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Nonlinear modes and traveling waves of parametrically excited cylindrical shells

R. Kochurov [b], K.V. Avramov [a,b,*]

[a] Department of Nonstationary Vibrations, A.N. Podgorny Institute for Mechanical Engineering Problems, National Academy of Sciences of Ukraine, 2/10 Dm. Pozharskoho St., 61046 Kharkiv, Ukraine
[b] Department of Gas and Fluid Mechanics, National Technical University "KhPI", Frunze St. 21, Kharkiv 61002, Ukraine

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ABSTRACT

Donnel's equations are used to predict nonlinear vibrations of cylindrical shells, which are excited by parametric dynamical load. A multi-degree-of-freedom dynamical system of cylindrical shells is derived. The nonlinear modes of the parametrically excited system are treated. The analyses have been carried out both with and without dissipation, using the Harmonic Balance Method. These nonlinear modes correspond to the standing waves in the shell. Traveling waves are also analyzed in detail. We come to the conclusion that the behavior of the nonlinear modes and the traveling waves are similar.

1. Introduction

Thin-walled structures are widely used in aerospace, nuclear, civil and mechanical engineering. Longitudinal periodic loads usually act on the shells and leads to complex dynamical behavior of the systems. Many efforts were made to study this behavior. Yao [1] reduced parametric oscillations of cylindrical shells to the well-known Mathieu equation and studied the stability boundaries. Bolotin [2] developed a general approach to analyze parametric oscillations of shells. Vijyaraghavan and Evan-Iwanowski [3] studied theoretically and experimentally the dynamic instability of clamped-free seismically excited cylindrical shells. Parametric oscillations of simply supported cylindrical shells were modeled by two interacting modes (asymmetric and axisymmetric ones) in [4]. Hsu [5] considered oscillations of a seismically excited clamped-free cylindrical shell. Koval [6] took into account longitudinal, bending and torsional oscillations to study shell parametric vibrations in the regions of the main parametric and combination resonances. Donnell's shallow shell equations were used to study parametric oscillations of cylindrical shells [7] and the fundamental role of axisymmetric modes in evaluating the parametric instability bounds is treated. The effect of initial imperfections on the parametric oscillations of simply supported cylindrical shells was studied by Koval'chuk and Krasnopol'skaya [8]. Linear oscillations of clamped-free cylindrical shells under the action of the horizontal seismic excitation were analyzed in [9]. Bondarenko and Telalov [10] studied experimentally the dynamic instability domains and nonlinear vibrations. They obtained the hard frequency response in the region of the main parametric resonance for circumferential wavenumber $n=2$ and softening for $n>2$. Kubenko et al. [11] obtained theoretically and experimentally the frequency response and the region of the main parametric resonance of simply supported cylindrical shells. Parametric vibrations of a rotating cylindrical shell were investigated by Ng et al. [12]; the effect of Coriolis forces on the parametric instability domain was analyzed. Ng et al. [13]

* Corresponding author at: Department of Gas and Fluid Mechanics, National Technical University "KhPI", Frunze St. 21, Kharkiv 61002, Ukraine.
E-mail address: kavramov@ipmach.kharkov.ua (K.V. Avramov).

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studied parametric oscillations of the functionally graded simply supported cylindrical shells. Pellicano et al. [14] and Pellicano and Amabili [15] analyzed nonlinear oscillations and dynamic instability of simply supported cylindrical shells under the action of longitudinal dynamic forces. The dynamics of circular cylindrical shells carrying a rigid disk on the top and clamped at the base was investigated by Pellicano and Avramov [16]. Experimental data on the parametric instability of cylindrical shells are reported in [17], where a saturation phenomenon is treated. In this case the energy transfer from low to high frequencies modes is observed. The dynamic stability of cylindrical shells under the action of both static and periodic axial loads is treated [18]. The correlation of parametric instability with the shell collapse was investigated by using Sanders–Koiter nonlinear shell theory in [19]. Kamat et al. [20] used a finite element approach to study the dynamic stability of shells with complex shapes. This work treats linear model and Bolotin method is applied to determine the stability boundaries. A low dimension model for studying of nonlinear dynamics and stability of compressed shells was proposed in [21]. In [22], interesting studies on shell stability are presented. Shell was modeled using the nonlinear Donnell's shallow shell theory and a reduced order system was obtained from the PDE using a proper displacement expansion and Galerkin methods. Sanders–Koiter theory is used to develop a nonlinear analytical model for moderately vibrations of shell. Analysis of nonlinear modes of cylindrical shells, which are described by three mode model, is considered in the paper [23]. The cylindrical shell with a rigid disk on a top under the action of harmonic base excitation is considered in the paper [24]. It is shown that the increase of the excitation amplitude results in chaotic motions of the top mass. In a paper by Goncalves et al [25] basins of attraction are used to measure the reliability and safety of the cylindrical shell structure. Detailed reviews of cylindrical shell dynamics are presented in [26].

Nonlinear dynamics of cylindrical shells in the case of the main parametric resonance is treated in the present paper. Cylindrical shells have dense frequency spectrum. Therefore, the case, when the three eigenfrequencies of conjugate modes are close, is considered. This case occurs frequently in shell dynamics. These three conjugate modes are taken into account in the analysis of the main parametric resonance. Two kinds of motions (nonlinear modes and traveling waves) are treated in this paper. It is shown that the nonlinear modes correspond to standing waves in cylindrical shells. We come to the conclusion that the behavior of the nonlinear modes and traveling waves are qualitatively similar.

2. Problem formulation and main equations

The simply supported cylindrical shell without initial geometric imperfections is considered. The following periodic distributed parametric load acts on the shell (Fig. 1):

$$N_x(t) = N_0 + N_1 \cos 2\nu t,$$
$$N_0, N_1 = \text{const} > 0, \tag{1}$$

where ν is an excitation frequency. The amplitude of vibration is assumed to be moderate. Then the strains are small and displacements are moderate and the strains–displacement relations are nonlinear. The strains and stresses satisfy the Hooke's law. The following Donnel equations describe the shell vibrations adequately [4,15]:

$$\frac{D}{h}\nabla^4 w + \rho\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 F}{R\,\partial x^2} + \left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial^2 F}{\partial x\,\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}\right);$$

$$\frac{1}{E}\nabla^4 F = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right], \tag{2}$$

where w is displacement of the middle surface points in the radial directions; x is a longitudinal coordinate; y is circumferential coordinate; R is mean shell radius; ρ is material density; E, μ are Young's modulus and Poisson's ratio; F is an Airy stress function; $D = Eh^3/12(1-\mu^2)$ is the flexural rigidity.

