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Adaptive synchronization of hyperchaotic systems via passivity feedback control with time-varying gains

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ABSTRACT

A passivity feedback control scheme with time-varying gains is proposed for adaptive synchronization of hyperchaotic systems. By transforming the synchronization error dynamics into an equivalent passive system, a synchronization control law with time-varying gains is achieved and the convergence of the synchronization errors is guaranteed. The feasibility and effectiveness of the proposed scheme is demonstrated through its application to hyperchaotic Lü systems.

1. Introduction

Chaos synchronization has been studied since the pioneering work of Pecora and Carroll [1] was published. It has attracted great attention due to its superior potential applications, for examples, in communication, laser physics and optics, mechanics, and chemistry. Many methods have been developed for synchronizing of chaos such as, for example, linear control [2,3], LMI-based approach [4,5], backstepping design [6], sliding control [7], adaptive control [8], active control [9], and passivity-based control [10–15].

In recent years, the study of chaos synchronization has more focused on hyperchaotic systems because of their rich chaos behaviors. The hyperchaotic systems are characterized by multiple positive Lyapunov exponents. The first example of the hyperchaotic systems was presented by Rössler in 1979 [16]. Since then, other hyperchaotic systems have been reported [17–20]. Nowadays, some methods have been proposed for synchronization of the hyperchaotic systems such as, for examples, backstepping design [21], sliding control [22], Lyapunov-based control [23,24]. When the parameters of the systems are uncertain or unknown, adaptive control techniques can be utilized to eliminate the influence of the uncertainty [25–27].

The passivity theory is considered as an alternative tool for analyzing stability of nonlinear systems and designing controllers for nonlinear systems [28–32]. Recently, the concept of passivity has attracted new interest in chaos control and synchronization. For example, Chen and Liu [10] developed a passivity-based controller for chaos suppression of a unified chaotic system. A simple linear state feedback controller was obtained. In Ref. [11], Kemih made used of the stability properties of passive systems to derive a controller to suppress chaos of a nuclear spin generator system. Wang and Liu [12] presented a passivity feedback control design to synchronize unified chaotic systems. The knowledge of the bound of a system state is needed to derive the control law. The effectiveness of the design was illustrated using numerical simulation results for Lorenz, Lü, and Chen systems. Zhang and Lu [13] proposed a passivity-based control for chaos control and synchronization of the hyperchaotic Chen system. The approach can be applied to other hyperchaotic systems as well. However, like the work in Ref. [12], the approach needs to know the bound of a system state. Although the bound usually

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can be easily found from extensive numerical simulations, this might not be applicable for a system with unknown parameters.

As far as passivity-based adaptive control for chaos suppression and synchronization is concerned, Wei and Luo [14] presented a passivity-based adaptive control law to suppress chaos of a second order power system. There is only one parameter that is assumed to have uncertainty. The similar approach was also applied to a space-clamped FitzHugh-Nagumo neurons problem [15].

This paper proposes an adaptive passivity-based control design for synchronization of hyperchaotic systems with unknown parameters. The problem is very similar to the problems presented in Refs. [26,27]. However, based on the passivity concept, there are only two control signals needed, instead of four signals. The proposed design could also be considered as an extension of the work in Ref. [13]. The extension includes that the control design is adaptive and it does not require the knowledge of the bound of a system state. Computer simulations are used to illustrate the effectiveness of the proposed approach. Hyperchaotic Lü systems are taken as illustrative examples.

2. Passivity and passivity feedback control

Consider the nonlinear affine system

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \tag{1}$$

where the state $x \in \mathbf{R}^n$, the input $u \in \mathbf{R}^m$, and output $y \in \mathbf{R}^m$. f and the m columns of g are smooth vector fields and h is a smooth mapping. It is assumed that the origin $x=0$ is an equilibrium point of the system. The internal dynamics of system (1) which are consistent with the constraint $y=0$ is defined as the zero-dynamics. System (1) is said to be zero-state detectable if $u=0$ and $y=0$ imply $\lim_{t\to\infty} x(t) = 0$.

