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Sensitivity of footbridge vibrations to stochastic walking parameters

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ABSTRACT

Some footbridges are so slender that pedestrian traffic can cause excessive vibrations and serviceability problems. Design guidelines outline procedures for vibration serviceability checks, but it is noticeable that they rely on the assumption that the action is deterministic, although in fact it is stochastic as different pedestrians generate different dynamic forces. For serviceability checks of footbridge designs it would seem reasonable to consider modelling the stochastic nature of the main parameters describing the excitation, such as for instance the load amplitude and the step frequency of the pedestrian. A stochastic modelling approach is adopted for this paper and it facilitates quantifying the probability of exceeding various vibration levels, which is useful in a discussion of serviceability of a footbridge design. However, estimates of statistical distributions of footbridge vibration levels to walking loads might be influenced by the models assumed for the parameters of the load model (the walking parameters). The paper explores how sensitive estimates of the statistical distribution of vertical footbridge response are to various stochastic assumptions for the walking parameters. The basis for the study is a literature review identifying different suggestions as to how the stochastic nature of these parameters may be modelled, and a parameter study examines how the different models influence estimates of the statistical distribution of footbridge vibrations. By neglecting scatter in some of the walking parameters, the significance of modelling the various walking parameters stochastically rather than deterministically is also investigated providing insight into which modelling efforts need to be made for arriving at reliable estimates of statistical distributions of footbridge vibrations. The studies for the paper are based on numerical simulations of footbridge responses and on the use of Monte Carlo simulations for modelling the stochastic nature of actions of a single pedestrian traversing various pin-supported single-span footbridges with frequencies in vertical bending in the range of 1.6–2.4 Hz.

1. Introduction

Loads generated by pedestrians may cause resonance in footbridges, and this paper addresses vibration serviceability of slender footbridges focusing on their vertical response to walking excitation. It is often so that the governing design criterion for slender footbridges is vibration serviceability. Guidelines for a serviceability assessment were, for example, implemented in the Ontario Highway Bridge Design Code [1] in 1983 and in BS 5400 [2] in 1978 striving to avoid the construction of footbridges unfit for their intended use, for instance, by outlining procedures for calculation of vertical

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walking forces. Later a number of researchers [3–7] addressed the same topic either by publishing new data on the mechanism of walking or by suggesting guidelines for calculation of walking forces, and then the London Millennium Bridge incident occurred in the year 2000 [8]. The footbridge was closed on its opening day due to excessive vibrations caused by loads induced by pedestrians. Basically, the London Millennium Bridge incident raised a lot of questions related to modelling pedestrian loading and estimating vibration response of footbridges [8–10]. The focus of the present paper is on the basic philosophy employed for evaluating vibration serviceability of footbridges considering the random nature of walking.

1.1. A set of observations and reflections

When evaluating the ultimate limit state of structures, the random nature of a number of loads (e.g. wind loads) is built into design practice and codes, and thus acknowledged. However, when it comes to vibration serviceability of footbridges and walking loads, quite simple deterministic models are often suggested in design codes. For instance, a calculation of bridge response to walking loads might be performed by modelling the walking force (from a single (average) person) as a deterministic harmonic force moving across the bridge with an excitation frequency that is equal to the bridge resonance frequency [2]. The calculated bridge vibration level is then compared with a code-specified threshold, and the result of the comparison defines whether the bridge is serviceable or not. This is a very simplistic performance evaluation procedure, which neither accounts for details about actual bridge operation nor for the random nature of walking loads. For example, the bridge frequency might be in a range where resonant walking excitation would not occur very often considering the actual stochastic nature of pedestrian step frequency. This questions the evaluation approach, because it would seem logical to consider return periods of excessive vibrations in vibration serviceability assessments.

The step frequency of walking, mentioned above, is only one of the several parameters of the model describing the vertical load induced by a pedestrian. When studying research results presenting the outcome of experiments dealing with the mechanism of walking loads [4,5,11], it is obvious that also parameters as pedestrian stride length and load amplitude of the harmonic excitation are parameters which are fundamentally stochastic. Hence, the parameters of the load model (walking parameters) would vary from one pedestrians to another, which, however, are not accounted for in a number of code provisions. Sétra's technical guide for footbridges [12] is an exception as this guideline does at some level recognise the stochastic character of human-loading.

As would be obvious from the discussions above, a number of current codes do not in full recognise the complex nature of walking loads, which typically is modelled fully deterministically. Such approach does not provide any insight into the actual risk of encountering excessive bridge vibrations, as it does not acknowledge that different pedestrians excite the bridge differently. A probability-based framework for a footbridge vibration serviceability check was recently suggested by Živanovic [11], who, for example, considered the stochastic nature of dynamic load factor, step frequency, and stride length, which are some of the primary parameters describing the action of walking. Generally, such framework facilitates estimates to be made of the statistical distribution of footbridge vibration levels, which is considered a useful basis and approach for a discussion with bridge owners or operators prior to launching a footbridge design or in retrofit studies.

However, since there is no tradition of stochastic modelling of walking loads and walking parameters, there are no results, to the authors knowledge, that strive to identify which of the walking parameters are important to model stochastically for a probability-based estimation of bridge vibrations induced by a pedestrian. For instance, efforts might be spent on modelling a certain walking parameter stochastically, but it cannot be excluded that an almost similar statistical distribution of bridge response would be obtained had a deterministic model for the parameter been employed. Also no results are published that focus on the model uncertainties that may be associated with estimates of statistical distributions of footbridge vibration levels, considering that there are actually different proposals at hand for modelling the stochastic nature of the individual walking parameters. That different proposals are at hand becomes obvious when reviewing results of experimental efforts made by various researchers studying the mechanism of walking, and it would seem useful to consider their various results, which is possible employing a probability-based framework for prediction of bridge vibration levels.

1.2. The approach and scope of this paper

The reasoning behind the use of a stochastic approach for predicting vibration levels in footbridges is acknowledged and is therefore adopted for the studies of this paper focusing on the bridge response generated by a single pedestrian crossing the bridge. Still, however, a stochastic approach for estimating vibration levels requires input data and assumptions of the parameters describing the load (the parameters describing the harmonic excitation). To this end, it is required to settle on an approach for modelling the stochastic nature of dynamic load factors, of step frequencies and pedestrian weight, etc. Different research results on the stochastic nature of these parameters are available or can be derived from published experimental results dealing with the mechanism of walking. One of the purposes of this paper is to study the sensitivity of calculated statistical distributions of bridge vibration levels to some of the different assumptions that can be made about the stochastic nature of the different walking parameters. This is of interest because it reveals whether the engineer performing a probability-based calculation of bridge response would obtain a different result had he relied on another assumption about the stochastic nature of one of the walking parameters. A literature review lines up some of the assumptions that an engineer might rely on for a probability-based estimation of vibration levels and describes how they are implemented in a probability-based framework.

Živanovic et al. [13] employed a probability-based framework for calculation of the statistical distribution of bridge vibrations in a specific footbridge, but for this paper a general model of the dynamic characteristics of a pin-supported single-span footbridge is introduced. The model is somewhat simplistic as it assumes that parameters as bridge modal mass and bridge span length are deterministically linked with the flexural fundamental frequency of the bridge. But its main advantage is that it allows calculation of footbridge vibration levels under similar structural conditions for different assumptions made about walking parameters. By varying the bridge frequency in the range of 1.6–2.4 Hz, which corresponds to footbridges which are very sensitive to actions of walking, it is possible to conduct a comparative study investigating and illustrating how various load assumptions (various stochastic models for the parameters of the load model) affect predictions of vibration levels for a range of bridge frequencies. Hence, the studies of this paper investigate the sensitivity of footbridge dynamic performance to various ways of modelling the stochastic nature of parameters describing the action of walking. By varying the assumptions for walking parameters from stochastic to deterministic parameters (neglecting scatter), the study also allows for identifying the importance of modelling the stochastic nature of the various walking parameters.

The footbridge response considered for the study is the peak acceleration encountered at midspan of the assumed pin-supported footbridge and vibrations in the bridge are caused by single-person pedestrian traffic, but considering that one pedestrian to the next excites the bridge differently. The peak acceleration is determined numerically on different assumptions about the parameters employed for describing the action of walking. Statistical distributions of peak accelerations are derived by employing Monte Carlo simulation methods. In practise the loading induced by a single pedestrian might not be the worst case loading condition for a vibration serviceability check (although some current codes suggest that it is), but it is considered a useful reference case for exploring the sensitivity of calculated statistical distributions of footbridge response to various ways of modelling the stochastic nature of the action.

1.3. Outline of paper

Section 2 outlines the basic assumptions adopted for modelling walking loads, and presents the basic equation for the modal load on a pin-supported single-span footbridge, hereby illustrating which parameters are involved in a mathematical description of the action. Section 3 reviews research results published on the stochastic nature of the parameters of the basic equation describing the load (the walking parameters), thus outlining some of the modelling options available for a stochastic description of the action of walking. A set of deterministic models for the walking parameters are also identified. The options available for describing walking parameters can be combined in a variety of ways, and Section 4 outlines the combinations considered for the studies of this paper, and describes the general model of dynamic characteristics of footbridges employed for the studies. Furthermore, Section 4 describes some of the assumptions made for the numerical simulations, and Section 5 presents the results covering a presentation of the sensitivity of the statistical distribution of footbridge vibration levels to various ways of modelling parameters of the load model. Tendencies in results and conclusions are discussed in Section 6.

2. Modelling of walking loads and response calculation

This section outlines the basic model assumptions adopted for describing vertical walking loads and the basic equation describing the modal load of the action (Section 2.1) and furthermore, the section describes how the vertical response of a pin-supported single-span footbridge excited by pedestrian traffic is calculated (Section 2.2).

