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Chaos synchronization and parameter estimation of single-degree-of-freedom oscillators via adaptive control

Youming Lei [a,b,*], Kai-Leung Yung [a], Yan Xu [a]

[a] Department of Industrial and Systems Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
[b] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an 710072, PR China

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ABSTRACT

This paper addresses the problem of synchronizing a class of single-degree-of-freedom oscillators with uncertain parameters. A modified adaptive control scheme is proposed to achieve globally asymptotic stable synchronization between the master and slave oscillators with arbitrary different initial conditions based on the Barbalat's Lemma. One of the advantages of the method is that it requires only a scalar driving signal which may be easily designed in practical applications. On the other hand, the method can ensure that the unknown parameters in the slave oscillator would be completely estimated with an adaptive updating law. Numerical simulations are performed on two examples to verify the analytical results.

1. Introduction

Chaos synchronization, since the pioneering work of Pecora and Carroll [1], has attracted increasing interest due to its wide application potential in many fields such as physics, chemistry, engineering and biology [2–4]. Given the conditional Lyapunov exponents are all negative, Pecora and Carroll designed master system signals to slave the responding system to follow the dynamics of the master system for synchronization with chaotic systems that intrinsically defy synchronization. The condition, however, is only necessary, but not sufficient, for synchronization. In view of this, a wide variety of approaches have been proposed for the synchronization of chaotic systems such as the linear coupling feedback scheme, active-passive decomposition method [5], backstepping design [6], adaptive control [7–12], sliding mode control [13–17]. Among these methods, the adaptive control scheme can ensure globally asymptotically stable synchronization for almost all continuous chaotic systems. Unfortunately, to implement synchronization, almost all of the existing methods, besides the traditional adaptive control scheme, require several controllers that are more difficult to put into practice than a scalar one. Moreover, in practice the effective synchronization method has to be able to identify unknown parameters admitting environmental perturbations.

The single-degree-of-freedom (SDOF) oscillator is the simplest possible mechanical system but with very rich dynamics and complex phenomena such as chaos [18–21]. The SDOF oscillator includes a class of well known systems such as the Duffing oscillator, van der Pol oscillator and Ueda oscillator, and can be used to model the motion of a gyro, single pendulum, earthquake, Josephson junction, etc. Due to its wide range of applications, the control and synchronization of SDOF systems has been investigated by many scholars. Chen studied chaos control and synchronization of a symmetric

* Corresponding author at: Department of Industrial and Systems Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong. Tel.: +852 27667015.
E-mail addresses: leiyouming@nwpu.edu.cn, mfleiym@polyu.edu.hk (Y. Lei).

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nonlinear gyro with numerical methods [21]. The authors attained robust synchronization for two chaotic nonlinear gyros by using the adaptive control scheme based on Lyapunov stability theory and Routh–Hurwitz criteria [22]. With initial parameter mismatches, Zhang et al. designed feedback controllers to realize synchronization for parametrically excited chaotic pendulums and at the same time to estimate unknown parameters [23]. Wu et al. proposed a linear feedback control technique to synchronize two non-autonomous Duffing oscillators based on the Lyapunov direct method and linear matrix inequality [24]. In most of these studies, the feedback signals require two controllers. In Ref. [25], an adaptive sliding mode control, which is only scalar, was devised for synchronization of a class of chaotic systems with uncertainties. When the parameters of chaotic gyros are fully unknown, Yan et al. used a simple scalar sliding mode control with adaptive laws of parameters to achieve synchronization [26]. However, the sliding mode control would inevitably cause chattering, which leads to difficulties in estimating unknown parameters of chaotic systems, around the synchronization manifold due to delay in control switching.

