI_1(qr) \tag{22}$$

where $I_1(qr)$ and $K_1(qr)$ are the modified Bessel functions of the first and second kind of the first order, respectively; A, B, C and D are constants which can be obtained from the boundary conditions.

It is noted from Eq. (22) that B should vanish to zero to guarantee bounded displacements and stresses as $r\to\infty$. Substitution of boundary conditions that are given in Eqs. (10) and (11) into Eq. (21) results in

$$C=0 \tag{23}$$

$$h_m=\frac{(2m-1)\pi}{2H};\quad m=1,2,3,\ldots \tag{24}$$

Then the general solution of Eq. (8) can be written in a series expansion as

$$u_\theta(r,z)=\sum_{m=1}^{\infty}D_mK_1(q_mr)\cos(h_mz) \tag{25}$$

where

$$q_m^2=\frac{h_m^2}{\delta}-\frac{\omega^2}{G_{\mathrm{sv}}\delta}\left(\rho+\frac{n\rho_f\omega}{\mathrm{i}b/\rho_f-\omega}\right) \tag{26}$$

and D_m ($m=1,2,3,\ldots$) are a series of constants.

The circumferential shear stress amplitude of the interface of the pile and soil can be expressed as

$$\sigma_{r\theta}(r,z)|_{r=r_0}=-G_{\mathrm{sH}}\sum_{m=1}^{\infty}D_mq_mK_2(q_mr_0)\cos(h_mz) \tag{27}$$

where $K_2(q_mr)$ denotes the modified Bessel functions of the second kind of the second order.

It is mathematically convenient at this stage to introduce the dimensionless quantities:

$$\bar{r}=\frac{r}{H},\quad \bar{z}=\frac{z}{H},\quad \bar{r}_0=\frac{r_0}{H},\quad a_0=\sqrt{\frac{\rho}{G_{\mathrm{sv}}}}r_0\omega,\quad \bar{h}_m=Hh_m=\frac{\pi}{2}(2m-1),$$
$$\bar{q}_m=Hq_m,\quad \bar{\rho}_f=\frac{\rho_f}{\rho},\quad \bar{b}=\frac{br_0}{\sqrt{\rho G_{\mathrm{sv}}}}$$

Using the dimensionless variables, given above, Eq. (26) can be rewritten as

$$\bar{q}_m^2=\frac{\bar{h}_m^2}{\delta}-\frac{a_0^2}{\delta\bar{r}_0^2}\left(1+\frac{n\bar{\rho}_fa_0}{\mathrm{i}\bar{b}/\bar{\rho}_f-a_0}\right) \tag{28}$$

2.3. Impedance of a pile

By utilizing the stress continuity condition that is given in Eq. (15), substituting Eq. (27) into Eq. (9) yields

$$\frac{d^2\phi(z)}{dz^2}+\frac{\rho_p\omega^2\phi(z)}{G_p}=\frac{4G_{sH}}{G_p r_0^2}\sum_{m=1}^{\infty}D_m q_m K_2(q_m r_0)\cos(h_m z) \tag{29}$$

It is not difficult to show that the solution of Eq. (29) can be expressed as

$$\phi(z)=\alpha_1\cos\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)+\alpha_2\sin\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)+\sum_{m=1}^{\infty}\psi_m\cos(h_m z) \tag{30}$$

where α_1 and α_2 are the coefficients which remain to be determined later from the boundary conditions; and

$$\psi_m=-\frac{4G_{sH}q_m K_2(q_m r_0)D_m}{G_p r_0^2\left(h_m^2-\dfrac{\rho_p\omega^2}{G_p}\right)} \tag{31}$$

During the vibration, the pile and the soil interact, so the vibration characteristics of the pile should be expressed in two parts. The first part is free vibration characteristics of the pile, as shown in the first and second terms in the right-hand side of Eq. (30). Because of the reaction from the soil, the second part represents the forced vibration characteristics of the pile, as is shown in the third term in the right-hand side of Eq. (30).

