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Vibration control of two degrees of freedom system using variable inertia vibration absorbers: Modeling and simulation

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A R T I C L E I N F O

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A B S T R A C T

Variable inertia vibration absorbers (VIVA) are previously used for the vibration control of single degree of freedom (dof) primary systems. The performance of such absorbers is studied in many investigations. This paper presents the dynamic modeling and simulation of a proposed modified design of such VIVA's for the vibration control of two dof primary systems. Lagrange formulation is used to obtain its dynamic model in an analytical form. This model, which is highly nonlinear, is used to develop a computational algorithm to study the absorber performance characteristics. This algorithm is programmed and simulated in Matlab. The obtained results are numerically verified using SAMS2000 software. The effect of mass and stiffness of the proposed VIVA on its performance and tuning is discussed. An optimization algorithm is developed to select the best absorber parameters for vibration suppression of a specific primary system. The obtained results show a good agreement with those obtained using similar techniques. In addition, a linearized model of VIVA dynamics is developed, tested and simulated for the same data used in its nonlinear model. The relative deviation between results of the linear and nonlinear models is less than 1%, which confirms the realistic use of this linearized model. The experimental testing and verification of the simulation results of the proposed VIVA is the subject of another paper.

1. Introduction

The study of the dynamics and vibrations of mechanical systems is one of the important problems in industry. Suppression of unwanted vibrations is an important goal in many applications such as machines, tall buildings, bridges, offshore platforms, pipelines and aircraft cabins. A significant amount of work has been devoted to search for a suitable means to reduce the vibration level in these applications. Different concepts had been developed and employed in this research area. The most popular used concepts are vibration damping, isolation, and vibration absorption. Dampers dissipate system energy, and vibration isolations prevent vibration transmission, while vibration absorbers transmit the vibration energy to a secondary system. The vibration absorber is a mechanical device, consists mainly of a mass, spring and damper, designed to have a natural frequency equal to the frequency of the unwanted vibration of the primary system.

Variable inertia vibration absorbers (VIVA) were used for the vibration control of single degree of freedom (sdof) primary systems and their performance were studied in many investigations as detailed in the next section. Very few studies were done for multi-degrees of freedom (mdof) primary systems [1,2]. This paper presents the dynamic modeling and simulation of a proposed modified design of such VIVA's for the vibration control of two dof primary systems using

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Nomenclature

c_a	torsional damping coefficient of the absorber resilient element
c_p	damping coefficient of the primary system resilient element
D	system dissipating energy
f_n	natural frequency of the absorber
f	excitation frequency
f_e	excitation force applied on the primary mass
f_v	force maintain the sliding block in its position
g	gravitational acceleration
h_p	height of the primary mass measured from the reference ground
k_a	torsional stiffness of the absorber resilient element
k_p	stiffness of the primary system resilient element
l_1,l_2	distance between of primary mass resilient elements
l_s	absorber rod length
m_p	primary mass
m_s	mass of the absorber rod
m_v	mass of the sliding block
q_i	generalized coordinate
q_s	vector maps state space variables
r_s	distance between the absorber rod rotation centre and its centre of mass
r_v	moving block position.
y_p(dom)	dominator for variable y_p
θ_p(dom)	dominator for variable θ_p
t	time (s)
Y_p	primary mass linear displacement
Θ_p	primary mass angular displacement
θ_t	total inclination angle of the absorber rod
θ_f	inclination angel of the absorber rod at which no deflection occurs in the absorber torsional resilient element
θ_d	static angular deflection of the absorber resilient element
δ_p	static deflection of the primary system resilient element (no absorber)
ω_p	natural frequency of the primary system
φ_I	phase angles of system displacement variables

Lagrange formulation. The effect of the absorber mass and stiffness on its performance and tuning is discussed. An optimization algorithm is developed to select the best absorber parameters for vibration suppression of a specific primary system. After an introduction, this paper is organized in the following main sections: (Section 2) Literature survey, (Section 3) Proposed variable inertia vibration absorber, (Section 4) System mathematical modeling, (Section 5) VIVA parameters optimization, (Section 6) Computation procedure, (Section 7) Simulation results and (Section 8) Conclusions.

2. Literatur survey

Dynamic vibration absorbers DVA's can be classified into three main types: passive, active and semi-active (active–passive). Sun et al. [3] paper gives an excellent survey of these three types of DVA's. The next subsections give an overview of research done on these three classes of vibration absorbers as well the existing optimization techniques used to select their optimum parameters.

2.1. Passive DVA's [4–19]

The concept of a passive DVA device was outlined by Watts [4], when addressing a method to reduce the rolling of ships. However, the practical design of a vibration absorber was proposed by Frahm [5]. He designed a fluid tank system to reduce the rolling of ships. In his design, both the primary system and the absorber had no damping. The DVA's advantages include the ease of installation, simple design, and effectiveness over narrowband frequency vibration. The DVA could be selected such that the combined system has anti-resonance at the desired cancellation frequency. An undamped DVA allows the best suppression at a specified fixed design frequency. This is only effective if the excitation frequency remains constant. The disadvantage is that if the excitation frequency shifts, the response of the combined system may be higher than the primary system alone, to remedy this problem, Ormondroyd and Den Hartog [6] considered the case of a damped vibration absorber attached to the primary system. This resulted in a system effective over an extended frequency range by reducing the response at the two resonant frequencies of the combined system. However, the response at the primary system natural frequency can no longer be reduced to zero. Hence, a trade exists between the primary system's response and operating in a broadband. Den Hartog [7] described an optimization method for broadband applications to obtain the optimum tuning frequency and damping ratio. Esmail zadeh and Jalili [8] studied the optimization of the vibration absorber to reduce vibrations of a structurally damped Timoshenko beam. Al-Bedoor et al. [9] used the concept of the passive absorber to reduce the torsional vibrations during start up of systems driven by synchronous motor. Ertas et al. [10] studied the performance of passive pendulum type vibration absorber attached to a primary structure whose orientation varies. Cuvalci [11] numerically and experimentally determined the absorption region with respect to forcing amplitude, internal frequency ratio and mass ratio, for a system of a cantilever beam connected to a pendulum absorber. Anderson et al. [12] used a sloshing absorber instead of damped DVA, which needs less maintenance and makes use of existing water storage tanks to reduce structures vibration. Cuvalci et al. [13] had performed a parametric study to experimentally determine the effect of forcing frequency, forcing amplitude, mass ratio and frequency ratio on the displacement response of a system coupled to a passive DVA. Dayou and Brennan [14] used multiple DVA's to reduce the structural vibration at each single frequency in the frequency range of interest. Maes and Sol [15] presented a smart solution to reduce the noise generated by the standing wave generated due to the motion of a vehicle on a railway. Cha [16] investigated reducing the vibration in a general structure during harmonic excitation by proper choosing of mass-spring properties and locations. Yaman and Sen [17] had studied the nonlinear behavior of a slender beam with a tip mass attached to a pendulum used as a passive vibration. The slender beam is of varying orientation and is subjected to a sinusoidal base excitation. Fischer [18] compared the effect of pendulum, ball and sloshing liquid absorbers and assessed the effectiveness of all of them. Hitchcock et al. [19] presented the results of a full-scale installation of a passive vibration absorber, called a liquid column vibration absorber (LCVA), on a 67 m height steel frame communications tower.

2.2. Active DVA's [20–41]

To provide additional features to vibration suppression in many applications, active DVA's were introduced. Active DVA's have an arbitrary force generation mechanism in parallel with its spring and damper. The force generation mechanism adds more flexibility to incorporate better control to provide cancellation forces. These forces are frequently implemented with a voice coil actuator design. Chang and Soong [20] presented an approach for optimal design of an active DVA system. Summerfield et al. [21,22] had developed an adaptive control system applied to minimize the force transmitted through a two-stage vibration isolation mount. Zimmerman et al. [23] described an active DVA designed for structural control. Sato [24] had examined a unique configuration using a variable speed rotor that may be partially filled with a liquid. Stephens et al. [25] had developed a new direction of using a damped dynamic vibration absorber with an active control element. Olgac and Holm Hansen [26,27] had developed the delayed resonator concept to provide active control of a DVA. von Flotow et al. [28] presented a survey paper which covers a number of adaptively tuned DVA designs. Okada-Matsuda and Hashitani [29] presented a novel circuit to provide sensing and actuation in a voice coil design. Heilmann [30] and Burdisso and Heilmann [31] had studied a single-mass active dynamic absorber and a dual-mass active dynamic absorber for broadband control. Hyde-Tupper and Anderson [32] had developed an active actuator using the same fluid for damping as a hydraulic lever for a voice coil. Jalili and Olgac [33,34] had studied the use of multiple delayed resonator vibration absorber as well the sensitivity of an optimum delayed feedback vibration absorber. Pai et al. [35] used nonlinear vibration absorbers with higher order internal resonances to control structural vibrations. Lesieutre et al. [36] described a class of recently developed inertial actuators that is passed on mechanical amplification of an active piezoceramic element. Caneal et al. [37] presented an experimentation of two different tuning algorithms to minimize the radiated noise of simply supported plate. Lin [38] developed an approach for achieving a high-performance active piezoelectric absorber of a smart panel using adaptive networks in a hierarchical fuzzy control. Abakumov and Miatov [39] presented different control algorithms for active vibration isolation systems subject to random disturbances. Shang-Teh et al. [40] developed an active vibration absorber for a flexible plate boundary-controlled by a linear motor. Sun et al. [41] studied the control effort of an active resonator absorber.

2.3. Semi-active DVA's [42–67]

A semi-active DVA is a system implemented such that small energy expenditures can alter the system parameters. There are three types of semi-active DVA's, variable stiffness, variable damping and variable inertia DVA. A semi-active system only requires signal processing and low level power signals rather than full power electronics for an active system. Semi-active methods involve the use of passive elements that can be optimally tuned to perform over a certain frequency range. Lamancusa [42] described haw semi-active methods are used for narrowband applications with adaptive Helmholtz resonators. These resonators are able to control sound within a certain frequency range, over a number of different speeds, by varying the resonator neck dimensions or cavity volume or both. Graf et al. [43] explained the use of semi-active methods to vary the stiffness and damping of an engine mount in the area of broadband applications as well in structural control areas. Ryan et al. [44] proposed a semi-active vibration absorber using variable spring stiffness as the adaptive component mounted on a primary system. Slavicek and Bollinger [45] had produced a variable stiffness using nonlinear stiffness characteristics of plastic elements. Karnopp et al. [46] had implemented a semi-active electro-hydraulic damper. Hrovat et al. [47] had demonstrated semi-active hydraulically controlled damping modulation to damp wind-induced vibration in tall buildings. Rakheja and Sankar [48] studied the vibration isolation using a semi-active 'on–off' damper with a variable sized orifice which controls the damping. Miller [49] reported the use of passive, semi-active and active suspension system in tests of a quarter car model. Tanaka and Kirushima [50] studied the transient vibration at impact rather than steady state using a semi-active damper. Wang and Lai [51] had developed a control strategy during rotational system startup using variable stiffness vibration absorber. Ryan et al. [52] presented a variable stiffness DVA that changes the helical spring length for stiffness alteration. Nagaya et al. [53] used a variable stiffness vibration absorber to control principle modes. Jie Liu and Kefu Liu [54] presented an electromagnetic vibration absorber (EMVA) whose stiffness could be tunable on-line and has the capability suppressing a primary system vibration excited by a harmoni force with a variable frequency. Seung-Yong et al. [55] presented a semi-active fuzzy control technique to enhance the seismic performance of cable-stayed bridges using magneto-rheological (MR) dampers. Chooi and Olutunde [56] had modeled and tested the MR dampers using analytical flow solutions. Swevers et al. [57] presented a flexible and transparent model-free control structure based on physical insights in the car and semi-active suspension dynamics used to linearize and decouple the system. Erramouspe et al. [58] presented a modified resetting stiffness algorithm which is implemented and tested. Lin [59] developed an innovative approach for achieving a high performance adaptive piezoelectric absorber in which the concept of hierarchy for controlling fuzzy systems is applied.