Motivated by the discussions above, the aim of this work is to develop the adaptive control for synchronizing two chaotic SDOF oscillators with unknown parameters. The rest of this paper is organized as follows. In Section 2, the problem of synchronization for two chaotic SDOF oscillators is addressed at first, and then the scalar adaptive control is proposed to establish a principle of their synchronization and parameter estimation based on the Barbalat's Lemma. In Section 3, it is shown that two illustrative examples, synchronizing two chaotic nonlinear gyros with unknown parameters and two chaotic parametrically excited Duffing oscillators with unknown parameters, respectively, confirm the validity and feasibility of the proposed method. Finally, conclusions are drawn in Section 4.

2. Problem formulation and synchronization principle

Consider the SDOF oscillator described by

$$\ddot{\theta}+f(\theta,\dot{\theta},t)=0 \tag{1}$$

where $\theta\in R$ is the position variable with regard to time t, $\dot{\theta}$ and $\ddot{\theta}$ represents the corresponding velocity and acceleration variables, respectively, and f is a nonlinear function of θ, $\dot{\theta}$ and t. As stated above, one may check easily that system (1) includes various types of systems such as the Duffing oscillator, van der Pol oscillator and Ueda oscillator.

Denoting $x_1=\theta$, $x_2=\dot{\theta}$, system (1) is transformed into first-order ordinary differential equations with the form

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f(x,t)\end{cases} \tag{2}$$

where $x=(x_1,x_2)^{\mathrm{T}}\in R^2$. Here, system (1) is considered as the master system, and the slave system with the same form of Eqs. (2) with a scalar controller $u=u(t)\in R$ added to its right side is introduced as follows:

$$\begin{cases}\dot{y}_1=y_2\\ \dot{y}_2=f(y,t)+u(t)\end{cases} \tag{3}$$

where $y=(y_1,y_2)^{\mathrm{T}}\in R^2$ denotes the slave state vector and $f(y,t)$ is a nonlinear function with the form of $f(y,t)=g(y,t)+\sum_{i=1}^{n}p_ih_i(y,t)$. Here, $g(y,t)$ and $h_i(y,t)$, $i=1,2,\ldots,n$, are nonlinear functions, and p_i, $i=1,2,\ldots,n$, are unknown parameters to be estimated. To investigate synchronization, we assume that the master and slave systems have bounded unique solutions in the time interval $(-\infty,+\infty)$ given arbitrary initial conditions $x(t_0)=x_0, y(t_0)=y_0$, and that the master system is globally chaotic.

The synchronization problem is how to design the scalar controller $u(t)$, which is attached to the slave system, such that the states of both the master and the slave systems are synchronized for arbitrary different initial conditions. If we define the error vector as $e=y-x$, the dynamic equations of synchronization errors can be expressed as

$$\begin{cases}\dot{e}_1=e_2\\ \dot{e}_2=g(y,t)+\sum\limits_{i=1}^{n}p_ih_i(y,t)-f(x,t)+u(t)\end{cases} \tag{4}$$

Therefore, the objective of synchronization is to make $\lim_{t\to+\infty}\|e(t)\|=0$ where $\|\cdot\|$ represents the Euclidean norm. In this way, the problem of synchronization between the master and the response systems can be transformed into a problem of how to implement asymptotical stabilization of the error system (4). Thus, the prime purpose of this study is to design a controller $u(t)$ to make the dynamical system (4) asymptotically stable at the origin.

We introduce the scalar adaptive control function $u(t)$ as

$$u(t)=f(x,t)-g(y,t)-\sum_{i=1}^{n}\hat{p}_ih_i(y,t)-e_1-ke_2 \tag{5}$$

where k is a positive constant, and $\hat{p}_i$, $i=1,2,\ldots,n$, are the estimates of the unknown parameters such that

$$\dot{\hat{p}}_i=\alpha_i h_i(y,t)e_2 \tag{6}$$

where α_i, $i=1,2,\ldots,n$, are positive constants.

Now, we give our main results in the following theorem.

