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Estimation of tensile force in tie-rods using a frequency-based identification method

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ABSTRACT

A technique is developed to identify in-situ the tensile force in tie-rods which are used in ancient monumental masonry buildings to eliminate the lateral load exercised by the vaults and arcs. The technique is based on a frequency-based identification method that allows to minimize the measurement error and that is of simple execution. In particular, the first natural frequencies of the tie-rods are experimentally identified by measuring the frequency response functions (FRFs) with instrumented hammer excitation; four to six natural frequencies can be easily identified with a simple test. Then, a numerical model, based on the Rayleigh–Ritz method, is developed for the axially loaded tie-rod by using the Timoshenko beam theory retaining shear deformation and rotary inertia. Non-uniform section of the rod is considered since this is often the case for hand-made tie-rods in old buildings. The part of the tie-rod inserted inside the masonry wall is also modeled and a simple support is assumed at the extremities inside the walls. The constraints given to the part of the tie-rod inserted inside the masonry structure are assumed to be elastic foundations. The tensile force and the stiffness of the foundation are the unknowns. In some cases, the length of the rod inside the masonry wall can be also assumed as unknown. The numerical model is used to calculate the natural frequencies for a given set of unknowns. Then, a weighted difference between the calculated and identified natural frequencies is calculated and this difference is minimized in order to identify the unknowns, and in particular the tensile force. An estimation of the error in the identification of the force is given. The technique has been tested on five tie-rods at the ground floor of the famous castle of Fontanellato, Italy.

1. Introduction

Tie-rods were often used in ancient monumental masonry buildings to eliminate the lateral load exercised by the vaults and arcs. They give a fundamental contribution to the structural equilibrium. As a consequence of foundation settlements, the tensile force on tie-rods can surpass the yield strength of the material, which is not particularly high since old-time metallurgy was not able to obtain high-strength rods. Also corrosion can play a role in decreasing the strength of ancient tie-rods. For these reasons, it is important to identify the tensile force in tie-rods of masonry building, especially in case of evident deformation of arcs and vaults, in order to substitute them in dangerous cases.

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Unfortunately there is no non-destructive technique for a direct in-situ measurement of the force on the rod. Several techniques have been proposed for an indirect measurement of the force. Blasi and Sorace [1,2] and Sorace [3] introduced a technique based on a combination of static and dynamic identification. In particular, they modeled the tie-rod as a simply supported Euler beam with two identical rotational springs of unknown stiffness at the edges. The two unknowns, i.e. the tensile force and the stiffness of the two identical rotational springs, were identified by two equations: (i) a static equation giving the central deflection of the rod under a given load, and (ii) a dynamic equation giving the fundamental natural frequency of the rod. The method was tested in laboratory giving good results. Anyway, it requires two different in-situ experiments (measurement of the central static deflection under static load and of the fundamental natural frequency) and can give results with significant error in case of measurement error since the two unknowns are determined by only two data points.

Briccoli Bati and Tonietti [4] introduced a single static test to identify the force. It requires the measurement (i) of three vertical displacements under a concentrated static load, and (ii) of the strains variations at three sections of the rod. Also in this case good agreement with laboratory experiments has been found.

Lagomarsino and Calderini [5] developed an algorithm to identify the axial tensile force in ancient tie-rods by using the first three natural frequencies. The tie-rod was modeled as an Euler beam of uniform cross-section, neglecting the shear deformation and rotary inertia, and was assumed to be simply supported at the ends with additional rotational springs.

The identification of tension of cables based on the fundamental frequency has been studied by Ren et al. [6]. Fully dynamical identification of cable tension force has been recently proposed by Kim and Park [7]. It allows to identify the tension force, flexural rigidity and axial rigidity of the cable from measured natural frequencies. Anyway this technique is not immediately applicable to tie-rods since they cannot be modeled as cables and present uncertain constraints due to the portion of the rod inserted into the masonry wall or column.

Livingston et al. [8] identified the tensile force in prismatic beams of uniform section by using modal data and assuming rotational and vertical springs at each end of the beam. Shear deformation and rotary inertia were neglected (according to the Euler beam model).

