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Numerical study on reducing the vibration of spur gear pairs with phasing

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ABSTRACT

A new method of reducing gear vibration was analyzed using a simple spur gear pair with phasing. This new method is based on reducing the variation in mesh stiffness by adding another pair of gears with half-pitch phasing. This reduces the variation in the mesh stiffness of the final (phasing) gear, because each gear compensates for the variation in the other's mesh stiffness. A single gear pair model with a time-varying rectangular-type mesh stiffness function and backlash was used, and the dynamic response over a wide range of speeds was obtained by numerical integration. Because of the reduced variation in mesh stiffness and the double frequency, the phasing gear greatly reduced the dynamic response and nonlinear behavior of the normal gears. The results of the analysis indicate the possibility of reducing vibration of spur gear pairs using the proposed method.

1. Introduction

The reduction of gear vibration and noise has received much attention because gears are basic components in many common mechanical systems. Because the internal excitation caused by the variation in tooth mesh stiffness is a key factor in causing vibration, most of this attention has been directed to the modification of gear teeth. However, such passive methods have limitations due to the load dependence of the modifications [1]. The use of periodic struts for gearbox support systems [2], periodic drive shafts [3], and one-way clutches [4,5] for the passive reduction of vibration has been reported, but these methods cannot prevent vibration of the gears themselves. Active methods for adapting to changing operating conditions using piezoelectric actuators or magnetic bearings have also been proposed [6–8]. However, these methods also have limitations in that they require additional actuators, external power, and signal processing.

Thus, another method is proposed to reduce the vibration of the gear itself without requiring any additional energy or signal processing in a manner that is independent of load conditions. This method is based on the reduction of mesh stiffness variation using gear pairs with phasing (i.e., phasing gears).

2. Phasing gears

Because the variation of tooth mesh stiffness during meshing is a principal source of internal excitation force and vibration, modifications of the optimal tooth shape and contact ratio (CR) have been studied as ways of reducing the

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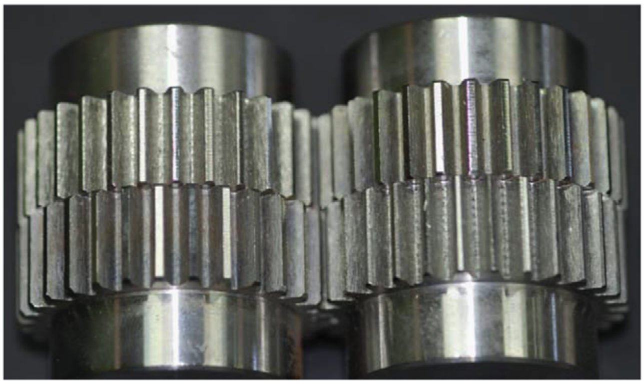

Fig. 1. Phasing gear conceptual model.

variation in mesh stiffness. Major variations in stiffness are caused by changes in meshing pair numbers, usually in the range 1.0–2.0 for normal spur gears. It is impossible to avoid this variation due to the integer numbers of gear teeth.

If another meshed and phased gear pair is added to reverse the stiffness functions of the two pairs, these phasing gears will complement the primary gears and reduce the mesh stiffness variation. The phasing gear pair is composed of two gears half the width and half the pitch phasing of the primary gears. Fig. 1 shows a conceptual model of phasing gears.

If the tooth mesh stiffness is assumed to be a rectangular waveform [9], then the mesh stiffness as a function of the distance s along the line of action (LOA) will be as shown in Figs. 2 and 3, were k_{max}, k_{min}, and k_m are the tooth mesh stiffness (assumed to be constant) of a two pair mesh, a one pair mesh, and the average of k_{max} and k_{min}, respectively.

The figures show that the phasing gear has a low amplitude variation with twice the frequency of the normal gear. The function $s(t)$ is the distance from the starting point to the contacting point along the LOA, while s_1 and s_2 are the distance from the starting point to the lowest point of single-tooth contact and the distance from the starting point to the highest point of single-tooth contact, respectively.

The mesh stiffness for both cases can be expressed as

For CR ≥ 1.5:

$$k(t)=\begin{cases} k_{max} & \text{when } s \leq s_c \text{ or } s_b \prec s \leq s_1 \\ k_m & \text{when } s_c \prec s \leq s_b \text{ or } s_1 \prec s \end{cases}$$

For $CR \prec 1.5$:

$$k(t)=\begin{cases} k_m & \text{when } s \leq s_1 \text{ or } s_b \prec s \leq s_d \\ k_{min} & \text{when } s_1 \prec s \leq s_b \text{ or } s_d \prec s \end{cases} \tag{1}$$

where

$$s_b=\tfrac{1}{2}s_2$$

$$s_c=s_1-\tfrac{1}{2}s_2$$

$$s_d=s_1+\tfrac{1}{2}s_2$$

If the average mesh stiffness value k_0 and constant stiffness value k_{min} of an individual tooth pair are assumed to be constant during an entire tooth meshing cycle, the relationship between k_0, k_{min}, and the CR can be expressed as [9]

$$k_0/k_{min}=\text{CR} \tag{2}$$

Fig. 4 shows the mesh stiffness of the normal and phasing gears for various values of CR. The maximum stiffness k_{max} increases as CR decreases. Typically, CR=1.5. If the stiffness function is rectangular, the mesh stiffness of the phasing gear for CR=1.5 is constant throughout the whole mesh cycle.

3. Mathematical model

The system in this study consisted of two gears mounted on well-aligned input and output shafts, as shown in Fig. 5. The gears were standard errorless unmodified involute spur gears, and the gear pair was modeled as a purely torsional vibration system. The torsional flexibility of both shafts was neglected, and the inertias of the input and output shafts and the load were lumped together. Gears 1 and 2 had base circles of radii r_1 and r_2, respectively, and mass moments of inertia I_1 and I_2. The driver and load mass moments of inertia were I_i and I_0, respectively. The pair of gears was modeled using two disks coupled with a nonlinear mesh stiffness and mesh damping, where $k(t)$ and c were the mesh stiffness and damping