System (1) is passive if there exists a positive semidefinite function $V(x)$, called a storage function, such that for $\forall t \geq 0$ satisfying

$$V(x) - V(x_0) \leq \int_0^t u(\tau)^T y(\tau) d\tau. \tag{2}$$

If the storage function $V(x)$ is differentiable, Eq. (2) can be expressed in the derivative form as

$$u^T y \geq \dot{V}. \tag{3}$$

Moreover, system (1) is output strictly passive if

$$u^T y \geq \dot{V} + y^T \sigma(y) \quad \text{and} \quad y^T \sigma(y) > 0, \quad \forall y \neq 0 \tag{4}$$

for some function σ. It is strictly passive if

$$u^T y \geq \dot{V} + \psi(y) \tag{5}$$

for some positive definite function ψ.

A passive system having a positive definite storage function is Lyapunov stable. If the system is strictly passive or output strictly passive and zero-state detectable, its origin is asymptotically stable. Furthermore, if the storage function is radially unbounded, the origin is globally asymptotically stable [28,29].

Passivity feedback control, also known as feedback passivation, is a problem of finding the feedback transformation or control law

$$u = \alpha(x) + \beta(x)v \tag{6}$$

such that the equivalent system

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v, \quad y = h(x)$$

of system (1) is passive. Here $\alpha(x)$ and $\beta(x)$ are smooth functions. $\beta(x)$ is also invertible for all x. v is the transformed input of the equivalent system. If system (1) has unknown parameters, the control law (6) may be replaced by an adaptive control law [30]

$$u = \alpha(x,k) + \beta(x,k)v, \quad \dot{k} = \gamma(x,v,k) \tag{7}$$

where k is the time-varying gain.

When system (1) has relative degree one at the origin and the distribution spanned by the vector fields $g_1(x), \ldots, g_m(x)$ is involutive, it can be represented as the normal form [28]

$$\dot{z} = f_0(z) + p(z,y)y, \quad \dot{y} = b(z,y) + a(z,y)u \tag{8}$$

where (z,y) is a new coordinate of the system, locally defined in the neighborhood of the origin, and $z \in \mathbf{R}^{n-m}$. $a(z,y)$ is nonsingular for all (z,y) in the neighborhood of the origin. Setting $y=0$ in (8) yields the zero dynamic

$$\dot{z} = f_0(z). \tag{9}$$

The stability of the zero-dynamics is a necessary condition for the passivity feedback control design. Therefore, system (8) cannot be made passive by the feedback control law if its zero-dynamics is unstable [28].

By considering (8), the passivity feedback control can also be restated as a problem of finding the feedback control law

$$u=\alpha(z,y)+\beta(z,y)\nu \tag{10}$$

$$u=\alpha(z,y,k)+\beta(z,y,k)\nu \quad \text{or} \quad \dot{k}=\gamma(z,y,\nu,k) \tag{11}$$

such that the system

$$\dot{z}=f_0(z)+p(z,y)y, \quad \dot{y}=b(z,y)+a(z,y)\alpha(\cdot)+a(z,y)\beta(\cdot)\nu$$

is passive.

3. Systems description

A Hyperchaotic Lü system will be taken as an illustrative example. The hyperchaotic Lü system [18] is described by

$$\begin{aligned}\dot{x}_1&=a(x_2-x_1)+x_4,\\ \dot{x}_2&=-x_1x_3+cx_2,\\ \dot{x}_3&=x_1x_2-bx_3,\\ \dot{x}_4&=x_1x_3+rx_4,\end{aligned} \tag{12}$$

where $x=[x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is the state variable, a, b, c, d are positive parameters, r is a parameter. When a=36, b=3, c=20, and $-0.35<r\leq 1.3$, system (12) becomes hyperchaotic.

4. Synchronization of hyperchaotic lü systems

From Eq. (12), the master and the slave hyperchaotic Lü systems are described, respectively, by the following equations:

$$\dot{x}_{m1}=a(x_{m2}-x_{m1})+x_{m4}, \quad \dot{x}_{m2}=-x_{m1}x_{m3}+cx_{m2}, \quad \dot{x}_{m3}=x_{m1}x_{m2}-bx_{m3}, \quad \dot{x}_{m4}=x_{m1}x_{m3}+rx_{m4}, \tag{13}$$

and

$$\dot{x}_{s1}=a(x_{s2}-x_{s1})+x_{s4}, \quad \dot{x}_{s2}=-x_{s1}x_{s3}+cx_{s2}+u_1, \quad \dot{x}_{s3}=x_{s1}x_{s2}-bx_{s3}, \quad \dot{x}_{s4}=x_{s1}x_{s3}+rx_{s4}+u_2, \tag{14}$$

where the subscripts m and s stand for the master and the slave, respectively, and u_1, u_2 are the only two control signals used to drive the slave system to synchronize the master system.