In the present study, a technique is developed to identify in-situ the tensile force in tie-rods in ancient monumental masonry buildings. The technique is based on a frequency-based identification method that allows to minimize the measurement error and that is of simple execution. In particular, the first natural frequencies of the tie-rods are experimentally identified by measuring the frequency response functions (FRFs) with instrumented hammer excitation; four to six natural frequencies can be easily identified with a simple test. Then, a numerical model, based on the Rayleigh–Ritz method [9], is developed for the axially loaded tie-rod by using the Timoshenko beam theory retaining shear deformation and rotary inertia. Non-uniform section of the rod is considered since this is often the case for hand-made tie-rods in old buildings. The part of the tie-rod inserted inside the masonry wall is also modeled and a simple support is assumed at the extremities inside the walls. The constraints given to the part of the tie-rod inserted inside the masonry structure are assumed to be elastic foundations. The tensile force and the stiffness of the foundation are the unknown parameters. In some cases, the length of the rod inside the masonry wall can be also assumed as unknown. The numerical model is used to calculate the natural frequencies for a given set of unknowns. Then, a weighted difference between the calculated and identified natural frequencies is calculated and this difference is minimized in order to identify the unknowns, and in particular the tensile force. An estimation of the error in the identification of the force is given. The technique has been tested on the five tie-rods at the ground floor of the north wing of the ancient castle of Fontanellato, Italy, famous for the frescos painted by the master Francesco Mazzolla, called *Il Parmigianino*, in 1524.

2. Analytical model

The tie-rod is modeled as a simply supported Timoshenko beam; the supports are assumed at the beam edges inside the masonry wall and the portion of the beam inside the wall is subjected to an elastic Winkler foundation simulating the interaction between the beam and the wall, as shown in Fig. 1(a). An isotropic beam of length L and non-uniform prismatic section A is considered. A Cartesian coordinate system (x, y, z) is assumed on the beam where the x-axis is coincident with the centroidal axis and the y and z axes are coincident with the principal axes of the root cross-section. It is assumed that the centroidal axis is coincident with the elastic axis so that bending-torsion coupling is negligible [10]. The analysis is here limited to the x–y plane, and the kinematic displacements u, v, along the x and y axes, respectively, are given by

$$u(x,y,t) = -y\theta(x,t), \quad v(x,y,t) = v(x,t), \tag{1a,b}$$

where v is the transverse displacement and of the centroidal axis and θ is rotation of the cross-section about the positive z-axis, as shown in Fig. 1(b). By using Eq. (1), the nonzero strain components of the beam are

$$\varepsilon_{xx} = -y\frac{\partial\theta}{\partial x}, \quad \gamma_{xy} = \frac{\partial v}{\partial x} - \theta. \tag{2a,b}$$

The flexural displacement of the simply supported beam is expanded by using the following series of admissible functions

$$v(x) = \sum_{i=1}^{N} a_i \sin\left(\frac{i\pi x}{L}\right). \tag{3}$$

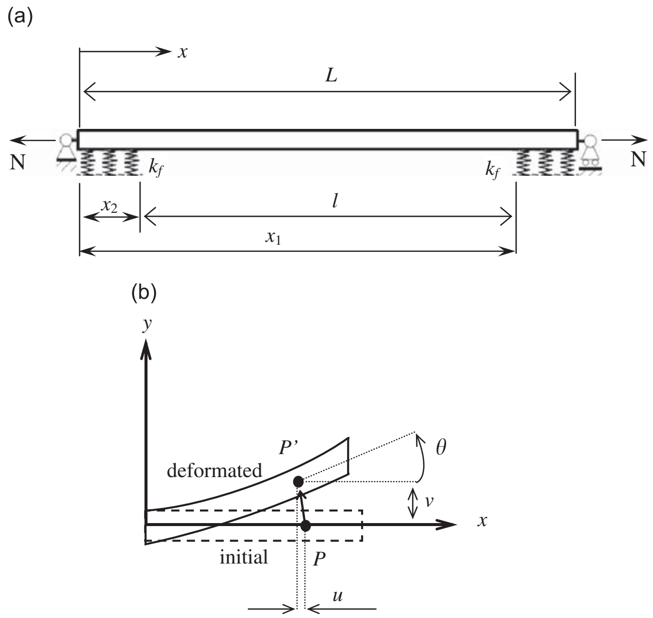

Fig. 1. (a) Model of the tie-rod; (b) variables describing the rod deformation.

