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Vibration-based damage identification of plate structures via curvelet transform

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ABSTRACT

This paper presents a new method based on curvelet transform to assess damage location in plate structures. The curvelet transform was developed over the last few years in an attempt to overcome inherent limitations of traditional multiscale representations such as wavelets. In this research, curvelet transform has been employed due to its favorable performance in detecting line feature. The formulation of discrete curvelet transform using unequally-spaced fast Fourier transforms for plate damage detection was investigated. The proposed method was applied to a four-fixed supported rectangular plate containing one or two damages with arbitrary length, depth and location. Thereafter, the damage was detected in the plate using the proposed method. By way of comparison between location obtained from the proposed method and simulation model, it was concluded that the method is sensitive to damage. Moreover, the performance of the method has been verified through using experimental modal data of a plate.

1. Introduction

Largely due to the increasing application of the structural damage detection, it has received noticeable attention over the last few years. Major infrastructures like bridges, tunnels, plants and other structures, as well as mechanical systems such as airplanes are a few to name where health monitoring becomes of strategic importance.

Damages cause some changes in the physical properties of structures like mass, stiffness and damping at damaged locations. Consequently, these changes make dynamic characteristics such as natural frequencies, mode shapes and damping ratio of the structure to deviate from its initial pre-damage condition [1].

For many years, damage identification methods have been studied by a number of researchers. Lynn and Kumbasar [2] applied Green's function to analyze free vibration behavior of cracked rectangular plates. Hirano and Okazaki [3] used Fourier series to study vibration characteristics of rectangular plates with crack. Cawley and Adams [4] extended a method for detecting imperfections in a frequency response function plate, on the basis of changes in frequency. Bayissa and Haritos [5] proposed a new damage identification technique based on the statistical moments of the energy density function of the vibration responses in the time-scale domain. Pandey et al. [6] stated that absolute alters in the curvature mode shapes are localized in the area of damage which can be applied to detect damage in a structure. Cornwell et al. [7] extended the modal energy method which was originally developed for damage detection in one-dimensional structures to plate structures.

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Wavelet analysis in the areas of damage identification and health monitoring has been widely employed. As such, many researchers devoted their efforts to present the applications of wavelet transform. Surace and Ruotolo [8] had studied vibration response signals of a damaged beam using wavelet transform. Yan and Yam [9] identified damage utilizing wavelet analysis in composite plates to decompose the dynamic responses. Chang and Chen [10] stated a technique to detect damage in rectangular plate with analyzing spatially distributed signals obtained by finite element using wavelet transform. Poudel et al. [11] proposed to use of digital video imaging for detecting damage in structures, mode shapes were obtained from the time series to find the mode shape difference functions between the damaged and the reference states. They were subjected to wavelet transformation for determining the damage locations. Rucka and Wilde [12] presented a method to estimate damage location in beam and plate using continuous wavelet transform, damage location is identified by a peak in the spatial variation of the transformed response. Bayissa et al. [13] proposed damage identification technique using the continuous wavelet transform to detect damage in a concrete plate model and in steel plate girder of a bridge structure.

Curvelet transform is a new multiscale pyramid representation with many directions and positions at each length scale and needle-shaped elements at fine scale. A curvelet transform gives better and sparser representation than traditional multiscale transforms. It should be mentioned that curvelet and wavelet transforms exhibits impressive performance in detecting line and point features, respectively.

Curvelets were first introduced in [14] and have been around for a little over eight years by now. Soon after their introduction, researchers developed numerical algorithms for their implementation [15] and scientists have started to report on a series of practical successes. Now these implementations are based on the original construction [14] which uses a pre-processing step involving a special partitioning of phase-space followed by the ridgelet transform [16] which is applied to blocks of data that are well localized in space and frequency. Candes and Donoho [17] has introduced new tight frames of curvelets and proved that curvelets provide optimally sparse representation of objects with singularities along C^2 edges. Ma and Plonka [18] has described some recent applications including image processing, seismic data exploration, turbulent flows and compressed sensing. Also, Ma and Plonka [19] has presented a review on the curvelet transform, involving its history beginning from wavelet transform, logical relationship to other multiresolution multidirectional methods, its basic theory and discrete algorithm and recent applications in engineering.

To summarize, the curvelet transform is mathematically valid and a very promising potential in traditional application areas for wavelet-like ideas such as image processing, data analysis and scientific computing clearly lies ahead. To realize this potential though and deploy this technology to a wide range of problems, one would need a fast and accurate discrete curvelet transform operating on digital data.

In the current study, a novel method based on the discrete curvelet transform using unequally-spaced fast Fourier transforms has been employed to identify damage location in plate structures. In addition, the performance and sensitivity of the proposed method have been investigated using numerical and experimental data.

2. Overview on curvelet transform

2.1. Continues curvelet transform

Curvelets are a recently developed multiscale system [17] in which the elements are highly anisotropic at fine scales, with effective support obeying the parabolic principle. A curvelet is indexed by three parameters which are: a scale a; an orientation θ and a location $x=(x_1, x_2)$. In curvelet transform, a curvelet mother defined by [20]:

$$\varphi_j(x) = U_j(r,\theta) \tag{1}$$

$$U_j(r,\theta) = 2^{-3j/4} W(2^{-j}r) V\left(\frac{2^{\lfloor j/2 \rfloor}\theta}{2\pi}\right) \tag{2}$$

where W and V are is radial window and angular window, respectively.