By defining the synchonization errors as e_1=$x_{s1}-x_{m1}$, e_2=$x_{s2}-x_{m2}$, e_3=$x_{s3}-x_{m3}$, e_4=$x_{s4}-x_{m4}$ and using (13) and (14), the synchronization error system is

$$\dot{e}_1=a(e_2-e_1)+e_4, \quad \dot{e}_2=-(x_{s1}x_{s3}-x_{m1}x_{m3})+ce_2+u_1, \quad \dot{e}_3=(x_{s1}x_{s2}-x_{m1}x_{m2})-be_3, \quad \dot{e}_4=(x_{s1}x_{s3}-x_{m1}x_{m3})+re_4+u_2. \tag{15}$$

Since $x_{s1}x_{s3}-x_{m1}x_{m3}$=$e_1e_3$+$e_1x_{m3}$+$e_3x_{m1}$ and $x_{s1}x_{s2}-x_{m1}x_{m2}$=$e_1e_2$+$e_1x_{m2}$+$e_3x_{m2}$,

$$\dot{e}_1=a(e_2-e_1)+e_4, \quad \dot{e}_2=-e_1e_3-e_1x_{m3}-e_3x_{m1}+ce_2+u_1, \quad \dot{e}_3=e_1e_2+e_2x_{m1}+e_1x_{m2}-be_3, \quad \dot{e}_4=e_1e_3+e_3x_{m1}+e_1x_{m3}+re_4+u_2 \tag{16}$$

By choosing the system output y=[$y_1 \quad y_2$]=[$e_2 \quad e_4$]^T and defining a new coordinate z=[$z_1 \quad z_2$]=[$e_1 \quad e_3$]^T, system (16) can be written in the normal form (8) as

$$\dot{z}_1=-az_1+ay_1+y_2, \quad \dot{z}_2=z_1x_{m2}-bz_2+(z_1+x_{m1})y_1, \quad \dot{y}_1=-z_1z_2-z_1x_{m3}-z_2x_{m1}+cy_1+u_1, \quad \dot{y}_2=z_1z_2+z_1x_{m3}+z_2x_{m1}+ry_2+u \tag{17a}$$

or

$$\dot{z}=f_0(z)+p(z,y)y, \quad \dot{y}=b(z,y)+a(z,y)u, \tag{17b}$$

where

$$u=\begin{bmatrix}u_1\\u_2\end{bmatrix}, \quad f_0=\begin{bmatrix}-az_1\\z_1x_{m2}-bz_2\end{bmatrix}, \quad p(z,y)=\begin{bmatrix}a & 1\\z_1+x_{m1} & 0\end{bmatrix}, \quad b(z,y)=\begin{bmatrix}-z_1z_2-z_1x_{m3}-z_2x_{m1}+cy_1\\z_1z_2+z_1x_{m3}+z_2x_{m1}+ry_2\end{bmatrix}, \quad a(z,y)=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

The stability of the zero-dynamics must be verified prior to performing the passivity feedback control design. By setting y_1-y_2=0 in (17), the zero-dynamics is

$$\dot{z}_1=-az_1, \quad \dot{z}_2=z_1x_{m2}-bz_2. \tag{18}$$

Note that, for the hyperchaotic Lü systems, $a>b$ and $a>1$. Define

$$W(z)=\frac{1}{2}z_1^2+\frac{1}{2}(z_1x_{m2}+z_2)^2+\frac{1}{2}(a-b)z_2^2 \tag{19}$$

as a Lyapunov function candidate for the zero-dynamics. Taking its derivative with respect to time yields