By using the displacement continuity condition that is given in Eq. (14), substituting Eqs. (30) and (25) into Eq. (14) results in

$$r_0\left\{\alpha_1\cos\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)+\alpha_2\sin\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)+\sum_{m=1}^{\infty}\psi_m\cos(h_m z)\right\}=\sum_{m=1}^{\infty}D_m K_1(q_m r_0)\cos(h_m z) \tag{32}$$

By invoking the orthogonality of eigenfunctions $\cos(h_m z)$ $(m=1,2,3,\ldots)$, multiplying $\cos(h_m z)$ on both sides of Eq. (32), and then integrating over the interval $z=[0,H]$, the undetermined coefficient D_m is found to be

$$D_m=\frac{1}{L_m E_m}\int_0^H P\cos(h_m z)\,dz \tag{33}$$

where

$$L_m=\int_0^H\cos^2(h_m z)\,dz=\frac{H}{2},\quad P=\alpha_1\cos\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)+\alpha_2\sin\left(\sqrt{\frac{\rho_p}{G_p}}\omega z\right)$$

$$E_m=\frac{1}{r_0}\left[K_1(q_m r_0)+\frac{4G_{sH}q_m K_2(q_m r_0)}{G_p r_0\left(h_m^2-\dfrac{\rho_p}{G_p}\omega^2\right)}\right]$$

It is again mathematically convenient at this stage to introduce the following dimensionless quantities:

$$\overline{\rho}_p=\rho_p/\rho,\quad \overline{\mu}=\frac{G_p}{\sqrt{\delta}G_{sv}},\quad \sqrt{\frac{\rho_p}{G_p}}\omega z=\frac{1}{\bar{r}_0}\sqrt{\frac{\overline{\rho}_p}{\overline{\mu}\sqrt{\delta}}}a_0\bar{z}=\overline{\lambda z},\quad \bar{E}_m=r_0E_m=K_1(\bar{q}_m\overline{r_0})+\frac{4\bar{q}_m\sqrt{\delta}K_2(\bar{q}_m\overline{r_0})}{\overline{\mu r_0}(\bar{h}_m^2-\bar{\lambda}^2)}$$

The amplitude of the twist angle of the pile is then given by

$$\phi(\bar{z})=\alpha_1\left\{\cos(\bar{\lambda}\bar{z})+\sum_{m=1}^{\infty}\xi_m\cos(\bar{h}_m\bar{z})\right\}+\alpha_2\left\{\sin(\bar{\lambda}\bar{z})-\sum_{m=1}^{\infty}\zeta_m\cos(\bar{h}_m\bar{z})\right\} \tag{34}$$

where

$$\begin{aligned}
\xi_m&=v_m\left[\frac{1}{\bar{\lambda}-\bar{h}_m}\sin(\bar{\lambda}-\bar{h}_m)+\frac{1}{\bar{\lambda}+\bar{h}_m}\sin(\bar{\lambda}+\bar{h}_m)\right]\\
\zeta_m&=v_m\left[\frac{1}{\bar{\lambda}+\bar{h}_m}\{\cos(\bar{\lambda}+\bar{h}_m)-1\}+\frac{1}{\bar{\lambda}-\bar{h}_m}\{\cos(\bar{\lambda}-\bar{h}_m)-1\}\right]\\
v_m&=-\frac{4\bar{q}_m\sqrt{\delta}K_2(\bar{q}_m\bar{r}_0)}{\bar{r}_0\bar{\mu}(\bar{h}_m^2-\bar{\lambda}^2)\bar{E}_m}
\end{aligned} \tag{35}$$

Based on the boundary conditions of the pile, then substituting Eqs. (12) and (13) into Eq. (34), the variables α_1 and α_2 are obtained:

$$\alpha_1 = \frac{T_0 H}{G_p I_p \bar{\lambda}} \tan \bar{\lambda} \tag{36}$$

$$\alpha_2 = -\frac{T_0 H}{G_p I_p \bar{\lambda}} \tag{37}$$

so that the dimensionless torsional impedance at the top end of the pile is given by

$$k_T = \frac{3T_0}{16 G_{sv} r_0^3 \phi(\bar{z}=0)} = \frac{3\pi \bar{r}_0 \bar{\lambda} \bar{\mu}}{32\left[\left(1 + \sum\limits_{m=1}^{\infty} \xi_m\right) \tan \bar{\lambda} + \sum\limits_{m=1}^{\infty} \zeta_m\right]} \tag{38}$$

where the real and imaginary parts of k_T denote the real stiffness and dynamic damping, respectively.

The frequency response function of twist angle of the pile top can be written as

$$H_\theta(\omega) = \left.\frac{\phi(\bar{z})}{T_0}\right|_{\bar{z}=0} = \frac{r_0}{G_p I_p} H'_\theta(\omega) \tag{39}$$

where the non-dimensional frequency response function of twist angle of the pile top is

$$H'_\theta(\omega) = \frac{\left[\tan \bar{\lambda}\left(1 + \sum\limits_{m=1}^{\infty} \xi_m\right) + \sum\limits_{m=1}^{\infty} \zeta_m\right]}{\bar{\lambda} \bar{r}_0} \tag{40}$$