The rotation of the cross-section is expanded as

$$\theta(x)=\sum_{i=1}^{N} b_i i\cos\left(\frac{i\pi x}{L}\right). \tag{4}$$

Eq. (3) respects the geometric boundary condition v=0 at x=0, L, while Eq. (4) respects the zero bending moment condition $M=EJ(\partial\theta/\partial x)=0$ at x=0, L, where EJ is the flexural rigidity of the rod. In Eqs. (3) and (4) the same number of terms N has been used for the expansions of v and θ; in general, this number of terms can be different. The potential strain energy of the beam is given by [10]

$$V_B=\frac{1}{2}\int_0^L\int_A \boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\varepsilon}\,\mathrm{d}A\,\mathrm{d}x, \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the strain vector and $\boldsymbol{\sigma}$ is the stress vector. By using Hooke's stress–strains relationships, Eq. (5) can be rewritten as [10,11]

$$V_B=\frac{1}{2}\int_0^L\left[EJ\left(\frac{\partial\theta}{\partial x}\right)^2+kGA\left(\frac{\partial v}{\partial x}-\theta\right)^2\right]\mathrm{d}x, \tag{6}$$

where kGA is the shear rigidity, with E being the Young modulus, J the second moment of inertia of the beam cross-section about the y-axis, k the shear coefficient [12] and G is the shear modulus. In particular, J, k and A are functions of x.

The potential energy associated with the axial load F (positive is for traction) is expressed by

$$V_F=\frac{1}{2}F\int_0^L\left(\frac{\partial v}{\partial x}\right)^2\mathrm{d}x=\frac{F\pi^2}{4L}\sum_{i=1}^{N} i^2 a_i^2. \tag{7}$$

The potential energy associated with the elastic foundation is given by

$$V_W=\frac{1}{2}\left(\int_0^{x_2}k_f v^2 dx+\int_{x_1}^{L}k_f v^2 dx\right)=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_i\,a_j\left[\int_0^{x_2}k_f\sin\left(\frac{i\pi x}{L}\right)\sin\left(\frac{j\pi x}{L}\right)dx\right.$$
$$\left.+\int_{x_1}^{L}k_f\sin\left(\frac{i\pi x}{L}\right)\sin\left(\frac{j\pi x}{L}\right)dx\right], \tag{8}$$

where k_f is the stiffness of the foundation, assumed uniform for simplicity, and the portion of the beam inserted in the walls is comprised between x_1 and x_2. In particular, x_1=0 for the left edge; for the right edge, x_2=L. Both energies associated to elastic foundations at the left and right ends of the beam must be included.

The global potential energy is

$$V=\frac{1}{2}\int_0^L\left[EJ(x)\left(\frac{\partial\theta}{\partial x}\right)^2+k(x)GA(x)\left(\frac{\partial v}{\partial x}-\theta\right)^2+F\left(\frac{\partial v}{\partial x}\right)^2\right]\mathrm{d}x+\frac{1}{2}k_f[\int_0^{x_2}v^2\,\mathrm{d}x+\int_{x_1}^{L}v^2\,\mathrm{d}x]. \tag{9}$$

The kinetic energy of the beam is given by

$$T_B = \frac{1}{2}\int_0^L \int_A \rho\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2\right] \mathrm{d}A\,\mathrm{d}x, \tag{10}$$

where ρ is the mass density. By using equations (1a,b) and integration over the cross-section A, Eq. (10) can be rewritten as [10]

$$T_B = \frac{1}{2}\int_0^L \left[\rho J\left(\frac{\partial \theta}{\partial t}\right)^2 + \rho A\left(\frac{\partial v}{\partial t}\right)^2\right] \mathrm{d}x. \tag{11}$$

The reference kinetic energy of the beam, i.e. the maximum kinetic energy divided by ω^2, is given by

$$T_B^* = \frac{1}{2}\int_0^L [\rho J(x)\theta^2 + \rho A(x) v^2]\,\mathrm{d}x. \tag{12}$$

By introducing the following vectorial notation

$$\mathbf{q}^{\mathrm{T}} = (a_1, \ldots, a_N, b_1, \ldots, b_N)^{\mathrm{T}}, \tag{13a}$$

$$V = \mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{q}, \tag{13b}$$

$$T_B^* = \mathbf{q}^{\mathrm{T}}\mathbf{M}\mathbf{q}, \tag{13c}$$

the natural circular frequencies ω of the tie-rod are obtained by solving the following eigenvalue problem

$$\mathbf{K} - \omega^2\mathbf{M} = 0. \tag{14}$$

The shear coefficient k in Eqs. (6, 9) for rectangular cross-section of Timoshenko beams is given by [12]

$$k = -\frac{2(1+v)}{\left[\frac{9}{4a^5 b}C_4 + v\left(1 - \frac{b^2}{a^2}\right)\right]}, \tag{15a}$$

where

$$C_4 = \frac{4}{45}a^3 b(-12a^2 - 15va^2 + 5vb^2) + \sum_{n=1}^{\infty} \frac{16v^2 b^5\left[n\pi a - b\tanh\left(\frac{n\pi a}{b}\right)\right]}{(n\pi^5)(1+v)}, \tag{15b}$$

v is the Poisson's coefficient, $2a$ is the depth of the beam in y-direction and $2b$ is the width of the beam in z-direction; a and b are functions of x.

