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Discussion

Comments on "Free vibration of super elliptical plates with constant and variable thickness by Ritz method"

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In a recent paper [1] Çeribaşi and Altay have presented a detailed study for the free vibration of super elliptical plates with constant and variable thickness, which is based on the Ritz method with two sets of trial functions. The discussers welcome their valuable contribution to the vibration analysis of super elliptical plates.

The intention of this discussion is to provide some remarks and corrections on the matter.

As it is known, the shape of the plate in the x–y plane can be defined by the super elliptical function as

$$\left[\frac{x}{a}\right]^{2n}+\left[\frac{y}{b}\right]^{2n}=1;\ \text{with}\ n=1,2,\ldots,\infty \tag{1}$$

where the maximum dimensions of the plate are $2a$ and $2b$ in the x and y directions, respectively. The coefficient n is the power of the super ellipse and the limiting cases of super elliptical shapes are an ellipse when $n=1$ and for larger values of n, the curve gets gradually more rectangular shapes, until for $n\rightarrow\infty$ the curve takes up a rectangular shape.

In their study, Çeribaşi and Altay have calculated the perimeters and areas of different super ellipses, and they are presented in their Table 2 [1]. They wrote in their work: "Any deviation from the area of the super ellipse causes extra error in the results. Calculating the area and perimeter of the region may give an idea about the convergence of the integration".

The discussers have found that some differences appeared in those basic results when they recalculated the perimeters and areas of the same geometries. The perimeters calculated by Çeribaşi and Altay have slight differences with the recalculated values, but the differences are more important for the recalculated areas of the super elliptical plates. To be sure about the good precision of their values, the discussers also obtained the areas and perimeters using a finite element code [2]. For example for $n=2$ the difference in the area is about 6%: 3.7081 (discussers') vs. 3.4961 [1].

The frequency coefficients calculated by Çeribasi and Altay also differ from results published by Wang et al. [3]. Wang and his co-workers had presented an excellent piece of work about buckling and vibration for super elliptical plates in 1994.

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Table 1
Comparison of the fundamental frequency coefficients for simply supported super elliptical plates. $\Omega_1=\omega_1 b^2(\rho h/D)^{1/2}$, ν=0.30.

		a/b			
		1	1.5	2	3
n=1	Present study	4.935	3.681	3.303	3.009
	Wang et al. [3]	4.935	3.681	3.303	3.009
	Çeribaşi-Altay [1]	4.935	3.687	3.314	3.035
n=2	Present study	4.633	3.399	3.005	2.740
	Wang et al. [3]	4.634	3.400	3.005	2.740
	Çeribaşi-Altay [1]	4.736	–	3.108	2.836
n=4	Present study	4.803	3.486	3.037	2.723
	Wang et al. [3]	4.804[a]	3.486[a]	3.038[a]	2.723[a]
	Çeribaşi-Altay [1]	–	–	–	–
n=8	Present study	4.894	3.540	3.069	2.735
	Wang et al. [3]	–	–	–	–
	Çeribaşi-Altay [1]	5.066	3.775	3.150	2.783
n=10	Present study	4.908	3.548	3.074	2.737
	Wang et al. [3]	4.910	3.550	3.076	2.738
	Çeribaşi-Altay [1]	5.181	–	3.216	2.826
$n\to\infty$	Present study	4.9348	3.5640	3.0843	2.7416
Rectangular	Leissa[4]	4.9348	3.5640	3.0843	2.7416

[a] These Wang's coefficients correspond to n=4 and not to n=8.

Table 2
Natural frequency coefficients $\Omega_i=\omega_i b^2(\rho h/D)^{1/2}$ for super elliptical plates. ν=0.30; n=8, 10.

		Present results							**[1]**		
	n	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6		λ_1^2	λ_2^2	λ_3^2
Simply supported											
a/b=1	8	4.895	12.278	12.278	19.605	24.560	24.674		5.0655	14.7932	28.6231
	10	4.908	12.297	12.297	19.647	24.593	24.675		5.1810	15.4828	30.2322
		4.910	*12.303*	*12.303*	*19.653*	*24.595*	*24.679*	[3]			
a/b=1.2	8	4.148	9.267	11.540	16.614	17.833	23.882		4.2904	11.1921	21.9421
	10	4.159	9.284	11.554	16.648	17.850	23.893		4.3883	11.6943	23.5105
a/b=2	8	3.074	4.911	7.990	10.476	12.304	12.306		3.2158	5.9419	11.3608
	10	3.074	4.911	7.990	10.476	12.304	12.306		3.2158	5.9419	11.3608
		3.076	*4.916*	*7.991*	*10.480*	*12.308*	*12.339*	[3]			
a/b=3	8	2.735	3.544	4.906	6.822	9.293	10.139		2.7832	3.9315	6.5759
	10	2.737	3.551	4.916	6.831	9.303	10.140		2.8259	4.0779	7.0114
		2.738	*3.556*	*4.919*	*6.863*	*9.355*	*10.143*	[3]			
Clamped											
a/b=1	8	8.997	18.349	18.349	27.059	32.896	33.056		9.3005	26.7632	44.5642
	10	8.997	18.349	18.349	27.057	32.897	33.054		9.3763	27.6621	48.4207
		8.986	*18.343*	*18.345*	*27.048*	*32.829*	*32.975*	[3]			
a/b=1.2	8	7.689	13.903	17.303	23.032	23.917	32.038		7.9395	19.2351	34.4356
	10	7.689	13.902	17.303	23.030	23.917	32.038		8.0022	20.8522	38.0785
a/b=2	8	6.145	7.958	11.195	15.837	15.997	17.773		6.2766	7.0101	11.3081
	10	6.145	7.957	11.195	15.836	15.997	17.772		6.3068	10.2140	19.1815
		6.138	*7.956*	*11.185*	*15.880*	*15.993*	*17.769*	[3]			
a/b=3	8	5.799	6.466	7.688	9.528	12.005	15.099		5.8716	7.0101	11.3081
	10	5.799	6.465	7.687	9.526	12.004	15.101		5.8826	7.1564	12.3145
		5.793	*6.465*	*7.684*	*9.555*	*12.075*	*15.653*	[3]			