The concept of variable inertia vibration absorber VIVA was proposed by Jalili et al. [60]. The main idea is to change the position of a mass sliding along a hinged rod to change the inertia of the absorber and consequently its natural frequency to be equal to the primary system excitation frequency. VIVA devices can be adaptively tuned to suppress the unwanted vibrations according to that measured from the original system. Takita and Seto [61] used a variable length pendulum to adaptively tune the absorber frequency by adjusting its moment arm. This was a creative implementation which provides a tuning mechanism for the anti-resonance. Moyka [62] examined a semi-active system using a variable inertia pendulum through a stepper motor. The frequency response for various pendulum lengths on a cantilever beam primary system was demonstrated. Williamas et al. [63] used shape memory alloys to construct and test a variable stiffness DVA's. Morgan and Wang [64] presented an active–passive piezoelectric absorber configuration that can track and suppress multiple harmonic excitations. Fallahi et al. [65] examined VIVA dynamics as adaptive tuned vibration absorbers. A linearized dynamic model is used for its tonal (pure harmonic) tuning with a comparison of the tuned and detuned responses of the primary system. Megahed et al. [66] had proposed different VIVA designs and their dynamic models were analytically obtained and simulated. El-kabbany [67] had studied and simulated the dynamics of different VIVA designs with experimental verifications for sdof primary systems. As mentioned before, Megahed and Abd El-Razik [1] and Abd El-Razik [2] had extended the use of VIVA for two dof primary systems in simulation and experimentation.

2.4. DVA performance optimization [6,68–81]

Many studies had been investigated about the optimization of the performance of DVA absorbers. For a DVA attached to a sdof undamped primary system, the used optimization techniques are based on the existence of two fixed invariant points [6]. This concept is not valid for damped primary systems. The optimum values of stiffness and damping coefficients of a DVA may be determined using the technique presented in the famous book of Den Hartog [68]. Since this pioneer work, several contributions had been achieved, which consider sdof and mdof undamped primary systems. Few studies were done on damped vibration absorbers attached to sdof damped main systems. Randall et al. [69] used a numerical search method to find the optimal damping and stiffness values of an absorber attached to a damped main system. Thompson [70] used the frequency locus method for the same problem but considering a primary system with associated viscous damping. The construction of the frequency loci for a general system leads to the determination of graphical criteria for the optimization problem. Kitis et al. [71] employed a numerical optimization method for finding the optimal parameters of a damped vibration absorber attached to a damped mdof primary system. The method is applied to twenty two dof primary systems. Snowdon et al. [72] proposed a cruciform dynamic vibration absorber that comprises two free–free beams loaded with masses at their free ends. Each branch of the beam is tuned to the fundamental frequency and second or third natural resonances of the primary system. Vakakis and Paipetis [73] studied the effect of a viscously damped dynamic absorber on the behavior of a linear vibration system with mdof. The optimum values of the absorber parameters are obtained with minimizing the transmissibility of the system over the whole frequency range. Ozer and Royston [74] presented an extension of the classical Den Hartog's approach to mdof undamped main system. Analytical expressions of the optimal absorber parameters are obtained using the Sherman–Morrison matrix inversion formula [75]. Rice [76] used the modal data and finite element methods for multiple discrete vibration absorber systems in broadband applications. Rade and Steffen [77] proposed a general methodology for the optimum selection of DVA parameters so as to guarantee the efficiency of those devices over a previously selected frequency band. Zuo and Nayfeh [78], an efficient numerical approach based on the descent-sub gradient method was proposed to maximize the minimal damping of modes in a prescribed frequency range for general viscous or hysteretic mdof tuned mass systems. Many other applications on the optimization of absorber performance can be found in [79–81].

3. Proposed variable inertia vibration absorber

Very few studies were done for mdof primary systems [1,2]. Fig. 1 presents a schematic sketch of the proposed VIVA attached to a two dof primary system. The primary system has a mass (m_p) and mass moment of inertia (I_p) mounted on a support of linear stiffness (k_p) and linear damping coefficient (c_p).The primary system is subjected to an excitation force (f_e) at its centre of mass and is described by two independent variables (y_p, θ_p). The proposed VIVA consists of a hinged rod of mass (m_s) and mass moment of inertia of (I_s), sliding block of mass (m_v) and mass moment of inertia of (I_v). The hing is assumed to have a linear tortional stiffness k_a and a linear damping coefficient c_a. The sliding block can be positioned at any distance (r_v) along the rod as shown in Fig. 1. The absorber geometric dimensions (h_P, r_s) are chosen to satisfy the system constraints. The absorber is described by two independent variables (θ, r_v). The whole system has four degrees of freedom (y_p, θ_p, θ, r_v).

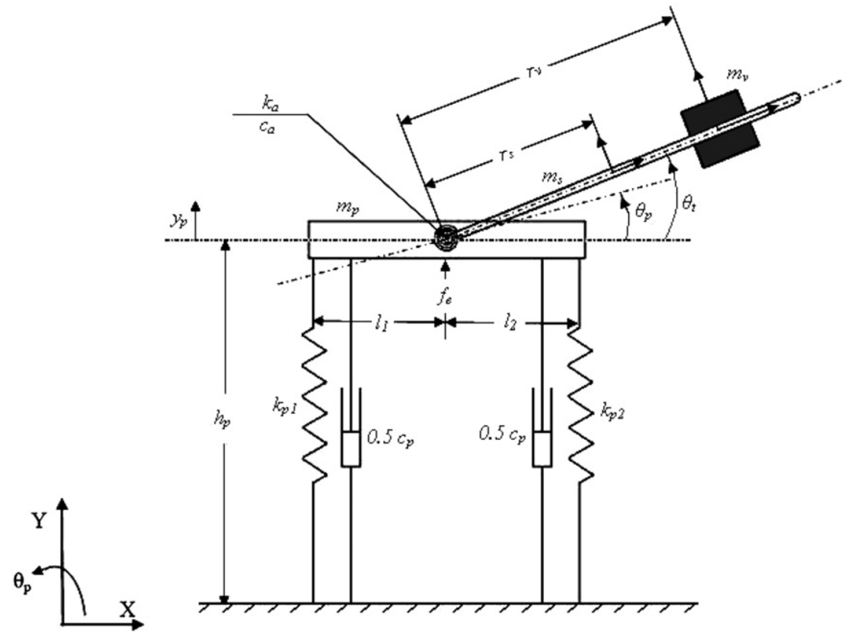


Fig. 1. Proposed VIVA attached to 2 dof primary system.

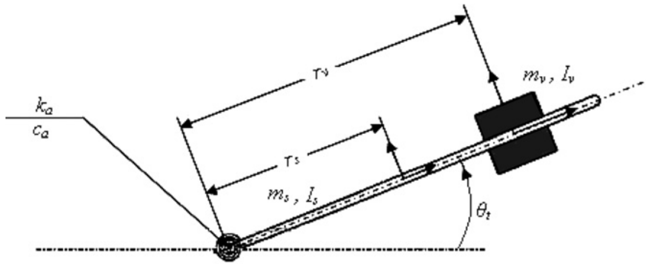


Fig. 2. VIVA absorber.

4. System mathematical modeling

The main idea to reduce the vibration of the primary system is to position the sliding block at most suitable location on the absorber rod (r_v) to minimize (y_p,θ_p). To achieve this goal, the dynamics of the whole system is studied using Lagrange equations of motion and the tonal tuning of the proposed VIVA is obtained as function of the absorber parameters (k_a and c_a).

The general form of Lagrange formulation in which damping is linear is as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}+\frac{\partial D}{\partial \dot{q}_i}=\boldsymbol{\Gamma}^i, \quad q_i=[y_p\theta_p\theta r_v]^{\mathrm{T}} \tag{1}$$

where T and U are, respectively, the total kinetic and potential energy of the system, D is the dissipated energy in the linear damping elements, while Γ^i, q_i and $\dot{q}_i$ are, respectively the ith variable generalized force, generalized coordinate and generalized velocity. All calculations are done [2] and the following four equations of motions are obtained:

$$\begin{aligned}&(m_p+m_s+m_v)\ddot{y}_p+(m_s r_s+m_v r_v)\cos(\theta_t)\ddot{\theta}_p-(m_s r_s+m_v r_v)\cos(\theta_t)\ddot{\theta}+m_v\sin(\theta_t)\ddot{r}_v+m_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})\cos(\theta_t)+m_v(\dot{\theta}_p-\dot{\theta})(\dot{r}_v\cos(\theta_t)\\&\quad-r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))-m_s r_s(\dot{\theta}_p-\dot{\theta})^2\sin(\theta_t)+k_{p_1}(y_p-l_1\theta_p)+k_{p_2}(y_p+l_2\theta_p)+c_p\dot{y}_p=f_e\end{aligned} \tag{2a}$$

$$\begin{aligned}&(m_s r_s+m_v r_v)\cos(\theta_t)\ddot{y}_p+(m_s r_s^2+m_v r_v^2+I_s+I_p+I_v)\ddot{\theta}_p-(m_s r_s^2+m_v r_v^2+I_s+I_v)\ddot{\theta}+m_v\dot{y}_p(\dot{r}_v\cos(\theta_t)-r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))\\&\quad+4m_v r_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})-m_s r_s(\dot{\theta}_p-\dot{\theta})\sin(\theta_t)+c_a(\dot{\theta}-\dot{\theta}_p)+k_a(\theta-\theta_p)-k_{p_1}(y_p-l_1\theta_p)(l_1)+k_{p_2}(y_p+l_2\theta_p)(l_2)=0\end{aligned} \tag{2b}$$

$$\begin{aligned}&-(m_s r_s+m_v r_v)\cos(\theta_t)\ddot{y}_p-(m_s r_s^2+m_v r_v^2+I_s+I_p+I_v)\ddot{\theta}_p+(m_s {r_s}^2+m_v r_v^2+I_s+I_v)\ddot{\theta}+m_v\dot{y}_p(\dot{r}_v\cos(\theta_t)\\&\quad+r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))-4m_v r_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})+m_s r_s(\dot{\theta}_p-\dot{\theta})\sin(\theta_t)+c_a(\dot{\theta}-\dot{\theta}_p)+k_a(\theta-\theta_p)=0\end{aligned} \tag{2c}$$

$$m_v\sin(\theta_t)\ddot{y}_p+m_v\ddot{r}_v-m_v r_v(\dot{\theta}_p-\dot{\theta})^2-2m_v r_v(\dot{\theta}_p-\dot{\theta})^2+c_v\dot{r}_v+m_v g\sin(\theta_t)=f_v \tag{2d}$$

where h_p is the height of the primary mass at its static equilibrium position (see Fig. 1), f_v the applied force to maintain the moving block in its position (input data), θ_p the primary system angular displacement (see Fig. 1), θ the absorber rod angular displacement measured from its static equilibrium given by

$$k_a\theta_d-(m_sr_s+m_vr_v)g\cos(\theta_f-\theta_d+\theta_p)=0 \tag{3a}$$

where θ_d is the absorber rod angular static deflection at static equilibrium position, θ_f the absorber rod installation angle with zero torsional deflection (input data) and θ_t the absorber rod total inclination angle (see Fig. 1).