2.1. Modelling of vertical excitation from a pedestrian

The dynamic part of the load-time history of a single pedestrian, $f(t)$, is modelled using the Fourier series expansion shown as follows (as suggested in [3–7]):

$$f(t) = W \sum_{i=1}^{M} \alpha_i \cos(2\pi i f_s t + \varphi_i), \tag{1}$$

where α_i represent dynamic load factors (Fourier coefficients) for the various load harmonics, and the load factors basically describe the amplitudes of the M harmonics chosen to describe the action. W represents the weight of the pedestrian (mass times gravity), and φ_i are the phase lags associated with the various harmonics. The step frequency is assigned f_s and represents the frequency of heel impacts during the locomotion. For the studies of this paper, it is considered sufficient to account for the dynamic part of pedestrian loading (the load beyond the static weight W).

If the pedestrian traverses a pin-supported footbridge of length L and of uniform cross-section, the modal load associated with the first vertical flexural mode of the bridge is

$$q(t) = \Phi(t) f(t), \tag{2}$$

where $\Phi(t)$ represents the value of the mode shape function at the position of the excitation force $f(t)$, which is

$$\Phi(t) = \sin\left(\pi \frac{vt}{L}\right), \quad 0 < t < t_e, \ \ t_e = \frac{L}{v}, \tag{3}$$

provided that the pedestrian starts his locomotion at one end of the bridge and continues to the other end walking at a constant pacing speed, v, which is assumed for the studies of this paper. For $t > t_e$, where t_e is the point in time at which the pedestrian leaves the bridge, $\Phi(t) = 0$ is assumed. A schematic of the situation considered is shown in Fig. 1.

Another parameter that can be employed in the description of the excitation is the stride length (or step length), l_s, of the pedestrian, which can be calculated using

$$l_s = \frac{v}{f_s}. \tag{4}$$

At first sight, it may seem superfluous to introduce this parameter, as it is described by two other parameters already introduced for describing the action (v and f_s). However, it proves useful to introduce l_s as from some experiments dealing with modelling the action, knowledge about l_s rather than v is gained.

For the studies of this paper, focus will be on bridges with fundamental frequencies in vertical bending in the range of 1.6–2.4 Hz and a later section will demonstrate that this is a bridge frequency range for which it is likely that the first harmonic of walking excitation will coincide with the bridge frequency and generate resonance in the bridge. Only the first harmonic of the excitation (i=1) will be considered and the bridge will be idealised as a SDOF system (single-degree-of-freedom system) represented by the dynamic features of the first vertical flexural mode of the bridge, which is fairly reasonable for pin-supported single-span bridges. On these assumptions, the modal load associated with the first flexural mode reduces to

$$q(t) = W\alpha_1 \cos(2\pi f_s t + \varphi_1) \sin\left(\pi \frac{vt}{L}\right), \quad v = f_s l_s, \tag{5}$$

which is a result obtained by inserting Eqs. (1) and (3) into Eq. (2). For the studies, the phase φ_1 will be assumed equal to 0°. For short bridges, bridge vibration levels would to some degree depend on which value is chosen for φ_1, but as the bridges studied in this paper generally are quite long, it is considered reasonable simply to assume φ_1 to be zero. Taking this approach, the governing parameters in terms of excitation generated by a single pedestrian are:

- The step frequency, f_s.
- The pacing speed, v.
- The dynamic load factor, α_1.
- The weight of the pedestrian, W.

For simplicity, the dynamic load factor α_1 is throughout the remaining part of the paper denoted α as there is no need to keep track of the harmonic component of the load (the subscript) since only the first harmonic is considered. The stride length, l_s, is not listed above, but stride length will be considered as information about stride length is required for calculating pacing speed v. The parameters mentioned above represent the walking parameters considered in this paper. As can be seen in Eq. (5), a geometrical parameter describing the bridge also influences the modal load, namely the length of the bridge, L.

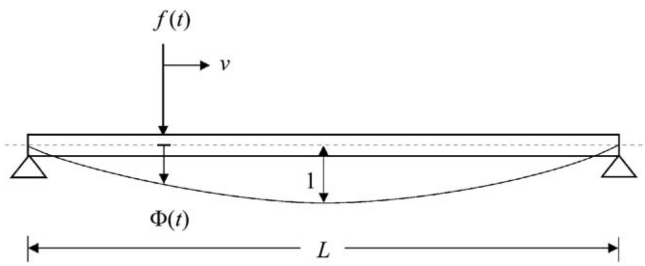

Fig. 1. Model of a pedestrian crossing a bridge.

2.2. Response of structural system to the excitation

On the assumption that the system excited by the pedestrian is a linear SDOF-system (viscously damped and linear elastic) and that it represents the first flexural mode of a footbridge, the equation of motion is

$$ma(t)+cw(t)+kx(t)=q(t), \tag{6}$$

where

$$c=4\pi m f_0 \zeta, \quad k=m(2\pi f_0)^2, \tag{7}$$

and where $a(t)$, $w(t)$, and $x(t)$ represent the time history of vertical footbridge displacement, velocity, and acceleration of bridge midspan, respectively. The parameters c and k are the damping coefficient and the stiffness of the studied mode of vibration. Knowing $q(t)$ and the dynamic characteristics of the system (modal mass m, undamped natural frequency f_0, and the damping ratio ζ), the response time histories ($x(t)$, $w(t)$, $a(t)$) can be computed for instance by employing a Newmark time integration scheme. For the computations, the bridge is assumed at rest when the pedestrian enters the bridge (t=0 s), i.e. $x(0)$=0 m, and $w(0)$=0 ms^{-1} is assumed. For a vibration serviceability check of a bridge, the acceleration response is of interest and need to be calculated. First, however, the walking loads need to be known or assumed. The next section deals with walking loads and the parameters that describe the action.

3. Parameters describing walking loads—a literature review

This section describes the results of a literature review focusing on different manners in which the stochastic nature of the vertical excitation from a single pedestrian may be described. The models in focus are those describing the statistical distribution of the parameters: (i) step frequency; (ii) pacing speed; (iii) dynamic load factor; and (iv) pedestrian weight. The review is reported dealing with the models in the line of order listed above; each model is assigned a separate subsection. At the end of each subsection a summary is given outlining which models will be considered for further study and thus for estimating statistical distributions of footbridge vibrations and for studying how sensitive these distributions are to the choices made about how to model the different walking parameters.

3.1. Models for step frequency

Dictating the probability of resonant excitation, the step frequency is a central parameter in probability-based estimation of bridge vibration levels. Proposals for the probability density of the step frequency of walking are reviewed below.

The first comprehensive investigations dealing with step frequencies of pedestrians in a statistical context were published by Matsumoto et al. [14]. Test subjects of 505 traversed a structure one by one, and the step frequency of each individual was identified. Matsumoto fitted a normal probability density function, $f_s \sim N(\mu_f, \sigma_f)$, to his results where μ_f and σ_f represent the mean value and the standard deviation of f_s, respectively. The notation N indicates that a normal (Gaussian) distribution is assumed and this way of indicating the assumption of a normal distribution (with mean value and standard deviation defined in the bracket after N) is used throughout the paper. Matsumoto obtained his best fit when the mean value (μ_f) and standard deviation (σ_f) of f_s were given the values of 1.99 and 0.173 Hz, respectively. The results obtained by Matsumoto date back to 1978, but since then other researchers have recorded step frequencies of pedestrians and arrived at somewhat different results in terms of values of μ_f and σ_f. Results obtained by Matsumoto and by others are listed in Table 1.

Actually, not all researchers mentioned above argued or verified that their data fitted well to a normal distribution, and some of the mean values and standard deviations listed in Table 1 thus represent empirical statistical parameters rather than parameters obtained by fitting a normal distribution to collected data sets. However, the quite substantial data sets collected by Matsumoto and by Živanovic suggest that the normal distribution is a useful distribution for describing the scatter in data, and, therefore, the normal distribution is assumed for the studies of this paper using the parameters μ_f and

Table 1
Mean value and standard deviation of step frequency reported by different researchers.

Reference	μ_f (Hz)	σ_f (Hz)	Subjects tested
Matsumoto et al. [14]	1.99	0.173	505
Schultze [15]	2.0	0.13	–[a]
Kramer et al. [16]	2.2	0.3	–[a]
Živanovic et al. [13]	1.87	0.186	1976
Kerr [5]	1.90	–[a]	40
Kasperski et al. [17]	1.82	0.12	250

The number of subjects tested is also indicated.
[a] The authors of this paper have not been able to identify the figure.

σ_f in Table 1 to describe the statistical distribution. For simplicity, the quality of the fitted normal distributions is not presented in this paper (comparing best fits to actual data sets), but the interested reader may find such comparisons in the stated references.

The manner in which the step frequency of a pedestrian is determined varies from one research effort to the next. Also in the different test efforts, the test environment differs as do the methods employed for determining step frequency, and by nature the tested population is not the same from one research effort to the next. It cannot be excluded, and it is actually likely that some of these factors have a bearing on the end result (the parameters of the fitted normal distribution). For details about the different tests, reference is given to the stated references. However, the bottom line is that there are different proposals in the literature on the likelihood of step frequencies that one may choose to rely on when using a probability-based approach to estimate vibrations in footbridges.

For the further studies of this paper (employing different step frequency models for predicting statistical distributions of bridge vibration), it is considered relevant to focus on distributions that are obtained by fitting to substantial amounts of data. However, at the same time it is of interest to consider distributions that reflect the diversity in mean values of step frequencies obtained in the different research works. These considerations have led to the decision of employing the distributions indicated below for the further studies of this paper.

