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Discussion

Comment on paper "the bulk modulus and Poisson's ratio of "incompressible" materials" by P.H. Mott, J.R. Dorgan, C.M. Roland

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ABSTRACT

A widespread error related to Poisson's ratio and its limiting value 0.5 provokes complex explanations of fully coherent experimental data.

The authors of [1] analyze and discuss experimental data on temperature variation in the elastic properties of some materials and provide their original explanation. The presented data for two polymers can be summarized as follows: while the sample temperature increases and approaches the experimentation limit, the shear modulus G drops dramatically, the bulk modulus B decreases gradually showing no tendency to an abrupt change, and Poisson's ratio ν approaches its theoretical limit 0.5. The subsequent discussion in [1] is aimed at devising a way to resolve a "paradox", "apparent conflict", or "contradiction" between the experimental data and "classical elasticity".

What are the theoretical postulates which seem to be in contradiction with the experiment? The authors quote them: "when ν=1/2 the ratio of the bulk modulus to shear modulus B/G is infinite (*Statement* 1) and the system is described as incompressible (*Statement* 2)". The absolutely correct conclusion that at ν=1/2 one faces a liquid is discarded without a serious analysis or motivation.

Consider the above *Statements* 1 and 2 separately. The first statement that "B/G is infinite when ν=1/2" is practically correct with the reservation that division by zero is impossible and hence "G/B becomes zero when ν=1/2" looks much better.

The second statement represents a widespread error. There is neither physical nor mathematical reason for the bulk modulus to tend to infinity. Instead, according to the established relations between the elastic constants (see e.g. [2]),

$$E = 3(1-2\nu)B, \tag{1a}$$

$$G = \frac{3(1-2\nu)}{2(1+\nu)}B, \tag{1b}$$

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the Young modulus E and the shear modulus G become zero when $\nu=1/2$, which corresponds to the liquid state of a material with no resistance to shear. As to the probable residual value of G in liquid polymers, it would mean that G/B is not exactly zero, hence $\nu \neq 1/2$, and that the medium can be treated either as a solid with extremely low plasticity limit, or as a respective non-Newtonian liquid capable of supporting a certain shear stress at zero velocity. In either case, there is no need for any hypotheses on singular behavior of the bulk modulus. Otherwise, following same logic, one could find a singularity in kinematics of steady motion

$$V = \frac{S}{\Delta t}. \tag{2}$$

As soon as the distance S is assumed finite for some reason, we must conclude that velocity V should jump to infinity while time interval Δt approaches zero. Then, to resolve this "paradox", we may note that no measurement can actually register a null time interval, which explains why we never encounter the infinite speed in reality… Of course, finite distance is an improper assumption in this case, and (2) is better presented as

$$S = V \cdot \Delta t$$

with no singularity at $\Delta t=0$ regardless of the velocity value. Similarly, there is no place for a singularity at $\nu \to 1/2$ in Eqs. (1a,1b).

To conclude, the terms "incompressible" and "elastic" contradict one another. If a material is considered elastic it cannot be incompressible. No material is known exhibiting infinite longitudinal speed of sound. The transversal sound speed decreases down to zero while materials approach the liquid state, and finally, true liquids ($\nu=1/2$) do not at all transmit transversal acoustic waves.

The above considerations could be regarded purely theoretical and having no actual importance, but numerous papers report practical struggle with measurements and computations at ν approaching or equal to 1/2. The measurement problems typically appear for soft materials, as for instance biological tissue, and could be resolved by calculating the bulk modulus from the longitudinal sound speed together with direct dynamic or static measurements of shear modulus (e.g. by torsion, if only the sample can be considered isotropic).

A typical example of computational troubles is represented by so-called "volume locking" at ν approaching 1/2, which requires specific "regularization" techniques to enforce convergence of the numerical solution. This trouble results from erroneous problem setting (assumption of finite E at $\nu=1/2$, ill-posed displacement-free formulations) and poor programming (allowing zero in the denominator). Using B and ν for elastic constants and preserving displacements or displacement velocities in the governing equations (see e.g. [3–5]) helps to avoid artificial singularity of the linear elastic problem at $\nu \to 1/2$ and permit computations for both elastic solids ($-1 < \nu < 1/2$) and liquids ($\nu=1/2$).

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