$$\theta_t=\theta_f-\theta_d-\theta+\theta_p \tag{3b}$$

The four equations of motion of the whole system (Eq. (2)) can be arranged in matrix form as follows:

$$M\ddot{q}=N\begin{bmatrix}(m_p+m_s+m_v) & (m_sr_s+m_vr_v)\cos(\theta_t) & -(m_sr_s+m_vr_v)\cos(\theta_t) & m_v\sin(\theta_t)\\ (m_sr_s+m_vr_v)\cos(\theta_t) & m_sr_s^2+m_vr_v^2+I_s+I_v+I_p & -(m_sr_s^2+m_vr_v^2+I_s+I_v) & 0\\ -(m_sr_s+m_vr_v)\cos(\theta_t) & -(m_sr_s^2+m_vr_v^2+I_s+I_v) & m_sr_s{}^2+m_vr_v^2+I_s+I_v & 0\\ m_v\sin(\theta_t) & 0 & 0 & m_v\end{bmatrix}\begin{bmatrix}\ddot{y}_p\\ \ddot{\theta}_p\\ \ddot{\theta}\\ \ddot{r}_v\end{bmatrix}=\begin{bmatrix}A_1\\ A_2\\ A_3\\ A_4\end{bmatrix} \tag{4a}$$

where

$$\begin{aligned}A_1&=-m_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})\cos(\theta_t)-m_v(\dot{\theta}_p-\dot{\theta})(\dot{r}_v\cos(\theta_t)-r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))+m_sr_s(\dot{\theta}_p-\dot{\theta})^2\sin(\theta_t)-k_{p_1}(y_p-l_1\theta_p)\\&\quad-k_{p_2}(y_p+l_2\theta_p)-c_p\dot{y}_p+f_e\\ A_2&=-m_v\dot{y}_p(\dot{r}_v\cos(\theta_t)-r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))-4m_vr_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})+m_sr_s(\dot{\theta}_p-\dot{\theta})\sin(\theta_t)-c_a(\dot{\theta}-\dot{\theta}_p)-k_a(\theta-\theta_p)\\&\quad+k_{p_1}(y_p-l_1\theta_p)(l_1)-k_{p_2}(y_p+l_2\theta_p)(l_2)\\ A_3&=-m_v\dot{y}_p(\dot{r}_v\cos(\theta_t)+r_v(\dot{\theta}_p-\dot{\theta})\sin(\theta_t))+4m_vr_v\dot{r}_v(\dot{\theta}_p-\dot{\theta})-m_sr_s(\dot{\theta}_p-\dot{\theta})\sin(\theta_t)-c_a(\dot{\theta}-\dot{\theta}_p)-k_a(\theta-\theta_p)\\ A_4&=-m_vr_v(\dot{\theta}_p-\dot{\theta})^2-2m_vr_v(\dot{\theta}_p-\dot{\theta})^2+c_v\dot{r}_v+m_vg\sin(\theta_t)+f_v\end{aligned} \tag{4b}$$

The whole system equations of motion (Eq. (4)) can be arranged in state space form by defining the following state variables:

$$q_s=\begin{bmatrix}y_p & \dot{y}_p & \theta_p & \dot{\theta}_p & \theta & \dot{\theta} & r_v & \dot{r}_v\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 & q_8\end{bmatrix}^{\mathrm{T}} \tag{5a}$$

$$\dot{q}_s=\begin{bmatrix}q_2 & \ddot{y}_p & q_4 & \ddot{\theta}_p & q_6 & \ddot{\theta} & q_8 & \ddot{r}_v\end{bmatrix}^{\mathrm{T}} \tag{5b}$$

As tuning process starts by positioning the sliding block location (r_v) on the absorber rod, it is normal to assume (r_v) and its derivatives as known input and the whole system equations of motion are reduced to three equations as given after in state space form:

$$\begin{bmatrix}(m_p+m_s+m_v) & ((m_sr_s+m_vq_7)\cos(\theta_f-\theta_d+q_3+q_5)) & -(m_sr_s+m_vq_7)\cos(\theta_f-\theta_d+q_3+q_5)\\ (m_sr_s+m_vq_7)\cos(\theta_f-\theta_d+q_3+q_5) & m_sr_s^2+m_vq_7^2+I_v+I_s+I_p & -(m_sr_s^2+m_vq_7^2+I_v+I_s)\\ -(m_sr_s+m_vq_7)\cos(\theta_f-\theta_d+q_3+q_5) & -(m_sr_s^2+m_vq_7^2+I_v+I_s) & m_sr_s^2+m_vq_7^2+I_v+I_s\end{bmatrix}\begin{bmatrix}\dot{q}_2\\ \dot{q}_4\\ \dot{q}_6\end{bmatrix}=\begin{bmatrix}A''_1\\ A''_2\\ A''_3\end{bmatrix} \tag{5c}$$

where

$$\begin{aligned}A''_1&=-m_vq_8(q_4-q_6)\cos(\theta_f-\theta_d+q_3-q_5)-m_v(q_4-q_6)(q_8\cos(\theta_f-\theta_d+q_3-q_5)-r_v(q_4-q_6)\sin(\theta_f-\theta_d+q_3-q_5))\\&\quad+m_sr_s(q_4-q_6)^2\sin(\theta_f-\theta_d+q_3-q_5)-k_{p_1}(q_1-l_1q_3)-k_{p_2}(q_1+l_2q_3)-c_pq_2+f_e\\ A''_2&=-m_vq_2(q_8\cos(\theta_f-\theta_d+q_3-q_5)+q_7(q_4-q_6)\sin(\theta_f-\theta_d+q_3-q_5))-4m_vq_7q_8(q_4-q_6)\\&\quad+m_sr_s(q_4-q_6)\sin(\theta_f-\theta_d+q_3-q_5)-c_a(q_6-q_4)-k_a(q_5-q_3)+k_{p_1}(q_1-l_1q_3)(l_1)-k_{p_2}(q_1+l_2q_3)(l_2)\\ A''_3&=-m_vq_2(q_8\cos(\theta_f-\theta_d+q_3-q_5)+q_7(q_4-q_6)\sin(\theta_f-\theta_d+q_3-q_5))+4m_vq_7q_8(q_4-q_6)\\&\quad-m_sr_s(q_4-q_6)\sin(\theta_f-\theta_d+q_3-q_5)-k_a(q_5-q_3)-c_a(q_6-q_4)\end{aligned} \tag{5d}$$

These equations will be used later for the simulation process of the proposed VIVA for specific values of its parameters and their effects on its performance.

5. VIVA parameters optimization

5.1. Absorber tonal tuning

Tonal tuning means that the absorber natural frequency is set equal to the excitation frequency of the primary system which causes the vibrations. The proof of this fact in the case of a two dof primary system is presented in Appendix A. Using VIVA, this can be achieved by positioning the moving block to the exact location (r_v) that suppresses the undesirable frequency and this can be calculated as follows [2]:

- getting the absorber equation of motion (see Fig. 2) and/or by putting y_p=0 and θ_p=0 in Eq. (2c) then the equation of motion of the absorber rod alone will be as follows:

$$(m_s r_s^2 + m_v r_v^2 + I_s + I_v)\ddot{\theta} + c_a\dot{\theta} + k_a\theta = 0 \tag{6a}$$

- calculate the absorber natural frequency ω_n which is

$$\omega_n(\text{absorber}) = \sqrt{\frac{k_a}{(m_s r_s^2 + m_v r_v^2 + I_s + I_v)}}$$
$$f_n(\text{absorber}) = \frac{\omega_n}{2\pi} \tag{6b}$$

It is noticed that the absorber natural frequency is where the equilibrium position of the absorber rod is θ=θ_0=0.

5.2. Absorber parametric study

Referring to Eq. (6b), the frequency range of VIVA depends on the selection of its parameters [m_s, r_s, I_s, r_v, k_a, m_v and I_v]. The optimum selection of such parameters has a good effect on its performance as outlined after:

- **Effect of absorber torsional stiffness (k_a):** Using Eq. (6), the effect of spring stiffness coefficient (k_a), in the tunable frequency range, is simulated for specific values of its other parameters. Fig. 3 presents the absorber natural frequency (Hz) versus the displacement of the moving block r_v (m) for different values of the stiffness coefficient

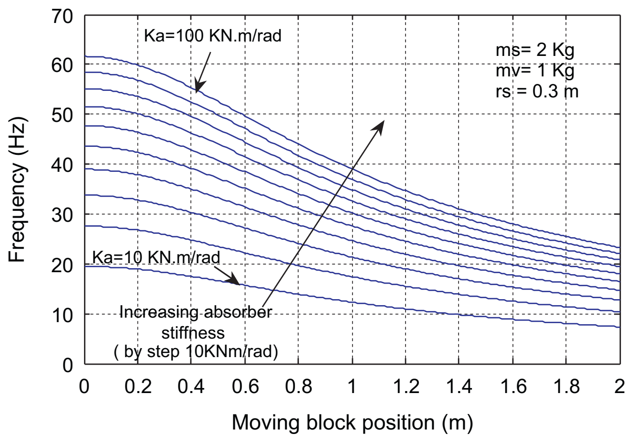


Fig. 3. Effect of absorber torsional stiffness (k_a).

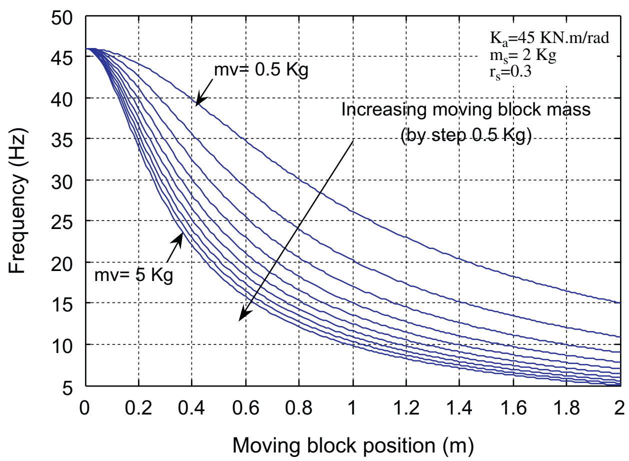


Fig. 4. Effect of absorber mass (m_v).

k_a (k_a=10, 20, 30,… and 100 kN m/rad) with m_s=2 kg m_v=1 kg, r_s=0.3 m. The obtained results show that increasing the spring stiffness coefficient (k_a) is more effective at high frequency range with small values of r_v.