A1: $f_s \sim N(\mu_f, \sigma_f) = N$ (1.87, 0.186 Hz), Živanovic et al. [13].
A2: $f_s \sim N(\mu_f, \sigma_f) = N$ (1.99, 0.173 Hz), Matsumoto et al. [14].
A3: $f_s \sim N(\mu_f, \sigma_f) = N$ (1.82, 0.120 Hz), Kasperski et al. [17].
A4: $f_s \sim N(\mu_f, \sigma_f) = N$ (2.20, 0.300 Hz), Kramer et al. [16].

Also it has been chosen to employ the deterministic model:

A5: $f_s \sim D(f_s) = D(f_s = f_0)$,

which basically assumes that the step frequency, by default, coincides with the resonant frequency of the bridge, f_0. The letter D is used throughout the paper for indicating when a model is deterministic rather than stochastic. In the result section of this paper, the calculation assumption made concerning step frequency of the pedestrian is specified by referring back to either model A1, A2, A3, A4, or A5.

Fig. 2 visualises the normal probability density functions for the models A1, A2, A3, and A4.

The models emphasise that for bridges with frequencies in a certain region around 2 Hz, there is a relatively high probability of resonant excitation, which is one of the main reasons why the further studies of this paper considers bridges with natural frequencies in the frequency range of 1.6–2.4 Hz.

Models for the pacing speed of the pedestrian are addressed next.

3.2. Models for pedestrian pacing speed

For the calculations of this paper, pedestrian pacing speed ($v = f_s l_s$) is assumed to be constant during the locomotion of a pedestrian, but it is assumed to vary from one pedestrian to another. Section 3.1 outlined different stochastic models for the step frequency (f_s), but models for the stride length (l_s) will also have a bearing on the pacing speed (v). Hence, this

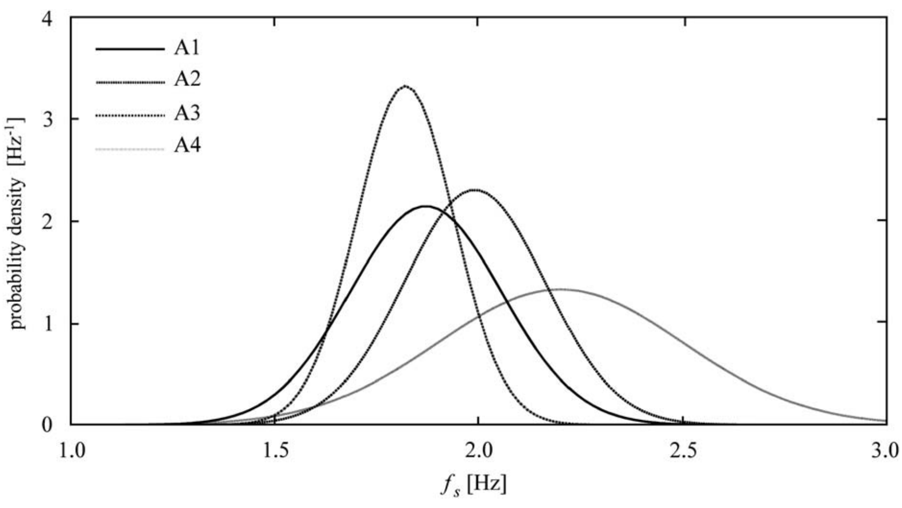

Fig. 2. Probability density functions for step frequency of walking.

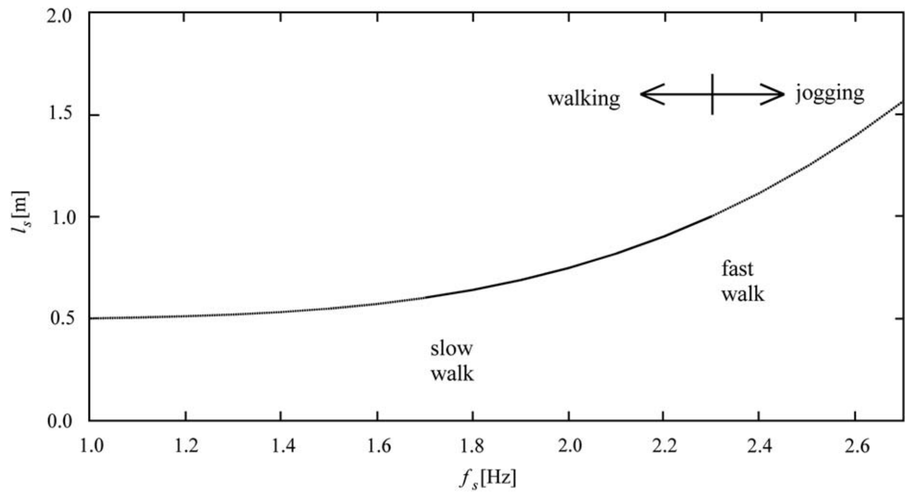

Fig. 3. Relationship between step frequency and stride length in the 1–2.7 Hz range (after Wheeler [18]).

section reviews different models proposed in literature on the stride length of pedestrians thus completing the basis necessary for computing pacing speed.

Wheeler [18] published results on stride length which suggested that the stride length is correlated with step frequency. Wheeler suggested the relationship shown in Fig. 3.

Actually, the relationship shown in Fig. 3 is a polynomial approximation to the suggestion by Wheeler, who did not describe the relationship by a mathematical expression, but only presented it graphically by a continuous branch in a (f_s, l_s)-plot. The mathematical expression in Eq. (8) is thus one made by the authors of this paper, but it fits well to the relationship (branch) suggested by Wheeler:

$$l_s(f_s) = 0.2011 f_s^3 - 0.6021 f_s^2 + 0.6462 f_s + 0.2547, \quad 1 < f_s < 2.7\,\text{Hz}. \tag{8}$$

Inserting f_s in Hz (s^{-1}) gives l_s in m. In the work by Wheeler [18] it was mentioned that "a degree of intuitive reasoning was applied to produce continuous curves" (to produce the branch in the (f_s, l_s)-plot). As his data basis encompassed walking, jogging, and running (the excitation frequency range of 1–5 Hz), it might suggest some shortcomings in the relationship in the frequency range of 1–2.7 Hz. This is because curves needed to be "smoothed" to obtain a simplistic and continuous relationship for the entire frequency range of 1–5 Hz. If the relationship suggested by Wheeler [18] solely reflects how a specific person adjusted his stride length to the step frequencies tested, it would not fully reflect the mechanism which one aims to model for probability-based estimation of footbridge response. This is because it cannot be ruled out that different pedestrians would choose somewhat different stride lengths for identical values of the step frequency. Regardless of these considerations, the model by Wheeler is employed for the further studies of this paper, as basically it may be a model chosen by engineers for modelling pacing speeds of pedestrians. From Eq. (8), the pacing speed of the pedestrian may be calculated using the basic relationship $v = f_s l_s$.

Contrary to Wheeler's results, it is suggested in works by Živanovic [11], that stride length and step frequency are independent random variables. At least this is the model assumed by Živanovic for constructing a probability-based model for estimating footbridge vibrations. Živanovic [11,13] proposes a normal distribution for the stride length, $l_s \sim N(\mu_l, \sigma_l)$, in which a mean stride length of $\mu_l = 0.71$ m and a standard deviation of $\sigma_l = 0.071$ m are assumed. Kasperski et al. [17] reported results for the average stride length of 0.75 m with an empirical standard deviation of 0.07 m, which are results that agree reasonably well with the distribution assumed by Živanovic.

More research results on stride length could be added, but for the further studies of this paper it is considered sufficient to employ models for stride length ranging from a model in which it is independent of step frequency (the model by Živanovic [11]) and a model in which the stride length is deterministically linked with step frequency (the model by Wheeler [18]). Thus, in the latter model, the pacing speed of the pedestrian is basically just a function of step frequency.

So for the further studies of this paper, the following relationships are considered for modelling the pacing speed, v, of pedestrians:

B1: $v = f_s l_s$, where f_s and l_s are independent random variables modelled as: $l_s \sim N(\mu_l, \sigma_l) = N(0.71\,\text{m}, 0.071\,\text{m})$ and $f_s \sim N(\mu_f, \sigma_f)$,
B2: $v = f_s l_s$, where $l_s(f_s)$ is determined according to Eq. (8) in which $f_s \sim N(\mu_f, \sigma_f)$,

where values of μ_f and σ_f depend on which research result is considered for the step frequency of walking, refer Section 3.1. As reference, a third model (B3) is considered in which the scatter of stride length is neglected:

B3: $v = f_s l_s$, where f_s and l_s are independent variables modelled as: $l_s \sim D(l_s) = D(l_s = \mu_l = 0.71\,\text{m})$ and $f_s \sim N(\mu_f, \sigma_f)$.

Basically, model B3 is a deterministic variant of model B1 as far as stride length is concerned (always equal to 0.71 m, which is the mean value for stride length employed in model B1). The use of model B3 for the further studies of this paper allows for examining implications of disregarding scatter of stride length in computations of statistical distributions of bridge responses.

3.3. Models for the dynamic load factor

Various researchers have published results on dynamic load factors describing the action of walking. Some of the results are reviewed below. As previously mentioned focus of this paper is on the first harmonic of the action.