Theorem 1. *Suppose that the slave system* (3) *is driven by the master system* (2) *under the scalar controller* (5) *with the parameters updating adaptive law* (6), *and the nonlinear functions* $h_i(y,t)$, $i=1,2,\ldots,n$, *are linearly independent and not invariant. Then the error dynamical system* (4) *is globally asymptotically stable at the origin, that is* $\lim_{t\to+\infty}\|e(t)\|=0$, *and the availability of the estimation of the unknown parameters can be guaranteed, that is* $\lim_{t\to+\infty}\hat{p}_i=p_i$, $i=1,2,\ldots,n$.

Proof. Combine the error system (4) between the master and slave systems and the system (6) which estimates the unknown parameters as an augmented non-autonomous system. To achieve global synchronization between the master and slave systems is to achieve global stability of the origin in the augmented system. Construct a non-negative function as follows:

$$V(t)=\frac{1}{2}\sum_{i=1}^{2}e_i^2+\frac{1}{2}\sum_{i=1}^{n}\frac{1}{\alpha_i}(\hat{p}_i-p_i)^2 \tag{7}$$

where $\hat{p}_i$, $i=1,2,\ldots,n$, are the estimates of unknown parameters p_i, $i=1,2,\ldots,n$.

By differentiating $V(t)$ with respect to time t along the solution of the augmented system, we have

$$\begin{aligned}\dot{V}(t)&=e_1\dot{e}_1+e_2\dot{e}_2+\sum_{i=1}^{n}\frac{1}{\alpha_i}(\hat{p}_i-p_i)\dot{\hat{p}}_i=e_1e_2+e_2\Big(g(y,t)+\sum_{i=1}^{n}p_ih_i(y,t)-f(x,t)+u(t)\Big)\\&\quad+\sum_{i=1}^{n}(\hat{p}_i-p_i)h_i(y,t)e_2=e_2\Big(\sum_{i=1}^{n}p_ih_i(y,t)-\sum_{i=1}^{n}\hat{p}_ih_i(y,t)-ke_2\Big)\\&\quad+\sum_{i=1}^{n}(\hat{p}_i-p_i)h_i(y,t)e_2=-ke_2^2\le 0\end{aligned} \tag{8}$$

Therefore, the non-negative function $V(t)$ is monotonically non-increasing and converges as $t\to+\infty$. Consequently, we can obtain

$$\int_{t_0}^{t}ke_2^2\,\mathrm{d}t=-\int_{t_0}^{t}\dot{V}(t)\,\mathrm{d}t=V(t_0)-V(t)$$

so that

$$\lim_{t\to+\infty}\int_{t_0}^{t}ke_2^2\,\mathrm{d}t$$

exists and is finite; besides, ke_2^2 is uniformly continuous in the range $[t_0,+\infty)$. Using Barbalat's Lemma [27], it follows that $\lim_{t\to+\infty}ke_2^2=0$, which is equivalent to $\lim_{t\to+\infty}e_2=0$. Note that substituting Eqs. (5) and (6) into the second equation of system (4) yields $\dot{e}_2=\sum_{i=1}^{n}(p_i-\hat{p}_i)h_i(y,t)-e_1-ke_2$. Since $h_i(y,t)$, $i=1,2,\ldots,n$, are linearly independent and not invariant, it follows that $\lim_{t\to+\infty}\hat{p}_i=p_i$ and $\lim_{t\to+\infty}e_1=0$, $i=1,2,\ldots,n$, by taking the limit $t\to+\infty$ of this equation. Moreover, since all the above derivations hold globally, we can conclude that the two systems (2) and (3) are globally asymptotically synchronized and the unknown parameters can be estimated with arbitrary initial conditions. This completes the proof. □

3. Illustrative examples

In this section, the effectiveness of the scalar adaptive control is demonstrated by two illustrative examples and numerical simulations are carried out to verify the proposed method. Throughout the simulations, the sixth-order Runge–Kutta method is used to solve ordinary differential equations with adaptive step-size algorithm.