The admittance function of angular velocity of the pile top can be further expressed as

$$H_v(\omega) = \mathrm{i}\omega H_\theta(\omega) = \frac{\mathrm{i}}{v_{ps} I_p \rho_p}\left[\tan \bar{\lambda}\left(1 + \sum_{m=1}^{\infty} \xi_m\right) + \sum_{m=1}^{\infty} \zeta_m\right] \tag{41}$$

By applying the inverse Fourier transform into Eq. (41), the response function of unit pulse torque in time domain can be written as

$$h(t) = IFT[H_v(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{i}}{v_{ps} I_p \rho_p T_c}\left[\tan \bar{\lambda}\left(1 + \sum_{m=1}^{\infty} \xi_m\right) + \sum_{m=1}^{\infty} \zeta_m\right] \mathrm{e}^{\mathrm{i}\varpi \bar{t}} \mathrm{d}\varpi \tag{42}$$

where $\varpi = \omega T_c$ and $T_c = H/v_{ps}$ denote the non-dimensional frequency and propagation time of elastic shear wave propagating from the pile top to pile tip, respectively; $\bar{t} = t/T_c$ is the non-dimensional time variable, respectively.

Table 1
Comparison of the torsional impedance of the pile top for an elastic isotropic soil.

a_0	$3T_0/[16G_{sv} r_0^3 \phi(\bar{z}=0)]$	
	Present study	Novak and Howell [6]
0	(22.05, 0)	(21.85, 0)
0.05	(22.03, 0)	(21.82, 0.01)
0.1	(21.98, 0)	(21.76, 0.05)
0.15	(21.88, 0)	(21.68, 0.11)
0.2	(21.74, 0.07)	(21.59, 0.19)
0.4	(21.25, 0.55)	(21.17, 0.66)
0.6	(20.76, 1.19)	(20.72, 1.27)
0.8	(20.28, 1.90)	(20.25, 1.97)
1.0	(19.77, 2.66)	(19.75, 2.72)
1.2	(19.22, 3.45)	(19.21, 3.50)
1.4	(18.62, 4.27)	(18.62, 4.31)
1.6	(17.96, 5.12)	(17.96, 5.16)
1.8	(17.23, 6.01)	(17.23, 6.04)
2.0	(16.42, 6.94)	(16.42, 6.97)

($\delta = 1.0$, $\rho_f = 0$, $n = 0$, $H/r_0 = 10$, $\bar{\mu} = 500$, $\bar{\rho}_p = 1.3$).

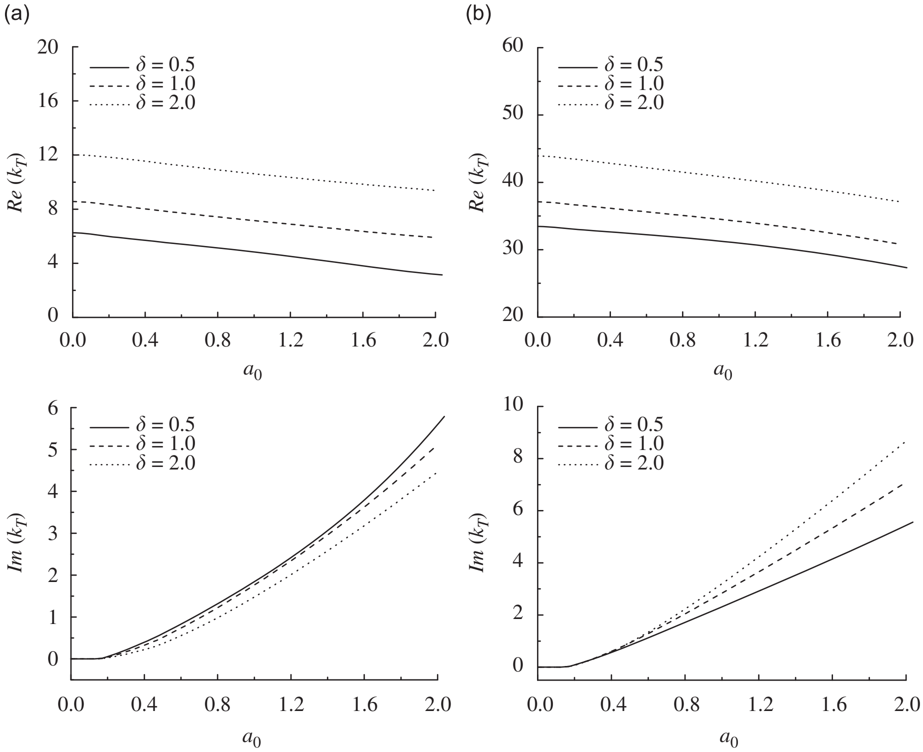

Fig. 2. Variation of torsional impedance of pile top with soil anisotropy ($H/r_0 = 10$): (a) $G_p/C_{44} = 100$ and (b) $G_p/C_{44} = 1000$.