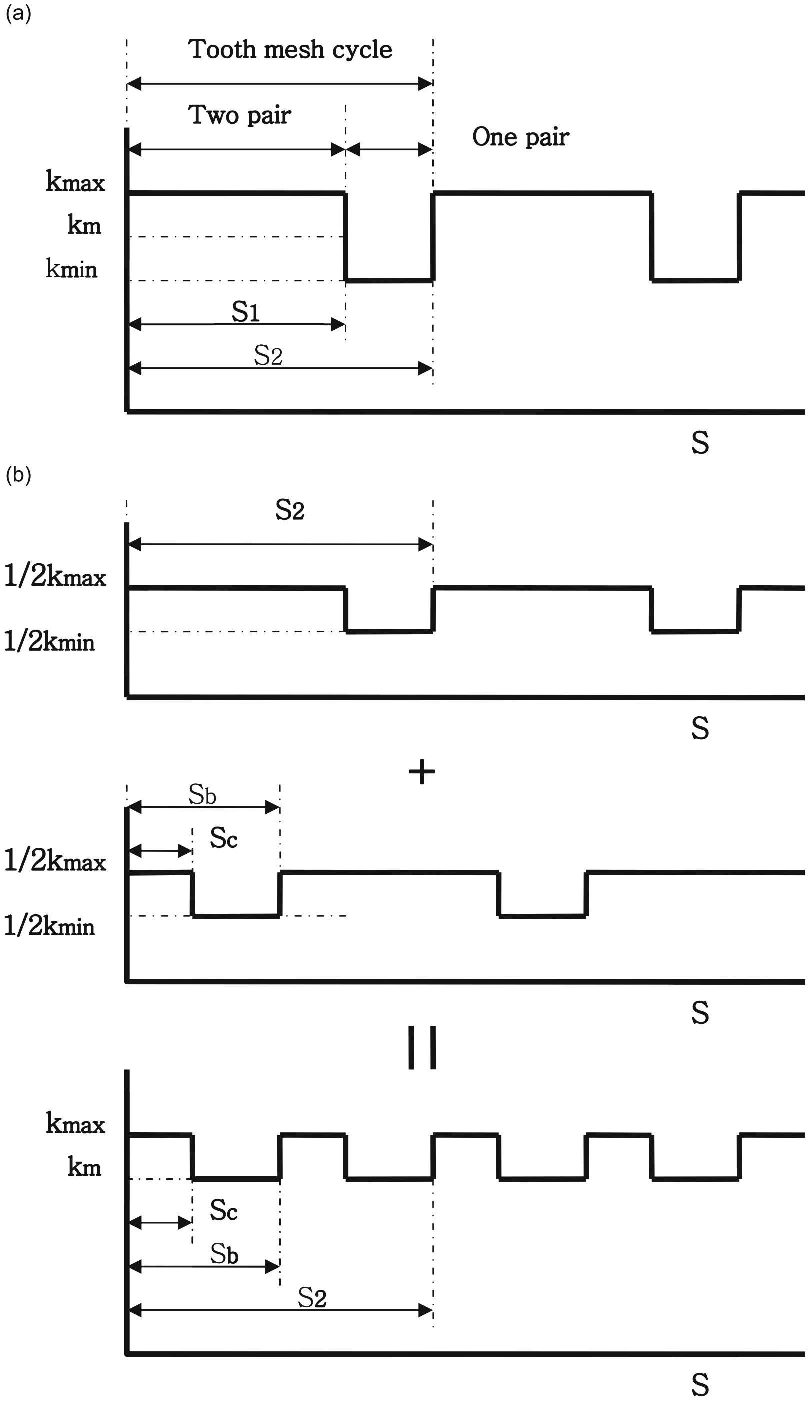

Fig. 2. Tooth mesh stiffness for CR ≥ 1.5 of (a) normal gear and (b) phasing gear.

coefficient of the gear pair, respectively. The total backlash was $2b$. The vibration of gears 1 and 2 about the nominal rigid body rotation were θ_1 and θ_2, respectively. The speed of the input shaft was assumed to be constant, and the static input and load torque were T_i and T_0, respectively.

The equations of motion of the two gears are:

$$(I_1+I_i)\ddot{\theta}_1+r_1c(r_1\dot{\theta}_1-r_2\dot{\theta}_2)+r_1k(t)\beta(t)=T_i$$

$$(I_2+I_0)\ddot{\theta}_2-r_2c(r_1\dot{\theta}_1-r_2\dot{\theta}_2)-r_2k(t)\beta(t)=-T_0 \quad (3)$$

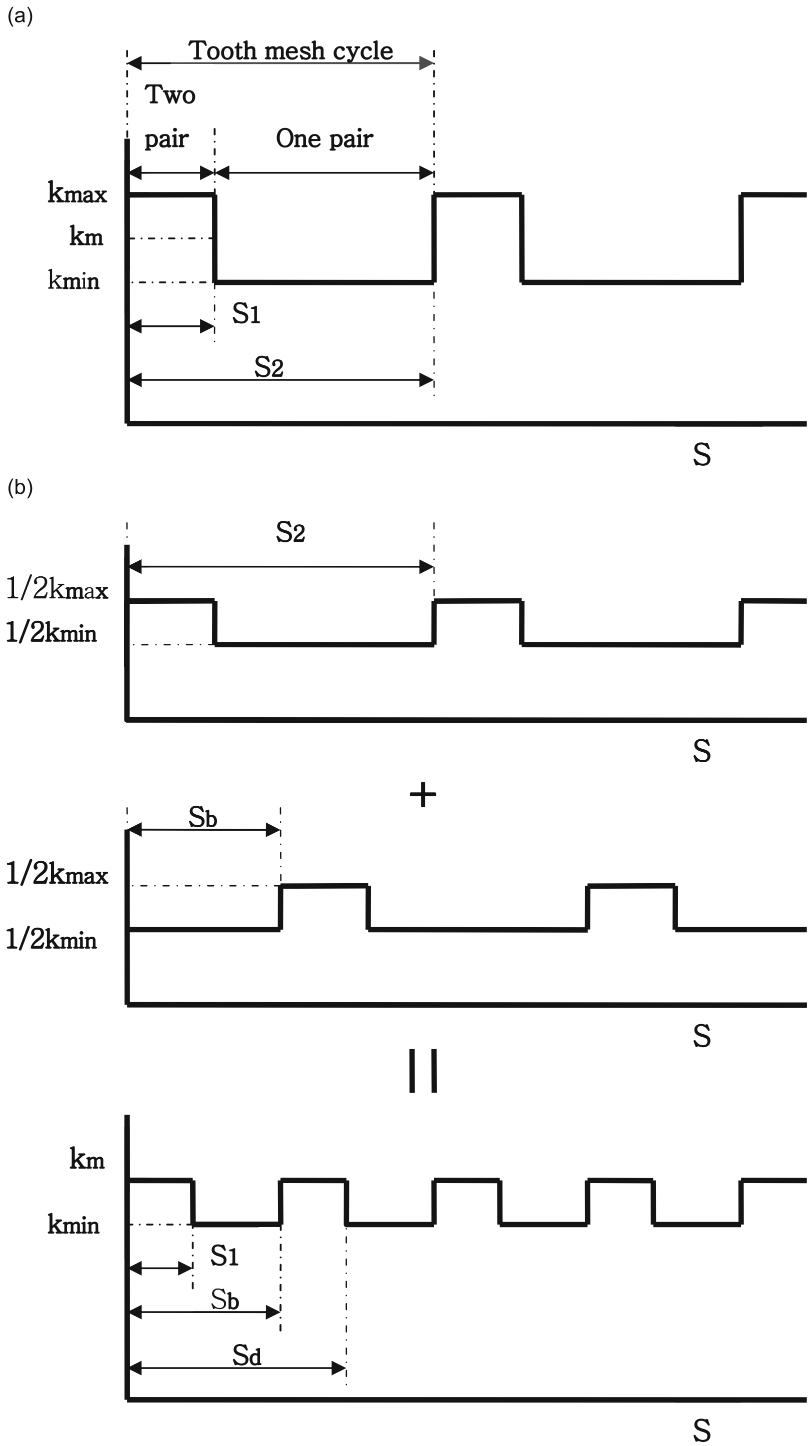

Fig. 3. Tooth mesh stiffness for CR < 1.5 of (a) normal gear and (b) phasing gear.