Kerr [5,19] employed an instrumented force plate to record the vertical load-time history of single footfalls. Tests were made with 40 different test subjects who approached the force plate using different step frequencies, f_s. From the footfall load-time history, Kerr identified the dynamic load factor, α. Around 1000 combinations of the set (f_s, α) were identified, and Kerr made the polynomial fit (shown in Eq. (9)) to the measured relationship between the dynamic load factor (μ_α, non-dimensional) and step frequency (f_s, inserted in Hz (s^{-1})).

$$\mu_\alpha(f_s) = -0.2649 f_s^3 + 1.3206 f_s^2 - 1.7597 f_s + 0.7613, \quad 1 < f_s < 2.7\,\text{Hz}. \tag{9}$$

Eq. (9) is a calibration result for values of f_s in the range of 1–2.7 Hz as step frequencies in this range were tested although it proved quite difficult to maintain a natural style of walking for step frequencies above 2.3 Hz. In Eq. (9), the notation μ_α is used for the dynamic load factor instead of simply α. The reason for this is that α is considered a random variable and because the best fit polynomial, $\mu_\alpha(f_s)$, is assumed to represent the conditional mean value for α at a given value of f_s. Kerr also addressed the scatter in α conditioned on f_s. This he did in the frequency range of 1.6–2.2 Hz. From his results and assuming a normal distribution of α for any step frequency in this range, the standard deviation of α can be identified to equal:

$$\sigma_\alpha(f_s) = 0.16\mu_\alpha(f_s). \tag{10}$$

For simplicity, this relationship will also be assumed to be valid beyond the frequency range of 1.6–2.2 Hz for the further studies of this paper. Eqs. (9) and (10) describe the parameters of the conditional normal distribution $N(\mu_\alpha(f_s), \sigma_\alpha(f_s))$ for the dynamic load factor constructed from the relationship between α and f_s measured by Kerr. For the entire set of data acquired by Kerr, reference is made to his publications.

Another suggestion to the relationship between α and f_s is seen in the work by Ellis [4]. Based on measurements of structural responses to single-person pedestrian traffic, Ellis [4] identified the dynamic load factor for step frequencies in the range of 1.7–2.4 Hz. The general approach was to have a test subject to walk across floors with known dynamic characteristics. Since the dynamic characteristics of the floors used in tests were known, characteristics about the excitation could be identified from recordings of floor response to the walking load. In total about 60 combinations of the set (f_s, α) were identified. A least square polynomial fit to the results of Ellis [4] is made by the authors of this paper and is shown in Eq. (11) and it is assumed to represent the conditional mean value for the dynamic load factor at a given value of f_s:

$$\mu_\alpha(f_s) = 4.1407 f_s^5 - 43.3815 f_s^4 + 181.0209 f_s^3 - 376.6609 f_s^2 + 391.8452 f_s - 163.16. \tag{11}$$

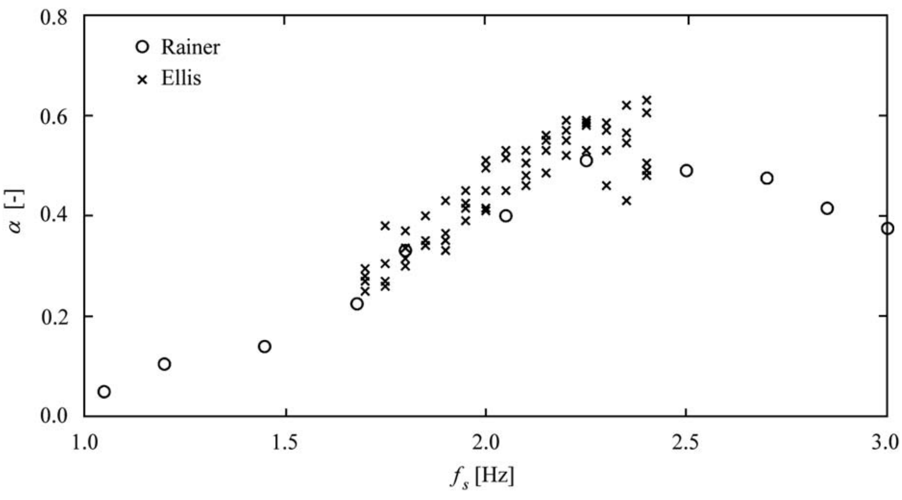

Fig. 4. Relationships between step frequency and dynamic load factor as measured by Rainer et al. [7] and Ellis [4] (as accurately reproduced as possible without access to the actual data).

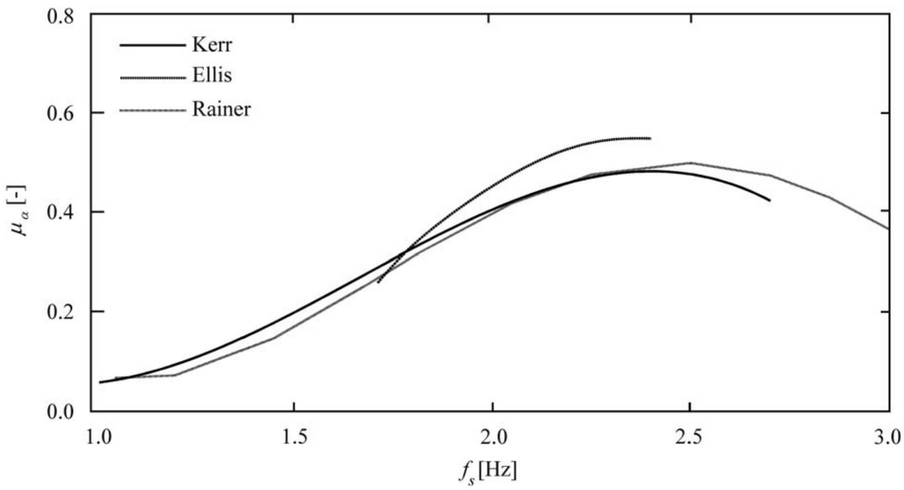

Fig. 5. The conditional mean value for the dynamic load factor, $\mu_\alpha\,(f_s)$, identified by polynomial fits to measurements made by Kerr [5], Ellis [4] and Rainer et al. [7].

Inserting f_s in Hz (s^{-1}) gives $\mu_\alpha(f_s)$; non-dimensional. The results of Ellis [4] showed some scatter for the specified values of f_s, as can be seen in Fig. 4, but for the studies of this paper it is chosen to focus only on the conditional mean value for the dynamic load factor and its variation with f_s.

Rainer et al. [7] also estimated dynamic load factors. By monitoring walkway reaction forces, they could derive dynamic load factors for individuals traversing a laboratory walkway. Their experimental data were few, as can be seen in Fig. 4. The authors of this paper have made a least square polynomial fit to the data, which is shown in Fig. 5 along with the polynomial fits to data measured by Ellis and Kerr, respectively.

As can be seen from the polynomial fits in Fig. 5, especially the variations measured by Rainer and Kerr do not deviate much, and therefore only results obtained by Kerr and Ellis will be considered for the further studies of this paper. The models considered are:

C1: $\alpha \sim N(\mu_\alpha(f_s),\ \sigma_\alpha\ (f_s))$ where $\mu_\alpha(f_s)$ and $\sigma_\alpha(f_s)$ are given by Eqs. (9) and (10), respectively. A stochastic model based on results of Kerr [5].
C2: $\alpha \sim D(\alpha\ = \mu_\alpha(f_s))$ where $\alpha = \mu_\alpha(f_s)$ is given by Eq. (9). A deterministic variant of the results of Kerr [5].
C3: $\alpha \sim D(\alpha\ = \mu_\alpha(f_s))$ where $\alpha = \mu_\alpha(f_s)$ is given by Eq. (11). A deterministic variant of the results of Ellis [4].

Model C2 is included so as to facilitate investigation of the implications of disregarding the scatter observed for the dynamic load factor around its conditional mean value in computations of statistical distributions of bridge responses. For simplicity a stochastic model based on results measured by Ellis [4] is not addressed, although it would be possible to construct such model.

Generally $f_s - \mu_\alpha$ relationships beyond the step frequency ranges used in experiments, $f_l < f_s < f_h$, are not known, but values of the dynamic load factor below f_l and above f_h are needed for the further studies of this paper. It is chosen to employ the value $\mu_\alpha(f_s{=}f_l)$ for $f_s < f_l$, and $\mu_\alpha(f_s{=}f_h)$ for $f_s > f_h$. This quite simplistic extrapolation of the relationship $\mu_\alpha(f_s)$ is considered overall reasonable in view of the limited likelihood of encountering step frequencies beyond the range $f_l < f_s < f_h$; at least for most of the probabilistic models discussed in Section 3.1. It was not mentioned in Section 3.2, but for model B2 it is also assumed for further calculations that the stride length assumes values corresponding to those at $f_s{=}1.0$ and 2.7 Hz, for values of f_s below or above these limits, respectively.

3.4. Models for pedestrian weight

Obviously, the weight of the pedestrian, *W*, has an impact on the excitation force, as was seen in Eq. (5). Generally, here is no restriction as to who transverses a bridge, and it is therefore reasonable to consider modelling the weight of the pedestrian as a random variable. Nevertheless, for evaluating magnitudes of vibrations in footbridges, it is common to consider the weight of the pedestrian as a deterministic parameter, for instance by using a weight of 750 N, but this figure might not be representative of the actual population of pedestrians.

For the further studies of this paper, the weight of the pedestrian is modelled as a random variable with a mean value and a standard deviation. The mean value of human body weight, however, is not a universal figure as indicated in Table 2 listing mean weights of males and females in three different countries.

Table 2
Average weights of males and females in different countries.

Country	Male (kg)	Female (kg)	Reference
USA	86	74	Wikipedia [20]
Canada	83	69	Wikipedia [20]
UK	80	67	Wikipedia [20]

Table 3
Mean human body weight, μ_W, and standard deviation, σ_W, for the four different populations considered.