- **Effect of absorber moving block inertia** (m_v and I_v): Increasing the inertia (m_v and $I_v = m_v r_v^2$) of the moving block increases the tunable frequency range as shown in Fig. 4. These two parameters **(**m_v and I_v**)** are more effective at lower frequency range with higher values of r_v.

5.3. VIVA optimization algorithm

After selecting the absorber constant parameters [m_s, r_s, I_s], its variable parameters [r_v, k_a, m_v, I_v] have to be optimized for better performance. Without loss of generality, the upper bounds of these parameters are relaxed to have a nonlinear constrained optimization problem. For VIVA better performance, its rod mass is always found to be the specified upper bound [77,78]. Though using a heavier mass reduces the dynamic response to a significant extent, the reduction may not be justified in terms of increasing mass of the system. In this section, a minimization problem is programmed to find the optimum values of VIVA variable parameters. One of the used methods is the large scale constrained nonlinear method that explores the searching space from a starting point [79]. This method failed to find the optimum values of absorber parameters and medium-scale method (line search) is used instead [2]. Fig. 5 presents the flow chart of a computation algorithm based on line search and using Eq. (6a) to find the minimum values of k_a, m_v, and maximum value of r_v. The absorber variable inertia I_v is calculated from the other parameters (m_v,r_v).

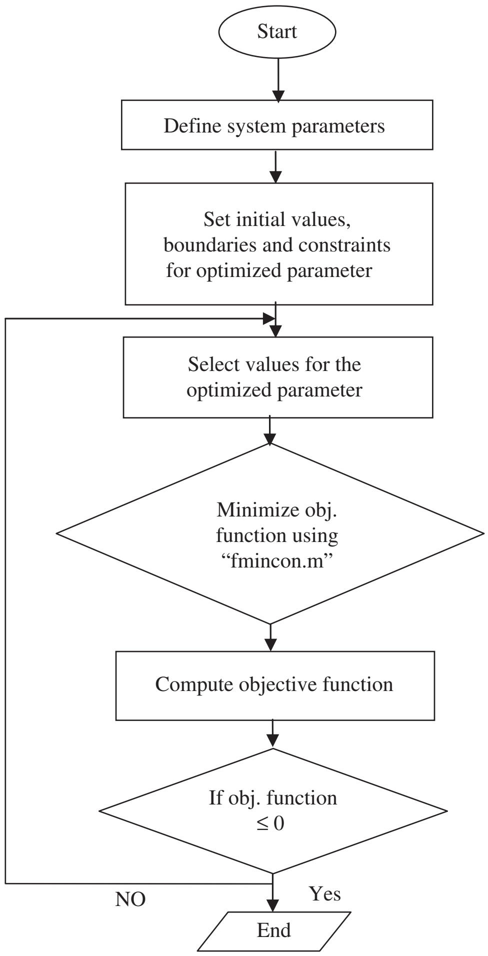


Fig. 5. Flow chart of the optimization process.

5.4. Numerical example (VIVA optimum parameters)

For simulation purpose, the developed optimization algorithm (Fig. 5), is tested using arbitrary input data for VIVA in two cases with two different operating frequency ranges (Case A: 24–40 Hz) and (Case B: 11–13.5 Hz). Table 1 includes the VIVA numerical values, operating frequency ranges and the obtained VIVA optimum parameters.

In the next sections, the computation algorithm for studying the performance of the proposed VIVA is outlined. Two explicit cases are simulated for the data given in Table 1 for the two specified frequency ranges.

Table 1
VIVA optimum parameters.

(a) Input data (Case A)	r_s=0.5 m	m_s=5 kg	I_s=0.416 kg m^2
	Operating frequency range is taken 24–40 Hz		
Output results (VIVA optimum parameters)	$r_{v(\max)}$=1.0 m	m_v=2.5977 kg	$I_v=m_v r_v^2$
	k_a=184.23 kN m/rad		
(b) Input data (Case B)	r_s=0.28 m	m_s=2.5199 kg	I_s=0.2634 kg m^2
	Operating frequency range is taken 11–13.5 Hz		
Output results (VIVA optimum parameters)	$r_{v(\max)}$=0.56 m	m_v=0.4379 kg	$I_v=m_v r_v^2$
	k_a=3.377212 kN m/rad		

Table 2
System parameters numerical values (see Fig. 2).

Primary system	Absorber rod	Sliding block	Other data
Case A			
m_p=30 kg	m_s=2 kg	m_v=1 kg	Excitation data f_{ex}=f_n=30 Hz f_e=50 sin($2\pi f_{ex}t$)
I_p=1.625 kg m^2	I_s=0.06 kg m^2	$I_v=m_v r_v{}^2$ kg m^2	
c_p=153.8 N s/m	c_a=0.5 N s/rad	C_v=20 N s/m	
k_{p1}=150 kN/m	k_a=20 kN m/rad		
k_{p2}=145 kN/m	θ_f=0		Simulation time t=3 s
l_p=0.8 m	r_s=0.3 m	r_v (variable)	
Case B			
m_p=16.406 kg	m_s=2.5199 kg	m_v=0.43788 kg	Excitation data f_{ex}=f_n=11 Hz f_e=2 sin($2\pi f_{ex}t$)
I_p=0.1275 kg m^2	I_s=0.2634 kg m^2	$I_v=m_v r_v{}^2$ kg m^2	
c_p=30 N s/m	c_a=0.5 N s/rad	C_v=5 N s/m	
k_{p1}=72 576 kN/m	k_a=3.377212 kN m/rad		
k_{p2}=72 576 kN/m	θ_f=0		Simulation time t=3 s
l_p=0.46 m	r_s=0.28 m	r_v (variable)	

6. Computation procedure

The VIVA optimum parameters obtained in the preceding sections (Table 1) are used to develop a computation algorithm for simulating a real VIVA attached to a two dof primary system. The goal of these simulations results is to assess the feasibility and performance of the proposed VIVA. The system response at any time (t) can be calculated using Euler's method. In this case the initial conditions and the time step (Δt) are known. The obtained system equations of motion (Eqs. (5) and (6)) are used to develop the computational algorithm described after:

- Set the system input data m_p, i_p, $\imath_p$, k_p, c_p and the excitation frequency f_{ex} (see Table 2).
- Set the suitable absorber parameters [m_s, r_s, I_s, r_v, k_a, m_v, I_v] and its inclination angle θ_f (see Table 2).
- Use the absorber natural frequency ω_n (Eq. (6)) to calculate the value of the tuned r_v and hence I_v.
- Using the system of equations (Eq. (5)), the system dynamics can be written in the form:

$$\ddot{q} = M^{-1}N \tag{5e}$$

- Set the initial values of all system variables and the desired time step (Δt)
- Use Euler Integration method q, q are obtained and plotted with time using Matlab.

7. Simulation results

The developed computation procedure is used to simulate the system presented in Fig. 1 in two cases: the obtained nonlinear model (Eq. (5)) and a linearized model [2]. A linearized model of the whole system is obtained using Eq. (5), as follows:

- absorber rod installation angle (θ_f) is taken as equal to zero (θ_f=0);
- total inclination angle (θ_t) is assumed as small as cos(θ_t)=1 and sin(θ_t)=θ_t.

The numerical values given in Table 2 are used for both cases and obtained simulation results are presented in the next section.

7.1. Simulation results of nonlinear model

Case #1: High frequency range:

Data#1: Using the data given in Tables 1(a) and 2 with a tuned VIVA where the sliding block is at its tuning position (r_v=0.2673) and excitation frequency of 30 Hz.

Simulation results #1 (Figs. 6–8): The primary mass linear and angular displacements (y_p,θ_p) and the developed VIVA absorber angular displacement (θ) are shown in Figs. 6–8, respectively, in time and frequency (FFT) domains at the sliding block tuned position (r_v=0.2673). The frequency domain shows the peak frequency at 30 Hz (Figs. 6 and 7), which is equal to the excitation frequency of the two dof primary system. Another peak appears at 15 Hz (Fig. 6), which is the natural frequency of the primary system. Fig. 8 shows that the absorber natural frequency (30 Hz) equal to the primary system excitation frequency.

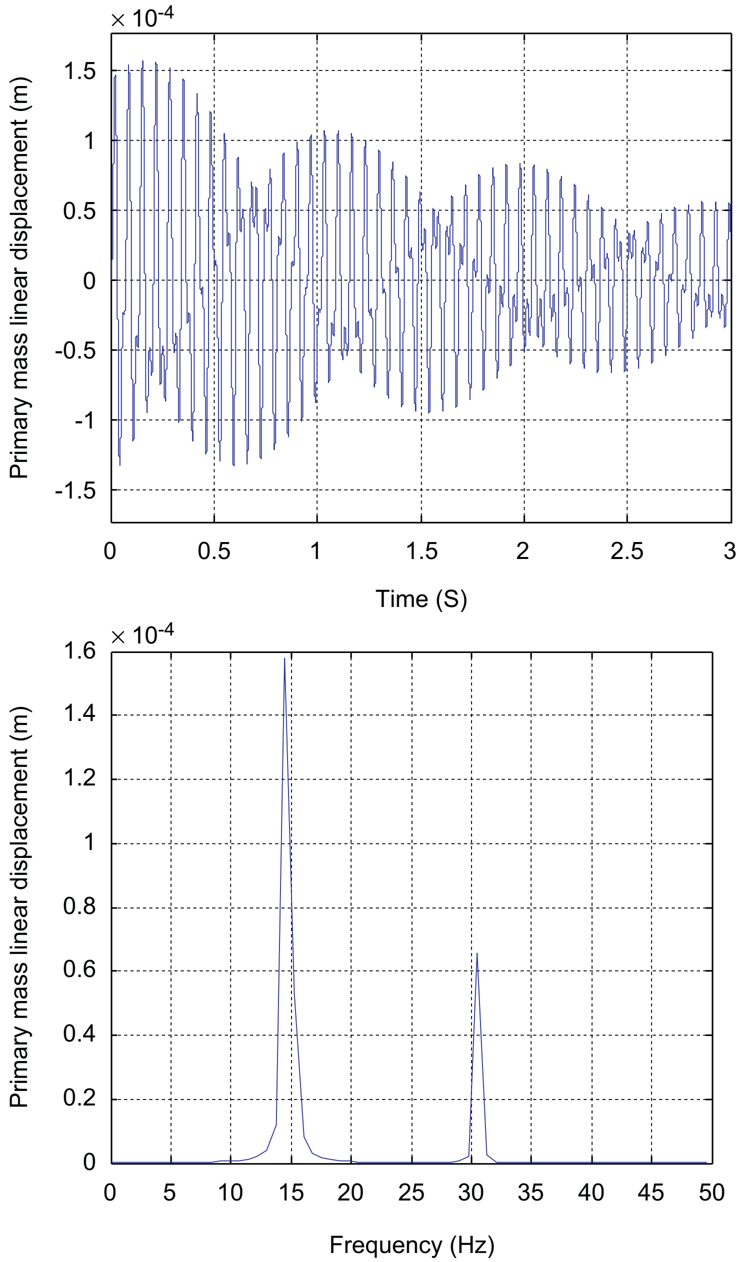


Fig. 6. Primary mass displacement (y_p) [Case #1: nonlinear model (r_v=0.2673 m)].

