Journal of Sound and Vibration 329 (2010) 2702–2705

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Discussion on "Lateral vibration of a composite stepped beam consisted of SMA helical spring based on equivalent Euler–Bernoulli beam theory" by C.-Y. Lee, H.-C. Zhuo and C.-W. Hsu, *Journal of Sound and Vibration*, Vol. 324 (2009) 179–193

Sunil K. Sinha

Non-linear Dynamics, GE Aviation, MD G-26, General Electric Company, Cincinnati, OH 45215, USA

A R T I C L E I N F O

Article history:
Accepted 4 January 2010
Handling Editor: M.P. Cartmell
Available online 22 January 2010

1. Introduction

In the above mentioned paper, the authors (C.-Y. Lee, H.-C. Zhuo and C.-W. Hsu) start with the theoretical formulation of the dynamics of a helical spring subjected to lateral loadings $\boldsymbol{F}_0$ and $\boldsymbol{M}_0$ at the end and derive the natural frequencies for the first two fundamental modes of vibration. Unfortunately, their paper essentially rehashes the same results that have been known to exist and have been well-documented over the last 30 years but does not cite any of the relevant sources of that information. There are four key shortcomings in Lee, Zhuo and Hsu's paper, which this current discussion paper will address and bring to the attention of readers and other interested researchers in this field:

(1) To highlight the major weakness of Lee, Zhuo and Hsu's approach in treating the dynamics of a helical spring from linear modal natural frequency considerations.
(2) Missing references of other major contributions of archival nature in this field during the last 40 years.
(3) To focus on the importance of stress-wave behavior in determining transient response of helical springs.
(4) Effect of neglecting non-linear terms in dynamic characteristics of helical springs.

Each of the 4 items mentioned above are further elaborated in the following section nos. 2, 3, 4 and 5, respectively.

2. Major drawbacks of Lee, Zhuo and Hsu's formulation

Lee and his coworker's approach of characterizing a helical spring as an Euler–Bernoulli's beam of equivalent stiffness and computing the first two fundamental frequencies is too simplified which would miss the entire dynamical response

DOI of original article: 10.1016/j.jsv.2009.01.055
E-mail address: sunil.sinha@ae.ge.com

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2010.01.001

due to the initial curvature of a helix. The helical spring has been modeled as a curved beam going as far back as 1966, when Wittrick [1] published his classic paper on the wave propagation in a helical spring. In 1974 Stokes [2] used a more refined formulation using Serret–Frenet geometrical relationship in the tangent-normal-binormal system to capture the dynamics of a helical coil using its initial curvature and the pre-twist of a naturally curved beam.

3. Missing references in Lee, Zhuo and Hsu's paper

In 1988, a very comprehensive list of nearly all the published literature, on this topic that was of any significance was listed in a review article titled 'Dynamic behavior of helical springs' by Pearson [3]. In the pioneering work, the derivations of force-deflection characteristics and the dynamical response of helical springs were studied and investigated in much more detail by the late Professor Costello and his coworkers and published in various journals [4–8]. Their original derivations dealt with even more complex problem of large deflection and forced transient dynamic response of helical springs including the coupled stress-wave solution versus small-deflection and natural frequency determination presented in the current paper of Lee, Zhuo and Hsu. Professor Costello and his Ph. D. students' original work, done during 1972–1977, became a basis for nearly all follow-up research investigation [10–16] in the area of dynamic behavior of helical springs. The natural frequency results presented in Lee's paper have also been known for over 20 years that were originally published by Pearson and Wittrick [9] in the International Journal of Mechanical Sciences.

The derivation reported in Lee's paper under Section 2 is almost a simplified version of the original formulation of Costello [4,7] of a more complex situation due to an upfront small-deformation assumption versus large-deformation non-linearity, which was included in Costello's derivation. For discussion purposes, we are presenting a short description of Costello's equations in Section 3.1 of this discussion paper. The following is a short list of some of the differences in the nomenclatures used by Costello [4,7] in his original work versus the notations used in the current paper by Lee et al.