At scale 2^{-j}, orientation $\theta_l = 2\pi \cdot 2^{-\lfloor j/2 \rfloor} \cdot l$ with $l = 0, 1, \ldots$ and position $x_k^{(j,l)} = R_{\theta_L}^{-1}(k_1 \cdot 2^{-j}, k_2 \cdot 2^{-j/2})$ with translation parameter $k=(k_1,k_2) \in Z^2$ the family of curvelets is generated by translations and rotations of a basic element φ_j:

$$\phi_{j,l,k}(x) = \phi_j(\mathbf{R}_{\theta_l}(x - x_k^{(j,l)})) \tag{3}$$

where $\mathbf{R}_\theta$ is rotation by θ radians:

$$\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4}$$

A curvelet coefficient is then simply the inner product between an element $f \in L^2(R^2)$ and a curvelet $\varphi_{j,l,k}$:

$$\mathbf{c}(j,l,k) = \langle f, \phi_{j,l,k} \rangle = \int_{R^2} f(x)\overline{\phi_{j,l,k}(x)}\,\mathrm{d}x \tag{5}$$

Since digital curvelet transforms operate in the frequency domain, it will prove useful to apply Plancherel's theorem and express this inner product as the integral over the frequency plane:

$$c(j,l,k)=\frac{1}{(2\pi)^2}\int \hat{f}(\omega)\overline{\hat{\phi}_{j,l,k}(\omega)}\,\mathrm{d}\omega=\frac{1}{(2\pi)^2}\int \hat{f}(\omega)U_j(\mathbf{R}_{\theta_l}\omega)\,\mathrm{e}^{\mathrm{i}\langle x_k^{(j,l)},\omega\rangle}\,\mathrm{d}\omega \tag{6}$$

2.2. Discrete curvelet transform

In discrete curvelet transform the family of Cartesian curvelets suggested [20,21]:

$$\tilde{\phi}_{j,l,k}(x)=2^{3j/4}\tilde{\phi}_j(\mathbf{S}_{\theta_l}^{\mathrm{T}}(x-S_{\theta_l}^{-\mathrm{T}}b)) \tag{7}$$

where b takes on the discrete values $b=(k_1\cdot 2^{-j},k_2\cdot 2^{-j/2})$ and S_θ is shear matrix:

$$\mathbf{S}_\theta=\begin{pmatrix}1 & 0\\ -\tan\theta & 1\end{pmatrix} \tag{8}$$

The coefficients given:

$$c(j,l,k)=\frac{1}{(2\pi)^2}\int \hat{f}(\omega)\tilde{U}_j(\mathbf{S}_{\theta_l}^{-1}\omega)\,\mathrm{e}^{\mathrm{i}\langle S_{\theta_l}^{-\mathrm{T}}b,\omega\rangle}\,\mathrm{d}\omega=\frac{1}{(2\pi)^2}\int \hat{f}(\mathbf{S}_{\theta_l}\omega)\tilde{U}_j(\omega)\,\mathrm{e}^{\mathrm{i}\langle b,\omega\rangle}\,\mathrm{d}\omega \tag{9}$$

where $\tilde{U}_j(\omega)$ is the "Cartesian" window:

$$\tilde{U}_j(\omega):=\tilde{W}_j(\omega)V_j(\omega) \tag{10}$$

$\tilde{W}_j(\omega)$ and $V_j(\omega)$:

$$V_j(\omega)=V(2^{\lfloor j/2\rfloor}\omega_2/\omega_1) \tag{11}$$

$$\tilde{W}_j(\omega)=\sqrt{\Psi_{j+1}^2(\omega)+\Psi_j^2(\omega)} \tag{12}$$

where Ψ is defined product of low-pass 1D window:

$$\Psi_j(\omega_1,\omega_2)=\phi(2^{-j}\omega_1)\phi(2^{-j}\omega_2) \tag{13}$$

where $2^j\le\omega_1\le 2^{j+1}$and $-2^{-j/2}\le\omega_2/\omega_1\le 2^{-j/2}$.

Finally, using unequally spaced fast Fourier transforms the coefficients of discrete curvelet transform become [20]:

$$\mathbf{c}(j,l,k)=\sum_{n_1,n_2}\hat{f}[n_1,n_2-n_1\tan\theta_l]\tilde{U}_j[n_1,n_2]\mathrm{e}^{\mathrm{i}2\pi(k_1n_1/L_{1,J}+k_2n_2/L_{2,j})} \tag{14}$$

where $\hat{f}[n_1,n_2]$ denotes 2D discrete Fourier transform of $f[t_1,t_2]$.

Further details about curvelet transform can be found in Candes and Donoho [17] and Ma and Plonka [19].

3. Damage identification method

This paper is aimed at presenting a new method based on curvelet transform to detect linear damage in plate structures. In the proposed method, discrete curvelet transform is calculated using unequally spaced fast Fourier transforms.

3.1. Mode shape estimation

In numerical study, the mode shapes of the rectangular plate with and without damage have been determined using finite element method. Geometry of a four-fixed rectangular plate with damage is shown in Fig. 1. The dimensions of the plate are $L\times B\times t$. The damage is represented as the elements with reduced thickness. The damage considered in the plate is the linear damage with the length of W and the width of D and started location coordinates of the damage is $L1$, $B1$ with directional angle α. The damage region is represented as the element with reduced thickness Δt.

The equation of free vibration of a plate with damage is as follows:

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=0 \tag{15}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively. Solving of the above equation leads to determine mode shape $\mathbf{\Phi}_i$, $i=1,2,\ldots,n$ where n is the number of structural mode. In this study, fundamental mode shape of the damaged plate $\mathbf{\Phi}_d$ and undamaged plate $\mathbf{\Phi}_{\mathrm{ud}}$ have been utilized.

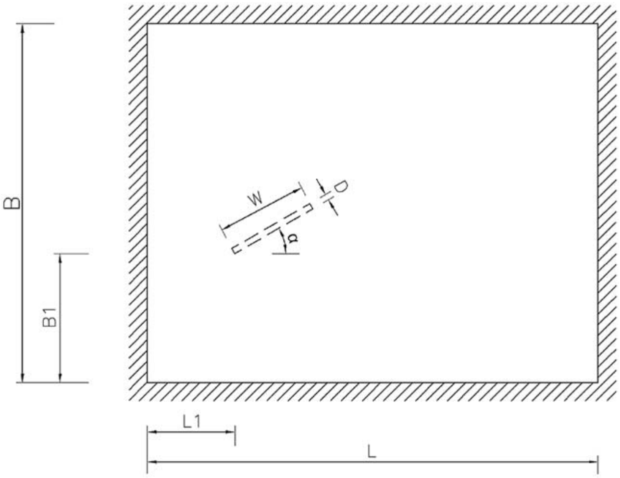

Fig. 1. Geometry of the plate structure with linear damage.