The gear backlash nonlinearity was modeled as a piecewise linear function.

$$\beta(t)=\begin{Bmatrix} r_1\theta_1-r_2\theta_2-b & \text{when } r_1\theta_1-r_2\theta_2 \succ b \\ r_1\theta_1-r_2\theta_2+b & \text{when } r_1\theta_1-r_2\theta_2 \prec b \\ 0 & \text{when } |r_1\theta_1-r_2\theta_2| \le b \end{Bmatrix} \tag{4}$$

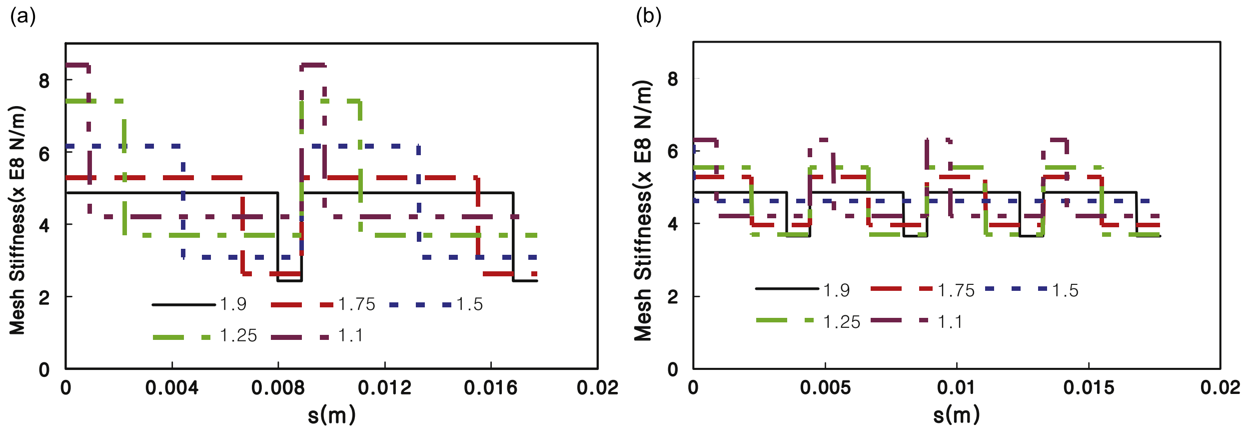

Fig. 4. Mesh stiffness of normal and phasing gears for various values of CR. (a) normal gear and (b) phasing gear.

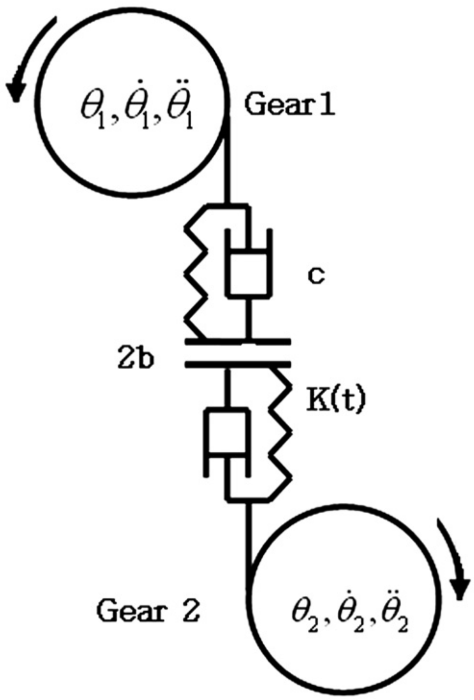

Fig. 5. Schematic diagram of gear pairs.

and the dynamic transmission error (DTE) was defined as $x=(r_1\theta_1-r_2\theta_2)$. The damping coefficient c of the tooth mesh was calculated as

$$c=2\xi\sqrt{k_0/[(r_1^2/I_1)+(r_2^2/I_2)]} \tag{5}$$

where ξ is the damping ratio of the tooth mesh.

The distance of the contact point s from the starting point can be expressed as a function of time to synchronize the time-varying stiffness with the tooth meshing phase. Because the magnitude of s varies periodically with the tooth mesh frequency f_m, it can be Fourier-transformed and expressed as

$$s(t)=\frac{p_b}{2}-\frac{p_b}{\pi}\sum_{i=1}^{L}\frac{1}{i}\sin(i2\pi f_m t) \tag{6}$$

where p_b is the transverse base pitch.

4. Parametric study

Table 1 shows the dimensions of an identical gear pair studied in previous research [9–12] and used here for the sake of comparison. The reference parameters were selected as follows: average mesh stiffness for mesh cycle $k_0=462.1\times10^6$ N/m, CR=1.75, tooth mesh damping ratio 0.07, input torque and load torque 150 N m, and $T_0=r_2/r_1\times T_i$. The inertias of the driver and load were kept the same as those of the gears. This study did not consider friction force [13].

To detect jump phenomena in the nonlinear model, sweeps with increasing and decreasing speeds were performed at a constant ratio for a wide range of speeds straddling the natural frequency. The calculated natural frequency was around 3250 RPM. To obtain stable data after a speed change, 10^5 samples were discarded before averaging. The time step was 1×10^{-5} s for all conditions.

The RMS of the DTE (RDTE) and the oscillating components of the DTE (ODTE) at a specific constant speed are defined as

$$\mathrm{RDTE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{DTE}_i)^2}$$

$$\mathrm{ODTE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{DTE}_{i+1}-\mathrm{DTE}_i)^2} \tag{7}$$

where $\mathrm{DTE}_i=r_1\theta_1(t_i)-r_2\theta_2(t_i)$ and N is the total number of time steps used for averaging, which was 2×10^4 in this study. The solutions were obtained using direct time-domain numerical integration (fifth-order Runge–Kutta algorithm). The effects of varying parameters such as CR, mesh damping ratio, backlash, torque, dynamic responses, such as ODTE and RDTE, and average mesh stiffness during averaging for a specific speed were evaluated. The CR was limited to the range 1.0–2.0 and L in Eq. (6) was set to 30.

Fig. 6 shows the dynamic response as a function of the normal and phasing gear rotation speeds. While the normal gear experienced the jump phenomena at the primary resonance frequency as well as at the superharmonic frequency, the phasing gear did not exhibit resonance at the primary resonance frequency, and only weak resonance at odd (e.g., first and third) superharmonic frequencies. The overlap range (multiple solution regions) was also almost undetectable in the phasing gear. Because the mesh stiffness of the phasing gear had a frequency twice that of the normal gear, it showed only odd-numbered superharmonic frequencies, and the amplitude decreased, mostly due to the decreased variation in mesh stiffness. The phasing gear had slightly lower average mesh stiffness than the normal gear over the whole speed range; that may have contributed to the decreased amplitude.

Based on experiments and analysis, Blankenship and Kahraman [10,11] showed that the existence of jumps was governed entirely by the harmonic stiffness variation and the damping ratio, and that the amplitude of the kth harmonic responses was determined almost entirely by the amplitude of the kth harmonic stiffness. They reported that the dynamic response and the multiple solution regions increased as the stiffness variation increased.

Because the stiffness variation frequency of the phasing gear was twice that of the normal gear, the primary stiffness harmonic amplitude ($k=1$) of the phasing gear corresponded to the first superharmonic amplitude of the normal gear. The results of this study correspond closely to the results of Blankenship and Kahraman.

These results indicated that the phasing gear would be effective for reducing vibration over a wide range of speeds, especially for avoiding primary resonance. Fig. 7 shows that for CR=1.9, the internal excitation due to mesh stiffness of twice the frequency excited only odd-numbered superharmonic resonances.

Fig. 8 shows the dynamic response of the phasing gear for various values of CR$>$1.5. Although the general behavior trends were similar, the amplitudes of the response for CR near to 2.0 or 1.5 were smaller than those near 1.75. This indicates that the dynamic response decreased as the difference between the duration of high stiffness T_h and low stiffness T_l (i.e., $|T_h-T_l|$) increased. Although the maximum value of the mesh stiffness k_{max} increased as CR decreased (Fig. 4), the average mesh stiffness for CR near to 2.0 or 1.5 was smaller than that for CR near 1.75.