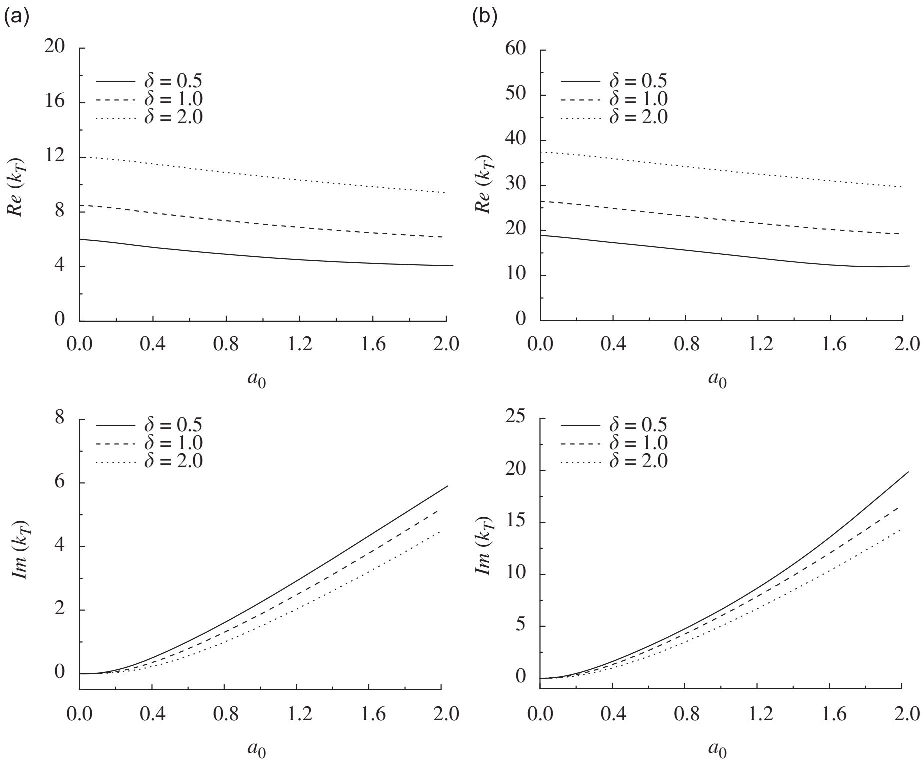

Fig. 3. Variation of torsional impedance of pile top with soil anisotropy ($H/r_0 = 40$): (a) $G_p/C_{44} = 100$ and (b) $G_p/C_{44} = 1000$.

If the Fourier transform of the arbitrary exciting torque $T(t)$ acting at the pile top is denoted by $T(\omega)$, then the velocity response of the pile top in the time domain is given via the inverse Fourier transform and convolution theorem as $V(t) = T(t) * h(t) = IFT[T(\omega)H_v(\omega)]$. In particular, when the exciting torque is a half-sine pulse,

$$T(t) = \begin{cases} T_{\max}\sin(\pi t/t_0), & t < t_0 \\ 0, & t \geq t_0 \end{cases} \tag{43}$$

where t_0 and $T_{\max}$ denote the duration of the impulse and the maximum amplitude of the exciting torque, respectively.

Then the velocity response of the pile top in the time domain can be written as

$$V(t) = T_{\max} IFT\left[\frac{1}{v_{\rm ps} I_p \rho_p} H_v'(\omega)\frac{\pi t_0}{\pi^2 - t_0^2\omega^2}(1 + \mathrm{e}^{-\mathrm{i}\omega t_0})\right] = \frac{T_{\max}}{v_{\rm ps} I_p \rho_p} V'(t) \tag{44}$$

To facilitate analysis, it is useful to introduce the dimensionless velocity response:

$$V'(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_v'(\omega)\frac{\pi\overline{T}}{\pi^2 - \overline{T}^2\varpi^2}(1 + \mathrm{e}^{-\mathrm{i}\theta\overline{T}})\mathrm{e}^{\mathrm{i}\varpi\overline{t}}\,\mathrm{d}\varpi \tag{45}$$

where $\overline{T} = t_0/T_c$ denotes the non-dimensional pulse width.

In the above, an analytical solution for the torsional impedance of the pile top in the frequency domain and a quasi-analytical solution for the velocity response of the pile top in the time domain have been derived. Through a comparison of solutions, it is evident that the solution developed in Ref. [12] is a special case of the present solution when fixing $\delta = 1.0 (G_{\rm sH} = G_{\rm sv} = G_s)$.