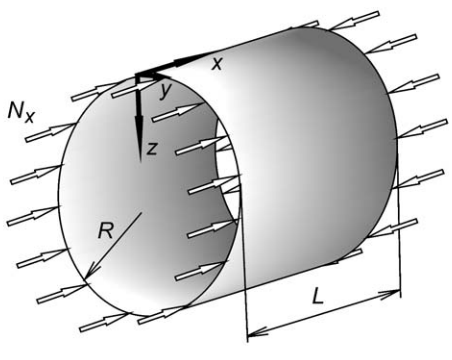

Fig. 1. Cylindrical shell.

The conjugation vibrations modes cos(*sy*)sin(*rx*) and sin(*sy*)sin(*rx*) have the same frequencies of cylindrical shells vibrations. If a shell performs nonlinear vibrations, these modes can be excited jointly. It is well-known [27] that, wide class of cylindrical shells has three close eigenfrequencies of conjugate modes. The main parametric resonance is considered $\nu \approx \omega_i$; $i = 1, 2, \ldots, 6$, where $\omega_{2i-1} = \omega_{2i}$; $i = \overline{1,3}$ are equal eigenfrequencies of conjugate modes. Three conjugate modes are taken into account in the expansion of the displacements in the radial directions. Then the flexural displacement w can be presented as

$$w = \sum_{i=1}^{3} (f_{2i-1} \cos s_i y + f_{2i} \sin s_i y) \sin rx + f_7 \sin^2 rx + f_8, \tag{3}$$

where $s_i = n_i/R$; $r = m\pi/L$; $i = 1, 2, 3$; n_i is numbers of waves in circumference directions; m is a number of half-waves in x direction. The summand $f_7 \sin^2 rx$ describes asymmetry of displacements with respect to a middle surface. The term f_8 describes displacements in radial directions of shell face sections points. This term does not depend on circumferential coordinate y. Therefore, the face sections can "breathe" [11].

The Airy stress function F is determined from the second equation of system (2). This function can be presented in the following form: $F = F_h + F_p$. Satisfying the periodicity conditions of the circumference displacements, the general solution of the second equation of system (2) F_h is determined as [11]

$$F_h = \frac{E}{16} \sum_{i=1}^{6} s_i f_i^2 x^2 - \frac{1}{4} \frac{E}{R} f_7 x^2 - \frac{1}{2} \mu N_x x^2 - \frac{1}{2} N_x y^2. \tag{4}$$

The particular solution of the second equation of system (2) F_p can be presented as

$$F_p = F_1^{(0)} \cos 2rx + \sum_{i=1}^{3} F_{i+2}^{(0)} \cos 2s_i y + \sum_{i=1}^{3} F_{i+5}^{(0)} \sin 2s_i y + F_1^* \sin rx + F_2^* \sin 3rx + F_3^* + F_4^* \cos 2rx, \tag{5a}$$

$$F_k^* = \sum_{i=1}^{3} (F_i^{(k)} \cos s_i y + F_{i+3}^{(k)} \sin s_i y), \tag{5b}$$

$$\begin{aligned} F_l^* = {} & F_1^{(l)} \cos(s_1+s_2)y + F_2^{(l)} \cos(s_1-s_2)y + F_3^{(l)} \cos(s_1+s_3)y + F_4^{(l)} \cos(s_1-s_3)y + F_5^{(l)} \cos(s_2+s_3)y + F_6^{(l)} \cos(s_2-s_3)y \\ & + F_7^{(l)} \sin(s_1+s_2)y + F_8^{(l)} \sin(s_1-s_2)y + F_9^{(l)} \sin(s_1+s_3)y + F_{10}^{(l)} \sin(s_1-s_3)y + F_{11}^{(l)} \sin(s_2+s_3)y + F_{12}^{(l)} \sin(s_2-s_3)y; \\ & k = 1, 2; \quad l = 3, 4. \end{aligned} \tag{5c}$$

Eqs. (5a)–(5c) are substituted into the second equation of system (2) and the amplitudes of the same harmonics are equated. As a result, the system of linear algebraic equations with respect to F_i is derived. The solution of this system F_i is not presented for brevity.

Solutions (4) and (5) are substituted into the first equation of system (2) and the Galerkin method is applied to the resulting equation. The system of nonlinear ordinary differential equations of the cylindrical shell vibrations is thus derived. This system is rewritten with respect to the dimensionless variables

$$t^* = \omega_0 t, \quad f_i^*(t) = h^{-1} f_i(t),$$

where ω_0 is the lowest eigenfrequency of shell vibrations, which is calculated according to the following formula:

$$\omega_0 = \frac{1}{\rho} \sqrt{\frac{D}{h} \left[\frac{\pi^2}{L^2} + \frac{25}{R^2} \right]^2 + \frac{E\pi^4}{R^2 L^4} \left[\frac{\pi^2}{L^2} + \frac{25}{R^2} \right]^{-2}}.$$

Dropping out asterisks in the notations, the finite-degree-of-freedom shell model with respect to the dimensionless variables and parameters has the following form:

$$\ddot{f}_i + \omega_i^2 f_i + f_i R_i(f_1, \ldots, f_7) + G_i(f_1, \ldots, f_6) + \chi_i N_x f_i = 0, \quad i = 1, 2, \ldots, 6, \tag{6}$$

$$\ddot{f}_7 + \frac{4}{3} \ddot{f}_8 + \omega_7^2 f_7 + \tilde{\omega}_8^2 f_8 + \sum_{j=1}^{6} \gamma_{7j} f_j^2 = 0, \tag{7}$$

$$\ddot{f}_8 + \frac{1}{2} \ddot{f}_7 + \omega_8^2 f_8 + \tilde{\omega}_7^2 f_7 + \sum_{j=1}^{6} \gamma_{8j} f_j^2 = 0, \tag{8}$$

where

$$\omega_7^2 = \frac{16Dr^4}{3\rho h} + \frac{E}{\rho R^2}; \quad \tilde{\omega}_7^2 = \frac{E}{2\rho R^2},$$

$$\omega_8^2 = \frac{E}{\rho R^2}; \quad \tilde{\omega}_8^2 = \frac{4E}{3\rho R^2}.$$

$$R_i(f_1,\ldots,f_7)=\sum_{j=1}^{6}\gamma_{ij}f_j^2+\lambda_i f_7+\zeta_i f_7^2,\quad i=1,2,\ldots,6,$$

$$G_{1,2}(f_1,\ldots,f_6)=\eta_{17}(f_{5,6}(f_{3,4}^2-f_{4,3}^2)+2f_{3,4}f_{4,3}f_{6,5}),$$

$$G_{3,4}(f_1,\ldots,f_6)=\eta_{37}(f_{1,2}f_{3,4}f_{5,6}+f_{1,2}f_{4,3}f_{6,5}+f_{2,1}f_{4,3}f_{5,6}-f_{2,1}f_{3,4}f_{6,5}),$$

$$G_{5,6}(f_1,\ldots,f_6)=\eta_{57}\left(f_{1,2}(f_{3,4}^2-f_{4,3}^2)+2f_{3,4}f_{4,3}f_{2,1}\right).$$