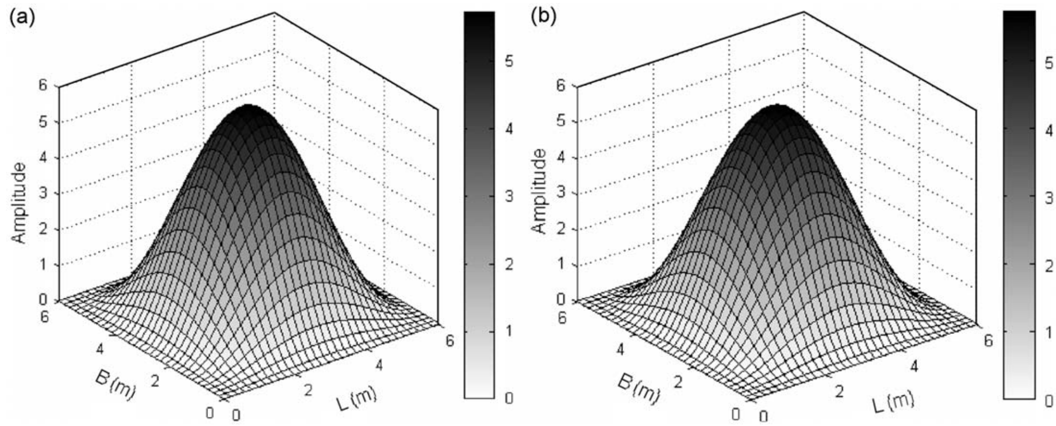

Fig. 2. Fundamental mode shape for the plate: (a) without damage and (b) with 20 percent damage.

3.2. *Denoising of mode shape*

The measured structural response data always contain noise. The existence of noise may have an appreciable influence on the accuracy of damage identification. A systematic investigation of the effects of noise on the performance of damage identification method is yet to be done. A dilemma is that it is not possible to know with any measure of certainty whether and how much the measured data are perverted by noise. If denoising techniques are indiscriminately applied to data with very little noise, then useful information may be removed from the data, leading to erroneous results [22]. Data denoising accomplished using the curvelet transform includes three following steps:

- Curvelet-based decomposition of the noisy signal.
- Thresholding the coefficients.
- Reconstructing the denoised signal.

3.3. *Application of curvelet transform*

The discrete curvelet transform via unequally spaced fast Fourier transforms is applied to the mode shape of the plate to determine the coefficients of discrete curvelet transform. Because, the curvelets are good for detecting of line feature and it is right for detecting of the liner damage in the plate.

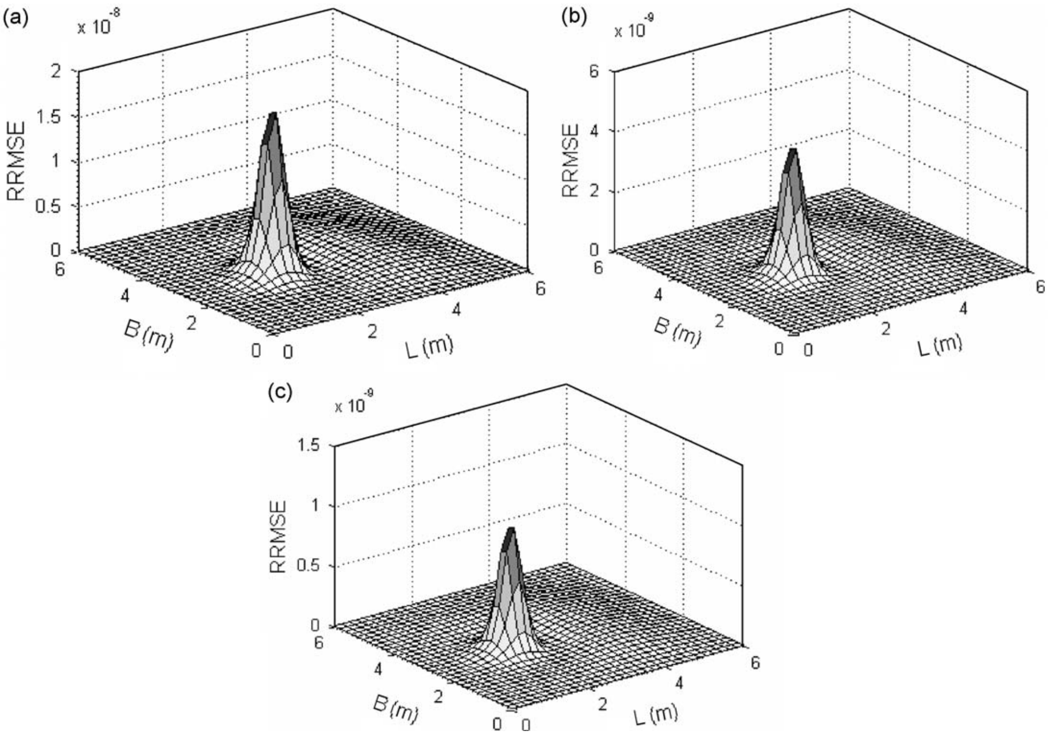

Fig. 3. Single damage identification in the first case using RRMSE: (a) 20 percent damage, (b) 10 percent damage and (c) 5 percent damage.

The coefficients of discrete curvelet transform for the fundamental mode shape of the damaged plate $\mathbf{\Phi}_d$ are given by

$$\mathbf{c}^d(j,l,k) = \sum_{n_1,n_2} \hat{\mathbf{\Phi}}_d[n_1, n_2 - n_1 \ \tan\ \theta_l]\tilde{U}_j[n_1,n_2]\mathrm{e}^{\mathrm{i}2\pi(k_1n_1/L_{1,j}+k_2n_2/L_{2,j})} \tag{16}$$

where $\mathbf{c}^d$ and $\mathbf{c}^{\mathrm{ud}}$ are coefficients of discrete curvelet transform of the damaged and undamaged plate, respectively.