Table 1
Gear dimensions.

Number of teeth	50
Module (m)	0.003
Pressure angle (deg)	20
Face width (m)	0.02
Modulus of elasticity (N/m^2)	207×10^9
Density (kg/m^3)	7600
Base radius (m)	0.07047
Backlash (2b) (m)	400×10^{-6}
Mass (kg)	2.8
Mass moment of inertia ($I_1=I_2$) (kg m^2)	7.875×10^{-3}

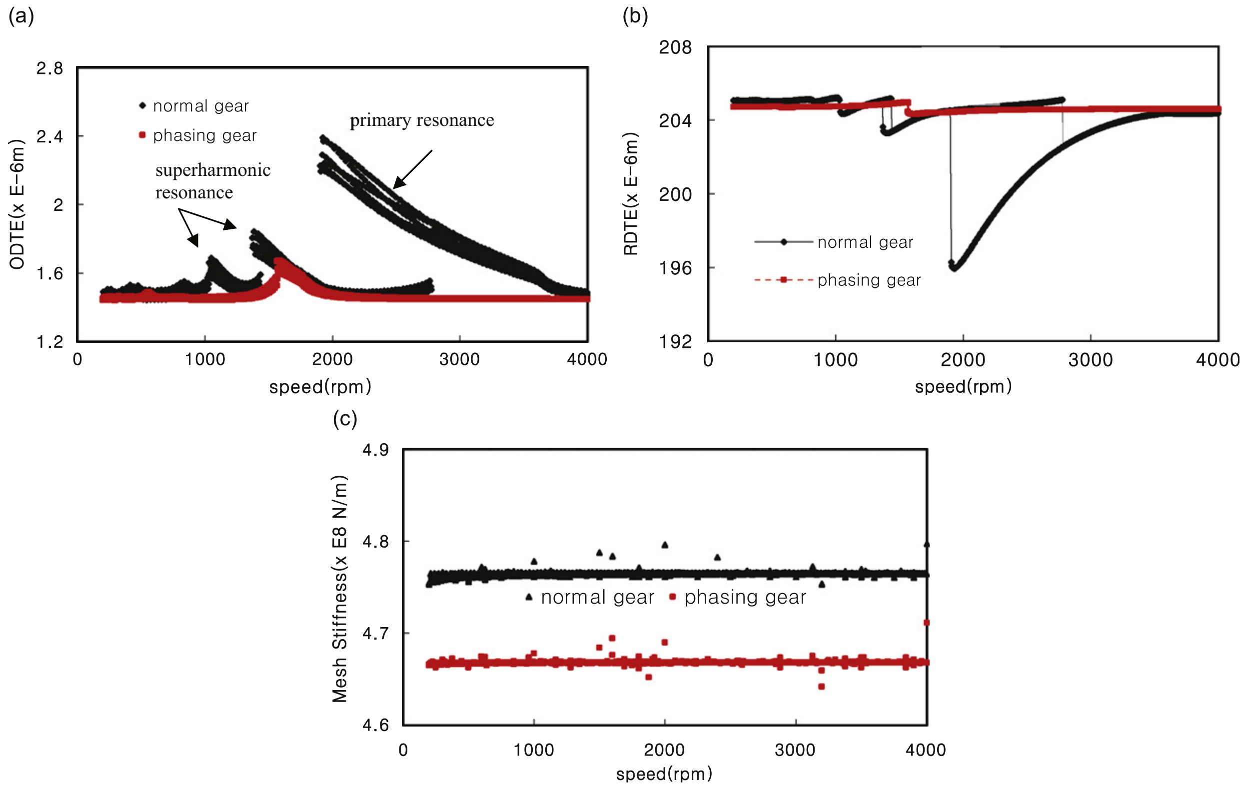

Fig. 6. Response of normal and phasing gears of reference dimensions: (a) ODTE, (b) RDTE, and (c) average mesh stiffness.

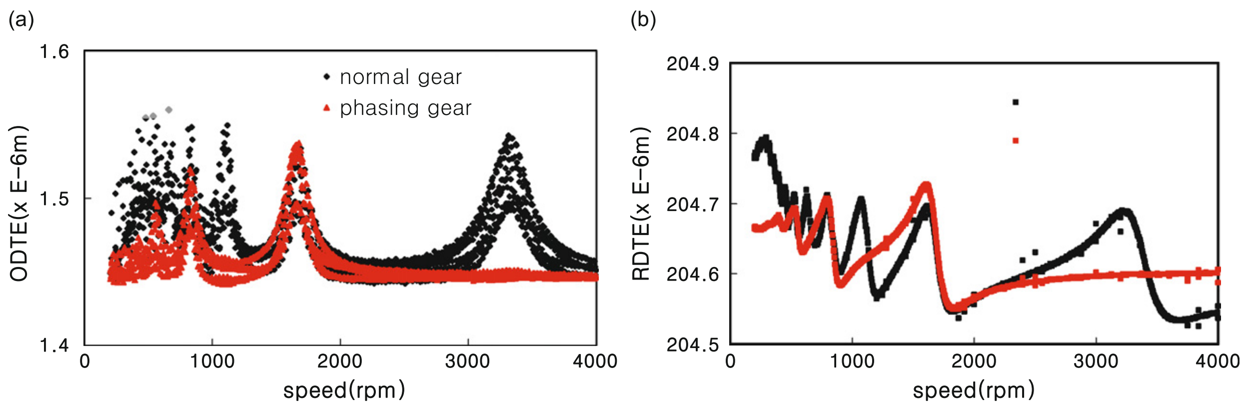

Fig. 7. Response of normal and phasing gears for CR=1.9: (a) ODTE and (b) RDTE.

Kahraman and Blankenship [9] reported that the value of CR affects the mesh stiffness $k(t)$ and dictates the harmonic content of $k(t)$. They showed that gears with integer values of CR were quietest, and a gear pair with CR=$n+m/r$ where m and n are integers, resulted in minimal rth harmonics. For the phasing gear, CR=1.5 satisfies this condition, because the second harmonic (r=2, first superharmonic) of a normal gear corresponds to the primary resonance of the phasing gear. Lin and Parker [14] reported that adjusting the contact ratios and mesh phasing of two-stage gear systems was a powerful way of eliminating or reducing the size of parametric instability regions. According to their study, while the most severe condition for primary instabilities was CR=1.5 with phasing=0, the secondary instability regions vanished for CR=1.5. They also showed that the primary instability region became even smaller as the CR was close to a whole number, and that instability regions increased as the variation in mesh stiffness increased. The results of this study are also consistent with these previous studies.

Fig. 9 shows the dynamic response of the phasing gear for CR < 1.5. Although the average mesh stiffness decreased for CR close to 1.5 or 1.0, the amplitudes of ODTE and RDTE were greatest for CR=1.1, not CR=1.25. This indicates that the CR should not be too small (i.e., not near 1.0).