List of parameters	Costello (1972)	Lee et al. (2009)
1. Helix angle	α	$\tan^{-1}(p/2\pi R)$
2. Mean radius of the spring	r	R
3. Number of coils	n	N_c
4. Axial length of spring	h	L_0

In the next section, we provide a detailed description of similarities in formulation of specific items in Lee, Zhuo and Hsu's paper in light of contributions by Costello [4,7,8]. As pointed out earlier that although there are certain differences in nomenclature between the original works of Costello (1972) and the current paper of Lee et al. (2009), for present discussion purposes we will be using the exact symbols and notations as used by Costello and his coworkers about 35 years ago.

3.1. On page-180, Section-2. Theoretical Formulation: (Lee, Zhuo and Hsu, 2009)

A similar derivation for the combined axial force and torque load of a helical spring with large deformation was published by Professor Costello [4] in 1972. With axial force F and twisting moment T for a spring of mean radius 'r' and helix angle 'α', the corresponding force-deflection relationships are given as [4]

$$F=\frac{EI}{r^2}[\beta\sin\alpha+\cos\alpha]\sin\alpha\left[-\frac{v}{1+v}(1+\varepsilon)(\beta\sin\alpha+\cos\alpha)-\frac{\cos\alpha}{1+v}+\frac{(1+\varepsilon)\cos^2\alpha}{(1-(1+\varepsilon)^2\sin^2\alpha)^{1/2}}\right] \tag{1}$$

$$T=\frac{EI}{r}\left[\frac{(1+\varepsilon)}{(1+v)}\sin^2\alpha[(1+\varepsilon)(\beta\sin\alpha+\cos\alpha)-\cos\alpha]+(\beta\sin\alpha+\cos\alpha)[1-(1+\varepsilon)^2\sin^2\alpha]-[1-(1+\varepsilon)^2\sin^2\alpha]^{1/2}\cos^2\alpha\right] \tag{2}$$

In his 1977 paper [5] he published for the first time the large deflection formulation of a helical spring due to end-bending-moment M_0. He showed that by using Love's curved beam theory with 'n' as the number of coils, the deformed curvature $(1/\rho)$ for large deflection of helical spring is given by [5],

$$\frac{1}{\rho}=\frac{v^2r^2}{6\sin\alpha}\left[\frac{3}{8}+\frac{(2\pi n)^2}{3}+\frac{5}{8}\sin^2\alpha\right]\frac{M_0^3}{EI}+\frac{(2+v\cos^2\alpha)}{2\sin\alpha}\frac{M_0}{EI} \tag{3}$$

In the case of small-deflection theory the above Eq. (3) reduces to,

$$\frac{1}{\rho}=\frac{(2+v\cos^2\alpha)}{2\sin\alpha}\frac{M_0}{EI} \tag{4}$$

Earlier in their 1972 paper Phillips and Costello [4] derived the various stiffness terms as,

$$\left(\frac{r^2}{EI}\right)\frac{\partial F}{\partial \varepsilon} = [\beta \sin\alpha + \cos\alpha]\sin\alpha\left[-\frac{v}{1+v}(\beta\sin\alpha+\cos\alpha)+\frac{\cos^2\alpha}{(1-(1+\varepsilon)^2\sin^2\alpha)^{1/2}}\right] \quad (5)$$

$$\left(\frac{r^2}{EI}\right)\frac{\partial F}{\partial \beta} = \left(\frac{r}{EI}\right)\frac{\partial T}{\partial \varepsilon} = \sin^2\alpha\left[\frac{(1+\varepsilon)\cos^2\alpha}{[1-(1+\varepsilon)^2\sin^2\alpha]^{1/2}} - \frac{\cos\alpha}{(1+v)} - \frac{2v}{(1+v)}(1+\varepsilon)(\beta\sin\alpha+\cos\alpha\right] \quad (6)$$

$$\left(\frac{r}{EI}\right)\frac{\partial T}{\partial \beta} = \sin\alpha\left[1-\frac{v}{1+v}(1+\varepsilon)^2\sin^2\alpha\right] \quad (7)$$

The above set of relations (5)–(7) for the linear case used in the current paper by Lee, Zhuo and Hsu simplifies to,

$$\frac{\partial F}{\partial \varepsilon} \cong \frac{EI}{r^2}\left[1-\frac{v}{1+v}\cos^2\alpha\right]\sin\alpha \quad (8)$$

$$\frac{\partial T}{\partial \beta} \cong \frac{EI}{r}\left[1-\frac{v}{1+v}\sin^2\alpha\right]\sin\alpha \quad (9)$$

$$\frac{\partial F}{\partial \beta} = \frac{1}{r}\frac{\partial T}{\partial \varepsilon} \cong \frac{EI}{r^2}\left[-\frac{v}{1+v}\sin^2\alpha\cos\alpha\right] \quad (10)$$