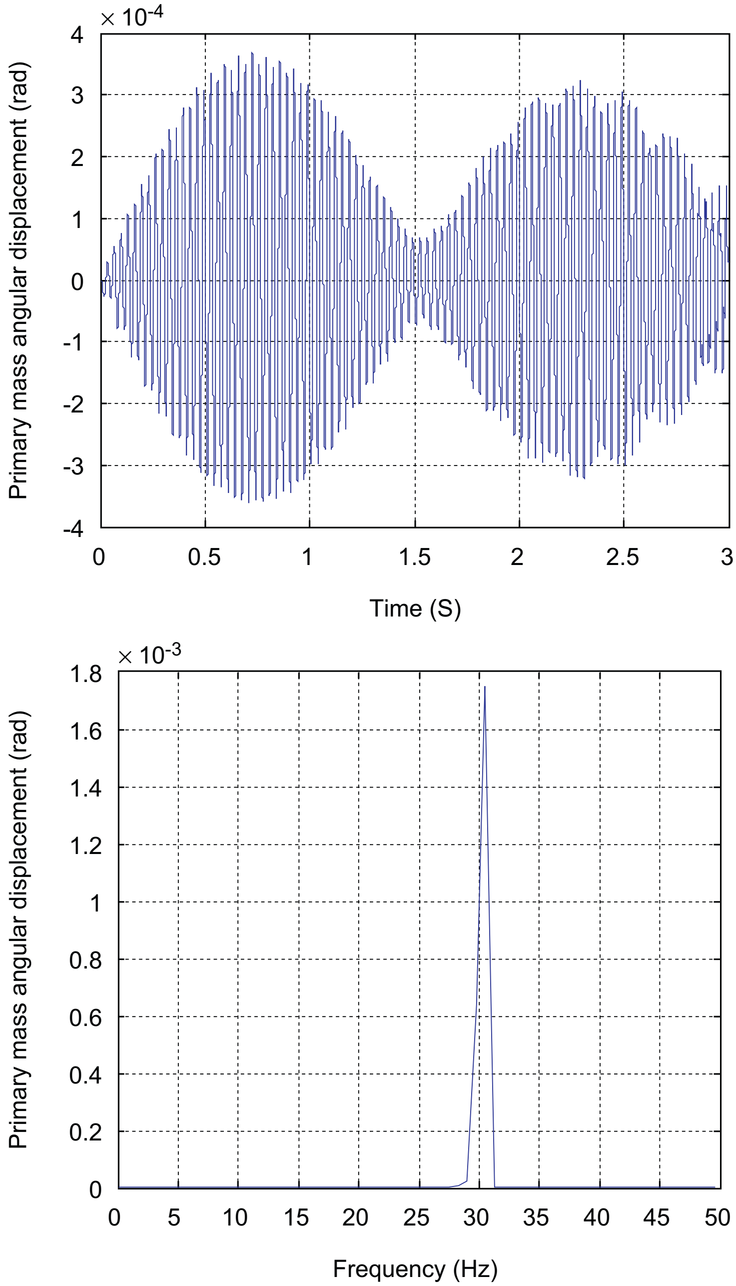


Fig. 7. Primary mass displacement (θ_p) [Case #1: nonlinear model (r_v=0.2673 m)].

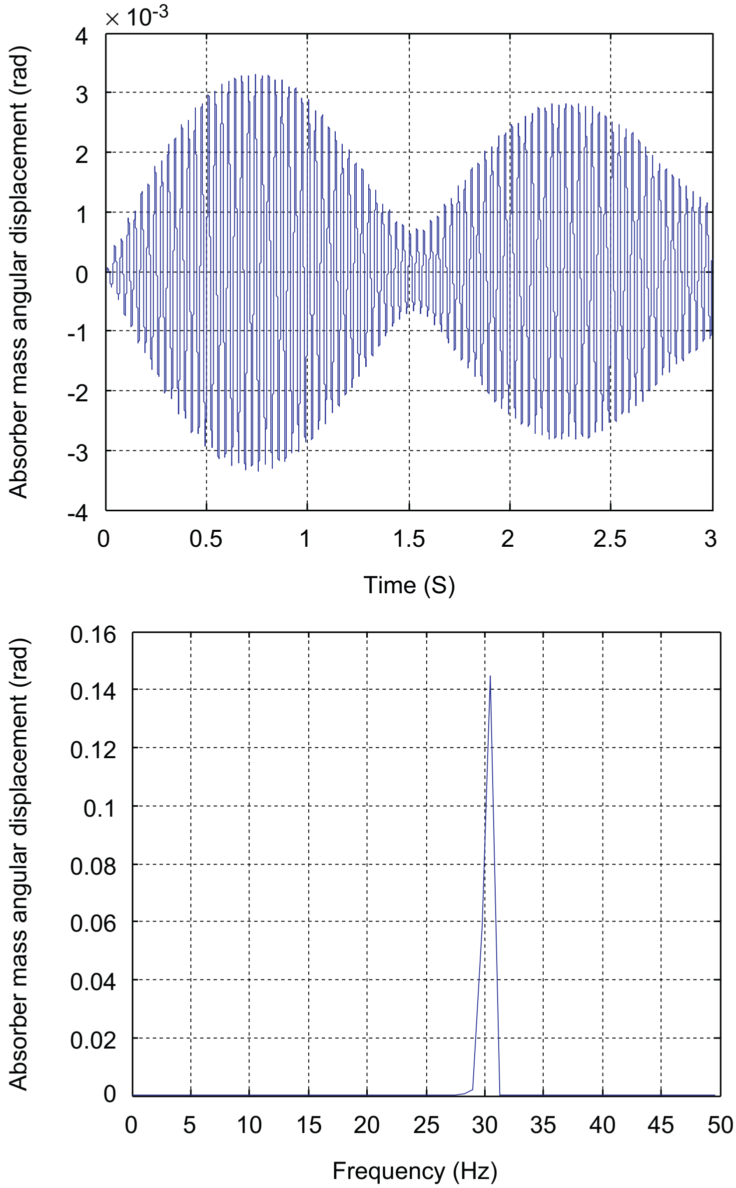


Fig. 8. Absorber displacement (θ) [Case #1: nonlinear model (r_v=0.2673 m)].

Case #2 Low frequency range:

Data#2: Using the data given in Table 2 with a tuned VIVA where the sliding block is at its tuning position (r_v=0.53) and excitation frequency of 11 Hz.

Simulation results #2 (Figs. 9–11): The primary mass linear and angular displacements (y_p,θ_p) and the developed VIVA absorber angular displacement (θ) are shown in Figs. 9–11, respectively, in time and frequency (FFT) domains at the sliding block tuned position (r_v=0.53). The frequency domain shows the peak frequency at 11 Hz (Figs. 9 and 10), which is equal to the excitation frequency of the two dof primary system. Other peaks appear at 14 Hz (Fig. 9) and 9 Hz (Fig. 10). Fig. 11 shows that the absorber natural frequency (11 Hz) equal to the primary system excitation frequency in addition to another peak at 8.5 Hz.

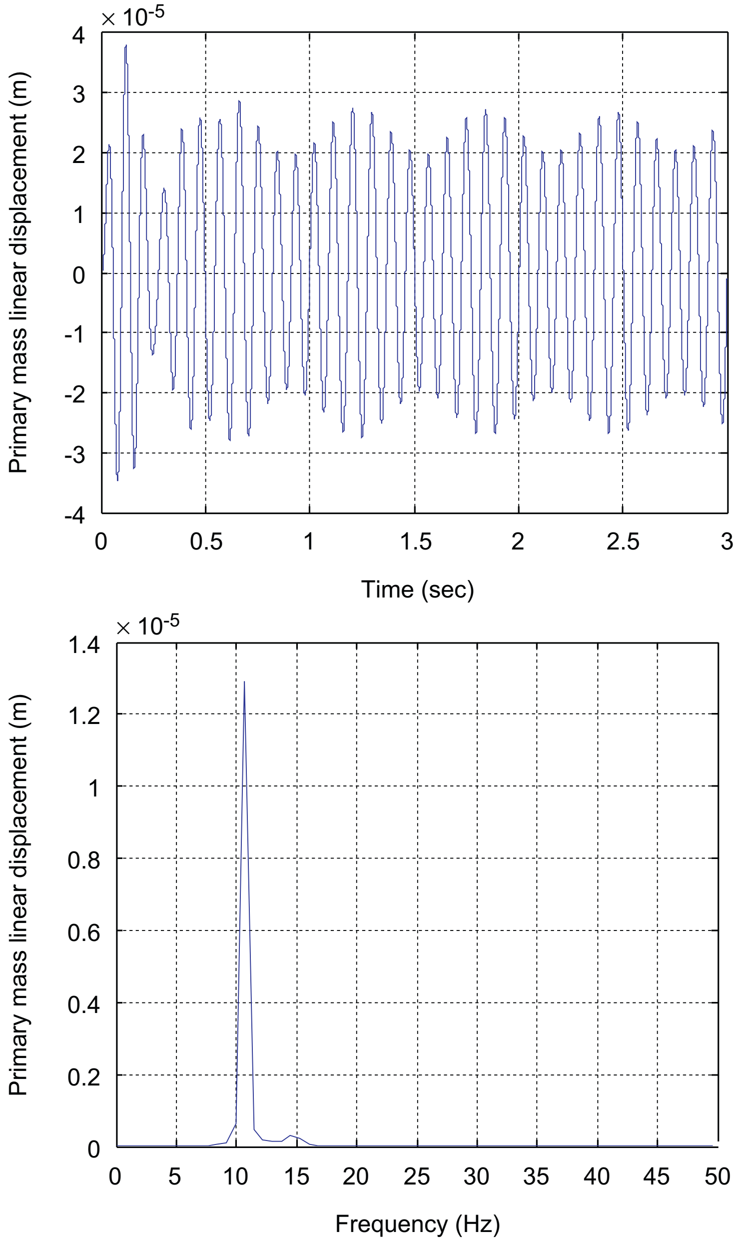


Fig. 9. Primary mass displacement (y_p) [Case #2: nonlinear model (r_v=0.53 m)].