3. Identification method

In order to identify the axial force F and the stiffness k_f of the elastic foundation, the weighted difference between the calculated and identified natural frequencies is introduced

$$err_{RMS}(F, k_f) = \sqrt{\frac{1}{M}\sum_{i=1}^{M} \delta_i^2}, \tag{16}$$

where M is the number of the natural modes included in the identification process (2–6) and δ_i is the difference between the i-th computed and experimentally measured natural frequency. The function given in Eq. (16) is minimized over the full set of the useful values of the two unknowns (F, k_f), defined in the pre-processing, giving finally the identification of the axial force F. In particular, the eigenvalue problem is computed on a bidimensional grid of points in the (F, k_f) plane, in order to identify the area where the global minimum is located; then, the grid is locally refined in order to identify the minimum. This minimization process requires the calculations of a large set of eigenvalue problems, Eq. (14). This time-consuming minimization process can be replaced by the Nelder–Mead method (also known as the downhill simplex method) in one of its improved versions in order to find the global minimum, or by the gradient descent method in order to find each one of the local minima. In order to speed-up the minimization process, the use of a self-developed code for the calculation of the natural frequencies of the beam based on a reduced size model is important for time saving with respect to a commercial FEM code, which requires a larger number of degrees of freedom. In the present case, the self-developed program has been validated by comparison to the ABAQUS commercial program, and the results obtained are practically coincident. In particular, the first two mode shapes of one of the tie-rods studied in Section 4 are shown in Fig. 2; they have been obtained by using the ABAQUS computer program and are coincident to those computed by the self-developed Rayleigh–Ritz program.

Since the two parameters F and k_f change the frequencies but not the order of modes, the correspondence between the measured and the computed mode shapes is kept during the process of minimization of the error function (16). In practice, since the rod has flexural mode shapes that increase in frequency with the number of half-waves along its length, there is

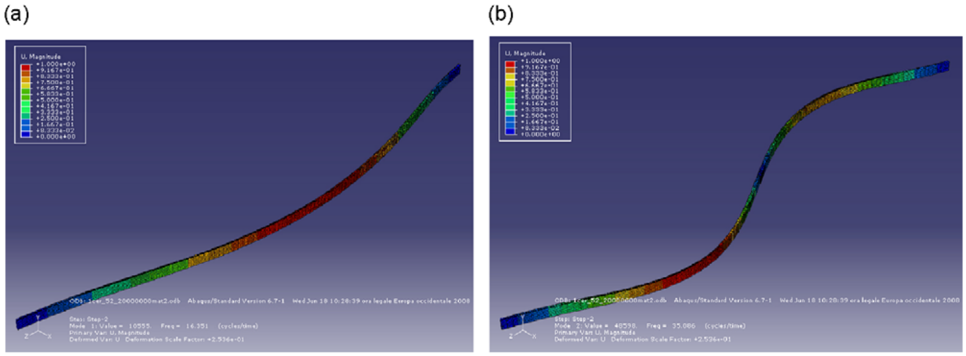

Fig. 2. First two mode shapes of the tie-rod "1" computed by using the commercial FEM program ABAQUS. (a) First mode; (b) second mode.