4. Coupled stress wave governing the dynamics of helical springs

In 1972, the governing equations of the Dynamics of a helical spring in the form of a fully-coupled set of non-linear partial-differential wave equations were derived by Phillips and Costello [4].

$$\left(\frac{\partial F}{\partial \varepsilon}\right)\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial F}{\partial \beta}r\right)\frac{\partial^2 \phi}{\partial x^2} = \frac{M}{h}\frac{\partial^2 u}{\partial t^2} \quad (11)$$

$$\left(\frac{\partial T}{\partial \varepsilon}\right)\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial T}{\partial \beta}r\right)\frac{\partial^2 \phi}{\partial x^2} = \frac{Mr^2}{h}\frac{\partial^2 \phi}{\partial t^2} \quad (12)$$

Since, the above set of wave equations are highly non-linear in nature, Phillips and Costello (1972) originally solved only the linearized form of these equations numerically and compared the time-dependent deformed shape of a 58-coiled helical spring under impact conditions with an experimentally obtained streak-photograph shown in Fig. 1.

5. Effect of non-linearity on the dynamic response of helical springs

To a common reader or new researcher in this field, the publication of Lee, Zhuo and Hsu's paper in 2009 gives the impression that a linear model that can predict the first two modes correctly is adequate enough to capture the correct dynamics of a helical spring. However, current generation of researchers needs to be made aware of two facts;

(a) What has been the historical state of research in this field and what facts are already available in open literature for the last 30 years.
(b) Why considering non-linearity is necessary to capture the true dynamic behavior of a helical spring.

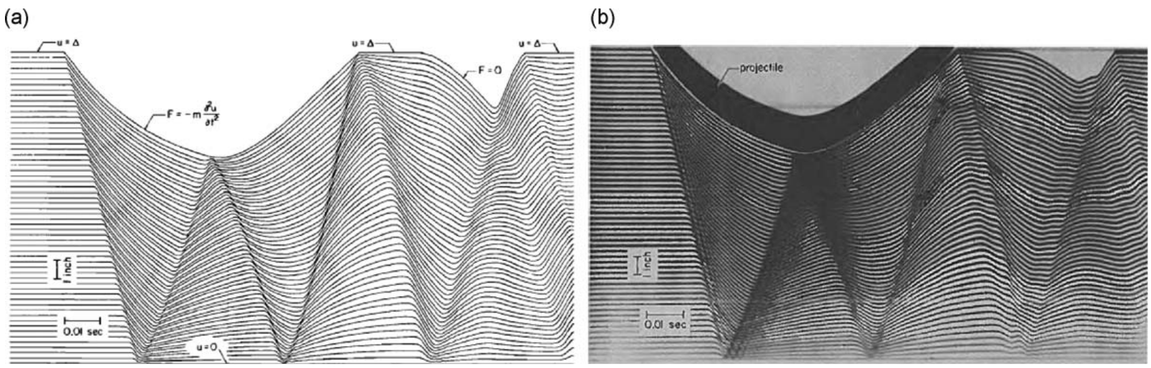

Fig. 1. Linear solution of Eqs. (11) and (12) and its comparison with the experimentally obtained streak photograph of a lightly compressed 58-coils helical spring (h=18.3 in.=46.48 cm) and impacted with projectile at x=0 [4]. (a) Linear analytical solution and (b) Experimental result.

The effect of non-linearity on the transient response has been investigated and the respective numerical results were reported in detail by Sinha and Costello [8] in their paper published in 1978. It is obvious that Lee, Zhuo and Hsu's model has limitations due to neglecting the effect of non-linearity. They may simply be unaware of the existence of a non-linear solution of the coupled stress-wave equation governing the dynamic response of a helical spring.

In short, the importance of non-linear terms in capturing the proper physics of helical spring dynamic behavior has been known in the published literature for over 30 years, which has also been supported by several other follow-up research work on this topic as well [10–16]. In light of well-established inadequacies and shortcomings of a linear model in representing the dynamics of a helical spring, the simplified equivalent Euler–Bernoulli's beam model used by Lee, Zhuo and Hsu fails to provide any new original information. The treatment of helical spring dynamics as an Euler-Bernoulli's beam of equivalent stiffness used in Lee and his coworkers' paper is very rudimentary.

