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Book Review

Truly Nonlinear Oscillations: Harmonic Balance, Parameter Expansions, Iteration, and Averaging Methods, Ronald E. Mickens, World Scientific, Singapore, 2010 (260pp., Hardcover) ISBN:978-981-4291-65-1

This book is the natural outcome of Ronald Mickens's work in the field described by its title, his numerous papers, and the impact that his publications had on other contributors. It was he who created the term 'truly nonlinear oscillator', which appeared officially for the first time in the title of his conference paper [1] and then in the title of his journal paper published in JSV [2], where he has a publication record of almost 80 papers.

The book comprises seven chapters and eight appendices. Chapter 1 sets the scene for the rest of the book by providing the background definitions and concepts for the methods presented subsequently. At the very beginning, truly nonlinear (TNL) functions and oscillators are defined. The former refers to a restoring force $f(x)$ that has no linear approximation in any neighborhood of $x=0$, and the latter to an oscillator with a restoring force of this form. Some general remarks are also given in this section, related to conservative oscillators and odd-parity oscillators. The subsequent section emphasizes the importance of scaling equations of motion by introducing dimensionless parameters, and is illustrated by several examples. Some TNL oscillator equations of motion can be solved exactly and four of them are presented here. Among them is a pure quadratic oscillator, the periodic solution of which is written down as a rational function of the Jacobi *cn* function. As far as I am aware this solution is original. An overview and a brief discussion of four techniques from the title that can be used to determine approximations to the periodic solutions of TNL solutions is given subsequently, but the details appear in the separate chapters devoted to each of them (Chapters 3–6) as well as in the chapter that contains their comparison (Chapter 7).

Chapter 2 deals with procedures that can be used to determine whether a TNL oscillator has periodic and/or oscillatory solutions without solving the equation of motion. The notions of phase space, fixed points, trajectories, nullclines and first integrals, among others, are introduced. These results are then used for a qualitative study of a linear oscillator and a number of TNL oscillators. Some non-conservative oscillators are also considered from this viewpoint.

The next four chapters are each devoted to one of the methods from the book title, presenting their basic methods and providing a range of worked examples that illustrate their use. Chapter 4 covers harmonic balance, comprising the direct harmonic balance procedure and the rational harmonic balance technique. The first-order harmonic balance procedure is used to calculate the approximation to the periodic solution of nonlinear third-order differential equations arising in the study of stellar oscillations. Chapter 4 moves on to parametric expansions. It starts with some background information on the method, which is then demonstrated on several worked examples. The next chapter introduces iteration methods. Two possible representations are discussed, the so-called direct and extended iteration methods and their application is illustrated on the same set of TNL oscillators. The penultimate chapter covers averaging methods. Unlike the previously presented methods that enable one to find a steady-state solution, this method also gives a transient solution. Three averaging methods are shown. Firstly, Mickens and Oyedi's procedure, in which the nonlinear term is truncated to the first term in the Fourier series for the assumed trigonometric-form solution is presented. This is followed by the combined linearization-averaging method, the basic idea of which is to replace the nonlinear term by an appropriate linear approximation with the unknown square of the constant frequency term in from of it. The third technique is Cveticanin's averaging method, which Mickens named in this book after its developer. This technique is the generalization of the original averaging method (Krylov–Bogoliubov) method for TNL non-conservative oscillators. It yields the first-order differential equations for the amplitude and phase of motion, while the frequency of vibrations is related to the amplitude in a way analogous to the relationship between the frequency and the amplitude of the corresponding conservative oscillator, defined by the energy-conservation law.

The final chapter comprises six TNL oscillator equations solved by all the methods applicable, with the results compared. Two conservative oscillators are solved by harmonic balance, parametric expansion and iteration. These oscillators are then set under the influence of a linear viscous force and the corresponding solution for motion found by three averaging techniques. Oscillators with the same type of restoring force are also considered for the case of van der Pol damping and both the transient and steady-state solution for motion are determined. This chapter concludes with some

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general comments regarding the results obtained and calculation strategies followed for conservative and non-conservative oscillators.

Eight appendices cover the topics relevant to understanding problems considered in the book. They range from basic mathematical relations, definitions and properties of the Gamma and Beta function to the basics of the second-order differential equations. In addition, brief outlines of the Lindstedt–Poincaré perturbation method and a standard averaging method are also presented. Based on the non-standard finite algorithm developed by the author, the last appendix gives the instructions on how to construct finite difference schemes for some equations.

The material covered in the book is presented in a well structured manner, with the consistent notation devised through the whole book. What makes this book outstanding are discussions, résumés or comments given at the end of each chapter. They stem from the author's impressive knowledge and experience, and many of them are real nuggets of wisdom, covering the essence of an issue they refer to. Among them are the description of the criteria or qualities that a good approximate analytical method should have, which are listed and explained at the end of Chapter 1 as well as the advantages and disadvantages of the methods discussed in the chapters devoted to them. Another exceptional feature is that each chapter takes a problem-solving approach, with many examples solved throughout, and with a list of problems closing each chapter, gently challenging the reader to check and cement his or her understanding. Many references directly linked to the topic under discussion are listed at the end of each chapter and serve also as a chronology of the methods presented.

The author wrote in the preface that 'it is assumed that the reader of this volume has a background preparation that includes knowledge of perturbation methods for standard oscillatory systems' and 'in particular,…an understanding of concepts of secular terms, limit-cycles, uniformly-valid approximations, and the element of Fourier series'. However, the methodology used and the comprehensive explanations given in the appendices enable the reader to go through the book smoothly, without any need to look elsewhere for background information, making this book appropriate for a wide range of readers interested in the techniques for a qualitative and quantitative analysis of nonlinear oscillators: students, researchers and educators. A positive and motivating approach used in the book, its structure and self-contained format are masterfully combined and are bound to inspire. Hopefully, this will happen to many young researchers, as has happened with Mickens's work in general and the author of this review.

References

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