Population	1	2	3	4
μ_W (N)	550	650	750	850
σ_W (N)	$0.18\mu_W$	$0.18\mu_W$	$0.18\mu_W$	$0.18\mu_W$

The variability of the mean value of human body weight would probably be even more pronounced than indicated in Table 2, if, for example, average weights of people in Asian countries were included in the table. Hence, walking loads might actually vary by several percent depending on the geographical location of the bridge. To the authors knowledge no attempts have previously been made to model the weight of the pedestrian as a random variable in a probability-based framework for a vibration serviceability check. Such attempt will be made here. For the construction of a probabilistic model of pedestrian weight for use in the studies of this paper it is chosen to consider four different imaginary populations of pedestrians. Each population is described by the model $W \sim N(\mu_W, \sigma_W)$ with the mean values and the standard deviations listed in Table 3.

Population 1 assumes a mean weight of approximately 55 kg whereas, for example, population 4 assumes a mean weight of approximately 85 kg. As it appears from Table 3, the standard deviation is assumed to be equal to $0.18\mu_W$ for the four populations. The value of $0.18\mu_W$ was obtained by weighing 77 students at Aalborg University, Denmark, and subsequently relating the empirical standard deviation of their weight to the average weight of individuals in the group. It must be stressed, however, that neither the measurements made at Aalborg University nor measurements of weights of human populations reported in [21] support that a normal distribution is ideal for modelling the stochastic nature of pedestrian weight. Although the modelling approach have shortcomings it is considered suitable as a first attempt to examining how the modelling of the stochastic nature of pedestrian weight influences statistical distributions of footbridge responses.

The models considered for the studies of this paper are summarised below:

D1: $W \sim N(\mu_W, \sigma_W) = N(750\,\text{N}, 18\ \text{percent of}\ 750\,\text{N})$.
D2: $W \sim N(\mu_W, \sigma_W) = N(550\,\text{N}, 18\ \text{percent of}\ 550\,\text{N})$.
D3: $W \sim N(\mu_W, \sigma_W) = N(650\,\text{N}, 18\ \text{percent of}\ 650\,\text{N})$.
D4: $W \sim N(\mu_W, \sigma_W) = N(850\,\text{N}, 18\ \text{percent of}\ 850\,\text{N})$.
D5: $W \sim D(W) = D(W = 750\,\text{N})$.

The models D1–D4 are stochastic models. As reference a deterministic model (model D5) is also considered in which the pedestrian weight is assumed to correspond to approximately 75 kg. This model is introduced so as to facilitate studying implications of modelling pedestrian weight deterministically (model D5) versus modelling it stochastically (model D1) in respect to the end result being the statistical distribution of vibrations in the bridge.

3.5. Supplementary comments and study assumptions

It would be possible to add even more research results to the literature review presented above, but as the paper is also to deal with a study employing the findings of models for the walking parameters, it is considered appropriate to keep the scope of the literature review at the current level. However, the interested reader may find more models and details on the mechanism of walking in Refs. [22–27].

For the studies to be made next, employing the models identified in the literature review, it is modelled that the dynamic load factor, α, depends on the step frequency, f_s, and that pacing speed, v, depends on the parameters f_s and l_s. It is common to assume that the variables f_s, l_s, v, and α are independent of pedestrian weight, W, and thus this assumption is made for the studies of this paper, although it cannot be ruled out that further studies into the mechanism of walking will reveal some degree of dependency between the parameters in question. For instance it might not be fully correct to assume that α is independent of W although this is often assumed when modelling walking loads.

For the study it is also assumed that pacing speed, v, is constant during the locomotion of the pedestrian. This is also an approximation traditionally used for calculations of vibrations in footbridges, and thus used here. It is recently suggested though that the load model (Eq. (5)) can be improved, as there are aspects of the load mechanism which it does not embrace, such as the inability of a pedestrian to produce a locomotion style that ideally matches the periodic excitation assumed for Eq. (5) [26,27]. For simplicity, these advanced aspects of the problem of modelling walking loads are not considered for the studies of this paper.

Basically, it is chosen to rely on a simple and traditionally used model structure for the load (Eq. (5)) well aware that over time, more refined models may be developed that more thoroughly describe all details of the complex action. The study focus is on the *sensitivity* of bridge response to model uncertainties associated with the parameters f_s, v, α and W. It is thus a comparative study where the objective is to look at the *degree of differences* in results of bridge vibration levels calculated on various model assumptions about the parameters in question, rather than a study into the *actually* calculated levels of bridge vibration. Had the aim of the paper been on state-of-the-art estimates of footbridge vibration levels it might be necessary to employ more advanced load and structural models, but for the comparative studies of this paper, the simplifying assumptions are considered useful.

4. Methods

A variety of models of walking parameters has been identified in the literature review, Section 3. In the further studies of this paper, statistical distributions of bridge vibrations are determined for different load assumptions, but it is useful to extract essential and condensed information from the distributions. Section 4.1 describes the approach taken. The various ways in which walking parameters are modelled to explore model uncertainty associated with estimating statistical distributions of bridge response are summarised in Section 4.2, and Section 4.3 outlines the dynamic configurations of the footbridges that are considered.

4.1. The parameter representing footbridge response

It is a quite novel approach to consider statistical distributions of bridge response for vibration serviceability checks, but it would seem sensible to focus on the likelihood of the most extreme response events and to explore how accurately these responses may be predicted (considering the various options available for modelling walking loads). Inspired by Ref. [13] focus is on the 95-th percentile (or 95 percent-quantile) of bridge accelerations (the vibration level with a 5 percent probability of being exceeded), here denoted a_{95}. For 1 out of 20 crossings, the value of a_{95} would be expected to be exceeded. The percentile a_{95} is extracted from the simulated statistical distribution and it is the sensitivity of the estimate of a_{95} that is examined in the paper. It is beyond the scope of the paper to address human acceptance levels of vibrations. Humans perceive vibrations differently and basically the acceptance level of vibrations is a random variable in itself. But it should be kept in mind that it is the model employed for the acceptance level (and the statistical basis it relies on) and how distant it is from a_{95} that determines the rate of crossings that are expected to be judged unacceptable by pedestrians. In some presentations of results (in Section 5), not only results for the percentile a_{95}, but also the results for the percentiles a_{75} and a_{50} are shown.

4.2. The load assumptions and combinations hereof

Considering that there are several options available for modelling walking loads, it is useful to establish a reference load case and to calculate the statistical distribution of footbridge response for this load case. The reference load case considered is defined in Table 4.

Recalling the models presented in Section 3, the reference load case assumes a fully stochastic model of walking parameters, in which all walking parameters are modelled as random variables.

It is considered useful to define a set of load cases that are different from the reference load case (implementing variability in the modelling of walking loads by considering different ways of modelling walking parameters) and to calculate statistical distributions of bridge vibration levels for these load cases, and finally to compare the results.

A variability in modelling walking loads is implemented firstly by assuming different models of step frequency (A1…A5) while models for pacing speed, dynamic load factor and pedestrian weight are kept constant (and identical to the models assumed for the reference load case: B1, C1, and D1, respectively). Then variability in the modelling of pacing speed

Table 4
The reference load case considered for this paper.

Walking parameter	Model	Reference to
Step frequency, f_s	A1	Section 3.1
Pacing speed, v	B1	Section 3.2
Dynamic load factor, α	C1	Section 3.3
Weight of pedestrian, W	D1	Section 3.4

Table 5
The load cases considered for this paper.

Variability in modelling	Models assumed for the walking parameters			
	f_s	ν	α	W
f_s	A1…A5	B1	C1	D1
ν	A1	B1…B3	C1	D1
α	A1	B1	C1…C3	D1
W	A1	B1	C1	D1…D5

The individual models are all described in Section 3.

(models B1–B3) is considered while other parameters are defined as in the reference load case, and so forth. A schematic showing the load cases considered for this paper is presented in Table 5.

Having defined the load cases, it is appropriate now to define the structural system.

4.3. The structural systems considered

As previously mentioned, the study focuses on the vertical response of footbridges to the action of walking, with particular focus on pin-supported single-span footbridges (modelled as SDOF systems) excited by the first harmonic of walking loads. For the study it is chosen to examine the response of such systems with natural frequencies, f_0, in the range of 1.6–2.4 Hz. In order to do that, a bridge model in which all relevant dynamic characteristics of the bridge are uniquely defined is required. It would be useful, if the bridge frequency, f_0, would provide information on the values of some of the other modal parameters describing the structural system.

A change in footbridge natural frequency would normally be associated with a change in footbridge modal mass, m, and/or length of the footbridge, L. It is quite difficult to generalise how f_0, m, and L interact as footbridge designs differ from one to the next, but nevertheless for this paper an attempt is made to model somewhat realistic relationships between f_0, m, and L. This has resulted in the following relationships:

$$m(f_0) = 158,000/f_0^2, \quad L(f_0) = 86/f_0, \tag{12}$$

which are assumed for the studies of this paper. Inserting f_0 in Hz (s^{-1}) gives m and L in kg and m, respectively. It can be verified that this approach to modelling relationships between the parameters m, L and f_0, is consistent with assuming equal flexural stiffness of all footbridges (regardless of the value of f_0).

When results are presented in the next section on response of different footbridges to pedestrian traffic, the bridges will only be identified by their natural frequency, f_0, and their damping ratio, ζ. The modal mass and the length of the bridge are calculated using Eq. (12). A damping ratio, ζ, equal to 0.5 percent of critical damping is assumed, which according to ranges of damping ratios reported in [6] could be a realistic value for footbridges made of steel, prestressed concrete and for composite footbridge designs as well. For the study, values of f_0 of 1.6–2.4 Hz are considered with incremental steps of 0.016 Hz.

The prime purpose of introducing the general footbridge model, Eq. (12), is to assure that a common structural basis is available for calculating structural vibrations on different load conditions (for different load assumptions) as this allows for examining the degree of difference in results that originate from the load assumption.