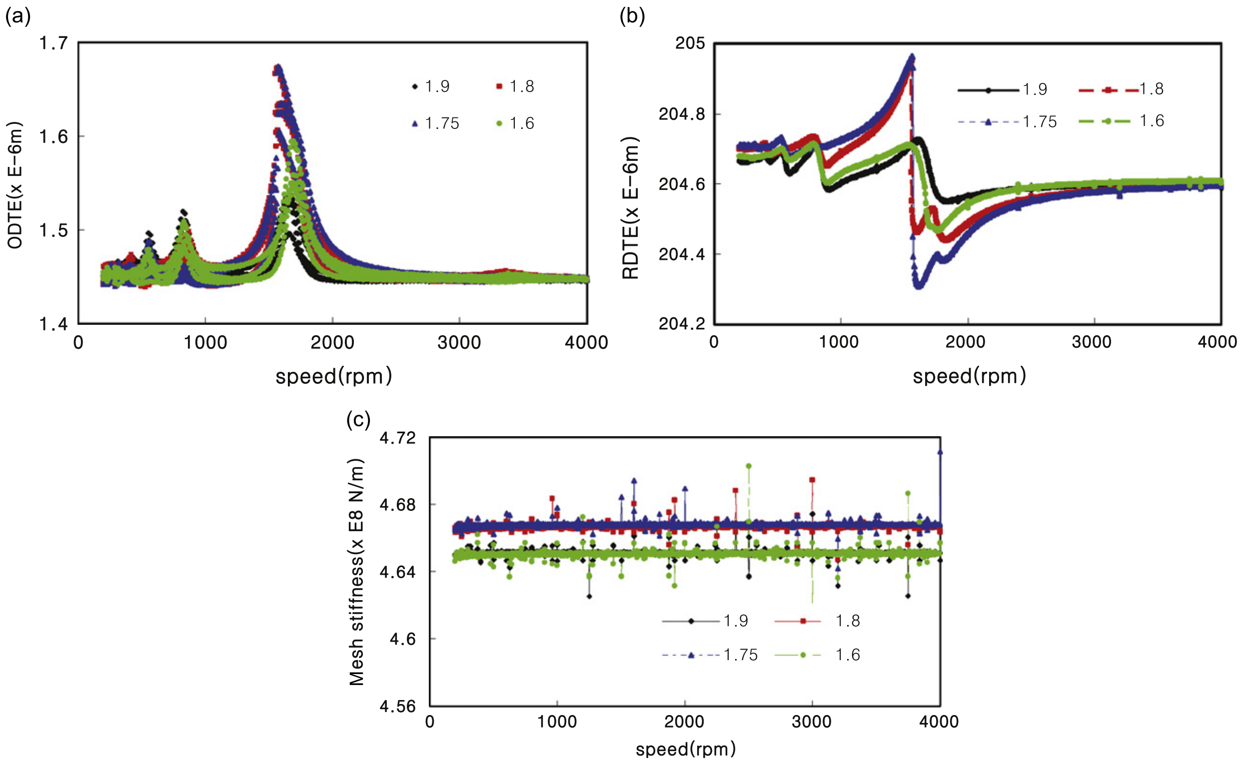

Fig. 8. Phasing gear response for various values of CR > 1.5: (a) ODTE, (b) RDTE, and (c) average mesh stiffness.

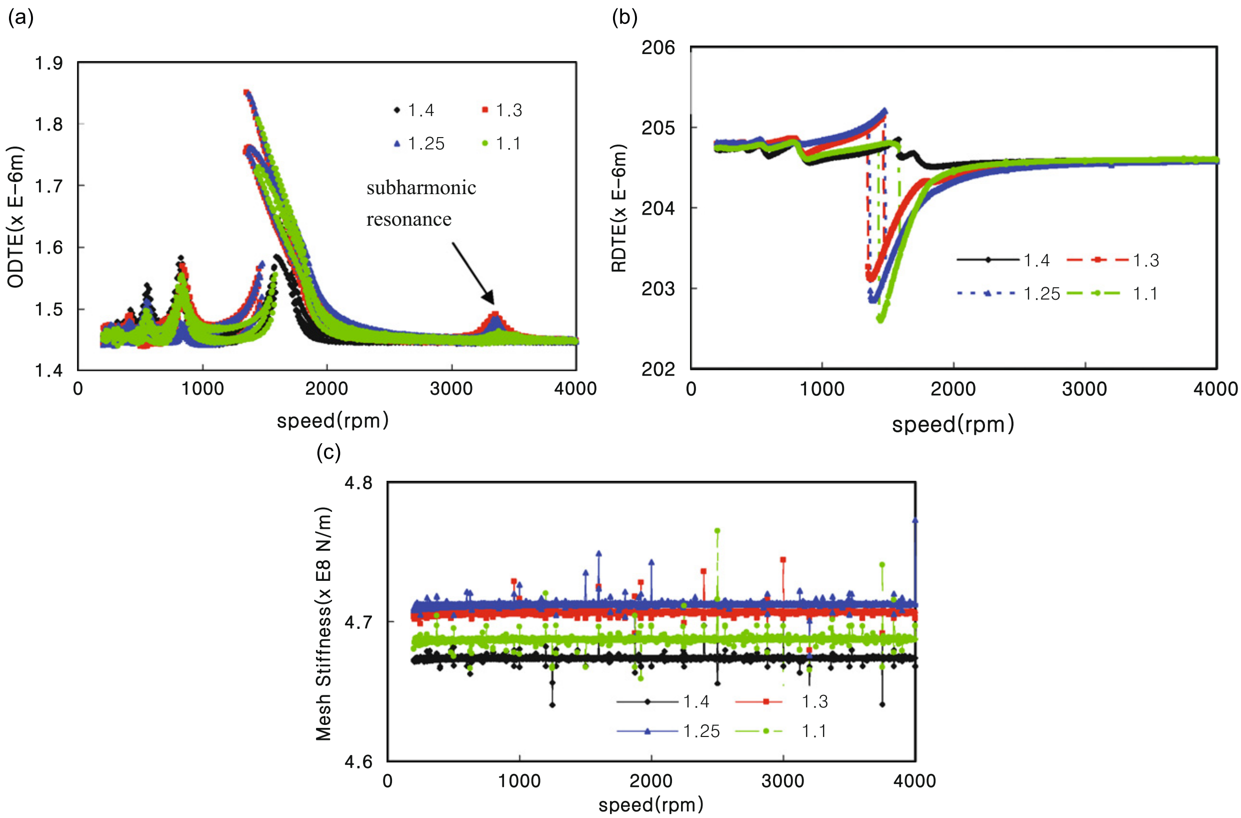

Fig. 9. Phasing gear response for various values of CR < 1.5: (a) ODTE, (b) RDTE, and (c) average mesh stiffness.