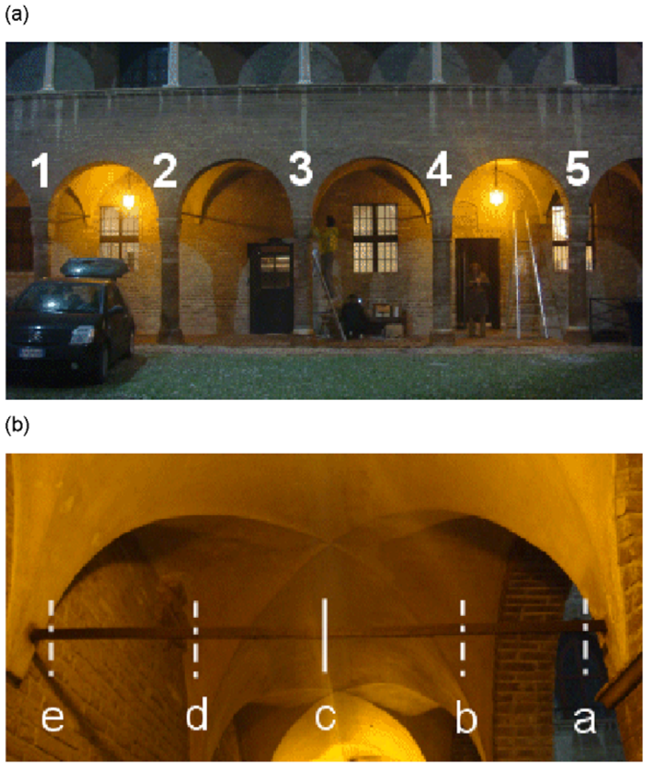

Fig. 3. Ground floor of the north wing of the Fontanellato castle viewed from the internal courtyard. (a) Photo of the arcs with the five tested tie-rods; (b) particular of a tie-rod with the measured sections.

correspondence of modes between the model and the experiment if no mode is missed during the experiments. The experimental results show that the peaks in the frequency response functions are clear enough to avoid this problem, at least in the present case.

The minimization process of function (16) can be implemented versus three unknowns F, k_f and the length of the elastic foundation, in case that the length of the tie-rod inserted into the masonry wall is unknown. In particular cases, a different stiffness k_f can be assumed at the right and left ends of the rod.

4. Experiments and identification at the castle of Fontanellato

Experimental measurement of the natural frequencies of the five tie-rods at the ground floor of the north wing of the ancient castle of Fontanellato, Italy (see Figs. 3 and 4), was performed in the afternoon of 11 December 2007. The used instrumentation is composed by an accelerometer *B&K* 4370, an instrumented hammer *B&K* 8202, two charge amplifiers

Fig. 4. Experimental measurement on tie-rod "1".

B&K 2635 and a *Pimento* 8-channel system by *LMS* for data acquisition and experimental modal analysis. The accelerometer has been placed about 20 cm away from the middle-length of the rod to measure the beam response in the horizontal plane (see Fig. 4) in order to detect all the low-frequency modes; the excitation has been applied with the instrumented hammer 3 cm away from the accelerometer (see Fig. 4). Since the studied rods are much stiffer in the vertical plane, tests in vertical direction were not performed since they could give a less accurate estimation of the axial load. Two of the five measured frequency response functions (FRFs), estimated on-line on the *Pimento* analyzer by using the *Hv* algorithm and 6 averages, are shown in Figs. 5(a,b) for the tie-rods "2" and "4", respectively.

Additional tests has been made hammering on the brick column next to each tie-rod, but the FRF measured by the accelerometer placed on the rod was very low, excluding a significant participation of the brick walls to the rod response.

The measured height and width of the rectangular section of the tie-rod "5" are given in Figs. 6(a,b), where also the interpolating polynomial functions are shown. The polynomial functions are used in the calculations. Fig. 6 shows that the width and height of the rectangular section are not constant since the tie-rods studied here are hand-made and very old. The dimensions of the five tie-rods are given in Table 1, where the length of the tie-rods inside the masonry is assumed being 0.2 m for all the rods and at both ends ($x_2=L-x_1$ in the equations). All the five tie-rods have similar dimensions and section. The assumed material characteristics are: $E=193\times10^9\,\mathrm{N/m^2}$, $G=74.8\times10^9\,\mathrm{N/m^2}$, $\rho=7870\,\mathrm{kg/m^3}$, $\nu=0.29$. From $M=2$ to $M=6$ natural modes are used in the identification process, and $N=25$ terms in the expansions (3) and (4). The tested rods are made of iron, which has mechanical characteristics that vary very little, even if compared to steel; moreover, the main parameters E and ρ appear both under square root in the frequency calculation, so their small variation has a minor effect on the natural frequencies. Therefore, the assumed material characteristics can be used with confidence in the present identification process. If the tie-rod behaves linearly for small-amplitude vibrations superimposed to its initial tension and its Young's modulus is not changed by the initial stress, then the present identification process can be applied. Since tie-rods in ancient buildings release load if they reach the plasticity limit of their material, these conditions are generally verified. The use of a geometrically nonlinear beam theory is not useful since the natural modes are identified with small excitation and the vibration is within the limit of small displacements and linear theory.