In italics Wang's results [3].

The authors state that Wang and his co-workers neglected the effect of Poisson's ratio. On the basis of this wrong consideration they explained part of their differences with Wang's results, which are shown in Table 4 of Ref. [1]. In Table 1, the discussers present results of simply supported super elliptical plates, which verify that Wang's results have taken into account the Poisson's ratio, $\nu=0.30$. The recalculated values are in excellent agreement with Wang's results [3].

Apart from the matter of the Poisson's ratio, a misunderstanding happened when the authors tried to compare their results with published ones in their Table 4 [1]. They have supposed that Wang's frequency coefficients 4.804, 3.486, 3.038 and 2.723, correspond to $n=8$ (marked with [a] in Table 1) when they really correspond to $n=4$.

Table 1 shows that there are no differences between Çeribaşi and Altay elliptical plates' results and the discussers' results for $n=1$, but differences appear for super elliptical ones, this means when n adopts values bigger than 1 (for example 2, 4, 8 or 10). On the other hand, as it might be expected, the discussers' results converge for $n\to\infty$ to the frequency factors of a rectangular plate [4].

The discussers present the natural frequency coefficients for simply supported and clamped super elliptical plates with uniform thickness, for the first six modes; they were determined for various aspect ratios $1 \le a/b \le 3$ and $n=8$, 10; (see Table 2, $\nu=0.30$). Those results were obtained by an approximate solution of the problem using the same method as Çeribaşi and Altay, the Ritz method, with an approximation for the transverse displacement amplitude W defined as a summation of functions, which were adopted as monomials functions selected from a set of monomials [5], of the form $x^{q-p}\cdot y^p$

$$W_a(x,y)=\sum_{i=1}^{N}C_i f_i(x,y)=\left[\left(\frac{x}{a}\right)^{2n}+\left(\frac{y}{b}\right)^{2n}-1\right]^{\beta}\sum_{q=0}^{s}\sum_{p=0}^{q}C_i\,x^{q-p}y^p$$
$$\text{with}\quad i=\frac{q(q+1)}{2}+(p+1);\quad N=\frac{(s+1)(s+2)}{2} \tag{2}$$

which, $\beta=1$, obviously, satisfies the boundary conditions at the simply supported edge and $\beta=2$, at the clamped edge. In this case the approximation, Eq. (2), was generated using a complete set of monomials of 136 terms ($N=136$).

The calculations have been performed for simply supported and clamped super elliptical plates with aspect ratio $a/b=1$, 1.2, 2, 3; with $n=8$ and $n=10$.

Table 2 shows that discussers' results for the first six frequencies are in excellent agreement with previous published results [3]. In the same Table, it can be seen that Çeribaşi and Altay's results do not have good precision for super elliptical plates ($n=8$ and $n=10$); a reason could be that they tested convergence for only some particular cases, circular and elliptical plates ($n=1$), and this was not enough to guarantee the convergence for the super elliptical plates′ coefficients $n=8$ and 10.

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References

[1] S. Çeribaşi, G. Altay, Free vibration of super elliptical plates with constant and variable thickness by Ritz method, *Journal of Sound and Vibration* 319 (2009) 668–680.
[2] ALGOR20, Linear Dynamic Analysis, Algor Inc., Pittsburgh, PA, 2007.
[3] C.M. Wang, L. Wang, K.M. Liew, Vibration and buckling of super elliptical plates, *Journal of Sound and Vibration* 171 (3) (1994) 301–314.
[4] A.W. Leissa, Vibration of Plates, *NASA S. P.* (1969) 160.
[5] S. Maiz, C.A. Rossit, D.V. Bambill, A. Susca, Transverse vibrations of a clamped elliptical plate carrying a concentrated mass at an arbitrary position, *Journal of Sound and Vibration* 320 (2009) 1146–1163.