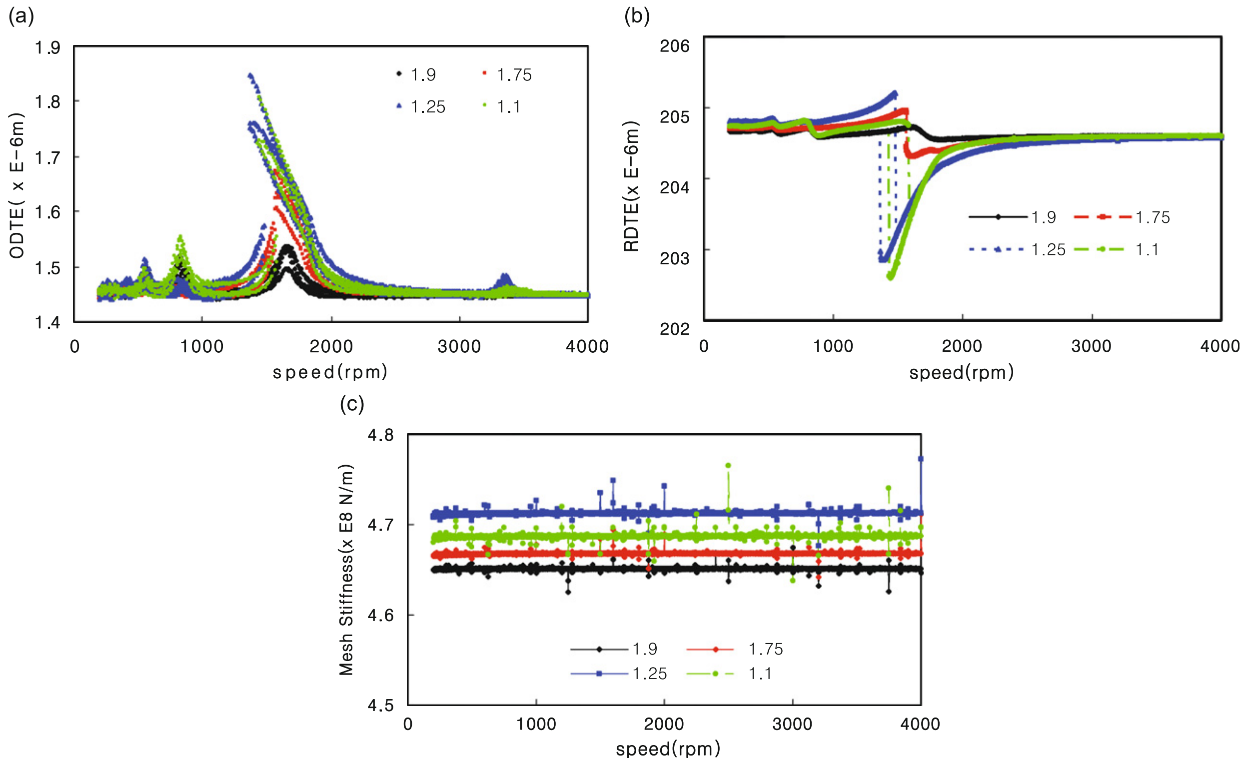

Fig. 10. Phasing gear response for various values of CR: (a) ODTE, (b) RDTE, and (c) average mesh stiffness.

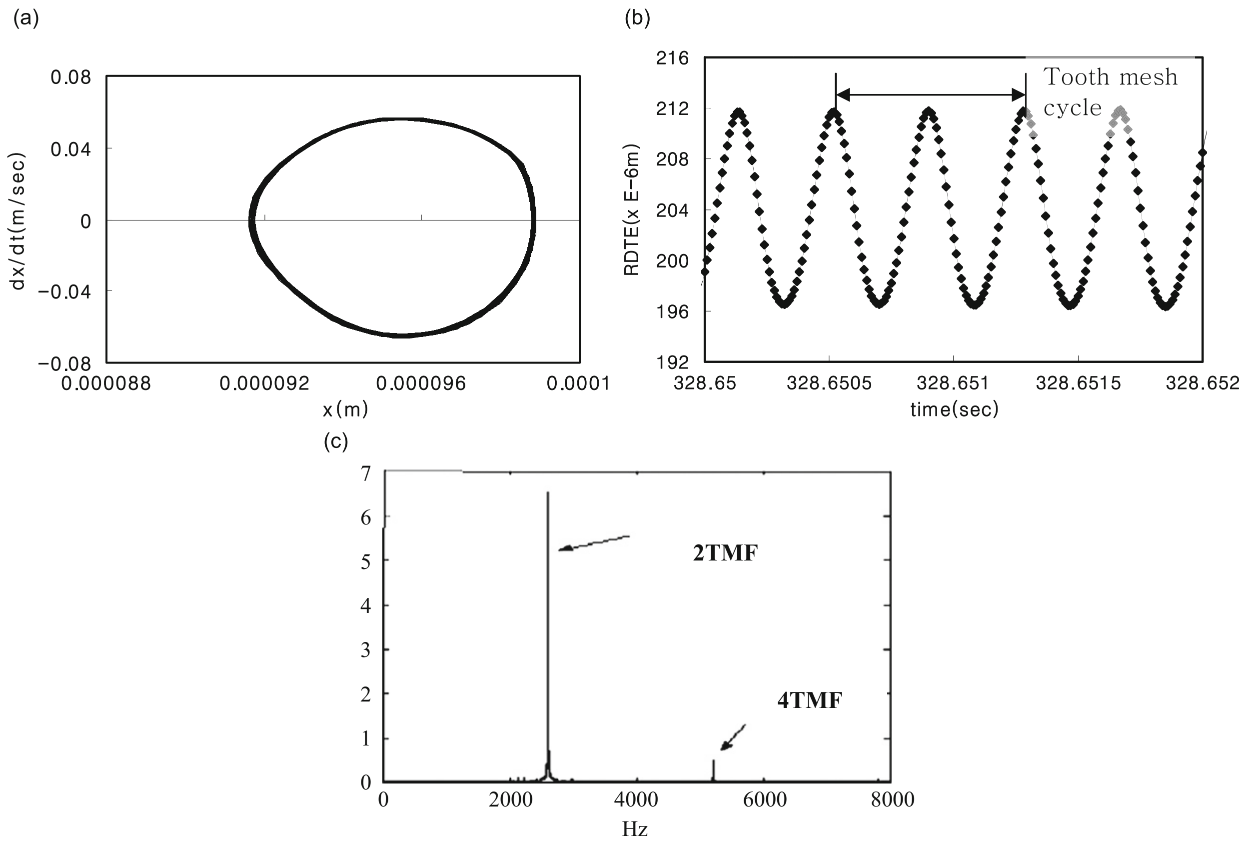

Fig. 11. Phasing gear DTE response for 1565 RPM with CR = 1.25 during an increase in speed: (a) phase diagram, (b) time trace, and (c) frequency spectrum.

Fig. 10 shows the dynamic response of the phasing gear for various values of CR greater than and less than 1.5. The amplitude of the dynamic response for CR$<$1.5 was larger than that for CR$>$1.5.

In Fig. 9, weak subharmonic resonances are evident around the primary resonance frequency for CR=1.25 and 1.3. Figs. 11 and 12 show the DTE phase diagram, frequency spectrum, and time trace of a phasing gear with CR=1.25 for 1565 RPM (in the multiple solution region around superharmonic resonance) and 3260 RPM (in the single solution region around primary resonance), respectively, during a speed increase. A high amplitude, at twice the tooth mesh frequency (TMF), which is a superharmonic resonance frequency of the normal gear, is evident in Fig. 11. A high amplitude at the tooth mesh frequency, which is a primary resonance frequency of the normal gear (a subharmonic resonance frequency of the phasing gear), is evident in Fig. 12.