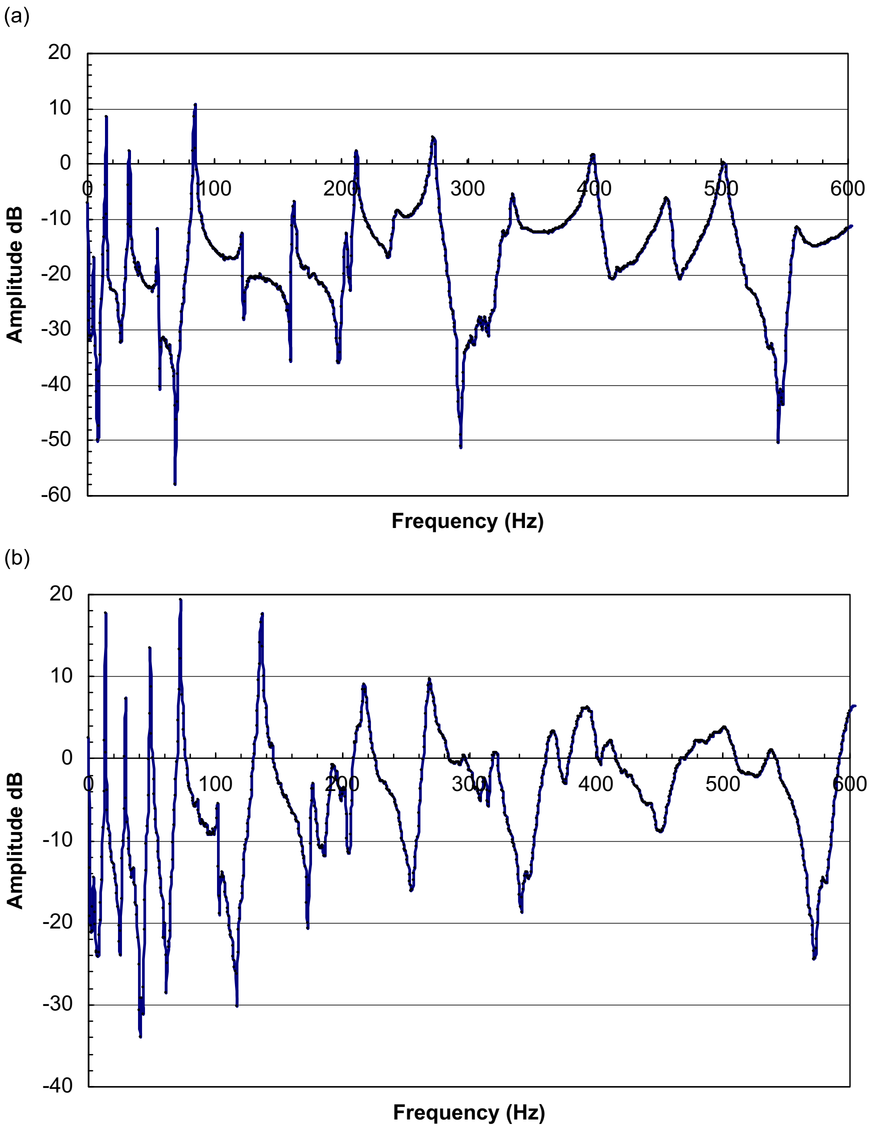

Fig. 5. Experimentally measured FRF. (a) Tie-rod “2”; (b) tie-rod “4”.

The effect of the axial force F on the first six natural frequencies of the tie-rod “3” is given in Fig. 7. This result shows that an increase in the axial force translates the curve interpolating the computed natural frequencies in the direction of higher frequency. Otherwise, the effect of the stiffness k_f of the elastic foundation is given in Fig. 8 and it changes the slope of the curve. Therefore, the two unknowns have two different types of effect on the dynamics of the tie-rod, simplifying the identification process.

The results of the identification process, i.e. the identified tensile force F and stiffness k_f of the elastic foundation, are given in Table 2. Results are presented for identification using $M=2$–6 modes and show a consistent identification of the tensile force F. In particular, the force identified with three or four modes is already quite accurate. Table 2 shows that the tie-rod “1” is the one subjected to the highest load, while the tie-rod “3” is the one subjected to the lowest load. The identified stiffness k_f is practically coincident for tie-rods “1”, “3” and “4”; for rods “2” and “5” higher stiffness has been identified, indicating possibly a longer part of the rod inside the masonry wall or a stiffer connection.