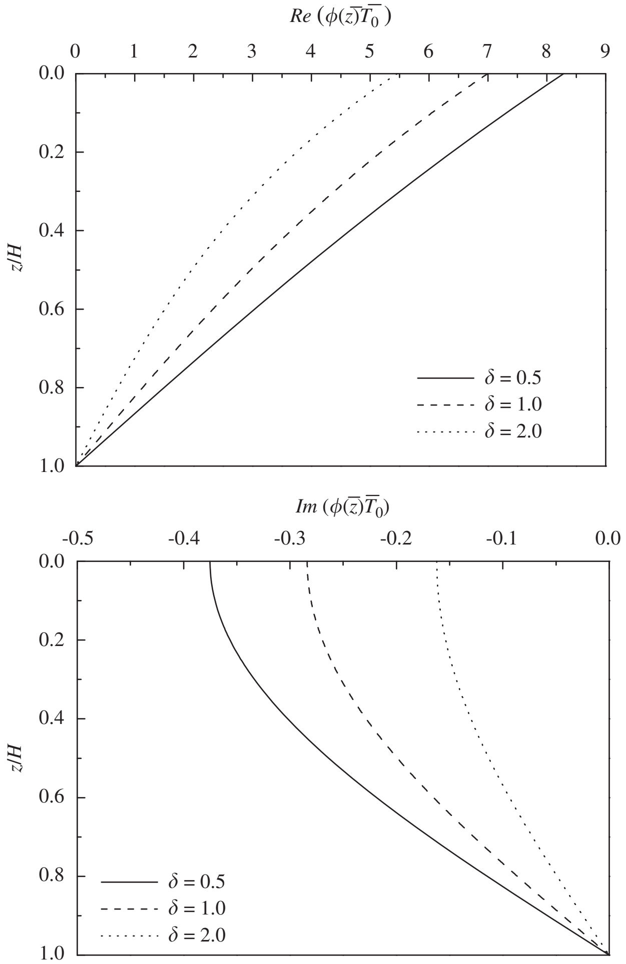

Fig. 4. Variation of non-dimensional twist angle of the pile with soil anisotropy ($a_0 = 0.5$).

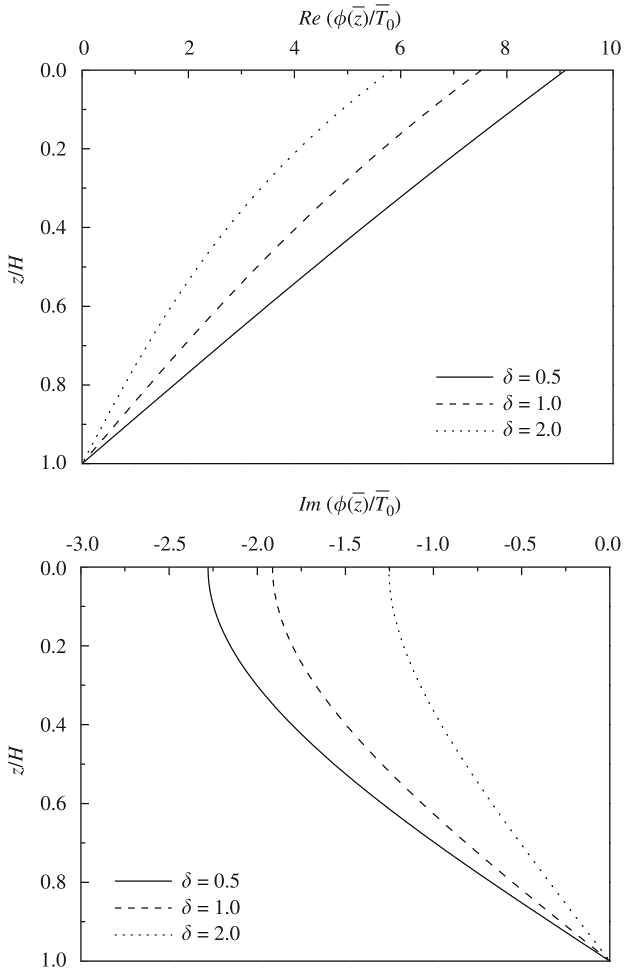

Fig. 5. Variation of non-dimensional twist angle of the pile with soil anisotropy ($a_0 = 1.5$).

3. Parametric study and discussion

In this section, the influence of soil anisotropy on the vibration characteristics of an end bearing pile is investigated and some numerical results from different solutions are compared and discussed.