$$\begin{aligned}\dot W=\frac{\partial W}{\partial z}f_0(z)&=-az_1(z_1+(z_1x_{m2}+z_2)x_{m2})+(z_1x_{m2}-bz_2)((z_1x_{m2}+z_2)+(a-b)z_2)\\&=-az_1^2-(b+ab-b^2)z_2^2-(a-1)z_1^2x_{m2}^2+(1-2b)z_1z_2x_{m2}\\&=-az_1^2-(b+ab-b^2)z_2^2-(a-1)\left[z_1x_{m2}-\left(\frac{1-2b}{2(a-1)}\right)z_2\right]^2+\left[\frac{(1-2b)^2}{4(a-1)}\right]z_2^2\\&=-az_1^2-(a-1)\left[z_1x_{m2}-\left(\frac{1-2b}{2(a-1)}\right)z_2\right]^2-\left[\frac{ab(a-b)}{a-1}\right]z_2^2<0.\end{aligned}\tag{20}$$

Therefore, the zero-dynamics is asymptotically stable at the origin.

If the system parameters a, b, c, r are known, take

$$V_0=W+\frac{1}{2}y_1^2+\frac{1}{2}y_2^2\tag{21}$$

as a storage function candidate for system (17). The function V_0 is positive definite and radially unbounded. Taking its derivative with respect to time yields

$$\dot V_0=\frac{\partial W}{\partial z}f_0(z)+\frac{\partial W}{\partial z}p(z,y)y+y_1\dot y_1+y_2\dot y_2.$$

Using (20) results

$$\begin{aligned}\dot V_0&\le y_1(a(z_1+(z_1x_{m2}+z_2)x_{m2})+(z_1+x_{m1})((z_1x_{m2}+z_2)+(a-b)z_2))\\&+y_1(-z_1z_2-z_1x_{m3}-z_2x_{m1}+cy_1+u_1)+y_2(z_1+(z_1x_{m2}+z_2)x_{m2})+y_2(z_1z_2+z_1x_{m3}+z_2x_{m1}+ry_2+u_2)\\&=y_1(az_1(1+x_{m2}^2+z_2)+z_1^2x_{m2}+z_1x_{m1}x_{m2}+az_2x_{m2}-bz_1z_2+az_2x_{m1}-bz_2x_{m1}-z_1x_{m3}+cy_1+u_1)\\&+y_2(z_1+z_1z_2+z_1x_{m2}^2+z_2x_{m2}+z_1x_{m3}+z_2x_{m1}+ry_2+u_2).=y_1((a-b)(z_1+x_{m1})z_2+a(z_1+z_1x_{m2}^2+z_2x_{m2})\\&+z_1(z_1x_{m2}+x_{m1}x_{m2}-x_{m3})+cy_1+u_1)+y_2(ry_2+z_1(1+z_2+x_{m2}^2+x_{m3})+z_2(x_{m1}+x_{m2})+u_2)\end{aligned}$$

Choosing

$$\begin{aligned}u_1&=-(a-b)(z_1+x_{m1})z_2-a(z_1+z_1x_{m2}^2+z_2x_{m2})-(c+\rho)y_1-z_1(z_1x_{m2}+x_{m1}x_{m2}-x_{m3})+v_1\\u_2&=-(r+\rho)y_2-z_1(1+z_2+x_{m2}^2+x_{m3})-z_2(x_{m1}+x_{m2})+v_2,\quad\rho>0\end{aligned}\tag{22}$$

yields

$$\dot V_0\le v_1y_1+v_2y_2-\rho y_1^2-\rho y_2^2=v^Ty-\rho y^Ty\tag{23}$$

or

$$v^Ty\ge\dot V_0+\rho y^Ty$$

which satisfies (4). Here, $v=[v_1\quad v_2]^T$ is the transformed input. Thus, when the parameters a, b, c, r are known, the feedback control law (22) renders system (17) passive.