Example 1. Consider an SDOF oscillator modeling the motion of gyros in which the states are described by

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=p_1x_2-p_2x_2^3+p_3\sin(2t)\sin x_1+p_4\sin x_1-g(x_1)\end{cases} \tag{9}$$

where $g(x_1)=(10-10\cos x_1)^2/\sin^3 x_1$ and p_1,p_2,p_3,p_4 are unknown parameters. The dynamics of this system has been investigated by Chen [21], and it is shown that for $p_1=-0.5$, $p_2=0.05$, $p_3=35.5$, $p_4=1$, the gyro exhibits chaotic behaviors. We select these parameters and take system (9) as the master system. Now, we introduce a gyro with identical parameters as the slave system

$$\begin{cases}\dot{y}_1=y_2\\ \dot{y}_2=p_1y_2-p_2y_2^3+p_3\sin(2t)\sin y_1+p_4\sin y_1-g(y_1)+u(t)\end{cases} \tag{10}$$

where $u(t)$ is a scalar control to be designed. According to Eqs. (5) and (6), we choose the control as

$$u(t)=f(x,t)+g(y_1)-\hat{p}_1y_2+\hat{p}_2y_2^3-\hat{p}_3\sin(2t)\sin y_1-\hat{p}_4\sin y_1-e_1-5e_2 \tag{11}$$

where $f(x,t)=p_1x_2-p_2x_2^3+p_3\sin(2t)\sin x_1+p_4\sin x_1-g(x_1)$, and the estimates of the unknown parameters satisfy the adaptive updating law as below

$$\begin{cases}\dot{\hat{p}}_1=y_2e_2\\ \dot{\hat{p}}_2=-y_2^3e_2\\ \dot{\hat{p}}_3=\sin(2t)\sin(y_1)e_2\\ \dot{\hat{p}}_4=\sin(y_1)e_2\end{cases} \tag{12}$$

With this choice, it follows from Theorem 1 that the two gyros (9) and (10) are globally asymptotically synchronized and the unknown parameters can be estimated in spite of arbitrary different initial conditions. In addition, we use numerical simulations to examine whether the method is feasible. Without loss of generality, the initial conditions for the master and slave systems (9) and (10) and for the adaptive law of parameters updating are set as $(x_1,x_2,y_1,y_2)=(0.6,0.12,1.5,-1.1)$ and $\hat{p}_i=0$, $i=1,2,3,4$, respectively. Fig. 1 shows temporal evolutions of the master system (9) and the error system between systems (9) and (10). It can be seen that the errors will converge to zero finally, thus implying that via the control (11) the two gyros with unknown parameters for different initial conditions can indeed achieve synchronization. At the same time, Fig. 2 shows that the unknown parameters can be dynamically estimated with the updating law (12). Compared with Ref. [22], this study uses only one scalar adaptive controller, which improves and extends the traditional adaptive control scheme, when synchronizing two chaotic gyros. On the other hand, compared with the sliding mode control [26], the proposed method, which avoids chattering, can identify unknown parameters in the chaotic gyros.

Example 2. Consider a Duffing oscillator subjected to a harmonic parametric excitation as follows:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-p_1x_2+p_2\sin(\omega t)x_1+p_3x_1+p_4x_1^3\end{cases} \tag{13}$$

where $\omega=1.0$ denotes the frequency of the parametric excitation and p_1,p_2,p_3,p_4 are unknown parameters. More details for the model can be seen in Ref. [28]. We select the parameters $p_1=0.2$, $p_2=0.5$, $p_3=1.0$, $p_4=-1.0$, with which the