To verify the solution developed in this paper, the soil is assumed to be an elastic homogeneous isotropic soil, and the corresponding complex impedance of the pile top is compared with that of Novak and Howell [6]. The comparison of these two solutions is presented in Table 1. The real part of the complex torsional impedance represents the real stiffness, while the imaginary part of the complex torsional impedance represents the dynamic damping which reflects the dissipation of the energy. It can be seen that the real stiffness and dynamic damping of this paper agrees well with Novak and Howell's solution generally. It can also be seen from Table 1 that a cut-off frequency for the dynamic damping part is obtained in the present solution. Below the cut-off frequency, the dynamic damping is zero and does not vary with frequency, which indicates that no damping is generated by the soil layer. Above the cut-off frequency, the dynamic damping increases quickly with the increase of exciting frequency. It is worth pointing out that Novak and Howell's solution did not produce the cut-off frequency. This can be attributed to the fact that the gradient of the shear stress $\sigma_{\theta z}$ in the vertical direction was neglected in their solution. Furthermore, the real stiffness given by the present solution is higher, and the dynamic damping calculated from the present solution is lower than those calculated by Novak and Howell below the cut-off frequency. Above the cut-off frequency, these two solutions agree very well.

In earlier studies [11,12], the influence of pile slenderness ratio, pile–soil modulus and poroelastic material properties on the dynamic behavior of the pile has been investigated. Given this, the remainder of this section will focus on the influence of soil anisotropy on the torsional impedance of the pile top, the distribution of non-dimensional twist angles and torque of the pile body along the depth direction and the velocity response of the pile top. In the calculation, the transversely isotropic poroelastic material properties are $G_{sv} = 1.94 \times 10^7$ Pa, $n = 0.4$, $k_h = 10^{-7}$ m/s, $\rho_f = 1000$ kg/m^3,

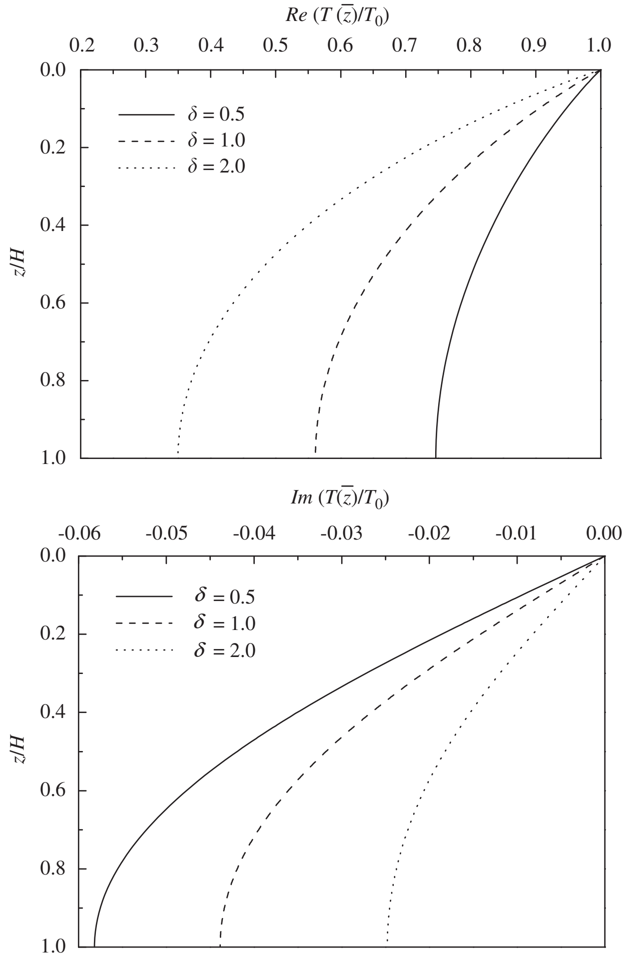

Fig. 6. Variation of non-dimensional torque of the pile with soil anisotropy ($a_0 = 0.5$).

$\rho_s = 2650\ \text{kg/m}^3$, $\delta = 0.5-2$, and pile's parameters are $r_0 = 1.0\,\text{m}$, $H/r_0 = 10-40$, $\rho_p = 2500\ \text{kg/m}^3$, $G_p/G_{\text{sv}} = 100-1000$, $t_0 = 2\,\text{ms}$.