6. Conclusions

In conclusion, there are several other highly relevant recently published works [10–16] in this area that are missing from the list of references presented by Lee, Zhuo and Hsu. In this discussion paper, an attempt has been made to provide a historical perspective on the dynamics of helical springs. The purpose is not just to draw the attention of readers to the shortcomings of Lee, Zhuo and Hsu's approach but rather to make the current academic community aware of already published research information that has been available out there in the print media for the last 30–40 years; but apparently has been overlooked by Lee and his coworkers. In order to make a real advancement in the dynamics of helical springs, it is felt that the seminal nature of contributions of giants in this field such as Wittrick, Costello, Stokes, Pearson and Kagawa are properly recognized, so that the new generation of researchers do not have to repeat the same old information that has already been done and has been in existence for more than 30 years. Keeping this goal in mind for the benefit of today's researcher, this discussion paper highlights and revisits the importance of various parameters such as the coupled wave equations governing the dynamics of helical springs, its transient response due to longitudinal impact, effect of non-linearity on the radial expansion etc., that have been overlooked in Lee, Zhuo and Hsu's paper. A technical paper in this field must start where distinguished researchers of the previous era, such as Wittrick, Costello and Pearson, have left their mark through their ground-breaking work, and gives proper credit to their accomplishments.

References

[1] W.H. Wittrick, On elastic wave propagation in helical springs, *International Journal of Mechanical Sciences* 8 (1966) 25–47.
[2] V.K. Stokes, On the dynamic radial expansion of helical springs due to longitudinal impact, *Journal of Sound and Vibration* 35 (1974) 77–99.
[3] D. Pearson, Dynamic behavior of helical springs, *The Shock and Vibration Digest* 20 (1988) 3–9.
[4] J.W. Phillips, G.A. Costello, Large deflection of impacted helical springs, *Journal of Acoustical Society of America* 51 (1972) 967–972.
[5] G.A. Costello, Large deflections of helical spring due to bending, *ASCE Journal of Engineering Mechanics Division* 103 (3) (1977) 481–487.
[6] G.A. Costello, J.W. Phillips, Contact stresses in thin twisted rods, *Transactions of ASME Journal of Applied Mechanics* 40 (Series E, 2) (1973) 629–630.
[7] G.A. Costello, Radial expansion of helical springs, *Transactions of ASME Journal of Applied Mechanics* 42 (Series E) (1975) 789–793.
[8] S.K. Sinha, G.A. Costello, The numerical solution of the dynamic response of helical springs, *International Journal for Numerical Methods in Engineering* 12 (1978) 949–961.
[9] D. Pearson, W.H. Wittrick, An exact solution for vibration of helical springs using Bernoulli–Euler model, *International Journal of Mechanical Sciences* 28 (2) (1986) 83–96.
[10] Y. Lin, A.P. Pisano, General dynamic equations of helical springs with static solution and experimental verification, *Transactions of ASME Journal of Applied Mechanics* 54 (1987) 910–917.
[11] W. Jiang, T.L. Wang, W.K. Jones, Study of coupled free vibration of helical springs, *AIAA Journal* 30 (5) (1992) 1443–1445.
[12] J. Lee, D.J. Thompson, Dynamic stiffness formulation, free vibration and wave motion of helical springs, *Journal of Sound and Vibration* 239 (2) (2001) 297–320.
[13] J. Lee, S.J. Heo, Wave motion of helical springs with circular sections, *Journal of Korean Mechanical Engineering Association (in Korean language)* 25 (5) (2001) 866–873.
[14] B. Temel, F.F. Calim, Forced vibration of cylindrical helical rods subjected to impulsive loads, *Transactions of ASME Journal of Applied Mechanics* 70 (2003) 281–291.
[15] S. Ayadi, E. Hadj-Taeb, Numeric simulation of the linear dynamic behavior of the helical springs, *Transactions of the Canadian Society of Mechanical Engineers* 30 (2) (2006) 192–208.
[16] M. Taktak, F. Dammak, S. Abid, M. Haddar, A finite-element for dynamic analysis of a cylindrical isotropic helical spring, *Journal of Mechanics of Materials and Structures* 3 (4) (2008) 641–658.