Therefore, damage identification indices like relative root mean square error (RRMSE) and difference error (DE) in coefficients of discrete curvelet transform were used for locating of the damage in a plate structure.

3.4. Damage index using RRMSE

The damage indices are calculated using relative root mean square error for an element node-based the coefficients of discrete curvelet transform for the damaged state to the undamaged state determined for each element node. The RRMSE damage index is defined as follows:

$$\mathbf{DI}^n(RRMSE) = \sqrt{\frac{\frac{1}{m}\sum_{j=1}^{m}(\mathbf{c}_j^d - \mathbf{c}_j^{\mathrm{ud}})^2}{\frac{1}{m}\sum_{j=1}^{m}(\mathbf{c}_j^{\mathrm{ud}})^2}} \tag{17}$$

where $\mathbf{DI}^n(RRMSE)$ is the element node-based damage index using RRMSE; m is number of nodes leads into the node n. $\mathbf{c}_j^d$ and $\mathbf{c}_j^{\mathrm{ud}}$ are the coefficients of discrete curvelet transform for the damaged and undamaged plate element node n, respectively.

3.5. Damage index using DE

In this damage index, the damage indices are computed using the deference of an element node-based the coefficients of discrete curvelet transform for the damaged state to the undamaged state determined for each element node using

$$\mathbf{DI}^n(DE) = \left(\frac{\frac{1}{m}\sum_{j=1}^{m}\mathbf{c}_j^d}{\frac{1}{n_n}\sum_{n=1}^{n_n}\mathbf{c}_n^d} - \frac{\frac{1}{m}\sum_{j=1}^{m}\mathbf{c}_j^{\mathrm{ud}}}{\frac{1}{n_n}\sum_{n=1}^{n_n}\mathbf{c}_n^{\mathrm{ud}}}\right) \tag{18}$$

where $\mathbf{DI}^n(DE)$ is the element node-based damage index using DE and n_n is total number of nodes used in the model.

4. Numerical analysis studies

The proposed method for damage detection in plates using discrete curvelet transform with unequally spaced fast Fourier transforms in this study has been applied to a four fixed rectangular plate containing one or two damages with arbitrary length, depth and location. For this purpose, a fixed support rectangular plate with dimensions $6 \times 6 \times 0.2$ m was considered. The material properties of the plate include Young's modulus of $E = 20$ GPa, mass density of $\rho = 2500\,\mathrm{kg/m^3}$ and Poisson's ratio of $\nu = 0.2$. ABAQUS software package has been employed to build model used in the finite element method. In finite element model, the damage is represented as the elements with reduced thickness. In order to damage detection, three different cases from the aspect of number and location of damage were considered.

In the first case of damage identification studies, a single damage induced in plate and the linear damage with the length of $W = 0.19$ m and the width of $D = 0.01$ m and started location coordinates of the damage was $L1 = 2.14$ m, $B1 = 2.71$ m with directional angle $\alpha = 90^\circ$. For different levels of damage in plate, 20 percent, 10 percent and 5 percent reduced thickness were considered.

Then the mode shapes of plate were determined using finite element mode and the fundamental mode shape should be employed in the proposed method. A comparison between the fundamental mode shapes of damaged and undamaged plates is depicted in Fig. 2. Since Fig. 2(a) and (b) seem completely similar in shape, it is quite difficult to observe the differences. Therefore, it is necessary to have method to find the differences for identification of damage.

Using the proposed method for damage identification through curvelet transform can determine the damage indices. Figs. 3 and 4 show the capability of the proposed method for identification of damage in plate using RRMSE and DE damage index, respectively. The single damage in plate is detected and localized using the damage index techniques for different levels of damage. The results show that the peak values of the damage indices are observed at the exact damage locations in model.

In second case of damage identification studies a single damage induced in plate and the started location coordinates of the damage is $L1 = 4.06$ m, $B1 = 4.45$ m with directional angle $\alpha = 135^\circ$ and the liner damage with the length of