However, the parameters a, b, c, r are actually unknown. Modify (22) as

$$\begin{aligned}u_1&=-k_1(t)(z_1+x_{m1})z_2-k_2(t)(z_1+z_1x_{m2}^2+z_2x_{m2})-k_3(t)y_1-z_1(z_1x_{m2}+x_{m1}x_{m2}-x_{m3})+v_1\\u_2&=-k_4(t)y_2-z_1(1+z_2+x_{m2}^2+x_{m3})-z_2(x_{m1}+x_{m2})+v_2\end{aligned}\tag{24}$$

where $k_1(t)$, $k_2(t)$, $k_3(t)$, $k_4(t)$ are time varying gains. Define $\theta_1(t)=k_1(t)-(a-b)$, $\theta_2(t)=k_2(t)-a$, $\theta_3(t)=k_3(t)-(c+\rho)$, and $\theta_4(t)=k_4(t)-(r+\rho)$ with the dynamics

$$\dot\theta_1(t)=\dot k_1(t)=\gamma_1(z,y),\quad\dot\theta_2(t)=\dot k_2(t)=\gamma_2(z,y),\quad\dot\theta_3(t)=\dot k_3(t)=\gamma_3(z,y),\quad\dot\theta_4(t)=\dot k_4(t)=\gamma_4(z,y)\tag{25}$$

where the functions γ_1, γ_2, γ_3, γ_4, will be determined later. Take

$$V=W+\frac{1}{2}y_1^2+\frac{1}{2}y_2^2+\frac{1}{2}\theta_1^2+\frac{1}{2}\theta_2^2+\frac{1}{2}\theta_3^2+\frac{1}{2}\theta_4^2\tag{26}$$

as a storage function candidate. The function V is positive definite and radially unbounded. Taking its derivative with respect to time yields

$$\dot V=\frac{\partial W}{\partial z}f_0(z)+\frac{\partial W}{\partial z}p(z,y)y+y_1\dot y_1+y_2\dot y_2+\theta_1\dot\theta_1+\theta_2\dot\theta_2+\theta_3\dot\theta_3+\theta_4\dot\theta_4.$$

Using (20) results

$$\begin{aligned}\dot V&\le y_1(a(z_1+(z_1x_{m2}+z_2)x_{m2})+(z_1+x_{m1})((z_1x_{m2}+z_2)+(a-b)z_2))\\&+y_1(-z_1z_2-z_1x_{m3}-z_2x_{m1}+cy_1+u_1)+y_2(z_1+(z_1x_{m2}+z_2)x_{m2})+y_2(z_1z_2+z_1x_{m3}+z_2x_{m1}+ry_2+u_2)\\&+\theta_1\gamma_1(z,y)+\theta_2\gamma_2(z,y)+\theta_3\gamma_3(z,y)+\theta_4\gamma_4(z,y)=y_1((a-b)(z_1+x_{m1})z_2+a(z_1+z_1x_{m2}^2+z_2x_{m2})\\&+z_1(z_1x_{m2}+x_{m1}x_{m2}-x_{m3})+cy_1+u_1)+y_2(ry_2+z_1(1+z_2+x_{m2}^2+x_{m3})+z_2(x_{m1}+x_{m2})+u_2)\end{aligned}$$

$$+(k_1(t)-(a-b))\gamma_1(z,y)+(k_2(t)-a)\gamma_2(z,y)$$
$$+(k_3(t)-(c+\rho))\gamma_3(z,y)+(k_4(t)-(r+\rho))\gamma_4(z,y) \quad (27)$$

Substituting the control law (24) yields

$$\dot{V} \le y_1((-k_1(t)+(a-b))(z_1+x_{m1})z_2+(-k_2(t)+a)(z_1+z_1x_{m2}^2+z_2x_{m2})$$
$$+(-k_3(t)+c)y_1+v_1)+y_2((-k_4(t)+r)y_2+v_2)+(k_1(t)-(a-b))\gamma_1(z,y)+(k_2(t)-a)\gamma_2(z,y)$$
$$+(k_3(t)-(c+\rho))\gamma_3(z,y)+(k_4(t)-(r+\rho))\gamma_4(z,y)=(k_1(t)-(a-b))(\gamma_1(z,y)-(z_1+x_{m1})z_2y_1)$$
$$+(k_2(t)-a)(\gamma_2(z,y)-(z_1+z_1x_{m2}^2+z_2x_{m2})y_1)+(k_3(t)-(c+\rho))(\gamma_3(z,y)-y_1^2)-\rho y_1^2+v_1y_1$$
$$+(k_4(t)-(r+\rho))(\gamma_4(z,y)-y_2^2)-\rho y_2^2+v_2y_2$$