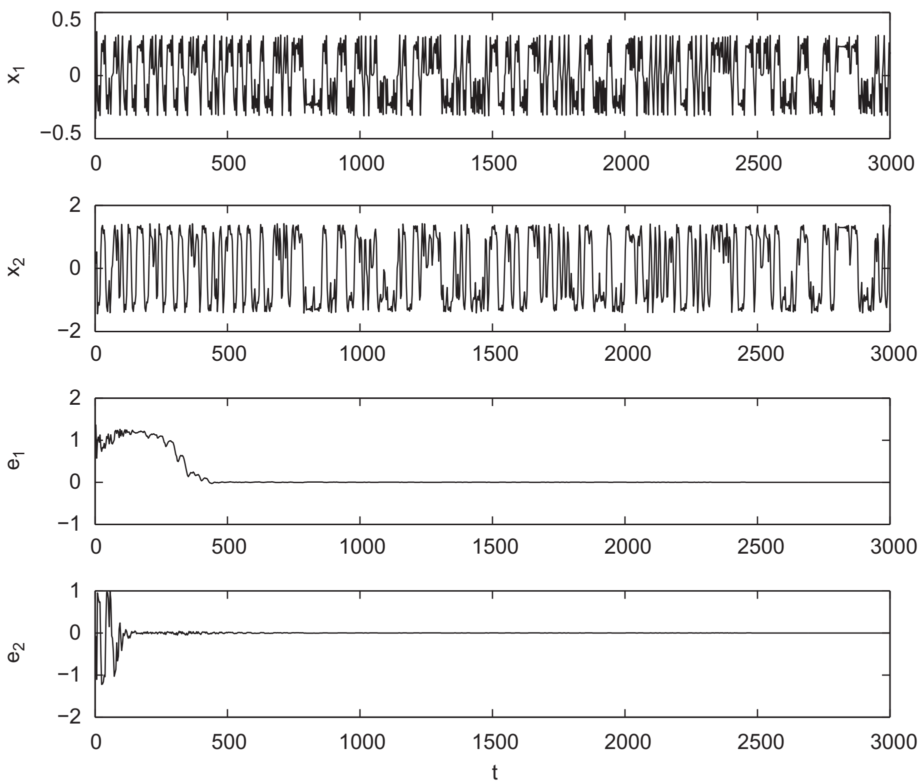

Fig. 1. Temporal evolutions of the master system (9) and the error system between systems (9) and (10), where $e_1=y_1-x_1$ and $e_2=y_2-x_2$.

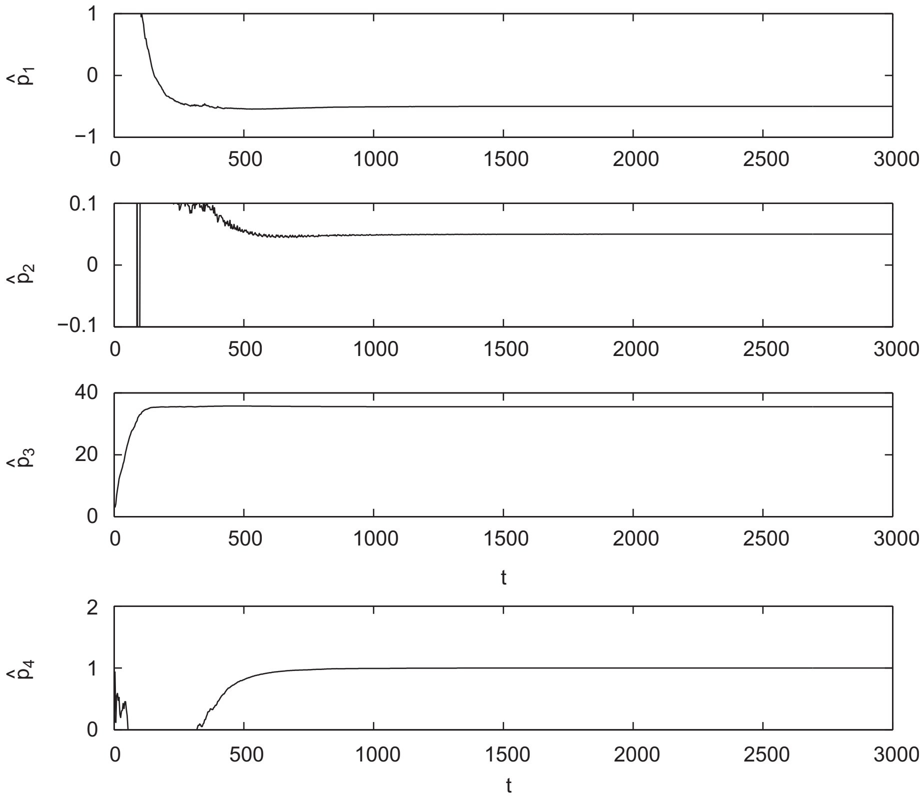

Fig. 2. Temporal evolutions of the adaptive parameter estimation (12).