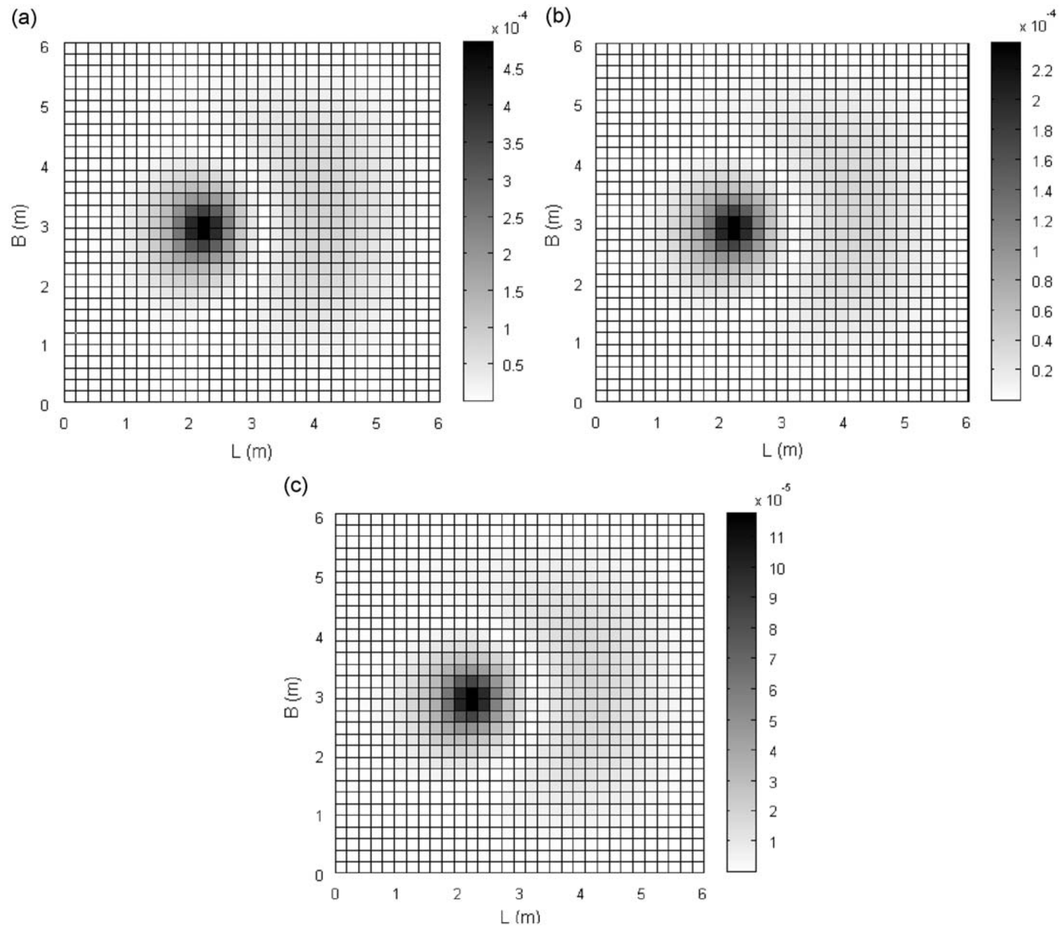

Fig. 4. Single damage identification in the first case using DE: (a) 20 percent damage, (b) 10 percent damage and (c) 5 percent damage.

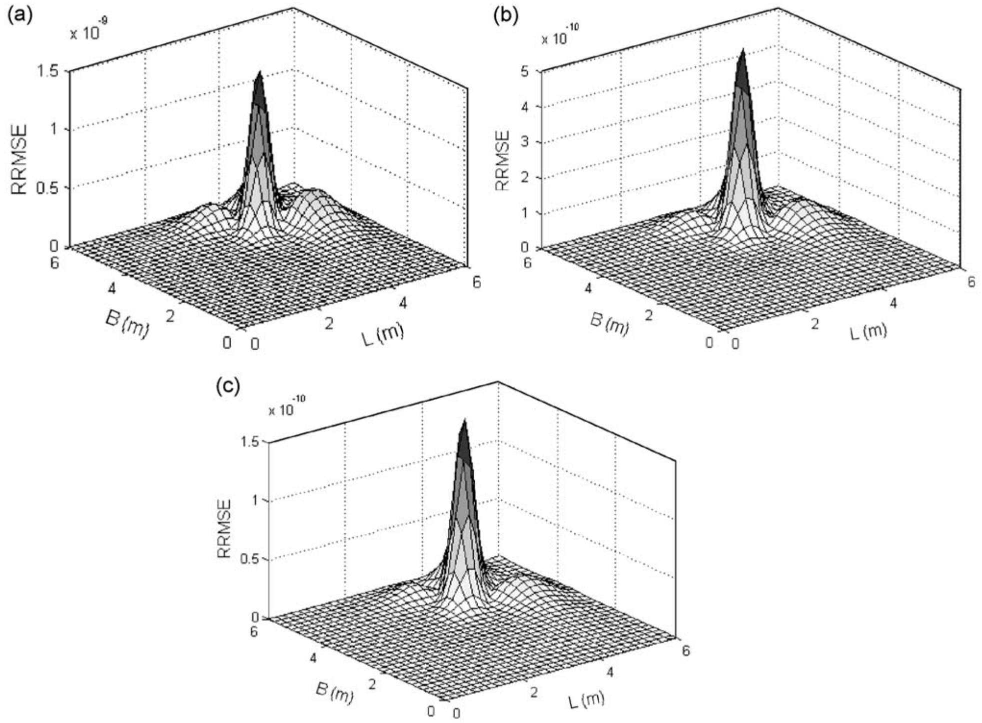

Fig. 5. Single damage identification in the second case using RRMSE: (a) 20 percent damage, (b) 10 percent damage and (c) 5 percent damage.