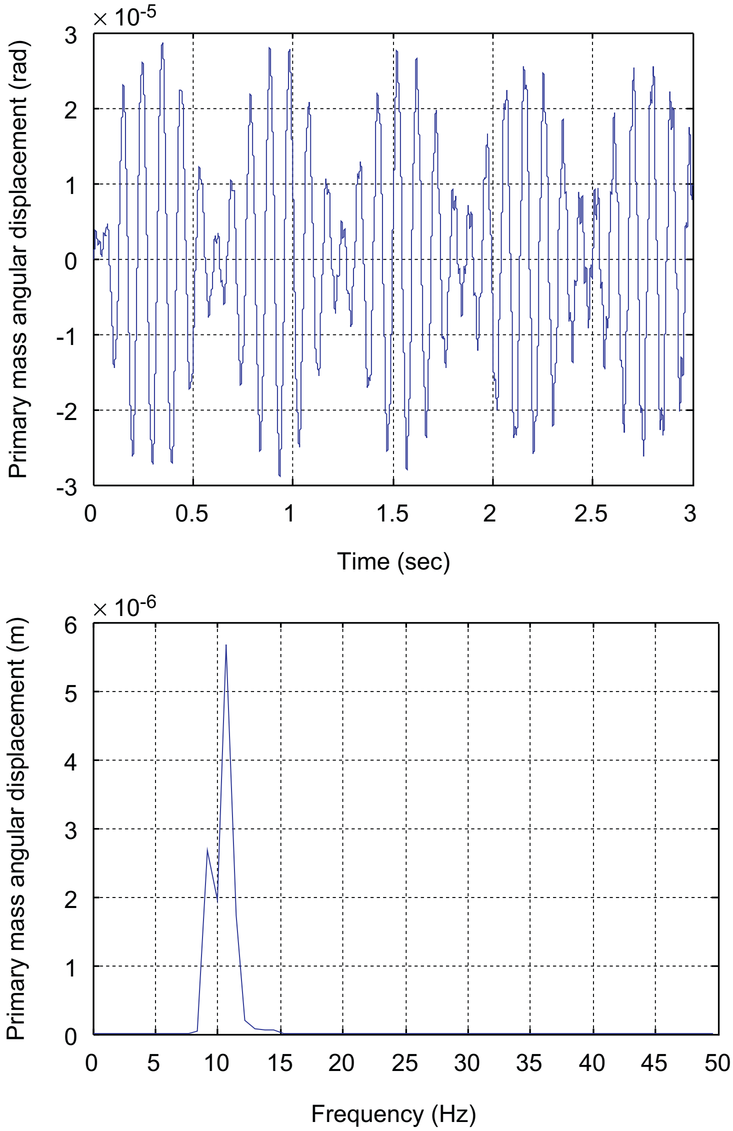


Fig. 10. Primary mass displacement (θ_p) [Case #2: nonlinear model (r_v=0.53 m)].

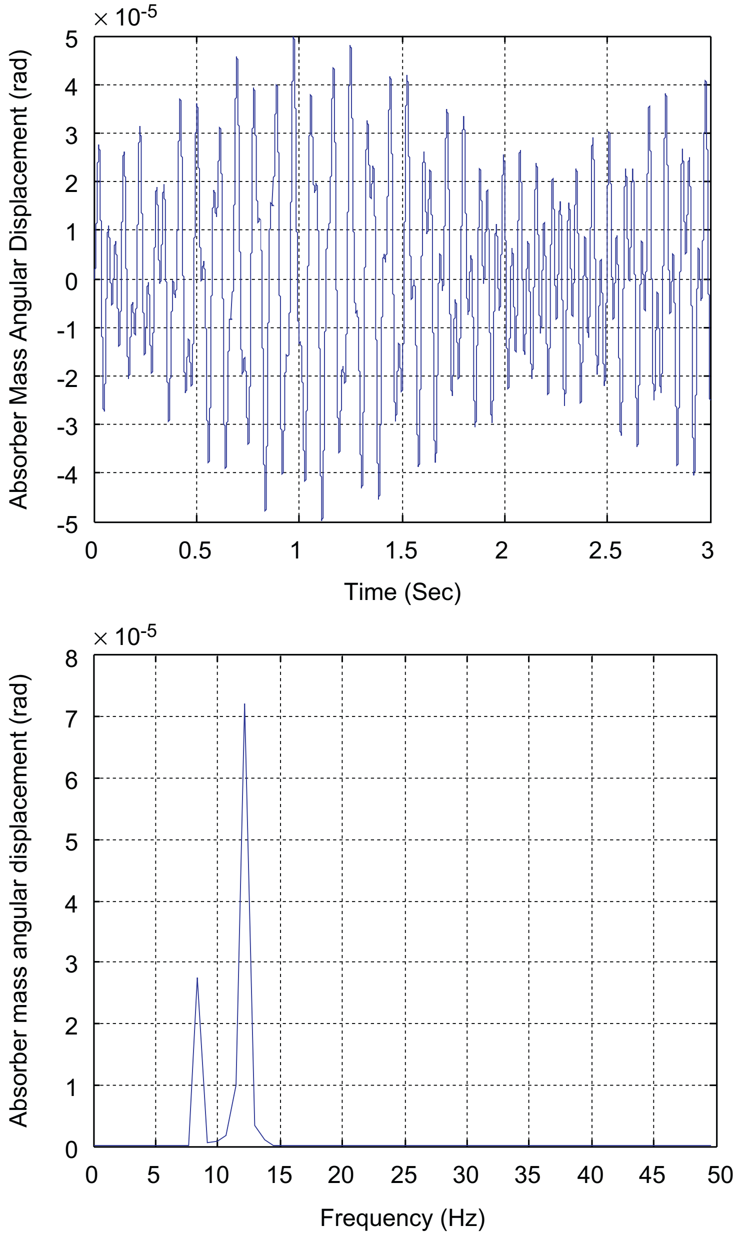


Fig. 11. Absorber displacement (θ) [Case #2: nonlinear model (r_v=0.53 m)].

7.2. Simulation results of llinearized model

The goals of the simulation of the linearized model are to assess:

- the feasibility and performance of the proposed VIVA design and
- the accuracy of the linearized model.

Applying the same procedure and same data in the cases simulated in Section 7.1 with the linearized equations, the obtained simulation results are presented as follows:

Case #3: High frequency range:

The obtained simulation results are presented in Figs. 12–14 when the sliding block tuning position (r_v=0.2673) and excitation frequency of 30 Hz.

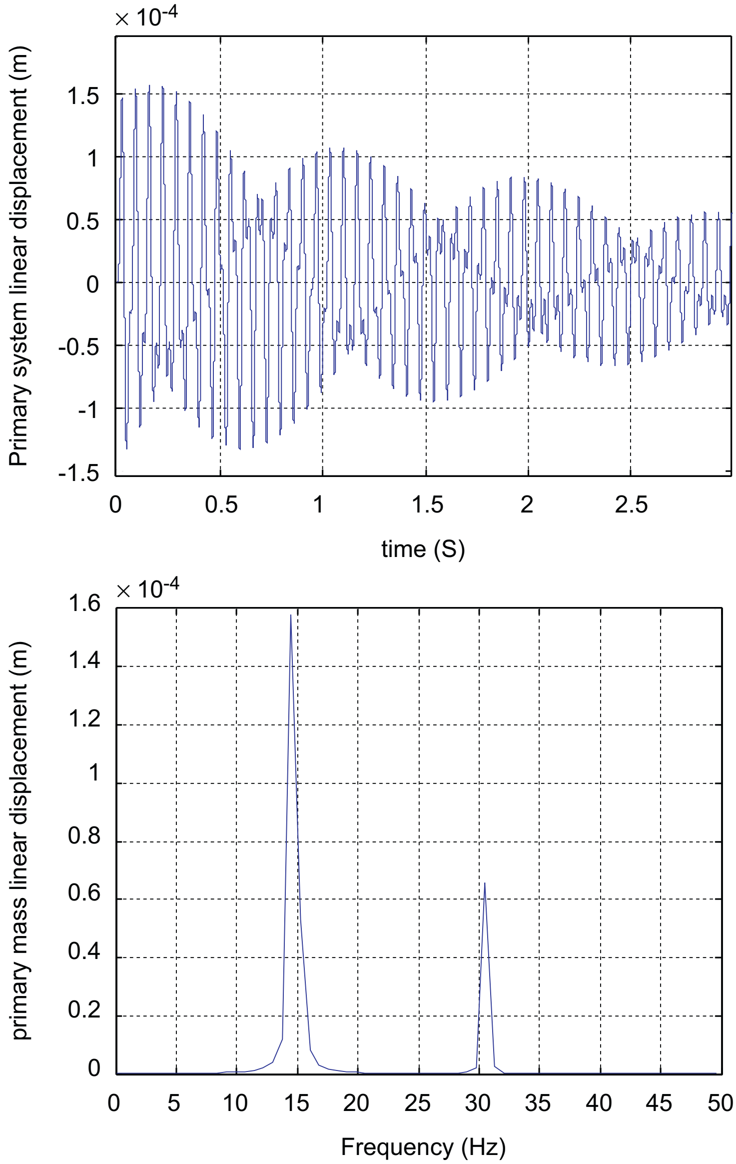


Fig. 12. Primary mass displacement (y_p) [Case #3: linearized model (r_v=0.2673 m)].

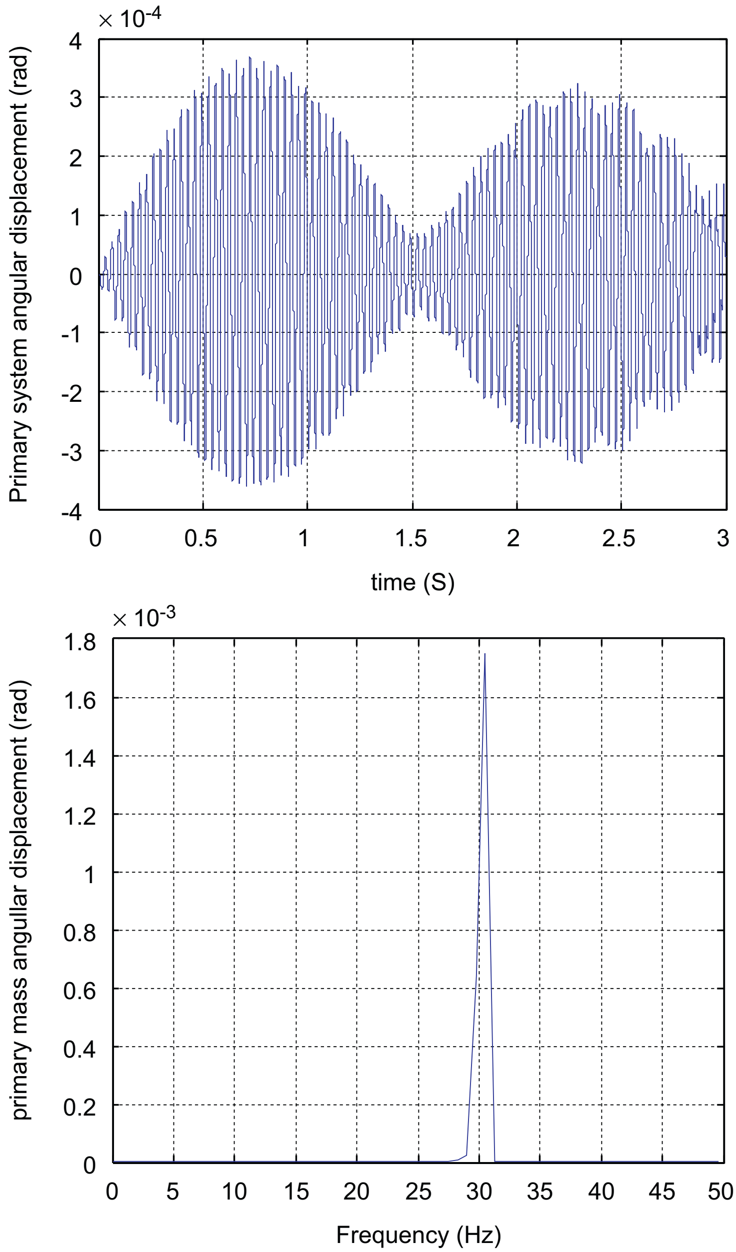


Fig. 13. Primary mass displacement (θ_p) [Case #3: linearized model (r_v=0.2673 m)].