The influence of soil anisotropy on the torsional impedance with the increasing dimensionless exciting frequency a_0 is shown in Figs. 2 and 3. It can be seen that soil anisotropy has a marked influence on the torsional impedance when the vertical shear modulus G_{sv} is considered as the reference modulus. For example, for flexible piles ($G_p/G_{\text{sv}} = 100$), the real stiffness markedly increases with increasing δ in the whole frequency range, indicating that the pile torsional impedance depends significantly on the soil horizontal shear modulus. As the pile becomes stiffer ($G_p/G_{\text{sv}} = 1000$), the increase of real stiffness becomes more pronounced. It can also be observed that the dynamic damping seems to be insensitive to soil anisotropy. For instance, for flexible and short piles ($G_p/G_{\text{sv}} = 100$, $H/r_0 = 10$), higher dynamic damping is associated with lower values of δ in the high frequency range. However, for stiffer piles ($G_p/G_{\text{sv}} = 1000$, $H/r_0 = 10$), higher dynamic damping is associated with higher values of δ in the high frequency range, and for stiffer and longer piles ($G_p/G_{\text{sv}} = 1000$, $H/r_0 = 40$) higher dynamic damping is associated with lower values of δ in the high frequency range.

Figs. 4 and 5 show the influence of soil anisotropy on the non-dimensional twist angle for $a_0 = 0.5, 1.5$, respectively. The non-dimensional twist angle of pile is defined as $\phi(\bar{z})/\overline{T}_0$, where $\overline{T}_0 = T_0 r_0/(G_p I_p)$. It can be seen that soil anisotropy has a marked influence on the twist angle of the pile in the low frequency range. The absolute value of the real and imaginary parts of twist angle of the pile top decreases with the increase of δ. It can also be observed that the absolute value of the real and imaginary parts of twist angle of the pile gradually decreases to zero with depth (z-direction). The range of the value of the twist angle of the pile along the pile length in the z-direction increases significantly with the decrease of δ.

Figs. 6 and 7 show the influence of soil anisotropy on the non-dimensional torque of the pile for $a_0 = 0.5, 1.5$, respectively. The non-dimensional torque of pile is defined as $T(\bar{z})/T_0$. It can be seen that soil anisotropy has a marked effect on the distribution of the torque of the pile along the pile length. The absolute value of the real and imaginary parts of the torque of pile decreases significantly with the increase of δ for a given z/H with the increase becoming more pronounced as z/H increases. This indicates that the soil around the upper part of the pile will bear a higher percentage of loading for a larger value of δ, as a consequence, a lower percentage of loading is transferred to the lower part of the pile.

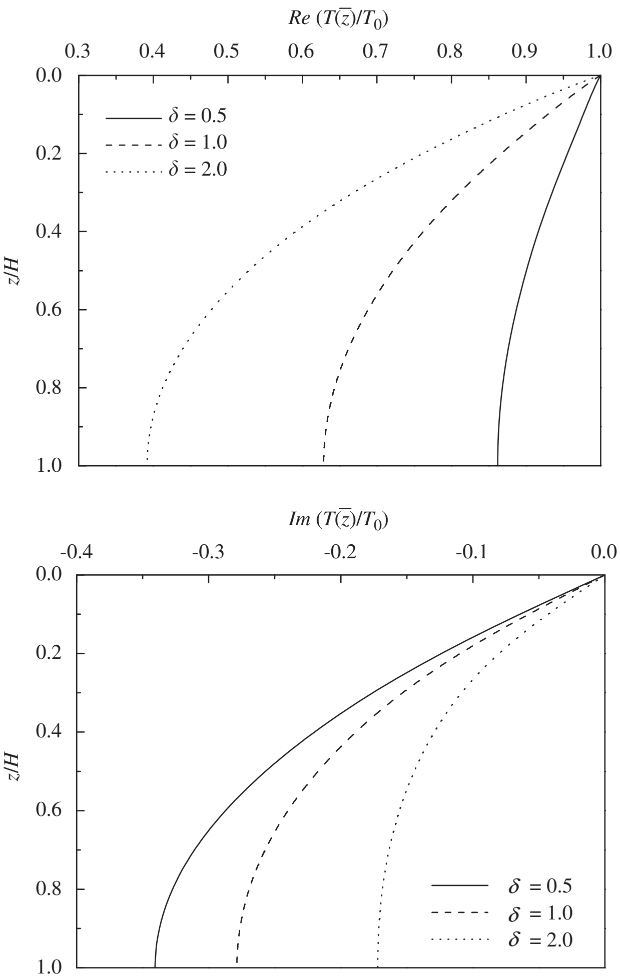

Fig. 7. Variation of non-dimensional torque of the pile with soil anisotropy ($a_0 = 1.5$).