The values γ_{ij}; ς_i; η_{ij} depend on the shell parameters and they are not presented here for brevity. The first and the second subscripts, which are detached by a comma, were taken in the last equations. The frequencies ω_7, ω_8 are significantly greater then $\omega_1,\ldots,\omega_6$. Therefore, it is assumed $\ddot{f}_7=0, \ddot{f}_8=0$. Then the following equations are derived from (7) and (8):

$$f_7=(\tilde{\omega}_8\sum_{j=1}^{6}\gamma_{8j}f_j^2-\omega_8\sum_{j=1}^{6}\gamma_{7j}f_j^2)\eta^{-1},$$

$$f_8=(\omega_7\sum_{j=1}^{6}\gamma_{7j}f_j^2-\tilde{\omega}_7\sum_{j=1}^{6}\gamma_{8j}f_j^2)\eta^{-1},$$

$$\eta=\tilde{\omega}_7\tilde{\omega}_8-\omega_7\omega_8. \quad (9)$$

Eqs. (9) are substituted into (6). As a result, the functions R_i, $i=1,2,\ldots,6$ of system (6) are derived in the following form:

$$R_i(f_1,\ldots,f_6)=\sum_{j=1}^{6}\eta_{ij}f_j^2,\quad i=1,2,\ldots,6. \quad (10)$$

Thus, the parametric vibrations of the shells are described by Eq. (6) with the functions R_i in the form (10). In future analysis, the parametric load is taken as $N_x(t)=N_1\cos 2\nu t$.

3. Nonlinear modes and harmonic balance analysis

The nonlinear dynamics of the system described by Eq. (6) is analyzed in this section. The equations

$$f_{2i-1}=\pm f_{2i},\quad i=1,2,3, \quad (11)$$

are exact solutions of system (6). If solutions (11) are substituted into (6), the following dynamical system is derived:

$$\ddot{f}_i+\omega_i^2 f_i+f_i\tilde{R}_i(f_1,f_3,f_5)+\tilde{G}_i(f_1,f_3,f_5)+\chi_i N_x f_i=0,\quad i=1,3,5, \quad (12)$$

where

$$\tilde{R}_i(f_1,f_3,f_5)=\sum_{j=1,3,5}2\eta_{ij}f_j^2,\quad i=1,3,5,$$

$$\tilde{G}_1(f_1,f_3,f_5)=2\eta_{17}f_3^2 f_5,$$

$$\tilde{G}_3(f_1,f_3,f_5)=2\eta_{37}f_1 f_3 f_5,$$

$$\tilde{G}_5(f_1,f_3,f_5)=2\eta_{57}f_3^2 f_1.$$

The solutions given by Eq. (11) are called nonlinear modes. These nonlinear modes are straight lines in configuration space (Fig. 2). The dynamical system (12) describes the motions on nonlinear modes. Note, that the method of nonlinear modes analysis of parametrically excited systems is suggested in the papers [28,29].

The harmonic balance method is used to study the motions on the nonlinear modes (12). As the nonlinear modes for the main parametric resonance are considered, the motions are presented as

$$f_i=A_i\cos(\nu t)+B_i\sin(\nu t),\quad i=1,3,5. \quad (13)$$

Now (13) is substituted into (12) and the amplitudes of harmonics $\cos(\nu t)$ and $\sin(\nu t)$ are equated. As a result the following system of nonlinear algebraic equations is derived:

$$A_i\left(\omega_i^2-\nu^2+\eta_{ii}A_i^2+\frac{1}{2}\sum_{j=1,3,5}\eta_{ij}(3A_j^2+B_j^2)+\frac{1}{2}\chi_i N_1\right)+G_i^{(A)}=0,$$

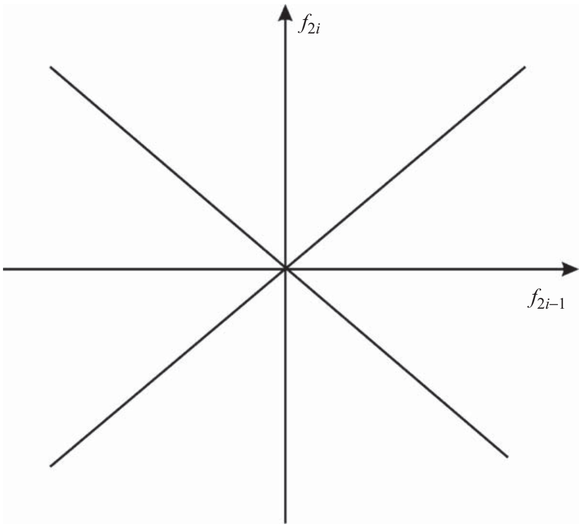

Fig. 2. Nonlinear modes in configuration space.

$$B_i\left(\omega_i^2-\nu^2+\eta_{ii}B_i^2+\frac{1}{2}\sum_{j=1,3,5}\eta_{ij}(3B_j^2+A_j^2)-\frac{1}{2}\chi_i N_1\right)+G_i^{(B)}=0,\quad i=1,3,5, \tag{14}$$

where

$$G_1^{(A)}=\eta_{13}A_3B_1B_3+\eta_{15}A_5B_1B_5+\tfrac{1}{2}\eta_{17}A_3B_3B_5+\tfrac{1}{4}\eta_{17}A_5(3A_3^2+B_3^2),$$

$$G_1^{(B)}=\eta_{13}B_3A_1A_3+\eta_{15}B_5A_1A_5+\tfrac{1}{2}\eta_{17}B_3A_3A_5+\tfrac{1}{4}\eta_{17}B_5(3B_3^2+A_3^2),$$

$$G_3^{(A)}=\tfrac{1}{2}(3\eta_{37}A_1A_3A_5+\eta_{37}A_1B_3B_5+\eta_{37}A_3B_1B_5+\eta_{37}A_5B_1B_3),$$

$$G_3^{(B)}=\tfrac{1}{2}(3\eta_{37}B_1B_3B_5+\eta_{37}B_1A_3A_5+\eta_{37}B_3A_1A_5+\eta_{37}B_5A_1A_3),$$

$$G_5^{(A)}=\eta_{53}A_3B_3B_5+\eta_{51}A_1B_1B_5+\tfrac{1}{2}\eta_{57}A_3B_1B_3+\tfrac{1}{4}\eta_{57}A_1(3A_3^2+B_3^2),$$

$$G_5^{(B)}=\eta_{53}B_3A_3A_5+\eta_{51}B_1A_1A_5+\tfrac{1}{2}\eta_{57}B_3A_1A_3+\tfrac{1}{4}\eta_{57}B_1(3B_3^2+A_3^2). \tag{15}$$