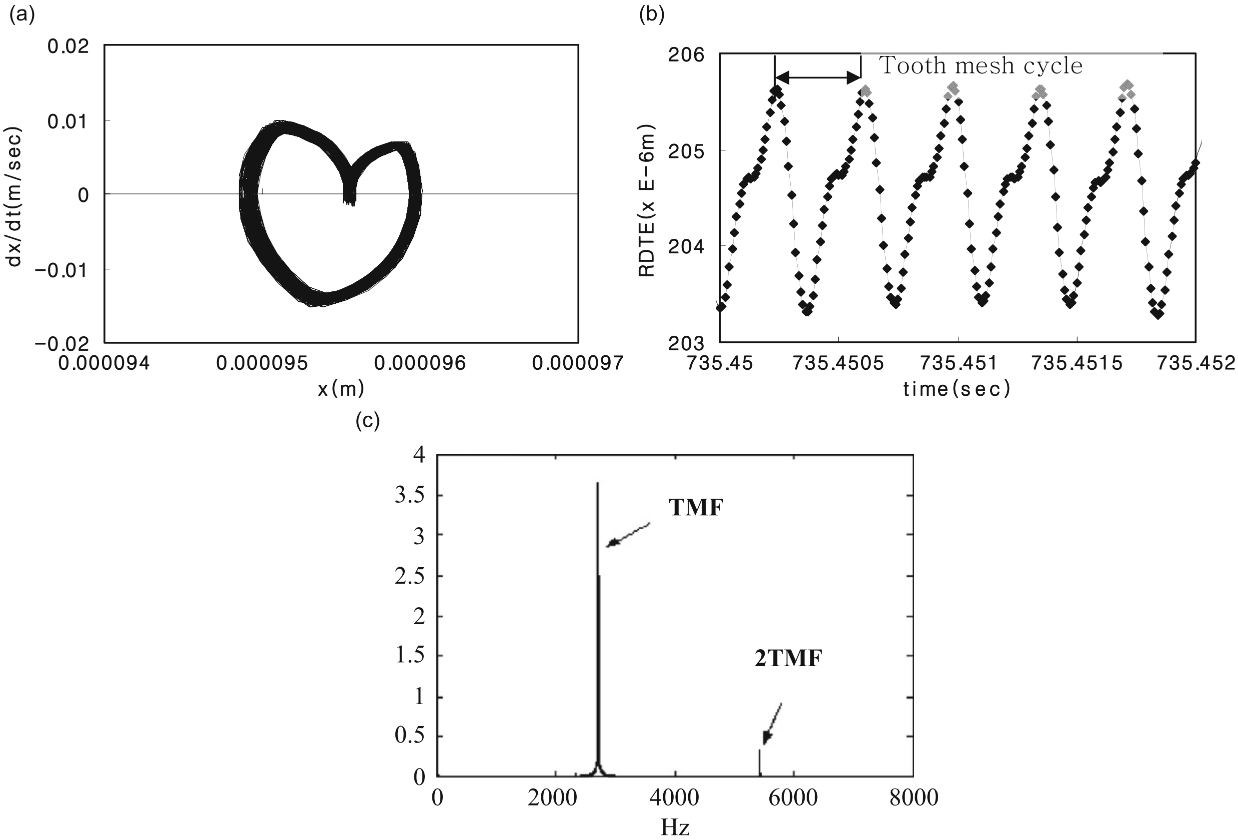

Fig. 12. Phasing gear DTE response for 3260 RPM with CR=1.25 during an increase in speed: (a) phase diagram, (b) time trace, and (c) frequency spectrum.

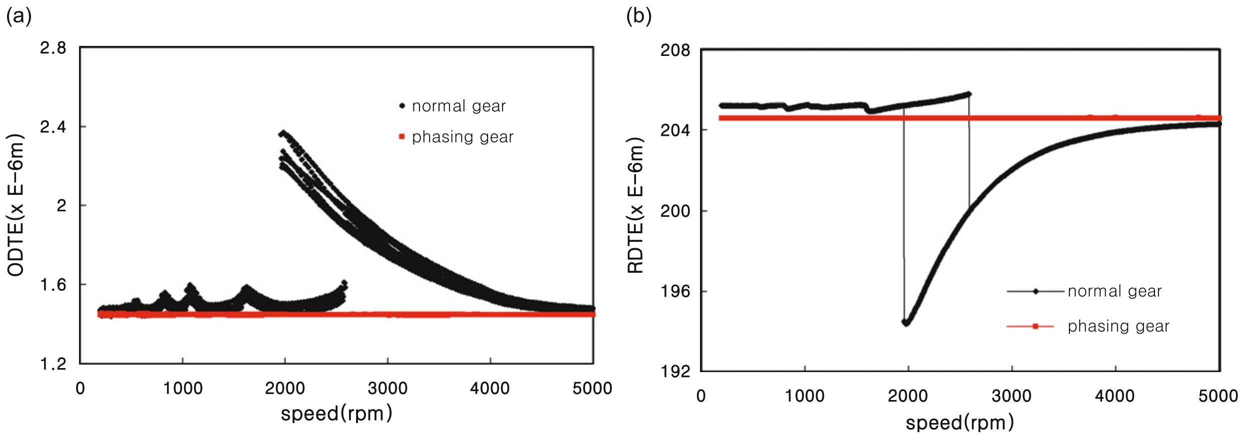

Fig. 13. Response of the phasing and normal gears for CR=1.5: (a) ODTE and (b) RDTE.

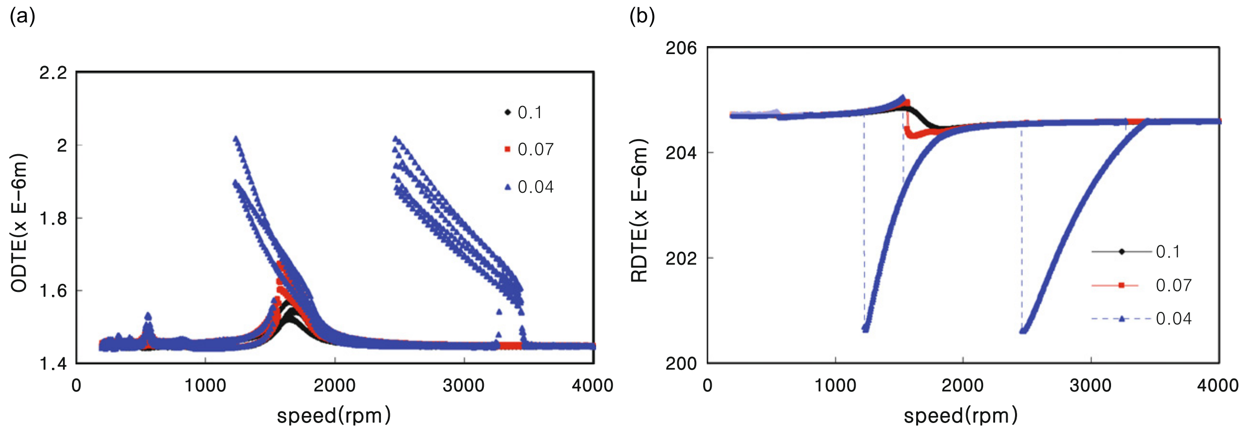

Fig. 14. Phasing gear response for various mesh damping ratio values: (a) ODTE and (b) RDTE.

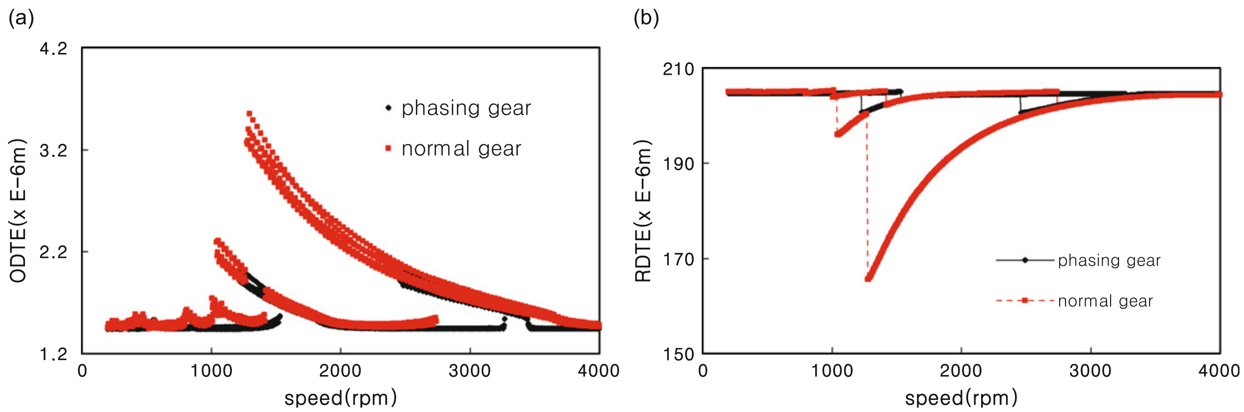

Fig. 15. Normal and phasing gear response for a mesh damping ratio value of 0.04: (a) ODTE and (b) RDTE.