With this purpose in mind, it is therefore not considered a requirement that the footbridge model is capable of reproducing actual relationships between dynamic characteristics of footbridges to a high level of detail and generally it would not be possible considering the variability of footbridge designs and the variability in use of materials for footbridges. But it is useful that it models the prime mechanisms of pin-supported footbridge designs, namely that overall the modal mass and span length drop, if bridge frequency increases. The model is in fact calibrated to as-built footbridges (to relationships between f_0 and L presented in [28]), and is a useful means for the studies of this paper.

4.4. Method used for simulation of footbridge response

For the study, footbridge acceleration-time histories are simulated using a Newmark time integration scheme using time steps of 0.005 s. For each value of f_0 and for each load case studied, a Monte Carlo simulation approach is adopted in order to establish realisations of peak midspan accelerations (recognising that realisations of these accelerations change from one crossing to the next due to the modelled randomness of the walking load). This provides estimates of statistical distributions of vibration levels, and from these distributions, the percentiles a_{95}, a_{75} and a_{50} are extracted. Convergence studies revealed that it was generally sufficient to employ 500,000 Monte Carlo simulation runs (emulating 500,000 different pedestrians separately passing the bridge) to provide a stable estimate of the statistical distribution of vibration levels and of the parameter a_{95}. Less Monte Carlo simulation runs than 500,000 might be used (for instance a fair stabilisation might appear after 50,000 or 100,000 simulations), but whether this is sufficient would depend on the intended use of the results and what they are to be compared with. In the present study as many as 500,000 simulation runs were used for any examination so as to provide confidence in the statistical results.

5. Results

This section presents results in terms of the variation of the footbridge response parameter a_{95} (95-th percentile of midspan peak accelerations) with footbridge frequency, f_0. First, results are presented for the reference load case (Section 5.1), and then for load cases A1–A5, B1–B3, C1–C3 and D1–D5 as defined in Table 5. A summarising section (Section 5.6) compares results from Sections 5.2–5.5 on how sensitive the footbridge response parameter a_{95} is to the different approaches considered for modelling walking parameters.

5.1. The reference load case

Calculated values for the percentiles a_{95}, a_{75}, and a_{50} for footbridge fundamental frequencies in the frequency range of 1.6–2.4 Hz are shown in Fig. 6.

The reference load case assumed for Fig. 6 assumes a mean value of the step frequency of 1.87 Hz and a standard deviation of the step frequency of 0.187 Hz. This partially explains why values of the percentiles are generally quite low in the upper and lower ranges of the 1.6–2.4 Hz range of bridge frequencies, as a step frequency coinciding with the bridge resonant frequency will not occur as often for these bridges. As for convergence of the simulated distributions, a necessary criterion is that the curves for a range of different percentiles are smooth, which is seen to be satisfied. Furthermore, any increase in the number of simulation runs would only cause insignificant changes in the curves in Fig. 6.

5.2. Variability in modelling step frequency

As described in Section 3.2, different models describing the likelihood of step frequencies (models A1–A5) are assumed for the study in order to explore how sensitive footbridge accelerations are to the modelling of step frequencies. Results in terms of the percentile a_{95} and its variation with the frequency of the bridge, f_0, are shown in Fig. 7.

Model A1 is the model employed for the reference load case. The first item to notice in Fig. 7 is that the location of the peak of the variation of a_{95} with bridge frequency is somewhat sensitive to the stochastic model of step frequency assumed for the calculations of bridge vibrations. The mean value of the step frequency is 1.82 Hz for model A3, 1.87 Hz for model A1, 1.99 Hz for model A2, and it is 2.2 Hz for model A4. It can be seen that as the mean value of step frequency increases it strives the peak of the a_{95}-variation to move further to the right in the (f_0, a_{95})-plot. For example, this has the effect that depending on which stochastic model for the step frequency is assumed, the parameter a_{95} for a bridge with a frequency of 2.2 Hz can be assessed to take on values ranging from 0.05 to 0.55 m s^{-2}. The deviation in results becomes less severe at the centre of the studied bridge frequency region.

Model A4 might not be feasible as probably it relies on calibration to a quite limited set of data, but for example models A1 and A2 both rely on model calibration to more than 500 observations, and still there is a noticeable difference in how the two models predict levels of a_{95}. This even though the mean value of the step frequency is not that far apart (A1: 1.87 Hz; A2: 1.99 Hz). If the bridge frequency is 1.92 Hz, the two models result in almost identical estimates of a_{95}. For bridges with a resonant frequency in the range of 1.75–2.15 Hz, model A2 provides estimates of a_{95} that deviate from the

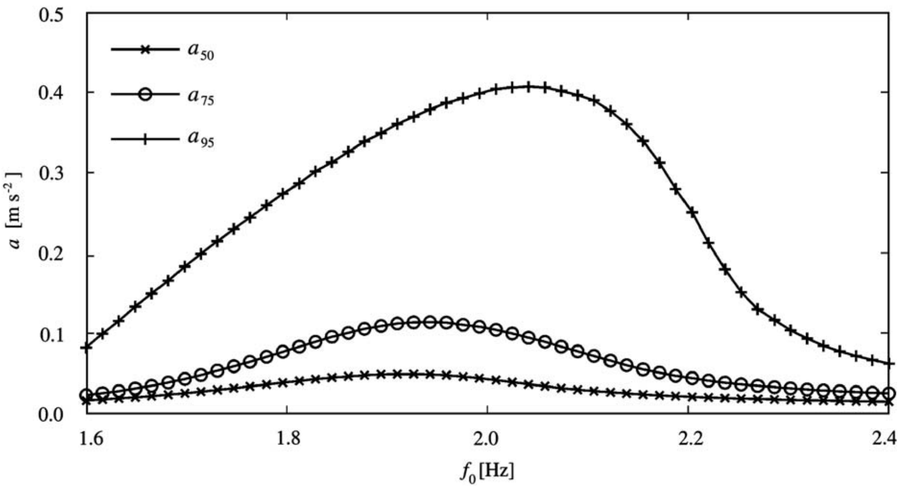

Fig. 6. The acceleration percentiles a_{95}, a_{75} and a_{50} as a function of bridge frequency f_0.

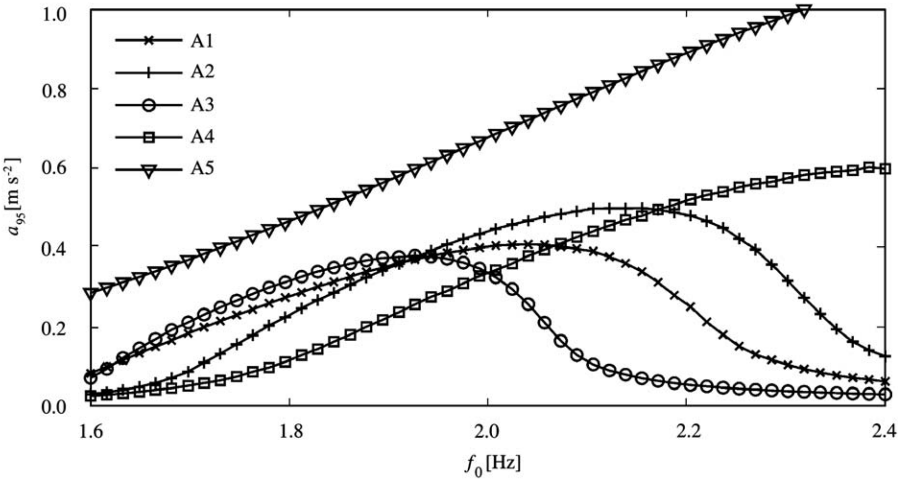

Fig. 7. The acceleration percentile a_{95} as a function of bridge frequency f_0 as calculated employing the stochastic models A1, A2, A3, and A4, and the deterministic model A5 for modelling step frequency of pedestrians.

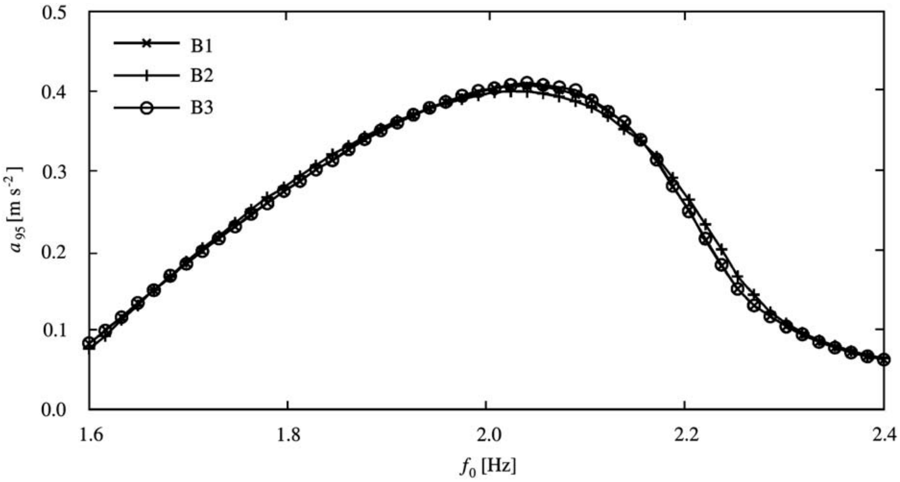

Fig. 8. The acceleration percentile a_{95} as a function of bridge frequency f_0 as calculated employing the models B1, B2, and B3 for modelling pedestrian stride length.

results obtained using model A1 by a factor 1.5 or less. In the range of 1.70–2.2 Hz, the deviation is a factor 2 or less, but beyond this frequency range, the two results deviate by more than a factor 2. It thus appears that there can be at least some model uncertainty (originating from the choice of stochastic model for the step frequency) associated with estimating a_{95}.