The following cases of solutions exist in system (14):

$$\text{Case 1.1}:\quad A_1\neq 0;\ A_3=A_5=0;\ B_i=0,$$

$$1.2.\quad B_1\neq 0;\ B_3=B_5=0;\ A_i=0,$$

$$2.1.\quad A_3\neq 0;\ A_1=A_5=0;\ B_i=0,$$

$$2.2.\quad B_3\neq 0;\ B_1=B_5=0;\ A_i=0,$$

$$3.1.\quad A_5\neq 0;\ A_1=A_3=0;\ B_i=0,$$

$$3.2.\quad B_5\neq 0;\ B_1=B_3=0;\ A_i=0,$$

$$4.1.\quad A_1\neq 0;\ A_5\neq 0;\ A_3=0;\ B_i=0,$$

$$4.2.\quad B_1\neq 0;\ B_5\neq 0;\ B_3=0;\ A_i=0,$$

$$5.1.\quad A_1\neq 0;\ A_3\neq 0;\ A_5\neq 0;\ B_i=0,$$

$$5.2.\quad B_1\neq 0;\ B_3\neq 0;\ B_5\neq 0;\ A_i=0, \tag{16}$$

where $i=1,3,5$.

Now every group of solutions is considered separately. At first, Cases 1.1 and 1.2 are considered. Fixing the value ν with a certain step size, the solutions are determined from the system of nonlinear algebraic equations (14). The vibrations amplitudes A_1 and B_1 are obtained analytically in the following form:

$$A_1^2=[2\nu^2-2\omega_1^2-\chi_1N_1]/3\eta_{11},$$

$$B_1^2=[2\nu^2-2\omega_1^2+\chi_1N_1]3\eta_{11}. \tag{17}$$

Cases 2.1, 2.2, 3.1 and 3.2 can be determined analytically too. The amplitudes A_3, B_3, A_5, B_5 are determined as

$$A_i^2=\Omega_i/3\eta_{ii};\quad B_i^2=\Omega_i/3\eta_{ii}\ \ i=3,5, \tag{18}$$

where $\Omega_i=2\nu^2-2\omega_i^2+\chi_iN_1$.

Altering the frequency ν with the certain step, Cases 4.1 and 4.2 are analyzed. The system of nonlinear algebraic equations (14) is solved for every value of ν. This system has the following analytical solution:

$$A_1^2=\theta^{-1}[\eta_{55}\Omega_1-\eta_{15}\Omega_5]/3;\quad B_1^2=\theta^{-1}[\eta_{55}\Omega_1-\eta_{15}\Omega_5]/3,$$

$$A_5^2=\theta^{-1}[\eta_{11}\Omega_5-\eta_{51}\Omega_1]/3;\quad B_5^2=\theta^{-1}[\eta_{11}\Omega_5-\eta_{51}\Omega_1]/3, \tag{19}$$

where $\theta=\eta_{11}\eta_{55}-\eta_{15}\eta_{51}$.

Let us consider Cases 5.1 and 5.2. Altering the frequency ν with certain step, the vibrations amplitudes are determined from the system of nonlinear algebraic equations (14), which is solved numerically by the Newton method with respect to A_1, B_1, A_3, B_3, A_5, B_5.

Now the nonlinear vibrations of cylindrical shells are considered accounting for energy dissipation. Then the linear damping is added into system (6). The resulted system has the following form:

$$\ddot{f}_i+\xi_i\dot{f}_i+\omega_i^2f_i+f_iR_i(f_1,\ldots,f_7)+G_i(f_1,\ldots,f_6)+\chi_iN_xf_i=0,\quad i=1,2,\ldots,6. \tag{20}$$

The functions R_i of system (20) are determined by Eqs. (10).

Note, that the equations $f_{2i-1}=\pm f_{2i}$; $i=1,2,3$ are exact solutions of the system expressed by Eq. (20). These solutions correspond to nonlinear modes. Moreover, these nonlinear modes coincide with the nonlinear modes of the system without dissipation (12). The harmonic balance method is used to study these nonlinear modes and the system motions are presented in the form of Eq. (13). Then the system of nonlinear algebraic equations with respect to amplitudes of harmonics (13) is derived as

$$A_i\left(\omega_i^2-\nu^2+\eta_{ii}A_i^2+\frac{1}{2}\sum_{j=1,3,5}\eta_{ij}(3A_j^2+B_j^2)+\frac{1}{2}\chi_iN_1\right)+B_i\xi_i\nu+G_i^{(A)}=0,$$

$$B_i\left(\omega_i^2-\nu^2+\eta_{ii}B_i^2+\frac{1}{2}\sum_{j=1,3,5}\eta_{ij}(3B_j^2+A_j^2)-\frac{1}{2}\chi_iN_1\right)-A_i\xi_i\nu+G_i^{(B)}=0,\quad i=1,3,5. \tag{21}$$

The functions $G_i^{(A)}$, $G_i^{(B)}$ are determined from Eqs. (15). The following groups of solutions exist in system (21):

6.1. $A_1\neq0;\quad B_1\neq0;\quad A_3=A_5=0;\quad B_3=B_5=0,$

6.2. $A_3\neq0;\quad B_3\neq0;\quad A_1=A_5=0;\quad B_1=B_5=0,$

6.3. $A_5\neq0;\quad B_5\neq0;\quad A_1=A_3=0;\quad B_1=B_3=0,$

6.4. $A_1\neq0;\quad A_5\neq0;\quad B_1\neq0;\quad B_5\neq0;\quad A_3=0;\quad B_3=0,$

6.5. $A_1\neq0;\quad A_3\neq0;\quad A_5\neq0;\quad B_1\neq0;\quad B_3\neq0;\quad B5\neq0.$ (22)

The solutions given by Eq. (22) of the system described in Eq. (21) are analyzed numerically. Setting the parameter ν with a certain step, system (21) is solved by the Newton method.