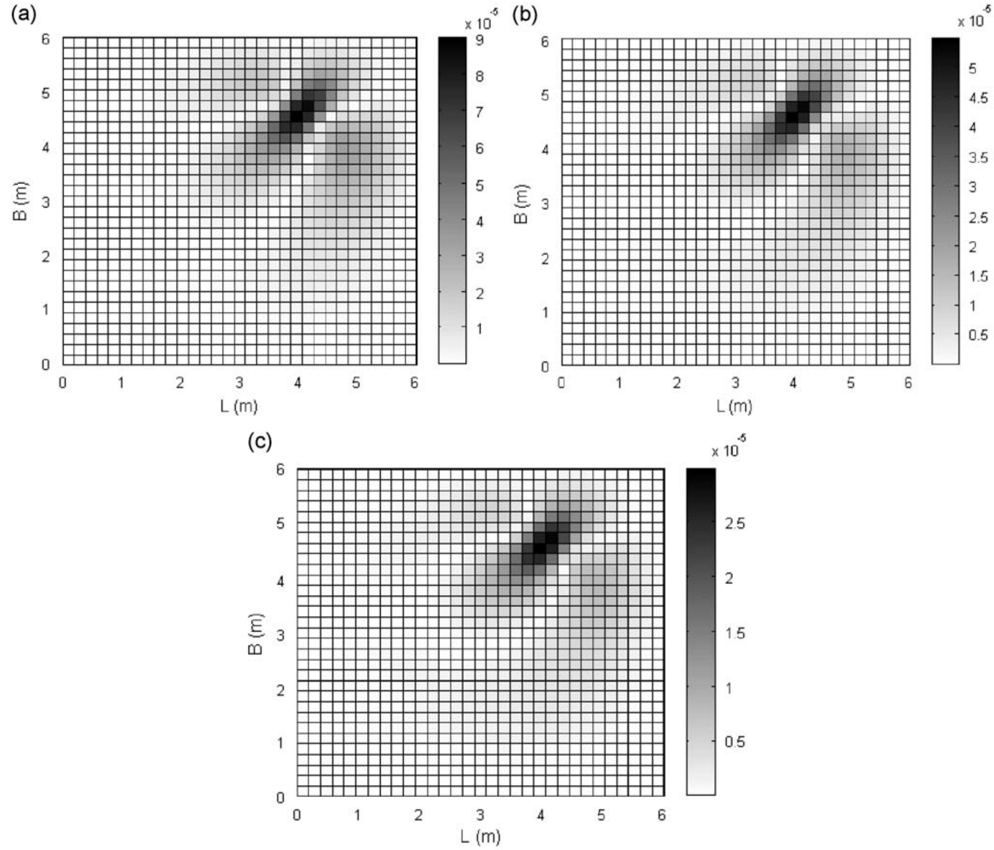

Fig. 6. Single damage identification in the second case using DE: (a) 20 percent damage, (b) 10 percent damage and (c) 5 percent damage.

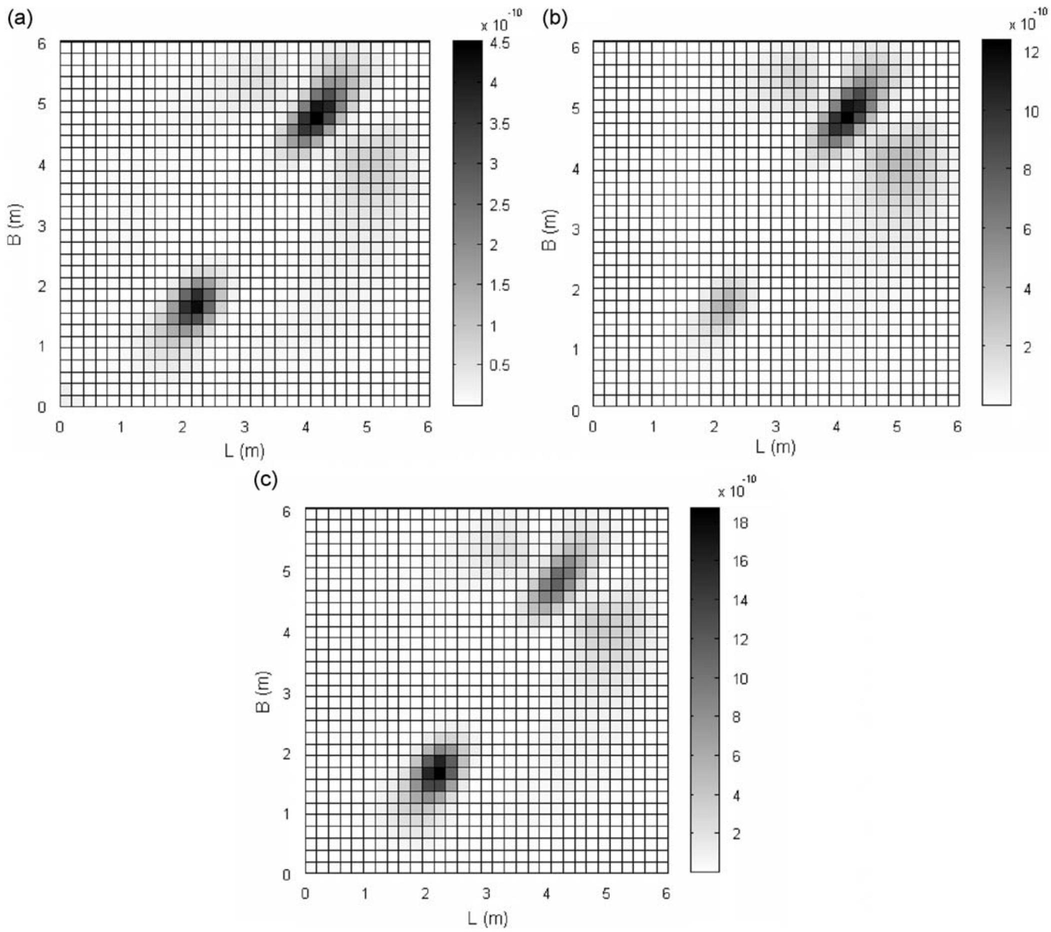

Fig. 7. Two damage identification in the plate using RRMSE: (a) 10 percent damage and 5 percent damage, (b) 20 percent damage and 5 percent damage and (c) 20 percent damage and 10 percent damage.

$W = 0.27$ m and the width of $D = 0.01$ m. This has been considered for different levels of damage in plate. The result of RRMSE and DE damage index for this case with different levels of damage is shown in Figs. 5 and 6. Results show that the maximum values of the damage indices occur at the damages in plate.

For third case, two damages are induced in plate. The first liner damage with the length of $W = 0.19$ m and the width of $D = 0.01$ m and the second liner damage with the length of $W = 0.27$ m and the width of $D = 0.01$ m. The location coordinates of the damage is $L1 = 2.14$ m, $B1 = 2.71$ m and $L2 = 4.06$ m, $B2 = 4.45$ m with directional angles $\alpha = 90^\circ$ and 135°, respectively. For different levels of damage in plate 20 percent and 10 percent, 10 percent and 5 percent, 20 percent and 5 percent reduced thickness are considered.

For this case the result of damage indices in various levels of damage in plate are shown in Figs. 7 and 8. Also, in this case the locations of damages in plate are identified using the results of RRMSE and DE damage index.

Finally, it can be concluded that this damage identification method is much more sensitive to damage. This is due to the use of discrete curvelet transform in the method for processing of data.

5. Experimental validation study

In the preceding section, the damage identification method was demonstrated through extensive numerical simulation studies with realistic levels of damage in plate. However, it is still useful to examine the empirical performance of the proposed method using measured data from an experimental study. Therefore in this section, the performance of the damage detection method using curvelet transform are validated thorough vibration response data measured from a steel plate tested by Rucka and Wilde [12].