Another item to notice is that the deterministic model (A5) that assumes resonant action for any pedestrian traversing the bridge (f_s=f_0 for any value of f_0, but stochastic models of other load parameters) results in the highest estimates of the parameter a_{95}. Generally, this approach to modelling the action gives values of a_{95} that are approximately twice as high as the highest values of a_{95} obtained using probabilistic approaches for modelling step frequency (models A1–A4). However, should a pedestrian adjust his pace a bit to the bridge movements once they are perceptible, then model A5 might not be as conservative as it would appear at first sight. However, the probability of this happening might be small.

5.3. *Variability in modelling pacing speed*

The pacing speed is defined by the equation $v=f_s l_s$ and for the stride length, l_s, three different models are considered: The stochastic model B1 and the deterministic models B2 and B3, respectively. Model B1 is the model employed for the reference load case. Fig. 8 shows how results in terms of a_{95} differ for different bridge frequencies on the different model assumptions.

It is obvious that fairly identical results are obtained for the three load models. Hence, in terms of an estimate of a_{95}, it does not seem to be important whether the stride length is modelled stochastically or deterministically, and in fact two different deterministic models for the stride length gave similar estimates of a_{95} for any bridge frequency.

5.4. Variability in modelling the dynamic load factor

The different models assumed for the dynamic load factor covers the stochastic model C1, which is the model employed for the reference load case, and the deterministic load models C2 and C3. Fundamentally, the model C2 is a deterministic variant of model C1 in which the basic mechanism is modelled similarly; only in model C2 any scatter associated with the dynamic load factor for a given value of step frequency is neglected. Model C3 represents another deterministic model relying on a different set of measurements for describing the mechanism being the relationship between dynamic load factor and step frequency. Employing the three different models for the estimation of a_{95} gave the results shown in Fig. 9 for various bridge frequencies.

As can be seen, the stochastic model C1 and the deterministic model C2 give results that are similar, suggesting that modelling the stochastic nature of the dynamic load factor is not that important for estimating a_{95}. Comparing results obtained assuming the deterministic load models (C2 and C3) suggest that the variation of mean value of the dynamic load factor with step frequency is a parameter influencing the estimates of a_{95}, as this variation is the prime difference between the two models. As was seen in Fig. 5, model C3 assumes a dynamic load factor which is slightly higher than that in model C2 for step frequencies above 1.75 Hz. From Fig. 9, it is seen that this has the effect that a_{95} calculated for model C3 is slightly higher than for model C2 for bridge frequencies above 1.75 Hz.

Generally, the choice of deterministic model for the dynamic load factor is seen to influence estimates of a_{95}, but for the two studied models, a maximum deviation in the estimate of a_{95} of a little less than 20 percent is noticed. On the one hand, this is judged as being a fairly insignificant model uncertainty considering the model uncertainties observed associated with the choice of model for the step frequency. On the other hand, a 15 percent deviation between results of models C2 and C3 is observed for bridge frequencies around f_0=1.92 Hz where models A1 and A2 (step frequency models) result in almost similar values of a_{95}. For bridges at this frequency, the model uncertainty associated with the choice of model for the dynamic load factor might not be considered insignificant. In order to add some perspective to this discussion, the influence of various models for pedestrian weight is addressed.

5.5. Variability in modelling the weight of the pedestrian

The pedestrian weight is modelled by the stochastic models D1–D4 assuming mean values of pedestrian weight of 750, 550, 650, and 850 N, respectively. Another model, D5, is also considered, and it is a variant of D1 in which the scatter in weight is neglected. Model D1 is the model assumed for the reference load case. Fig. 10 presents how the various model assumptions influence calculations of the parameter a_{95}.

One interesting item to observe in Fig. 10 is that whether the pedestrian weight, W, is modelled stochastically, (tenderly done by assuming a normal distribution (with a mean weight of 750 N, model D1)), or deterministically (750 N, model D5) does not seem to have much influence on the statistical distribution of footbridge response represented by a_{95}. Another noticeable item is that the assumption of the mean value of pedestrian weight has an impact on footbridge vibrations,

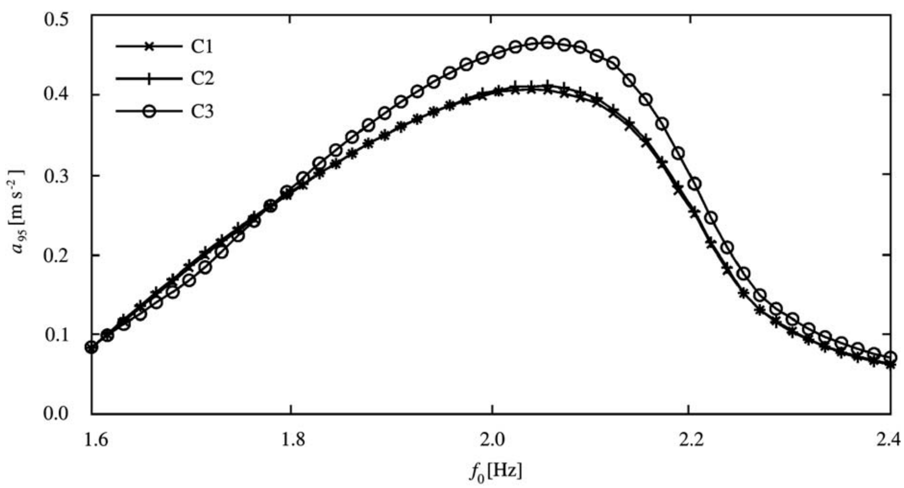

Fig. 9. The acceleration percentile a_{95} as a function of bridge frequency f_0 as calculated employing the stochastic model C1, and the deterministic models C2 and C3 for modelling the dynamic load factor of pedestrians.

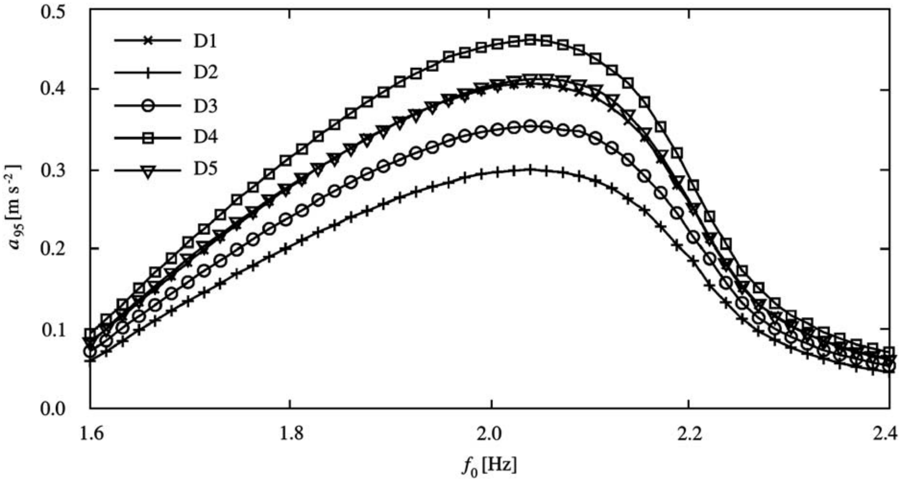

Fig. 10. The acceleration percentile a_{95} as a function of bridge frequency f_0 as calculated employing the stochastic models D1, D2, D3, and D4, and the deterministic model D5 for modelling pedestrian weight.

which is generally not surprising as it is in agreement with Eq. (5) predicting that walking loads increase with increases in pedestrian weight.

Hence, it seems important to have a good idea about the mean value of pedestrian weight when evaluating vibration serviceability of a particular footbridge. A mean weight of 850 N might apply in some countries, but in others and/or if the bridge is mostly used by school children, a weight of 850 N might be considerably over the top. Since the parameter a_{95} is affected almost proportionally by the choice of mean weight of the pedestrians for any bridge frequency, a misjudged choice of the mean value will generally impose relative errors in estimates of a_{95} of a scale proportional to the relative misjudgement of pedestrian weight.

5.6. *Summary of results*

From the sections above it appears that estimates of a_{95}:

- are not very sensitive to whether a stochastic or a deterministic model is assumed for the parameters: pedestrian weight, dynamic load factor or pedestrian stride length,
- are sensitive to the assumption made about the mean value of pedestrian weight,
- are sensitive to the assumption made about the mean value of the dynamic load factor, but for the models for the dynamic load factor studied, a maximum difference of less than 20 percent is noticed, and
- can be quite sensitive to the choice of stochastic model for step frequencies, however, depending on bridge frequency.

For a more detailed description of the sensitivity of estimates of a_{95} to the choice of step frequency model, reference is given to Section 5.2, but it is noticeable that the sensitivity of the estimate of a_{95} depends on the bridge frequency and on the choice of stochastic model for the step frequency. Additionally, that it is found important to model this parameter stochastically for a reliable prediction of a_{95}, whereas it immediately would seem to be of less importance to model the stochastic nature of other parameters describing walking loads. The reliability of this reasoning is considered next.

5.7. *Footbridge response to semi-stochastic load model*

In this section focus is on a comparison between a_{95} calculated assuming the reference load case (a fully stochastic load case) and a load case in which only the step frequency is modelled stochastically (a semi-stochastic load case). The motivation for this comparison is that the summary above suggests that it is more or less only the stochastic nature of step frequency (and not the stochastic nature of other walking parameters) that has an influence on estimates of a_{95}. However, this conclusion was drawn based on calculations of a_{95} for the reference load case (a fully stochastic load model), and then by calculating a_{95} when one of the other walking parameters was modelled deterministically rather than stochastically. These studies indicated that a_{95} did hardly change by modelling a single one of the other walking parameters deterministically. However, the exercise of calculating a_{95} by modelling deterministically all other walking parameters than step frequency was not done. Results of such calculation is presented in this section.