The traveling waves for the main parametric resonance, which are described by system (20), are considered taking into account dissipation. The harmonic balance method is used to study these motions and the system vibrations are presented as

$$f_i=A_i\cos(\nu t)+B_i\sin(\nu t),$$

$$f_{i+1}=A_i\sin(\nu t)+B_i\cos(\nu t),\quad i=1,3,5. \tag{23}$$

Then the amplitudes of harmonics in Eq. (23) are determined from the following system of nonlinear algebraic equations:

$$A_i\left(\omega_i^2-\nu^2+\eta_{ii}B_i^2+\sum_{j=1,3,5}\eta_{ij}(A_j^2+B_j^2)\pm\frac{1}{2}\chi_iN_1\right)\pm B_i\xi_i\nu+\tilde{G}_i^{(A)}=0,$$

$$B_i\left(\omega_i^2-\nu^2+\eta_{ii}A_i^2+\sum_{j=1,3,5}\eta_{ij}(A_j^2+B_j^2)\pm\frac{1}{2}\chi_iN_1\right)\pm A_i\xi_i\nu+\tilde{G}_i^{(B)}=0,\quad i=1,3,5, \tag{24}$$

where

$$\tilde{G}_1^{(A)}=\eta_{13}A_3B_1B_3+\eta_{15}A_5B_1B_5+\eta_{17}A_3B_3B_5+\tfrac{1}{2}\eta_{17}A_3^2A_5,$$

$$\tilde{G}_1^{(B)}=\eta_{13}B_3A_1A_3+\eta_{15}B_5A_1A_5+\eta_{17}B_3A_3A_5+\tfrac{1}{2}\eta_{17}B_3^2B_5,$$

$$\tilde{G}_3^{(A)}=\eta_{37}A_1A_3A_5+\eta_{37}A_1B_3B_5+\eta_{37}A_5B_1B_3+\eta_{31}A_1B_1B_3+\eta_{35}A_5B_3B_5,$$

$$\tilde{G}_3^{(B)}=\eta_{37}B_1B_3B_5+\eta_{37}B_1A_3A_5+\eta_{37}B_5A_1A_3+\eta_{31}B_1A_1A_3+\eta_{35}B_5A_3A_5,$$

$$\tilde{G}_5^{(A)}=\eta_{51}A_1B_1B_5+\eta_{53}A_3B_3B_5+\eta_{57}A_3B_1B_3+\tfrac{1}{2}\eta_{57}A_1A_3^2,$$

$$\tilde{G}_5^{(B)}=\eta_{51}B_1A_1A_5+\eta_{53}B_3A_3A_5+\eta_{57}B_3A_1A_3+\tfrac{1}{2}\eta_{57}B_1B_3^2. \tag{25}$$

The following groups of solutions exist in system (24):

$$(1)\quad A_1=B_1\neq 0;\quad A_3=A_5=B_3=B_5=0,$$

$$(2)\quad A_1=B_1\neq 0;\quad A_5=B_5\neq 0;\quad A_3=B_3=0,$$

$$(3)\quad A_1=B_1\neq 0;\quad A_5=B_5\neq 0;\quad A_3=B_3\neq 0. \tag{26}$$

Altering the frequency of the parametric load ν, system (24) is solved by the Newton method.

In order to analyze stability of periodic vibrations, the system of variational equations is derived and fundamental matrix is calculated numerically. Then the multipliers are obtained from the fundamental matrix [30].

Now the parameters of the vibrations are connected to the dynamic flexure w. Nonlinear modes $f_{2i-1}=\pm f_{2i}$, $i=1,2,3$ correspond to the following standing waves of the cylindrical shell:

$$w(x,y,t)=\sqrt{2}\sin\frac{m\pi x}{L}\sum_{i=1}^{3}(A_{2i-1}\cos(\nu t)+B_{2i-1}\sin(\nu t))\cos\left(\frac{n_iy}{R}\mp\frac{\pi}{4}\right)+C\sin^2\frac{m\pi x}{L}+E, \tag{27}$$

$$C=\eta^{-1}\sum_{j=1,3,5}[\tilde{\omega}_8(\gamma_{8j}+\gamma_{8j+1})f_j^2-\omega_8(\gamma_{7j}+\gamma_{7j+1})f_j^2],$$

$$E=\eta^{-1}\sum_{j=1,3,5}[\omega_7(\gamma_{7j}+\gamma_{7j+1})f_j^2-\tilde{\omega}_7(\gamma_{8j}+\gamma_{8j+1})f_j^2].$$

For the traveling waves (23) the shell dynamic flexure has the following form:

$$w(x,y,t)=\sin\frac{m\pi x}{L}\sum_{i=1}^{3}\left[A_{2i-1}\cos\left(\nu t-\frac{n_iy}{R}\right)+B_{2i-1}\sin\left(\nu t+\frac{n_iy}{R}\right)\right]+\tilde{C}\sin^2\frac{m\pi x}{L}+\tilde{E}, \tag{28}$$

$$\tilde{C}=\eta^{-1}\sum_{j=1,3,5}\{\tilde{\omega}_8[\gamma_{8j}f_j^2+\gamma_{8j+1}f_{j+1}^2]-\omega_8[\gamma_{7j}f_j^2+\gamma_{7j+1}f_{j+1}^2]\},$$

$$\tilde{E}=\eta^{-1}\sum_{j=1,3,5}\{\omega_7[\gamma_{7j}f_j^2+\gamma_{7j+1}f_{j+1}^2]-\tilde{\omega}_7[\gamma_{8j}f_j^2+\gamma_{8j+1}f_{j+1}^2]\}.$$

The summand $A_{2i-1}\cos(\nu t-(n_iy/R))$ of Eq. (28) describes the rotation of the shell deformation pattern about symmetry axis with angular velocity $\Omega_*=\nu/n_i$ in the direction of y increase. The second term $B_{2i-1}\sin(\nu t+(n_iy/R))$ of Eq. (28) describes the wave motions with the same angular velocity on the opposite direction.