A steel plate of length $L = 560$ mm, width $B = 480$ mm and height $t = 2$ mm is shown in Fig. 9. The material properties of the plate include Young's modulus of $E = 192$ GPa, mass density of $\rho = 7430$ kg/m^3 and Poisson's ratio of $v = 0.25$. The plate

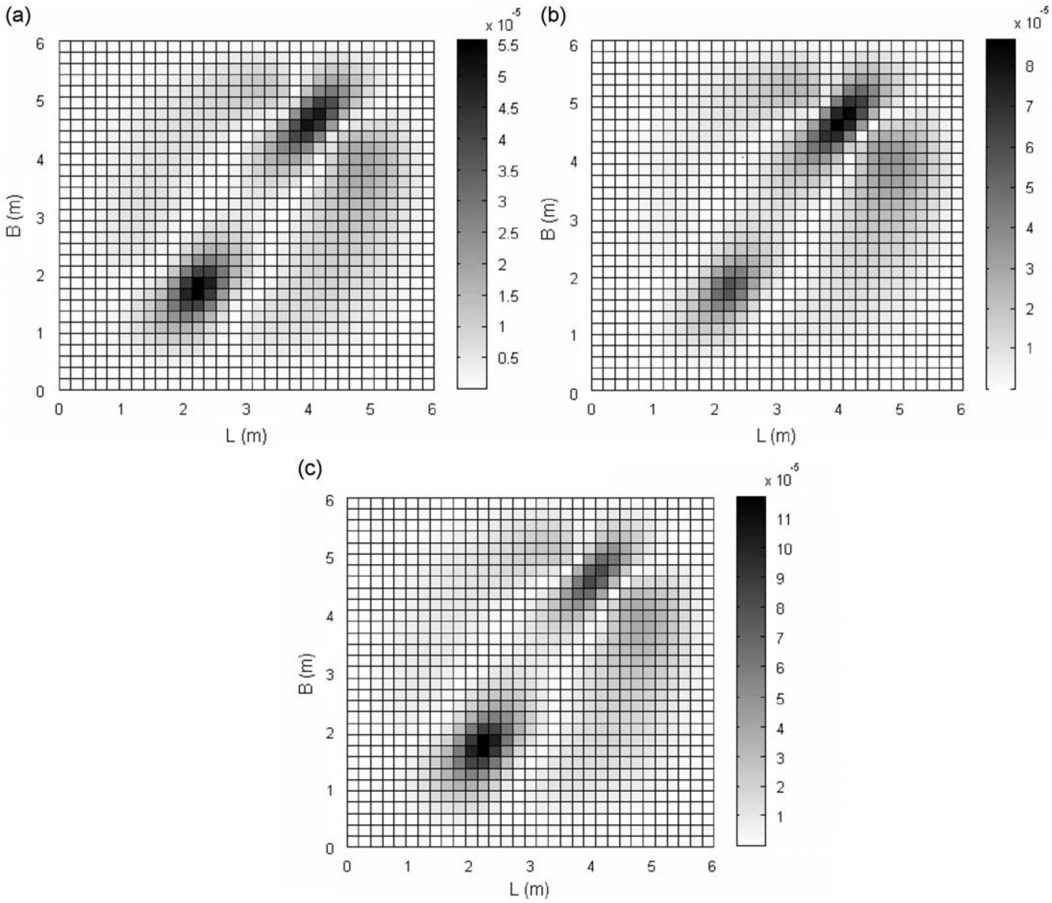

Fig. 8. Two damage identification in the plate using DE: (a) 10 percent damage and 5 percent damage, (b) 20 percent damage and 5 percent damage and (c) 20 percent damage and 10 percent damage.

contains a rectangular defect of length $W = 80$ mm, width $D = 80$ mm and height of $\Delta t = 0.5$ mm. The location coordinates of the damage is $L1 = 200$ mm and $B1 = 200$ mm.

Based on measured data, the mode shapes of plate were determined and then the fundamental mode shape of the plate was utilized for damage detection. In practice, experimentally collected data includes measurement noise. Therefore, it seems necessary to perform a noise reduction process. To achieve this, the proposed method in Section 3.2 was employed. In this method, the noisy mode shape is decomposed using curvelet transform. Then the coefficients are hard-thresholded and finally inverse curvelet transform is applied to reconstruct the denoised mode shape. The results of noise for experimental fundamental mode shape of the plate are illustrated in Fig. 10 and a comparison between the noisy mode shape and the denoised mode shape is shown in Fig. 11.

Finally, the proposed method was applied to detect the damage in the plate. Fig. 12 shows the capability of the proposed method for detection of damage in the experimented plate using DE damage index. The obtained results indicated that the proposed method can be characterized as a robust and viable method for damage detection of actual structures.

6. Conclusion

In this study, a new vibration-based structural damage detection method was proposed for identification of structural damage in plate structures based on discrete curvelet transform using unequally spaced fast Fourier transforms. The main advantages of curvelet transform are in proper performance in detecting linear damage and its capability of denoising recorded data prior to applying the damage detection method.

To verify the efficiency and applicability of the proposed method, a comprehensive study on the damage detection was conducted through simulated damage conditions. The obtained results from the numerical and experimental studies indicated that the proposed method is a strong and viable method to the problem of damage detection in the plate structures. In addition, conducted sensitivity analysis revealed that the method is strongly sensitive to the damage.