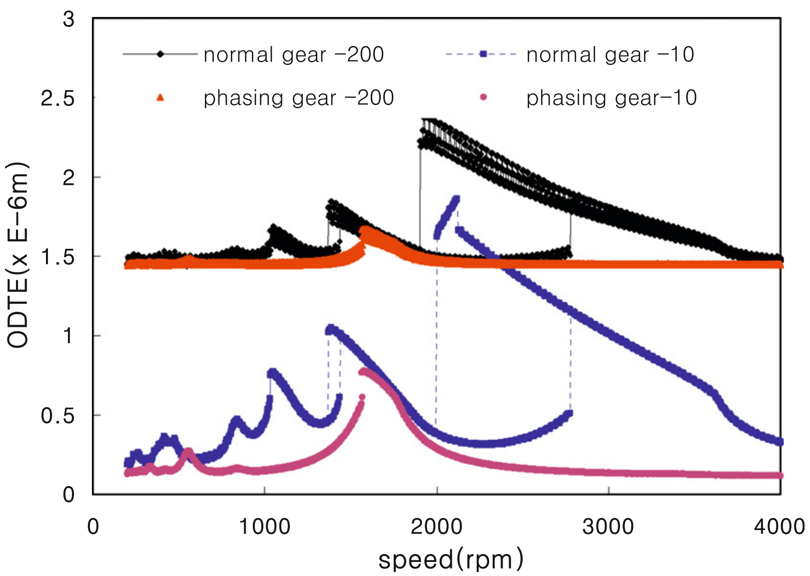

Fig. 16. ODTE of normal and phasing gears for various values of backlash (μm).

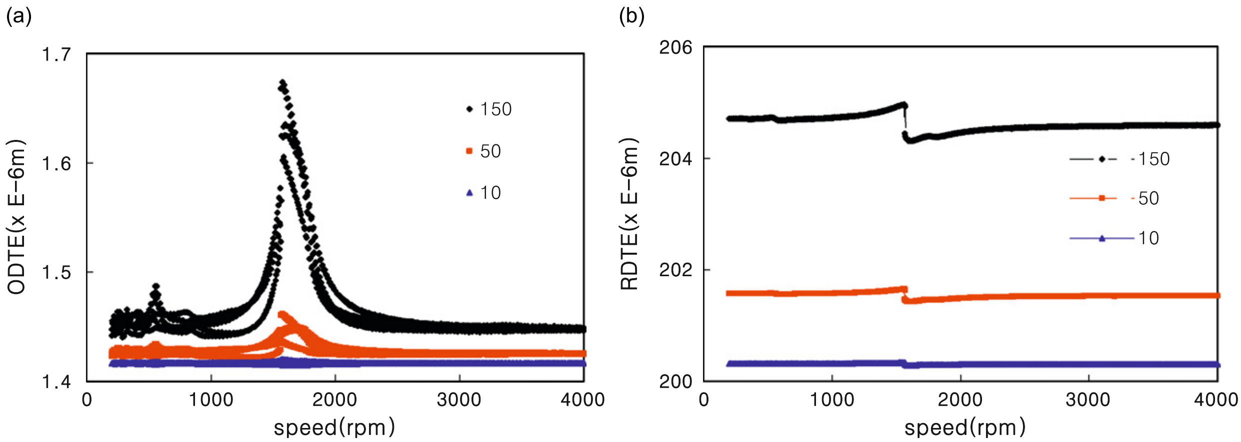

Fig. 17. Phasing gear response for various levels input torque (N m): (a) ODTE and (b) RDTE.

In the special case where CR=1.5, the mesh stiffness variation vanished throughout the whole mesh cycle. Fig. 13 shows the dynamic response of the phasing gear and normal gear for CR=1.5. Neither jump phenomena nor resonance was observed for the phasing gear, and the response was steady for all rotation speeds. The constant steady ODTE values converged to zero as the averaging number increased to allow transient vibrations to disappear after a speed change, but the number was limited to reduce the computation time in this study. Because the assumed rectangular mesh stiffness function was not realistic, it is unlikely that ideal conditions with no vibration will actually occur in real applications. However, a phasing gear with CR=1.5 will be the most effective for minimizing vibration. Instead of an identical pair with CR=1.5 (i.e., 1.5+1.5), combinations with different CR values totaling 3.0, e.g., 1.4+1.6 or 1.3+1.7 with appropriate phasing and pressure angle, should produce similar results.

Figs. 14 and 15 show the effects of mesh damping on the dynamic behavior of the phasing and normal gears. The average mesh stiffness was not affected by the mesh damping variance and is not shown here. Even so, the amplitude of the dynamic response decreased as the mesh damping increased. In general, damping decreases the vibration as well as nonlinear behavior. When the damping ratio decreased to less than some specific value (0.04 in this study), the subharmonic resonance (primary resonance of the normal gear) predominated and had wider multiple solution regions than superharmonic resonances in normal gears. Kahraman and Blankenship [12], Ma and Kahraman [15], and Wang et al. [16] studied the subharmonic resonance of gear systems and concluded that subharmonic motions were very sensitive to the damping ratio, time-varying mesh stiffness, and mean load. Thus, careful consideration of mesh damping is essential in gaining full advantage of the effectiveness of phasing gear. However, the inclusion of a phasing gear is still more effective for reducing vibration than just normal gears alone.

Fig. 16 shows the ODTE of normal and phasing gears for various values of backlash. Unlike the normal gear, the phasing gear did not exhibit a hardening type bend to the right, caused by double side contact even when the backlash was very small. Because the phasing gear did not cause a high-vibration primary resonance, it seemed to prevent double side contact.

Fig. 17 shows the dynamic response of the phasing gear for various values of input torque. The amplitude was proportional to the input torque, and the general trends were similar in all cases.

5. Conclusions

A new method is proposed for reducing the variation in mesh stiffness, which is a principal source of internal excitation in gear systems. A second gear pair meshing with half-pitch phasing was added to complement the primary gear pair and reduce the variation in mesh stiffness. The phasing gear was composed of two gears half the width and half the pitch phasing of the primary gears. The dynamic response of a single gear pair model with a rectangular-type mesh stiffness function and backlash were analyzed using numerical integration. Because of the decrease of mesh stiffness variation and the doubled variation frequency, the phasing gear eliminated the primary resonance with the highest vibration and instability, and reduced the amplitudes of the other superharmonic resonances. The phasing gear exhibited high-amplitude subharmonic resonance for very low mesh damping. In particular, for CR=1.5, the variation in tooth mesh stiffness disappeared and vibration was minimal. Despite the limitations imposed by assuming a rectangular-type stiffness function and the high sensitivity of the dynamic response to mesh damping, the use of a phasing gear is expected to mitigate vibration problems. The results of this study should be verified experimentally due to the limitations imposed by the assumed rectangular-type mesh stiffness model.

Acknowledgment

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