4. Numerical analysis of vibrations

The shell with the following parameters, the same as those used in [11] is considered:

$$h=0.002\,\text{m};\quad L=0.4\,\text{m};\quad R=0.2\,\text{m};\quad E=2.1\times10^{11}\,\text{N/m}^2;\quad \mu=0.3;\quad \rho=7850\,\text{kg/m}^3;$$

$$\omega_0=3165.03\,\text{rad/s};\quad N_1=0.6N_{\text{cr}}, \tag{29}$$

where $N_{cr} = Eh^2/R\sqrt{3(1-\nu^2)}$ is classical critical load per unit length [15], which has the following value for the parameters (29): $N_{cr} = 2.54 \times 10^6$ N/m. The frequencies of shell linear vibrations are the following (rad/s):

$$\omega_{3,1} = 5636.32; \quad \omega_{4,1} = 3745.32; \quad \omega_{5,1} = 3165.03; \quad \omega_{6,1} = 3437.18; \quad \omega_{7,1} = 4214.28; \quad \omega_{8,1} = 5289.51,$$

where the first subscript indicates the wavenumbers in circumference direction and the second subscript shows the number of half-waves in x directions (Fig. 1). In future nonlinear analysis the modes with the following parameters are taken: $n_1 = 4$; $n_2 = 5$; $n_3 = 6$; $m = 1$.

The numerical analysis of the nonlinear modes is carried out for the shell parameters (29). The dependence of the vibrations amplitudes A_1, B_1 on the frequency ν are presented on the frequency response (Fig. 3). The stable solutions are denoted by solid lines and the unstable solutions are shown by dashed lines. The branches of the frequency response (Fig. 3) are denoted by $A_1^{(1)}$ and $B_1^{(1)}$ for Cases 1.1 and 1.2 of Eq. (16). In this case only one pair of the conjugate modes from Eq. (3) is active. The branches $A_1^{(2)}$, $B_1^{(2)}$ (Fig. 3) describe the motions with two pairs of conjugate vibrations modes. These solutions correspond to Cases (4.1) and (4.2) of Eq. (16). The branches $A_1^{(3)}$, $B_1^{(3)}$ of the frequency response show the vibrations with three pairs of conjugate modes, which correspond to Cases (5.1) and (5.2) of Eq. (16).

The direct numerical integrations of the system in Eq. (12) at different values of frequency ν are carried out to confirm the analytical results. Using such an approach, only stable solutions are derived. The data of the calculations are shown by small squares in Fig. 3. The results of the direct numerical integration are very close to the data, which are obtained by the harmonic balance method.

Carrying out numerical integration on long time interval, the periodic solution is considered unstable, if the numerical trajectory escapes from the considered it to another trajectory. This behavior is explained by an error of numerical integration.

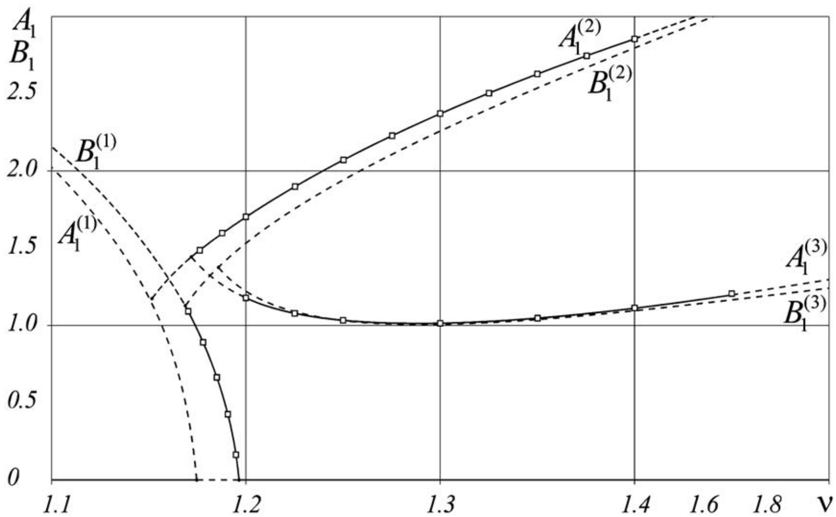

Fig. 3. Frequency response of parametric vibrations on nonlinear mode.

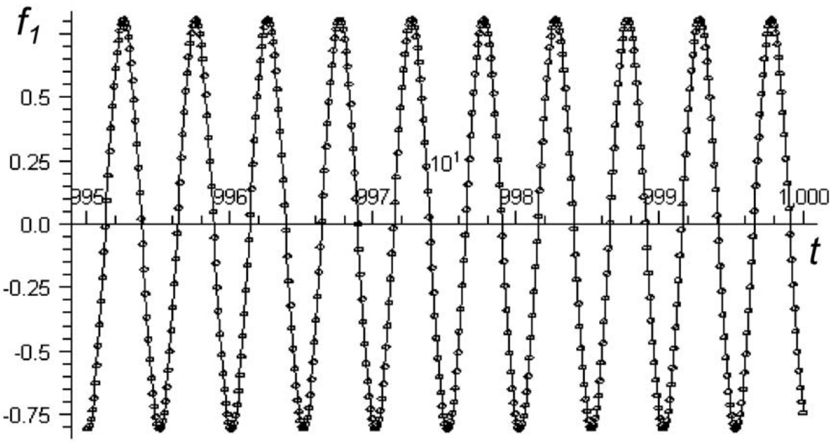

Fig. 4. Comparison of the results of the direct numerical integration with analytical solution.

To study stability of the parametric vibrations the direct numerical integration of the differential equations (6) is carried out on the time interval $t \in [0; 2000\pi\nu^{-1}]$. The initial conditions are determined from Eqs. (13, 24).

Fig. 4 shows the comparison of the data of the direct numerical integration of system (6) with the analytical solution of Eq. (13), which has the following parameters: $\nu = 1.25$, $A_1 = 0.8069$, $A_3 = 0.6872$, $A_5 = 0.6022$, $B_i = 0$, $i = 1, 3, 5$. The results of the analytical solution are shown by solid lines and the data of the direct numerical integration are represented by small squares.

Now loss of normal modes of stability of the dynamical system (6) is analyzed by the direct numerical integration of the system given in Eq. (6). The results of the calculations are presented in Fig. 5. Wave form (Fig. 5a) corresponds to Case (1.1) of (16) with the following parameters: $\nu = 1.18$, $A_1 = 0.3694$. The unstable solution (4.1) of Eqs. (16) is presented in Fig. 5b, where the system time history is shown. This solution is calculated from the periodic solution with the following parameters: $\nu = 1.43$, $A_1 = 2.8494$, $A_5 = 2.6091$.

The dynamics of the system with dissipation (20) on the nonlinear modes is presented on the frequency response (Fig. 6). The dependence of the vibrations amplitudes A_1, B_1 on the frequency ν is shown in this figure. The symbols $A_1^{(1)}$, $B_1^{(1)}$ denote the frequency response branches, which correspond to Case (6.1) of the solutions in Eq. (22). Then, only one pair of conjugate modes of the expansion (3) is active. The branches $A_1^{(2)}$, $B_1^{(2)}$ describe the excitation of two pairs of conjugate modes, which correspond to type (4) of Eq. (22). The branches $A_1^{(3)}$, $B_1^{(3)}$ describe the vibrations, when three pair of conjugate modes are excited. This corresponds to Case (5) of Eq. (22).