Choosing

$$\gamma_1(z,y)=(z_1+x_{m1})z_2y_1, \quad \gamma_2(z,y)=(z_1+z_1x_{m2}^2+z_2x_{m2})y_1, \quad \gamma_3(z,y)=y_1^2, \quad \gamma_4(z,y)=y_2^2 \quad (28)$$

results

$$\dot{V} \le -\rho y_1^2+v_1y_1-\rho y_2^2+v_2y_2=v^Ty-\rho y^Ty$$

or

$$v^Ty \ge \dot{V}+\rho y^Ty$$

which satisfies (4). Substituting (28) into (25) yields

$$\dot{k}_1(t)=(z_1+x_{m1})z_2y_1, \quad \dot{k}_2(t)=(z_1+z_1x_{m2}^2+z_2x_{m2})y_1, \quad \dot{k}_3(t)=y_1^2, \quad \dot{k}_4(t)=y_2^2 \quad (29)$$

Therefore, the feedback control law (24) with the time-varying gains (29) adaptively renders system (17) passive. By replacing $z_1=e_1$, $z_2=e_3$, $y_1=e_2$, $y_2=e_4$, and setting $v_1=v_2=0$, Eqs. (24) and (29) can be rewritten as

$$u_1=-k_1(t)(e_1+x_{m1})e_3-k_2(t)(e_1+e_1x_{m2}^2+e_3x_{m2})-k_3(t)e_2-e_1(e_1x_{m2}+x_{m1}x_{m2}-x_{m3})$$
$$u_2=-k_4(t)e_4-e_1(1+e_3+x_{m2}^2+x_{m3})-e_3(x_{m1}+x_{m2}) \quad (30)$$

and

$$\dot{k}_1(t)=(e_1+x_{m1})e_2e_3, \quad \dot{k}_2(t)=(e_1+e_1x_{m2}^2+e_3x_{m2})e_2, \quad \dot{k}_3(t)=e_2^2, \quad \dot{k}_4(t)=e_4^2 \quad (31)$$

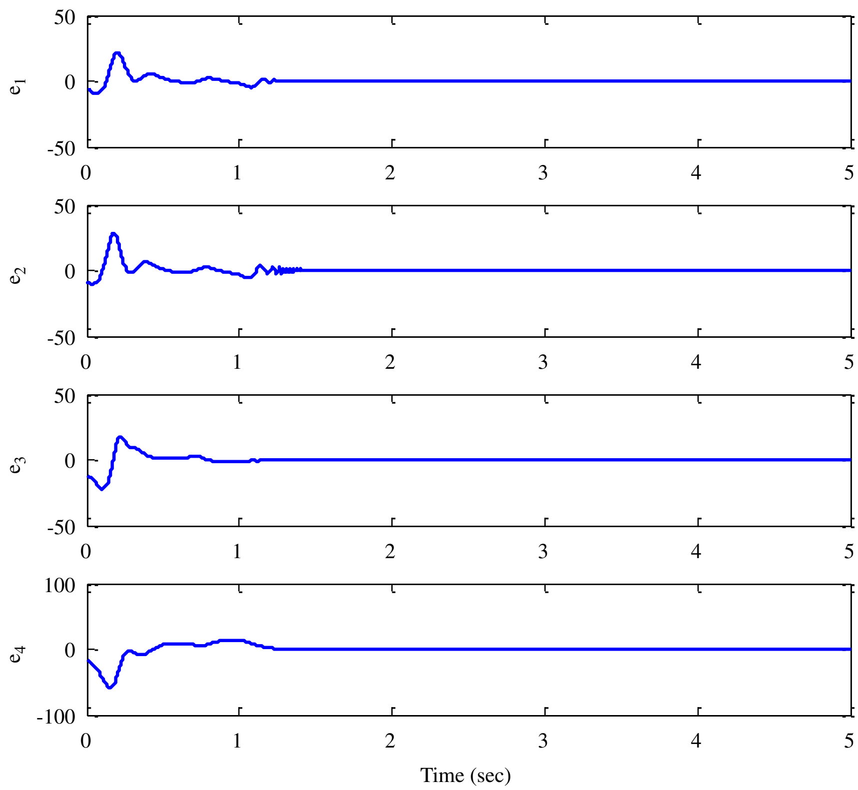

Fig. 1. Synchronous errors of the hyperchaotic Lü systems.