An estimation of the error in the identified force F is also introduced. The identification process minimizes the function (16); as a result, the points representing the experimental natural frequencies lie near the computed curve, as shown in Fig. 9. Since the force F gives a translation of the curve interpolating the computed natural frequencies, the force F corresponding to the translation of the curve to pass by the highest (i.e. the one more far from the curve in the high-frequency direction), and then the lowest (i.e. the one more far from the curve in the low-frequency direction) of all the experimental points gives the two extreme values of the force F. These two extreme values (keeping k_f constant) allow to calculate the maximum error on F; the positive and negative maximum errors are given in Table 3 together with the maximum axial stress (force F divided by the minimum cross-section of the rod) on each tie-rod. In the studied case, the maximum stress, 58.4 MPa, is below the admissible stress for the material of the tie-rods. The maximum error is for tie-rod "3", which is the one with the smallest load, and therefore the less dangerous.

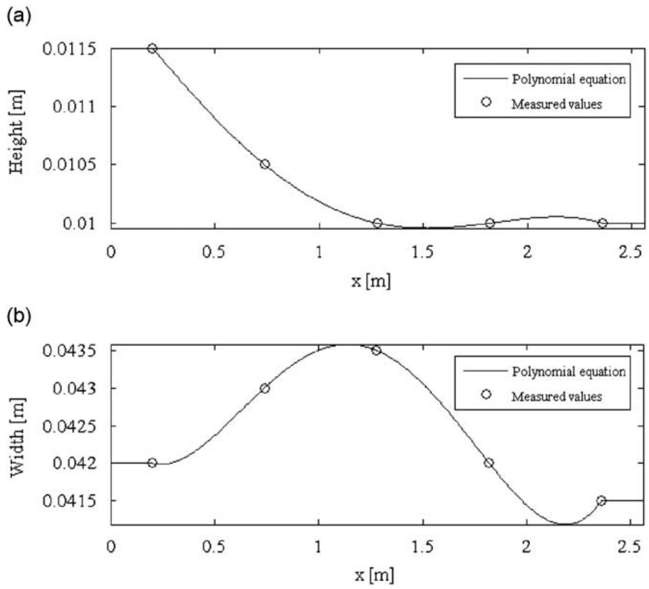

Fig. 6. Measured, ◦, and interpolated, —, dimensions of the section of the tie-rod "5". (a) Height of the rod; (b) width of the rod.

Table 1
Measured length l outside the masonry, height $2a$ and width $2b$ of the five tie-rods.

Tie-rod	Length l (m)	Position in Fig. 3(b)	$2a$ (mm)	$2b$ (mm)
1	2.135	a	9	46
		b	12	40
		c	10	38
		d	8	41
		e	10	40
2	2.130	a	13	46
		b	10	45
		c	8.5	40
		d	10	47
		e	12.5	44
3	2.105	a	9.5	41.5
		b	10	40.5
		c	8.5	42
		d	9.5	40
		e	11.5	38.5
4	2.169	a	10	42
		b	9	46
		c	9	48
		d	9.5	44
		e	11	45.5
5	2.159	a	11.5	42
		b	10.5	43
		c	10	43.5
		d	10	42
		e	10	41.5

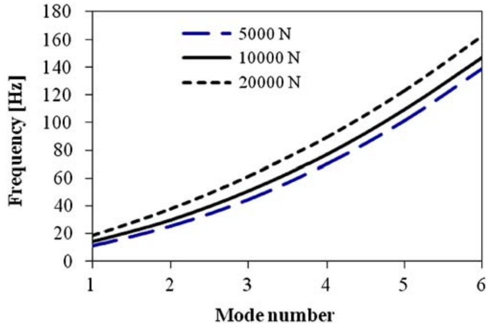

Fig. 7. Effect of the axial force F on the first six natural frequencies of the tie-rod "3".

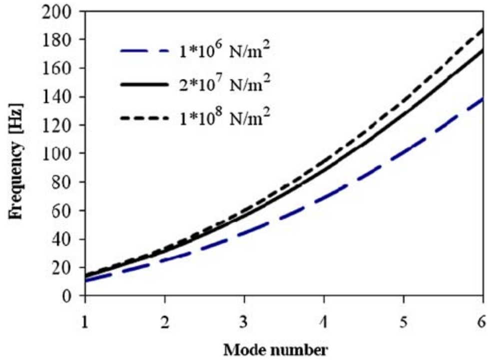

Fig. 8. Effect of the foundation stiffness k_f on the first six natural frequencies of the tie-rod "3".

Table 2
Identified tensile force F and stiffness k_f of the elastic foundation for the five tie-rods.

