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Discussion

Reply to comments on "A critical evaluation of closure methods by two simple dynamic systems"

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Certain mathematical proofs and numerical results were used in [2,3] to show that the performance of closure methods is determined by the structure of the moment equations rather than the closure techniques. The discussion in [1], including a mathematical proof of the lack of uniqueness of the solution of moment equations and numerical results, provides a very useful addition to developments in [2,3].

We would like to note that the statement "we present a mathematical explanation of the numerical results in [2]" in [1] seems to overlook some mathematical arguments in the paper and implies that the conclusions in [2] are solely based on numerical results. It was shown in [2] that the solution X of the Itô stochastic differential equation

$$\mathrm{d}X(t) = (\alpha X(t) + \beta X(t)^3)\,\mathrm{d}t + \sigma\,\mathrm{d}B(t), \quad t \geq 0, \tag{1}$$

with $\alpha, \sigma \in \mathbb{R}$ and $\beta < 0$ becomes stationary as $t \to \infty$, its stationary odd moments are 0, and its stationary even moments $\mu_{2k} = E[X(t)^{2k}]$ satisfy the recurrence formula

$$\mu_{2(k+1)} = a\mu_{2k} + (2k-1)b\mu_{2(k-1)}, \quad k = 1, 2, \ldots, \tag{2}$$

where $a = -\alpha/\beta$ and $b = -\sigma^2/(2\beta)$. The solutions $\{\mu_{2k},\ k = 1, 2, \ldots\}$ of the recurrence formula in Eq. (2) with $\alpha > 0$ are positive for $\mu_0 > 0$, so that they are acceptable moments of X. Hence, Eq. (2) delivers an uncountable number of valid moments $\{\mu_{2k},\ k = 1, 2, \ldots\}$ for X, one for each value of $\mu_0 > 0$. This simple mathematical argument was employed in [2] to show that the moment equations do not have a unique solution. Numerical results and plots were used to illustrate this finding. We also mention that additional proofs on the performance of closure methods are given in [3].

References

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