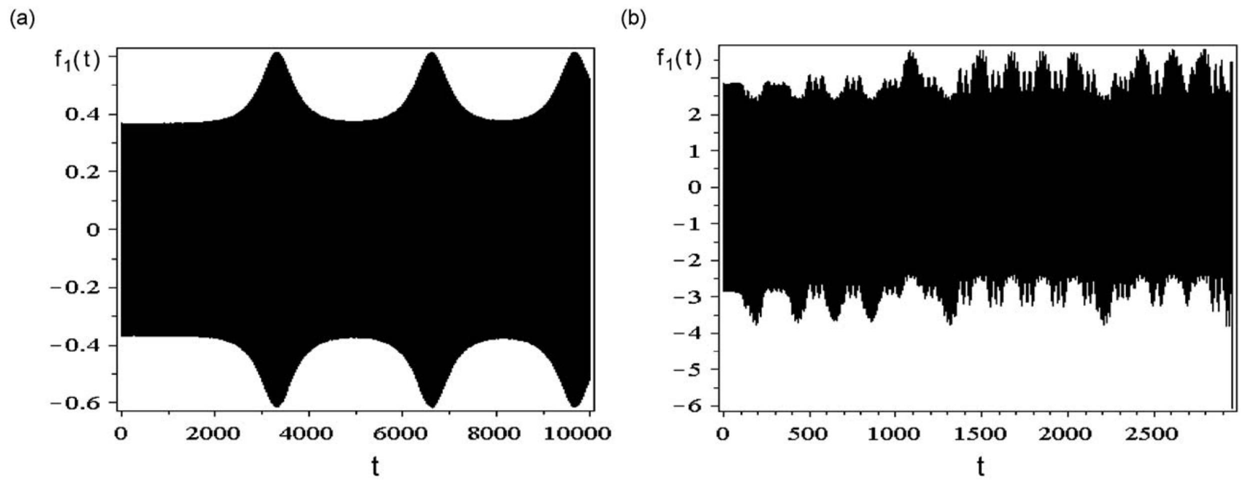

Fig. 5. The behavior of the system without dissipation, when stability of nonlinear mode is lost. (a). The nonlinear mode corresponds to the solutions (1.1) from formulas (15) with the following parameters: $\nu = 1.18$, $A_1 = 0.3694$. (b) Group (4.1) of Eqs. (15) with the following parameters: $\nu = 1.43$, $A_1 = 2.8494$, $A_5 = 2.6091$.

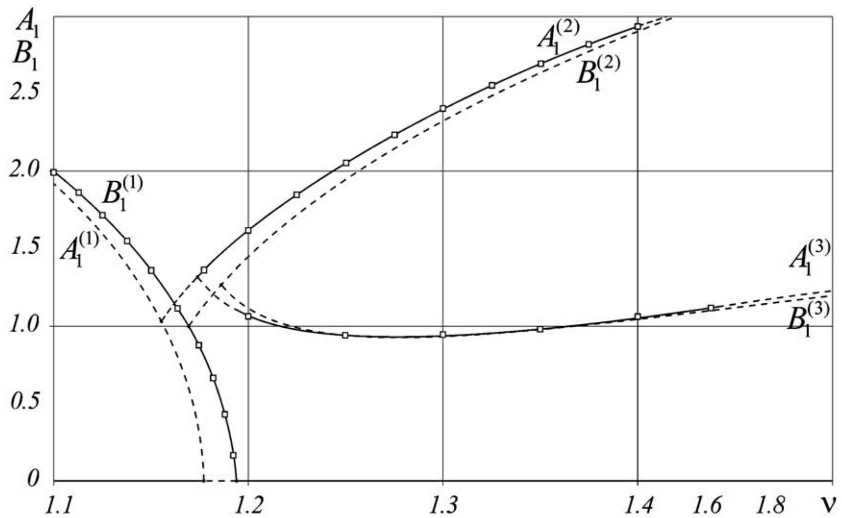

Fig. 6. Frequency response of nonlinear mode of parametric vibrations of the system with dissipation.

If the nonlinear modes of the system with dissipation (20) lose stability, the attraction of the trajectories to the stable solutions takes place. The motions, when the nonlinear mode losses stability, are shown in Fig. 7. These dynamics is obtained by the direct numerical integration from the initial conditions, which are determined from the solutions of the harmonic balance method with the following parameters:

(a) $\nu = 1.182$, $A_1 = 0.1941$, $B_1 = 0.0959$;
(b) $\nu = 1.45$, $A_1 = 2.7440$, $B_1 = 1.0756$, $A_5 = 2.5257$, $B_5 = 0.9921$.

Fig. 7a shows the trajectory attracted to the periodic motions and Fig. 7b shows the trajectory attracted to the stable trivial solutions.

The numerical analysis of the traveling waves is carried out. Fig. 8 shows the frequency response of the traveling waves. The dependence of the vibrations amplitudes A_1 on the frequency ν is shown in this figure. The branches of the frequency response describing Case (6.1) of the solutions in Eq. (24) are denoted by $A_1^{(1)}$. Then only one pair of conjugate vibrations modes of expansion (3) is active. The branches $A_1^{(2)}$ (Fig. 8) describe the vibrations with two pairs of conjugate modes. These motions are characterized by Case (2) of Eq. (26). The branches $A_1^{(3)}$ show the shell vibrations with three pairs of conjugate modes, which correspond to Case (6.3) of Eqs. (26).