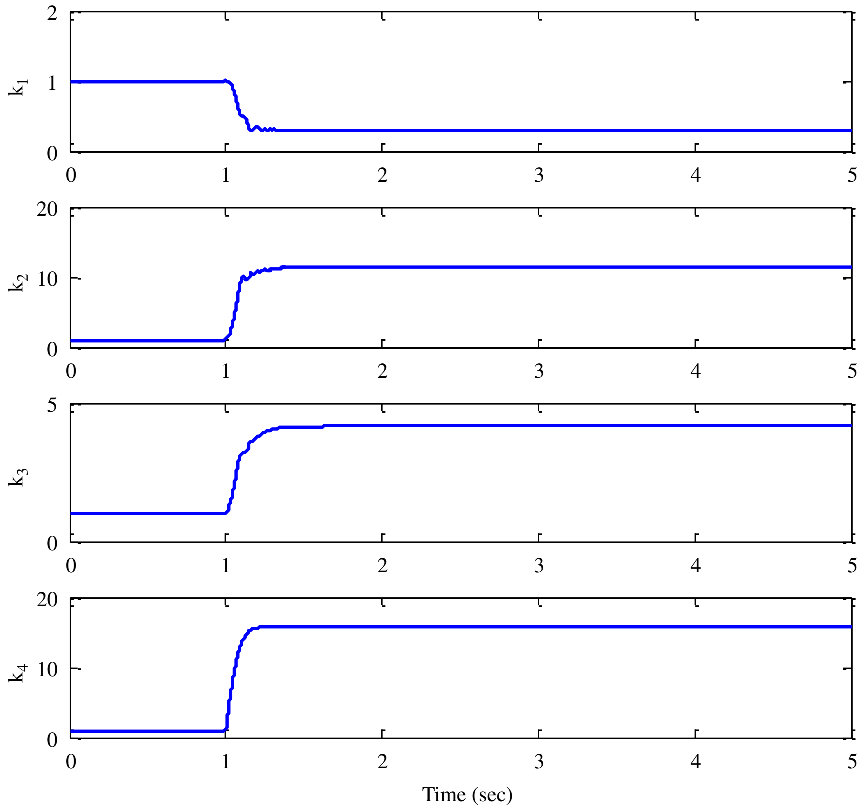

Fig. 2. Time varying gains for synchronizing the hyperchaotic Lü systems.

respectively. Thus, the synchronization error (17) with the adaptive control law (30) and the time varying gains (31) is asymptotically stable at the origin. Therefore, the synchronization errors converge to zero, resulting in two synchronized hyperchaotic Lü systems as desired.

Note that since the control law (30) is not simple the realization of the controller using hardware circuit may be too difficult. Thus, other realization technologies may be needed.

5. Simulation study

Numerical simulations are conducted in this section to confirm the effectiveness of the designed passivity-based adaptive controller. The parameters of the hyperchaotic Lü systems are selected as a=36, b=3, c=20, r=1.3, so the systems exhibit hyperchaotic behaviors. Fig. 1 shows the convergence of the synchronization errors. The initial conditions of the master system and the slave system are $x_m = [5 \quad 10 \quad 15 \quad 20]^T$ and $x_s = [1 \quad 2 \quad 3 \quad 4]^T$, respectively. The initial values of the gains $k_1(t)$, $k_2(t)$, $k_3(t)$, $k_4(t)$, are set to be one. The controller is activated at t=1 s. As it can be seen in Fig. 1, the adaptive control law (30) with the time varying gains (31) can effectively synchronize the slave system to the master system as desired. The convergence of the varying gains is shown in Fig. 2.

6. Conclusions

Adaptive synchronization control for hyperchaotic systems using a passivity feedback control with time-varying gains approach has been proposed. There are only two control signals needed, instead of four signals as done in Refs. [26,27]. By choosing a novel Lyapunov function for zero dynamics, the derived adaptive control law does not require the knowledge of the bound of a system state. This could be considered as an extension of the work in Ref. [13]. Numerical simulations have illustrated that the proposed adaptive controllers are able to drive the states of the slave hyperchaotic Lü system to asymptotically synchronize the states of the master hyperchaotic Lü system effectively.

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