For the semi-stochastic load case, the stride length is modelled using the deterministic model B3, the dynamic load factor is modelled using the deterministic model C2, and the weight of the pedestrian is modelled using the deterministic

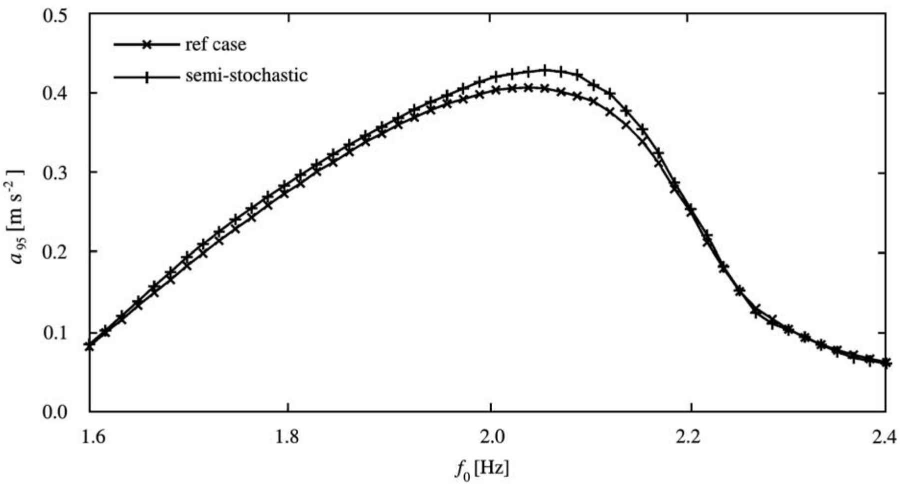

Fig. 11. The acceleration percentile a_{95} as a function of bridge frequency f_0 as calculated for the reference load case and the semi-stochastic load case.

model D5. These are models, which can be considered as deterministic variants of the models used for the reference load case (B1, C1, and D1) as they disregard scatter in stride length, dynamic load factor and pedestrian weight, respectively, and the models thus rely on mean values for these parameters to describe the action. Thereby only the step frequency is modelled stochastically and in the same way as in the reference load case (model A1).

Fig. 11 compares values of a_{95} calculated for the reference load case (fully stochastic load case) with the semi-stochastic load case where only the step frequency is modelled stochastically.

As can be seen, the reference load case and the semi-stochastic load case give values of a_{95} that are almost identical, suggesting that almost identical estimates of a_{95} would be obtained regardless of whether stride length, pedestrian weight and dynamic load factor are modelled stochastically or deterministically. Generally it suggests that if the first harmonic of the excitation induced by a pedestrian is modelled using Eq. (5), it may be useful to calculate statistical distributions of bridge vibration levels (a_{95}) by using the following models:

$$W \sim D(W = \mu_W), \quad \alpha \sim D(\alpha(f_s) = \mu_\alpha(f_s)), \quad f_s \sim N(\mu_f, \sigma_f), \quad l_s \sim D(l_s = \mu_l). \tag{13}$$

In this load representation only the step frequency is modelled stochastically, whereas in the reference load case all four parameters are modelled stochastically. Modelling 3 out of 4 parameters deterministically significantly reduces the complexity of the problem of estimating values of a_{95} in vibration serviceability checks.

The studies suggest that a realistic mean value for pedestrian weight (μ_W) should be used, and, for example, the reference load case gives proposals for the parameters $\mu_\alpha(f_s)$ and μ_l. What remains for describing the action is the choice of the stochastic model for step frequencies, $f_s \sim N(\mu_f, \sigma_f)$. Given the fact that the statistical distribution of bridge vibrations (represented by the parameter a_{95}) is found, in some cases, to be rather sensitive to the choice of parameters of this model, it would seem sensible for a vibration serviceability check using stochastic modelling, to compute vibrations based on various step frequency models available in literature. This would assist in understanding the degree of model uncertainty associated with estimating statistical distributions of the vibrations. One might, for example, choose to rely on the most critical estimate. Generally, it would be useful to have detailed information available about the mean value of pedestrian weight and the statistical distribution of step frequencies for the population of pedestrians expected to traverse the bridge, but often such information would not be available. Naturally it would also be useful to have accurate information available about modal characteristics of the bridge. These parameters are treated as deterministic (and well-defined) parameters in the present study which they basically are not, and some of the modal parameters are difficult to predict at the bridge design stage. For instance an overestimation of bridge damping would underestimate the liveliness of the bridge. As bridge damping can be as low as 0.2–0.3 percent of critical damping and thus lower than the damping of 0.5 percent assumed for the studies of the paper, bridge vibration levels would in the worst case be much higher than predicted here. Separate studies (not reported in the paper) suggest that the main conclusions of the investigations of the paper also apply to bridges with extremely low damping.

6. Concluding and discussion

The studies of the paper involved modelling walking parameters such as step frequency, stride length, dynamic load factor and pedestrian weight as random variables thus acknowledging that they will vary from one pedestrian to the next.

A literature review identified a range of different ways in which the walking parameters might be modelled. The result of the review might be useful for others interested in modelling the stochastic nature of walking.

The findings of the literature review were used as a basis for exploring how sensitive results of calculations of the 95th-percentile of midspan footbridge accelerations would be to the different assumptions that may be made about the walking parameters. These investigations were made for pin-supported single-span bridges with a fundamental frequency in the range of 1.6–2.4 Hz. The bridge model employed is simplistic but useful as it allows numerical calculation of bridge responses for different load assumptions on similar structural conditions.

The results of the study showed that the way in which the stochastic nature of *step frequency* is modelled can have a profound influence on the response parameter a_{95}. This is observed despite the fact that the step frequency models, considered for the studies of the paper, all rely on the same distribution; the normal distribution. Various parameters of the distribution, however, are proposed in literature (mean value and standard deviation of step frequencies of populations of walking persons) and the choice made concerning the parameters of the normal distribution appears to significantly influence estimates of a_{95} for some bridges in the 1.6–2.4 Hz range. This is basically because the choice can have a significant influence on the likelihood of bridge resonant excitation. Another observation from the study is that for one bridge frequency range, one stochastic model of step frequencies gives the highest estimates of a_{95}, whereas for another bridge frequency range, another stochastic model of step frequencies gives the highest estimates of a_{95}. Hence it is not immediately possible to identify the stochastic model that generally would give a conservative estimate of a_{95}. The model that would ensure a conservative estimate would depend on the frequency of the bridge.

The study also revealed that estimates of a_{95} are not very sensitive to whether the *dynamic load factor*, the *pedestrian weight* and the *pedestrian stride length* are modelled stochastically or deterministically. It seems more important to rely on deterministic models that describe these parameters well. Identifying a universal mean value of pedestrian weight representative for any footbridge is impossible, and identifying a representative mean value for a specific bridge is also not straightforward. Operational conditions might be more complex than just the passage of a single and well-defined population of pedestrians. If, for instance, the bridge conveys two well-defined populations with quite different average weights, already in this case, it would probably no longer be sensible to employ the mean value of pedestrian weight (computed from the average weights of the two populations) for computations of bridge vibrations. The fortunate circumstance is that it is fairly simple to decide on a generally conservative model for pedestrian weight; which might be 850 N irrespective of bridge location and bridge usage.

The results of the studies of this paper suggest that for many bridges it is the decision made about pedestrian weight combined with the assumptions made about the parameters describing the stochastic nature of the step frequencies that predominates the outcome of estimates of a_{95} and thus the outcome of vibration serviceability checks of footbridges. It is recommended that further work is made on the subject of exploring the stochastic nature of step frequencies. How such works would end up is not known; however, it seems useful to get an improved understanding of the variability of step frequency behaviour from one population to the next. Furthermore to study how environmental conditions might influence step frequency behaviour, as there might be differences in pedestrian behaviour in different laboratory environments as well as between pedestrian behaviour on a real structure vs. laboratory environment. Such studies might also reveal new or more advanced and reliable descriptions of relationships between walking parameters and load models than what is assumed for the studies of the present paper which to some extent is based on a gathering of different research efforts each focusing on different walking parameters separately.

For the numerical studies of this paper it has been chosen to employ reasonably simple and deterministic models for the bridge in order to place focus on a set of the primary parameters describing walking loads and how choices made about them influence estimates of the statistical distribution of bridge response. However, it must be stressed that the simplifying assumptions will not apply to all footbridges. Furthermore, that detailed knowledge is not always available about bridge modal characteristics, which might influence the accuracy of response predictions. As for the bridge excitation, there are aspects of the quite complex nature of walking excitation on real footbridges that are not addressed (for instance multi-person pedestrian traffic performed by groups of people, a potential interaction between the pedestrians, and between pedestrians and the bridge, and inability of a pedestrian to produce an ideal sinusoidal force). These aspects/phenomena are likely to influence the bridge response to some extent, although in some current codes focus on a number of these aspects are not required. Horizontal loading would also be an item to consider in a vibration serviceability check, as this, for some bridges, would be the critical action. Along the way, models of these phenomena may also be incorporated into a probability-based framework for predicting bridge vibrations, and with suitably calibrated acceptance criteria, the probability-based approach is believed to be a powerful and the most refined tool for vibration serviceability checks.

Although this paper has not addressed all aspects of the problem at hand, the authors of the paper believe that the findings of the paper (narrowing down the complexity of the problem) will be useful for further investigations into the problem of calculating statistical distributions of bridge response to the action of walking.

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