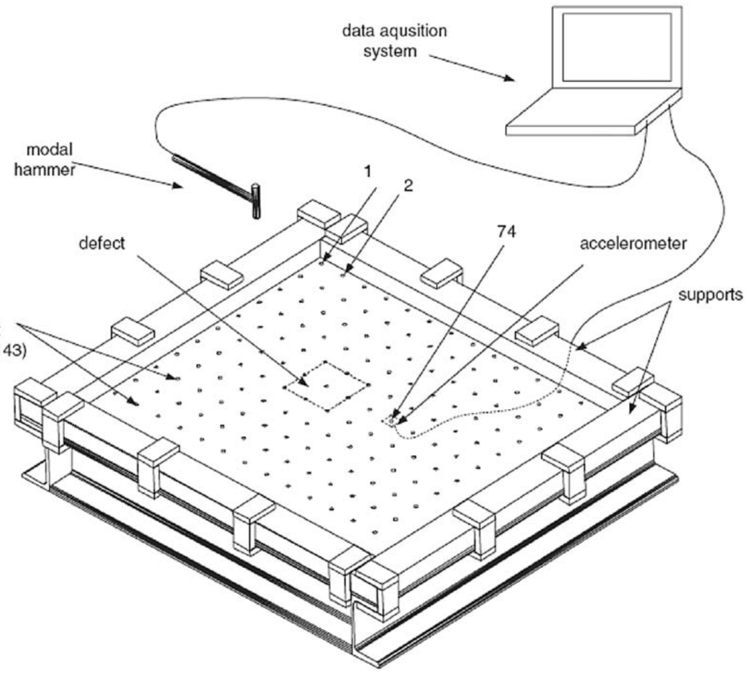

Fig. 9. Experimental set-up [12].

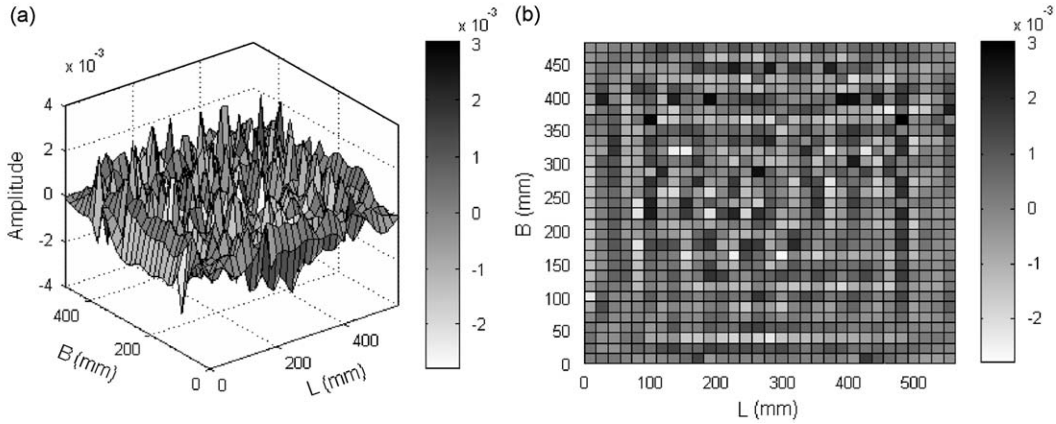

Fig. 10. Noise of experimental fundamental mode shape of the plate: (a) 3D view and (b) top view.

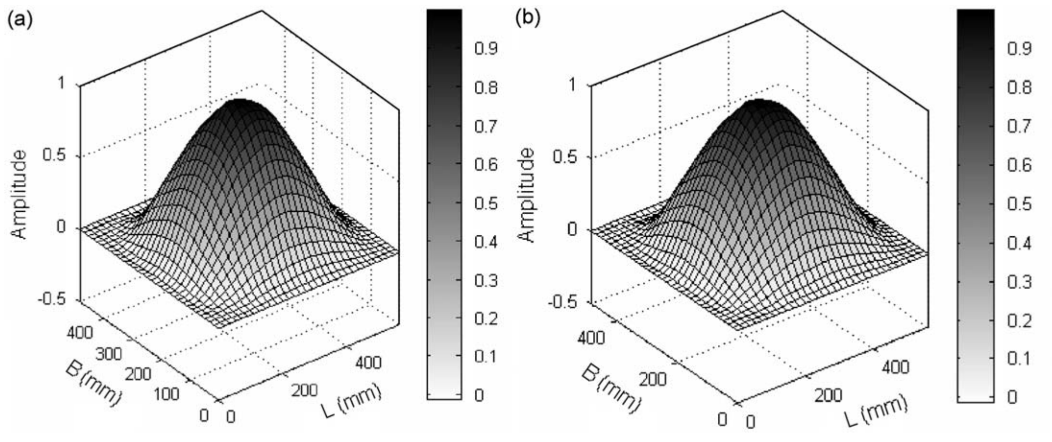

Fig. 11. Experimental fundamental mode shape for the plate: (a) recorded and (b) with denoising.

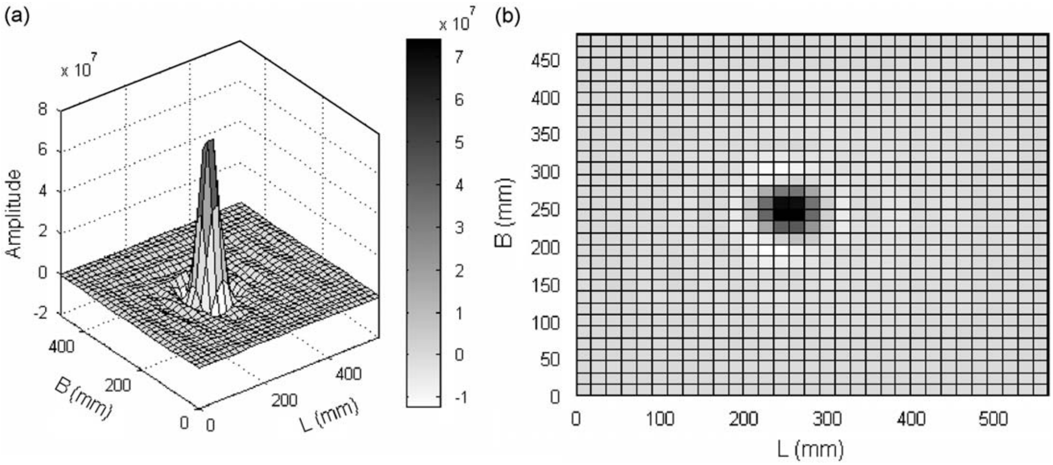

Fig. 12. Damage identification in the experimented plate using DE: (a) 3D view and (b) top view.

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