	2 modes	3 modes	4 modes	5 modes	6 modes
Tie-rod "1"					
Tensile force (N)	16 500	18 800	19 600	19 100	17 600
k_f (N/m^2)	2.46×10^6	1.21×10^6	0.75×10^6	0.86×10^6	1.22×10^6
Tie-rod "2"					
Tensile force (N)	5500	7100	7700	6800	7200
k_f (N/m^2)	2.5×10^7	1.1×10^7	0.94×10^7	1.15×10^7	1.06×10^7
Tie-rod "3"					
Tensile force (N)	2700	4500	4800	4400	5500
k_f (N/m^2)	4.72×10^6	2.02×10^6	1.80×10^6	2.07×10^6	1.26×10^6
Tie-rod "4"					
Tensile force (N)	10 900	11 500	12 100	11 100	10 100
k_f (N/m^2)	1.56×10^6	1.25×10^6	0.9×10^6	1.16×10^6	1.42×10^6
Tie-rod "5"					
Tensile force (N)	10 300	12 100	11 200	9500	10 000
k_f (N/m^2)	9.53×10^6	5.23×10^6	6.53×10^6	9.42×10^6	8.6×10^6

5. Conclusions

The fully dynamic identification method allows to determine with a simple experiment the axial force on tie-rods. Since the experimental apparatus is compact and no fixed and accurate reference is necessary, the technique is particularly suitable for in-situ measurement on monumental buildings. The technique presented here has the advantage of using redundant data with respect to the unknowns in the identification process, minimizing the measurement

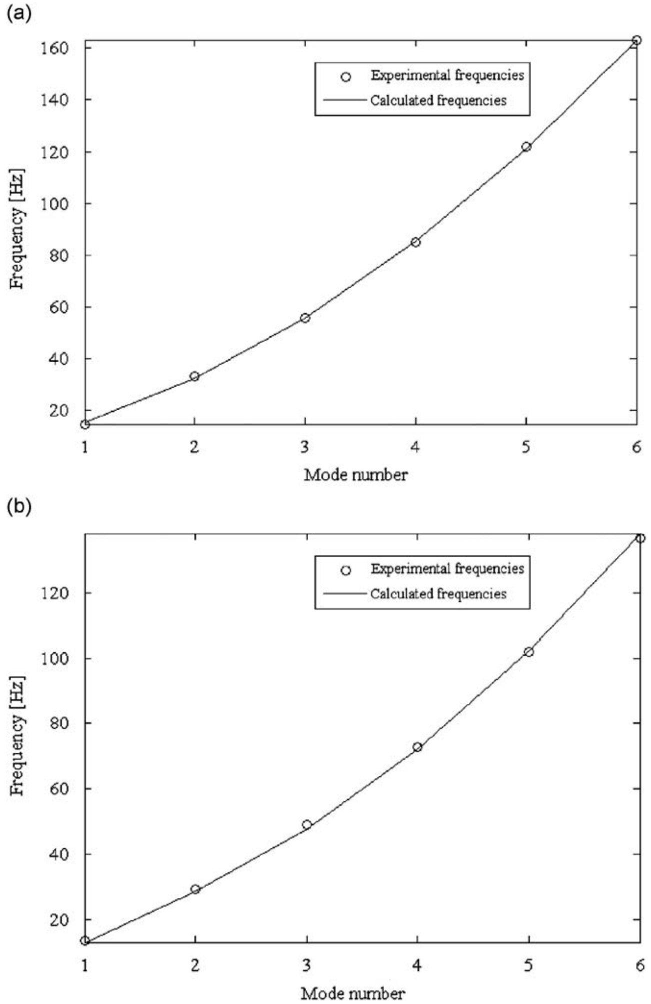

Fig. 9. Comparison of computed and experimental natural frequencies of tie-rods after identification of the parameters using M=6 modes. Measured, ◦, and computed, —, natural frequencies. (a) Tie-rod "2"; (b) tie-rod "4".

Table 3
Estimation of maximum error in the identified tensile force F and maximum of the axial stress σ_{xx} for the five tie-rods.

	Tie-rod "1"	Tie-rod "2"	Tie-rod "3"	Tie-rod "4"	Tie-rod "5"
Max σ_{xx} (MPa)	58.4	21.2	17.0	26.9	24.4
Max error on F (%)	+5.8	+17.6	+30.4	+10.7	+6.6
	−7.6	−13.0	−17.6	−10.5	−8.9

and modeling errors. Moreover, it allows to estimate the accuracy in the identification of the axial force acting on the tie-rod.

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