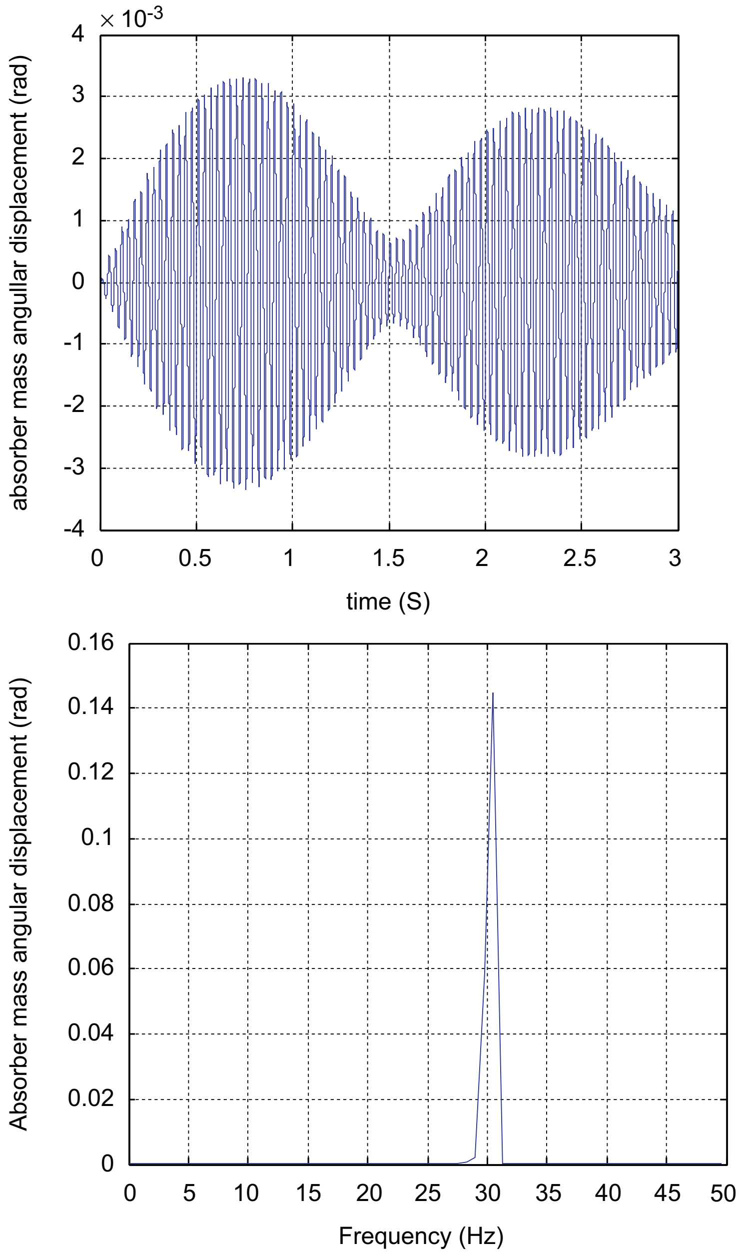


Fig. 14. Absorber displacement (θ) [Case #3: linearized model (r_v=0.2673 m)].

Case #4: Low frequency range:

The obtained simulation results are presented in Figs. 15–17 when the sliding block tuning position (r_v=0.53) and excitation frequency of 11 Hz.

The relative deviation between the linear and nonlinear models in cases #3 and #4 is less than 1%, which confirms the realistic use of this linearized model.

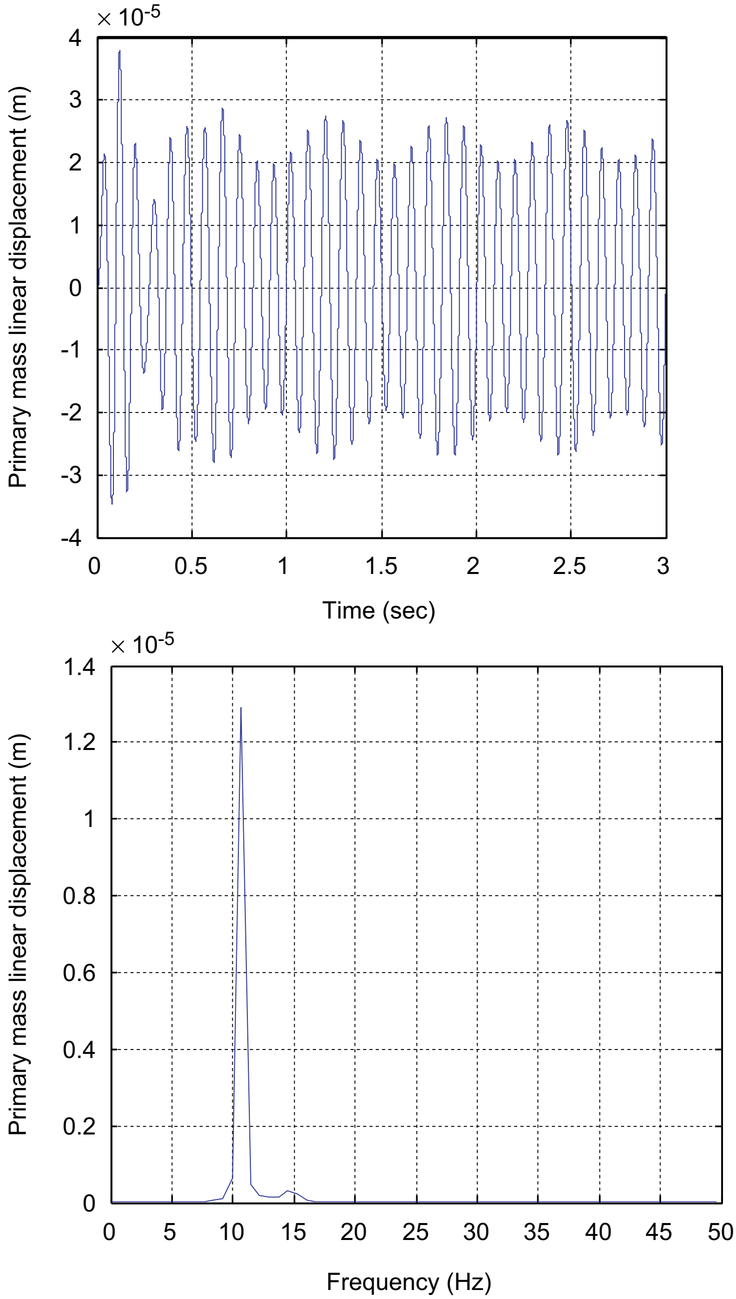


Fig. 15. Primary mass displacement (y_p) [linearized model (r_v=0.53 m)].

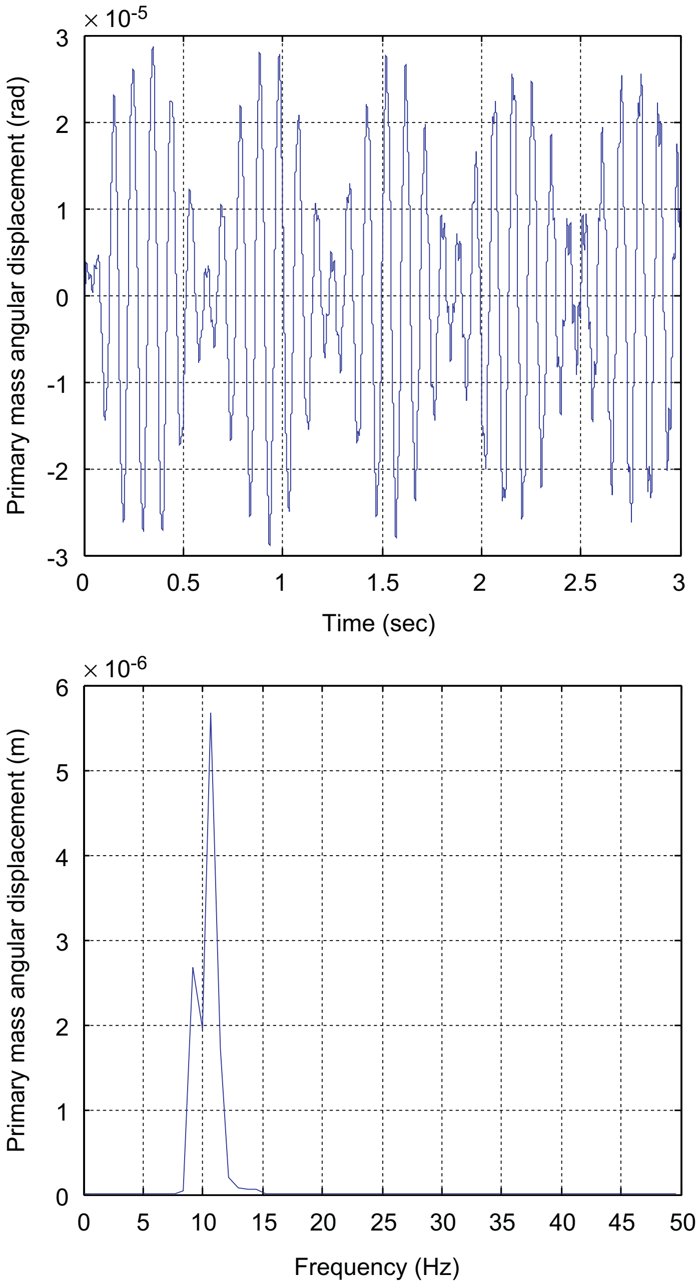
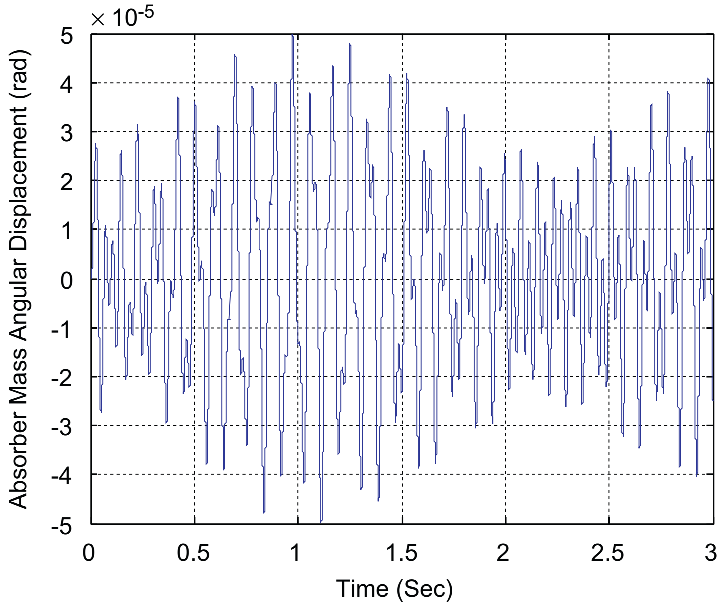


Fig. 16. Primary mass displacement (θ_p) [Case #4: linearized model (r_v=0.53 m)].