system is chaotic, and take it as the master system. Correspondingly, the slave system can be written as

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -p_1 y_2 + p_2 \sin(\omega t) y_1 + p_3 y_1 + p_4 y_1^3 + u(t) \end{cases} \tag{14}$$

where $u(t)$ is a scalar control to be designed. According to Eqs. (5) and (6), we choose the control as

$$u(t) = f(x,t) + \hat{p}_1 y_2 - \hat{p}_2 \sin(\omega t) y_1 - \hat{p}_3 y_1 - \hat{p}_4 y_1^3 - e_1 - e_2 \tag{15}$$

where $f(x,t) = -p_1 x_2 + p_2 \sin(\omega t) x_1 + p_3 x_1 + p_4 x_1^3$, and the estimates of the unknown parameters satisfy the adaptive updating law as below

$$\begin{cases} \dot{\hat{p}}_1 = -y_2 e_2 \\ \dot{\hat{p}}_2 = \sin(\omega t) y_1 e_2 \\ \dot{\hat{p}}_3 = y_1 e_2 \\ \dot{\hat{p}}_4 = y_1^3 e_2 \end{cases} \tag{16}$$

With this choice, it follows from Theorem 1 that the two Duffing oscillators (13) and (14) are globally asymptotically synchronized and the unknown parameters can be estimated in spite of arbitrary different initial conditions. In addition, we use numerical simulations to examine whether the method is feasible. Without losing any generality, the initial conditions for systems (13) and (14) and for the parameters updating law are set as $(x_1, x_2, y_1, y_2) = (2.0, -1.2, 0.5, 0.2)$ and $\hat{p}_i = 0$, $i = 1, 2, 3, 4$, respectively. Fig. 3 shows that for these initial conditions the master and slave systems (13) and (14) have been synchronized by the control (15); while, Fig. 4 shows that the unknown parameters have been simultaneously identified by the updating law (16).

4. Conclusions

In this paper, a modified adaptive controller, which requires only a scalar driving signal and could be easily designed in practical applications, is proposed for globally synchronizing two chaotic SDOF oscillators with uncertain parameters, based on the Barbalat's Lemma. Compared with the traditional sliding mode control, the proposed method avoids