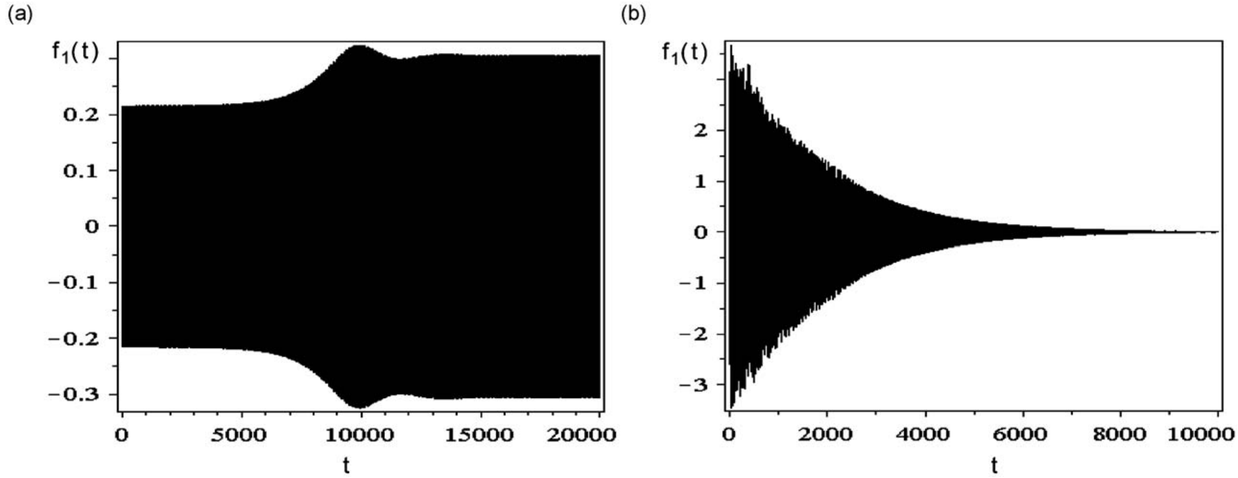

Fig. 7. The behavior of the system with dissipation, when the nonlinear mode is lost stability. The results of the direct numerical integration, which is started from the harmonic balance solution with the following parameters: (a) $\nu = 1.182$, $A_1 = 0.1941$, $B_1 = 0.0959$; (b) $\nu = 1.45$, $A_1 = 2.7440$, $B_1 = 1.0756$, $A_5 = 2.5257$, $B_5 = 0.9921$.

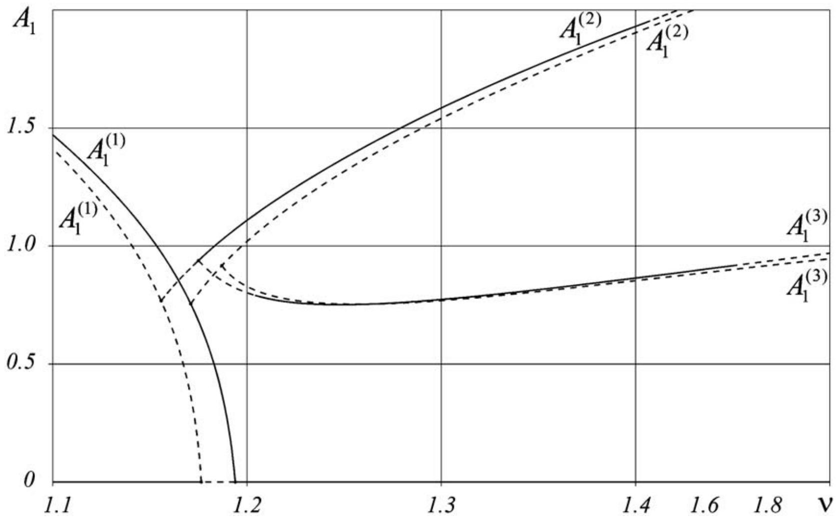

Fig. 8. Frequency response of the traveling waves of the system with dissipation.

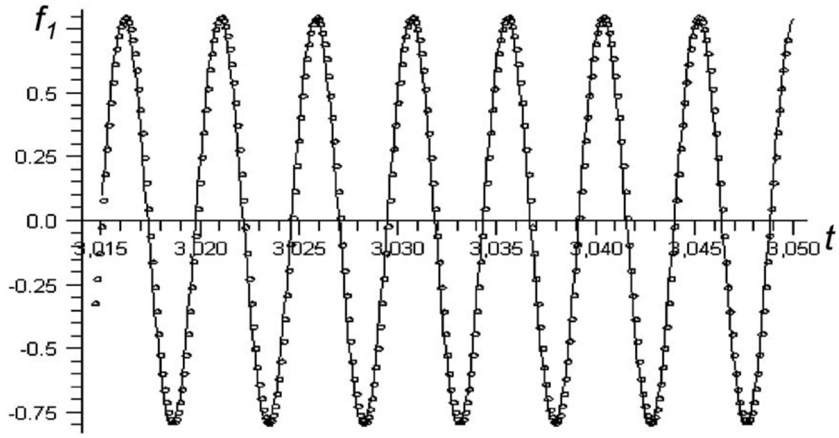

Fig. 9. Comparison the results of the direct numerical integration with analytical solution.

To check the results of the analysis of the traveling waves, the comparison of the motions, which are obtained by the direct numerical integration of system (20), with the results of the harmonic balance method is carried out. The steady vibrations, which are obtained by the direct numerical integration, are shown by small squares in Fig. 9. The analytical solution (23) with the parameters $\nu = 1.3, A_1 = B_1 = 0.5616, A_3 = B_3 = 0.6051, A_5 = B_5 = 0.5206$ is presented by solid lines in the same figure.

5. Conclusions

One and two conjugate modes approximations of shell vibrations are not enough to predict dynamics of a wide class of cylindrical shells. This is explained by closeness of the eigenfrequencies of the different conjugate modes. In this case, only a multimode model of shells can describe the parametric vibrations adequately.

The following types of vibrations are analyzed in this paper:

- one pair of conjugate modes is active;
- two or three pairs of conjugate modes are active.

The vibrations, which are characterized by one or two pairs of conjugate modes, can be described by a dynamical model with two and four degree-of-freedom, respectively.

Nonlinear modes, which are straight lines in a configuration space, are observed for multimode shells dynamics. We stress, that the same nonlinear modes exist both in the system without damping and in the system with damping. The existence of such normal modes is explained by axisymmetry of cylindrical shells.

Nonlinear modes and traveling waves are two possible forms of solutions of the dynamical system expressed by Eq. (6). The traveling waves are described by Eq. (23). As follows from the results of the analysis, the normal modes and traveling waves exist in the frequency bands $\nu \in [1; 1.6]$ and $\nu \in [1.1; 1.8]$, respectively. Thus, the frequency band ν with two kinds of motions exists. Any one of these motions has a basin of attraction. Therefore, if the initial conditions belong to the basin of attraction of nonlinear mode or traveling waves, then nonlinear mode or traveling waves take place.

All frequency responses of nonlinear modes and traveling waves are qualitative similar. This is explained by similarity of the systems of nonlinear algebraic equations with respect to amplitudes.

The periodic motions of cylindrical shells with geometrical imperfections are not presented as nonlinear modes considered in this paper. The vibrations of this system can be described by more complex nonlinear modes and others periodic motions. The analysis of nonlinear modes of the shell with imperfections may be a very interesting problem for future research.

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