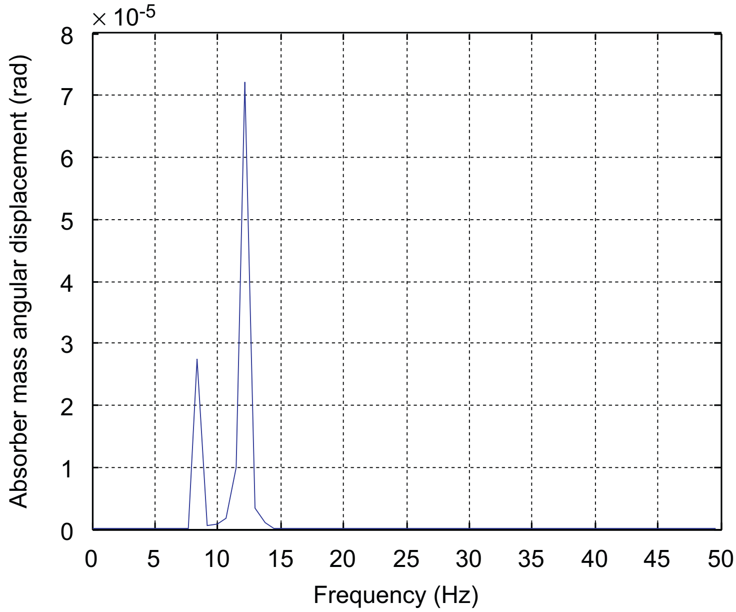


Fig. 17. Absorber displacement (θ) [Case #4: linearized model (r_v=0.53 m)].

7.3. Comments and discussion

The obtained simulation results prove that the proposed VIVA for two dof primary systems have the following advantages compared to other VIVA designs:

- It covers wider frequency range than other VIVA designs with shorter settling time [2].
- The absorber installation angle in both the vibration reduction and frequency range is effective.
- The proposed VIVA for two dof primary systems is more realistic in many applications [1].
- More vibration reduction in the primary system linear and angular displacements as presented in Figs. 18 and 19. These two figures show, respectively, the primary system linear and angular displacements at three different conditions: (1) response of primary system alone, (2) response of primary system with detuned VIVA and (3) response of primary system with tuned VIVA.

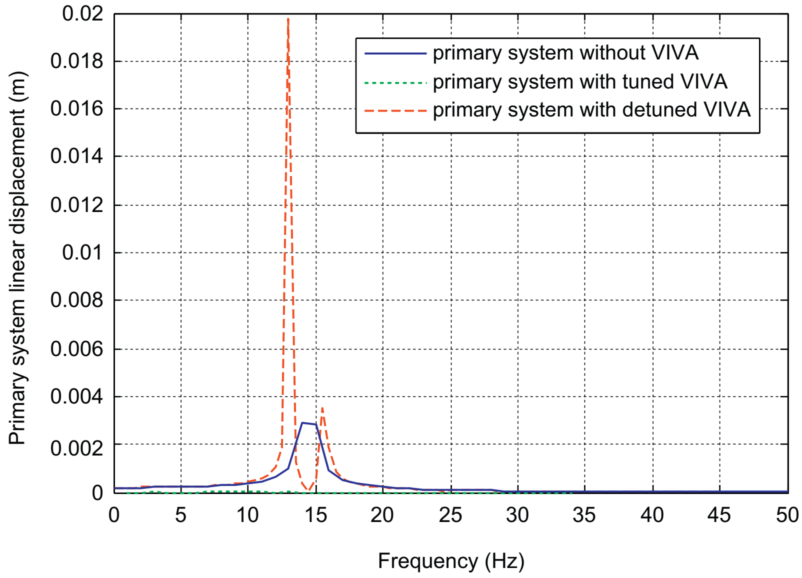


Fig. 18. Primary system linear displacement.

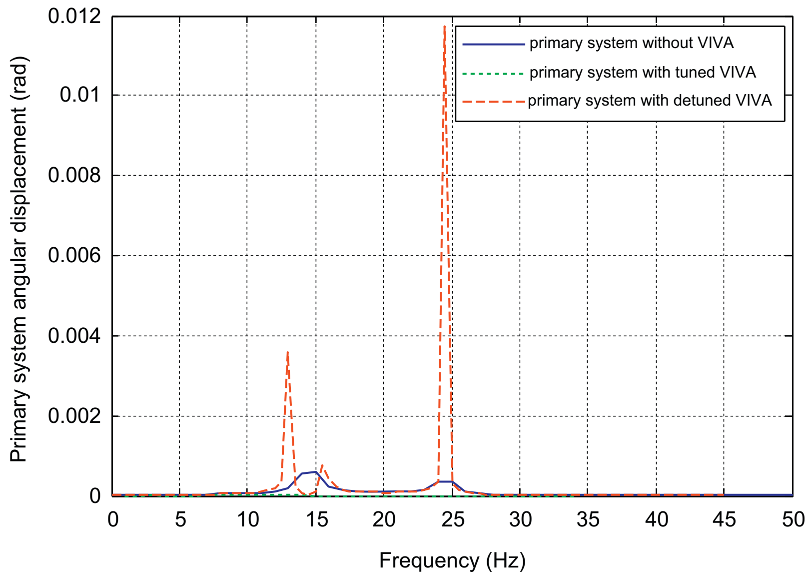


Fig. 19. Primary system angular displacement.

- The effectiveness of the proposed VIVA in reducing the primary system vibration to more than 95%.
- The minimum vibration always occurs at the tuned position.
- The primary system response with detuned VIVA may be higher than its own response alone.

8. Conclusions

This investigation presents the dynamic modeling and simulation of a proposed modified design of VIVA's for the vibration control of two dof primary systems. Lagrange formulation is used to obtain its dynamic model in an analytical form. An optimization algorithm is developed to select the best absorber parameters for better vibration suppression. A computation procedure is developed and programmed to simulate the absorber performance characteristics. The simulation results show better performance compared to previous VIVA designs. The effect of VIVA parameters m_v and I_v, which represent its size, and k_a, which represents its resilient element, on the vibration suppression and absorber performance and broadband frequency is studied. The obtained results can be used as a guide to design the proper VIVA for a certain application. In addition, a linearized model of VIVA dynamics is developed, tested and simulated for the same data used in its nonlinear model. The relative deviation between results of the linear and nonlinear models is less than 1% which confirms the realistic use of this linearized model. The proposed VIVA needs to be experimentally tested to verify the obtained simulated results about its performance, effectiveness, efficiency on vibration suppression of two dof primary systems. This is the subject of another paper.

Acknowledgements

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Appendix A

Using Eq. (5), the linearized model of the whole system is obtained as follows: the absorber rod installation angle (θ_f) is taken as equal to zero (θ_f=0). The total inclination angle (θ_t) is assumed as small as cos(θ_t)=1 and sin(θ_t)=θ_t. The following mass, damping and stiffness matrices are obtained:

$$\mathbf{M}=\begin{bmatrix} m_s+m_v+m_p & m_sr_s+m_vr_v & -(m_sr_s+m_vr_v) \\ m_sr_s+m_vr_v & m_vr_v^2+m_sr_s^2+I_v+I_s+I_p & -(m_vr_v^2+m_sr_s^2+I_v+I_s) \\ -(m_sr_s+m_vr_v) & -(m_vr_v^2+m_sr_s^2+I_v+I_s) & m_vr_v^2+m_sr_s^2+I_v+I_s \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} c_p & 0 & 0 \\ 0 & c_a & -c_a \\ 0 & -c_a & c_a \end{bmatrix}$$

$$\mathbf{K}=\begin{bmatrix} kp_1+kp_2 & -(k_{p_1}l_1-k_{p_2}l_2) & 0 \\ -(k_{p_1}l_1-k_{p_2}l_2) & k_a+k_{p_1}(l_1)^2+k_{p_2}(l_2)^2 & -k_a \\ 0 & -k_a & k_a \end{bmatrix}$$

$$\mathbf{D}=\mathbf{M}^{-1}\mathbf{K}$$

$$\mathbf{M}_d=\begin{bmatrix} \mathbf{M} & \mathbf{C} \\ 0 & \mathbf{M} \end{bmatrix}$$

$$\mathbf{K}_d\begin{bmatrix} 0 & \mathbf{K} \\ -\mathbf{M} & 0 \end{bmatrix}$$

$$\mathbf{D}_d=\mathbf{M}_d^{-1}\mathbf{K}_d$$

Using the dynamic matrix $\mathbf{D}_d$ [82] the forced vibration analysis, the **H** matrix elements may be calculated as follows:

$$h_{ij}=(k_{ij}-\omega^2m_{ij})+\mathrm{i}\omega c_{ij}\mathrm{i}, \quad j=1,2,\ldots,n$$

$$y_p\mathrm{e}^{-\mathrm{i}\varphi 1}=\frac{\begin{vmatrix} f_1 & h_{12} & h_{13} \\ f_2 & h_{22} & h_{23} \\ f_3 & h_{32} & h_{33} \end{vmatrix}}{\det(H)}$$

$$\theta_p\mathrm{e}^{-\mathrm{i}\varphi 2}=\frac{\begin{vmatrix} h_{11} & f_1 & h_{13} \\ h_{21} & f_2 & h_{23} \\ h_{31} & f_3 & h_{33} \end{vmatrix}}{\det(H)}$$

$$\theta e^{-\mathrm{i}\varphi 3}=\frac{\begin{vmatrix} h_{11} & h_{12} & f_1 \\ h_{21} & h_{22} & f_2 \\ h_{31} & h_{32} & f \end{vmatrix}}{\det(H)}$$

where φ_1, φ_2 and φ_3 are phase angles of the displacements y_p, θ_p and θ, respectively. Using Matlab the expressions of y_p and θ_p in symbolic form are obtained. The dominator for y_p is given by

$$y_p(\mathrm{dom}) = -w^2 I_p k_a - k_{p1} l_1^2 w^2 I_v - k_{p1} l_1^2 w^2 I_s - k_{p2} l_2^2 w^2 I_v - k_{p2} l_2^2 w^2 I_s + w^4 I_p m_v r_v^2 + w^4 I_p m_s r_s^2 - k_{p1} l_1^2 w^2 m_v r_v^2 - k_{p1} l_1^2 w^2 m_s r_s^2 - k_{p2} l_2^2 w^2 m_v r_v^2 - k_{p2} l_2^2 w^2 m_s r_s^2 + w^4 I_p I_v + w^4 I_p I_s + k_{p1} l_1^2 k_a + k_{p2} l_2^2 k_a$$

$$\theta_p(\mathrm{dom}) = k_{p1} l_1 k_a - k_{p2} l_2 k_a - k_{p1} l_1 w^2 m_v r_v^2 - k_{p1} l_1 w^2 m_s r_s^2 - k_{p1} l_1 w^2 I_v + k_{p2} l_2 w^2 m_v r_v^2 - k_{p1} l_1 w^2 I_s + k_{p2} l_2 w^2 m_s r_s^2 + k_{p2} l_2 w^2 I_v + k_{p2} l_2 w^2 I_s$$

Substituting (Eq. (6b)) in these expressions gives, y_p=0 and θ_p=0 in this configuration. This proves that to reduce the vibration of a two dof primary system due to a desired excitation frequency, the natural frequency of the absorber (Eq. (6b)) must be set equal to the excitation frequency that needs to be eliminated.

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