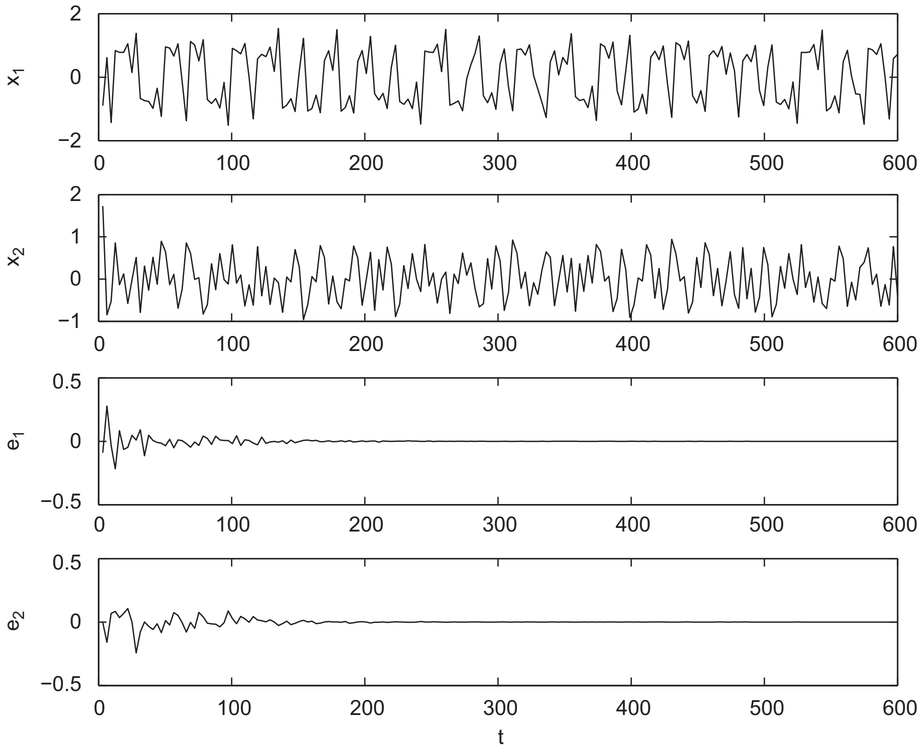

Fig. 3. Temporal evolutions of the master system (13) and the error system between systems (13) and (14), where $e_1 = y_1 - x_1$ and $e_2 = y_2 - x_2$.

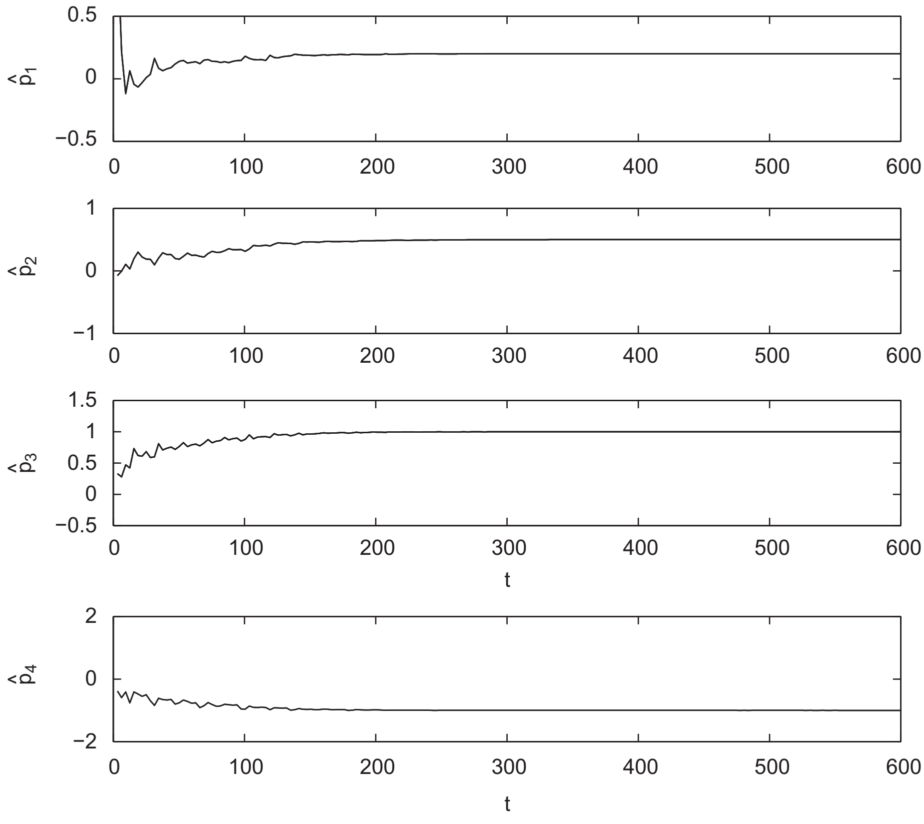

Fig. 4. Temporal evolutions of the adaptive parameter estimation (16).

chattering and can dynamically estimate the unknown parameters in the slave system with the adaptive law for parameters updating. Two examples have been used to demonstrate how to apply the proposed controller and numerical simulations, and further verified the feasibility and effectiveness of the proposed method.

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