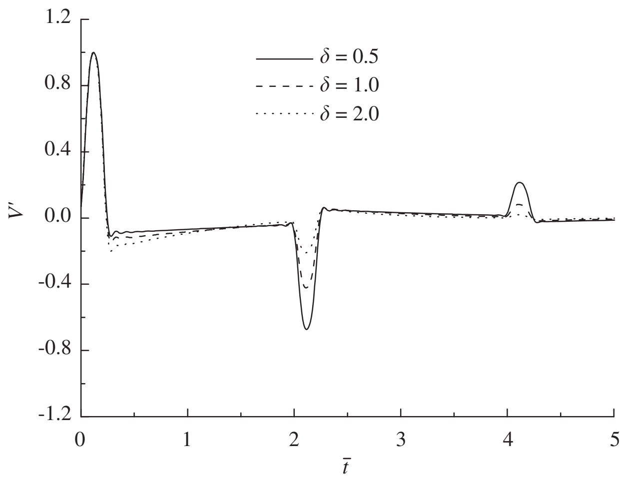

Fig. 8. Time histories of the non-dimensional velocity response of the pile top with different transverse isotropy.

Fig. 8 shows the influence of soil anisotropy on the velocity response of the pile top in the time domain. The velocity response curve can be regarded as the reflected curve which is useful for analyzing the integrity and verifying the length of the pile. It can be seen that the reflected signal of the pile tip firstly arrives at the pile top when $\bar{t} = 2$. For an embedded end bearing pile, the phase of the first reflected signal of the pile tip is opposite to that of the incident impulse, and the phase of the second reflected signal of the pile tip is the same as that of incident impulse. This can be explained as follows. The time of incident impulse propagating from the pile top to pile tip is equal to $T_c = H/v_{ps}$ and when the impulse reaches the interface of pile tip and bedrock, the opposite phase of reflected signal is generated due to the fixed boundary. The corresponding propagation time from the pile tip to pile top is also equal to $T_c = H/v_{ps}$ so accordingly, the impulse signal reaches the pile top at $t = 2T_c$ (that is $\bar{t} = t/T_c = 2$). Due to the great energy dissipation during the propagation of impulse signal, the intensity of the second reflected signal is significantly lower than that of the first reflected signal. Therefore, the length of an end bearing pile can be determined by multiplying the arrival time of the first reflected signal of the pile tip and shear wave velocity in the pile. It is also seen that soil anisotropy has a marked influence on the velocity response. The intensity of reflected signal of the pile tip decreases significantly with the increase of δ. This indicates that an increase of the soil horizontal shear modulus is not advantageous for identifying the arrival time of reflected signal of the pile tip and verifying the length of the pile.

4. Conclusions

By considering an end bearing pile embedded in transversely isotropic saturated soil as a dynamic pile–soil interaction problem, an analytical solution for the torsional impedance of the pile in the frequency domain and quasi-analytical solution for the velocity response in the time domain have been derived. It is seen that the solutions developed in this paper for the dynamic response of an end bearing pile in transversely isotropic soil are a generalization of our earlier work in Ref. [12]. An extensive parameter study has been conducted to investigate the influence of soil anisotropy on the torsional vibration characteristics of the single pile and the following conclusions have been obtained:

(1) Soil anisotropy has a marked influence on the torsional impedance in the low frequency range. The real stiffness increases markedly with increasing δ. As the pile becomes stiffer, the increase of real stiffness becomes more pronounced. However, the dynamic damping seems to be insensitive to soil anisotropy.
(2) Soil anisotropy has a marked influence on the twist angle of the pile in the low frequency range. The absolute value of the real and imaginary parts of twist angle of the pile top decreases significantly with the increase of δ. The range of the value of the twist angle of the pile along the pile length increases significantly with the decrease of δ.
(3) Soil anisotropy has a marked effect on the distribution of the torque of the pile along the pile length in the low frequency range. The absolute value of the real and imaginary parts of the torque of pile decreases significantly with the increase of δ for a given z/H with the increase becoming more pronounced as z/H increases. The soil around the upper part of the pile will bear greater percentage of loading for a larger value of δ, with a transfer of corresponding lower percentage of loading to the lower part of the pile.
(4) For an embedded end bearing pile, the phase of the first reflected signal of the pile tip is opposite to that of the incident impulse and the phase of the second reflected signal of the pile tip is the same as that of incident impulse. The intensity of the second reflected signal is significantly lower than that of the first reflected signal. Soil anisotropy also has a marked effect on the velocity response of the pile top in the time domain. The intensity of reflected signal of the pile tip decreases with the increase of δ.
(5) Finally, the length of an end bearing pile can be determined in a pile test by multiplying the arrival time of the first reflected signal of the pile tip and shear wave velocity in the pile.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (Grant no. 50879077) and by the Science and Technology Agency Program of Zhejiang Province of China (Grant no. 2007C13065).

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