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A particle filtering approach for structural system identification in vehicle–structure interaction problems

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ABSTRACT

The problem of identification of parameters of a beam-moving oscillator system based on measurement of time histories of beam strains and displacements is considered. The governing equations of motion here have time varying coefficients. The parameters to be identified are however time invariant and consist of mass, stiffness and damping characteristics of the beam and oscillator subsystems. A strategy based on dynamic state estimation method, that employs particle filtering algorithms, is proposed to tackle the identification problem. The method can take into account measurement noise, guideway unevenness, spatially incomplete measurements, finite element models for supporting structure and moving vehicle, and imperfections in the formulation of the mathematical models. Numerical illustrations based on synthetic data on beam-oscillator system are presented to demonstrate the satisfactory performance of the proposed procedure.

1. Introduction

The dynamic interactions between moving vehicles and supporting structures have been extensively studied in the existing literature [1]. The phenomenon of vehicle structure interaction (VSI) is rich with several interesting features that make its study both fascinating and challenging. The governing equations of motion, even when linear, here have time varying coefficients and, therefore, the notions of normal modes and natural frequencies are not valid for this class of problems; similarly, the frequency domain methods also are not applicable here. These difficulties arise essentially only when the moving vehicles can no longer be modeled as a set of moving forces but their mass, stiffness and damping characteristics also need to be accounted for. The passage of a single vehicle on the bridge leads to parametric excitation terms that are transient in nature while a stream of vehicles crossing the structure could lead to parametric resonances. The propensity for dynamic amplification of response here is influenced notably by the vehicle velocities with certain velocities producing unfavorable responses. The movement of vehicle mass on vibrating supporting structure results in Coriolis terms in the governing equations of motion. Furthermore, one needs to take into account several sources of uncertainties, such as, those associated with vehicular loads (number of vehicles, their velocities and arrival times, mass, stiffness and damping characteristics), other ambient loads, temperature and guideway unevenness. Presence of nonlinearities in vehicle and supporting structures, and, also, due to vehicle losing contact with the bridge while in motion, make the problem further complicated. The research literature on this subject is very vast with both the forward and

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inverse problems receiving extensive attention. The seminal book by Fryba [1] provides valuable overviews on modeling and phenomenological issues. Several studies provide computational perspectives, especially, on the application of the finite element method, on class of problems have also been conducted [2–11].

There also exist several studies that focus on identifying the moving vehicle characteristics (weight, axle loads, dynamic characteristics of the vehicle such as mass, stiffness and damping characteristics) and (or) to identify the parameters of model for the supporting structure based on a set of measurements made on moving vehicle and (or) supporting structure. The focus of the present study is on the inverse problem of identifying dynamic characteristics of the vehicle–structure system based on measurements made on beam displacements and bending strains. We clarify at the outset that the focus of the study is not on methods that are applicable when moving vehicles can be well approximated as a set of moving forces. The governing equations of motion in this case would not have time varying coefficients and the analysis is relatively more straightforward. In these situations there exist several options to identify the properties of the supporting structure [12,13].

The problem of identification of loads induced by moving vehicles based on response measurements on supporting structure and (or) moving vehicle has received wide research attention. Typically, these methods consist of formulating a set of over-determined equations based on the structural equilibrium equations (formulated either in the time or frequency domain), and measured structural strains, displacements and (or) accelerations. These equations are subsequently solved using matrix pseudo-inverse theory along with regularization tools [14–18]. Alternative formulations based on time domain sensitivity analysis and optimal state estimation methods also have been explored [19,20]. The recent paper by Yu and Chan [21] presents a review of relevant literature in this area of research.

The problem of identification of bridge parameters based on data from VSI-s has also been studied by a few authors. Hoshiya and Maruyama [22] have used the EKF with global iterations to identify beam-moving oscillator system based on measurements made on the moving oscillator and beam response. Majumder and Manohar [23,24] have developed a conceptual framework to detect local/distributed damages to beam structures when these structures are traversed by a test moving oscillators. A vector of damage indicator factors are shown to be governed by a set of over-determined nonlinear algebraic equations and these are solved iteratively. The idea of using a test vehicle as an actuator as well as a sensor to extract natural frequencies and detect structural damages of the bridge structure has been explored by a few authors [25–27]. The problem of identifying axle loads as well as the bridge prestressing force based on VSI data has been considered by Law et al. [28]. These authors employ finite element and wavelet analyses tools to achieve this and assume that the bridge response (strains and accelerations) are measured. Based on time domain response sensitivity analysis, Lu and Law [29] have investigated the problem of identifying input force and system parameters in linear dynamical problems with illustrations drawn from VSI problems. The application of wavelet analysis of measured beam response to identify cracks in the beam when the beam is traversed by moving time varying forces has been studied by Zhu and Law [30]. The application of Tikhonov regularization scheme in the identification of prestress in a beam based on beam response to moving loads has been discussed by Law and Lu [31].

In the present study we consider the problem of identification of dynamic characteristics of a beam-moving oscillator system based on measurements on beam response when the beam is traversed by a single degree of freedom (SDOF) oscillator. A new framework to tackle this problem is proposed based on the application of particle filtering based dynamic state estimation tools. The response measurements are limited to the beam response while the velocity (and acceleration) of the longitudinal motion of the oscillator is taken to be known. The solution method accounts for imperfections in measurements and mathematical modeling and also spatial incompleteness of measurements. The formulation does not require any model reduction as is often needed in problems of system identification based on limited response measurements. The mathematical foundations of the method lie in the theory of Bayesian filtering [32–34]. This consists of the definition of state and measurement vectors; typically the time variable is discretized. The elements of state vector are taken to be unobservable and this vector is governed by a process equation and the measured quantities are related to the system states through a measurement equation. Both the process and measurement equations are taken to contain noise terms that allow for imperfections in underlying mathematical modeling and random errors in measurements. The noise processes are modeled as sequences of independent random variables and, consequently, the system state vector possesses Markov property. The problem on hand consists of determining the posterior probability density function (pdf) of the system states conditioned on the measurements made. For linear process and measurement equations with additive Gaussian noises, this problem is amenable for an exact solution and this was provided by Kalman [32]. The celebrated Kalman filter provides an exact set of recursive relations for time evolution of conditional mean and covariance of system states conditioned on measurements made. For more general class of problems involving nonlinear systems and (or) multiplicative/additive non-Gaussian noises, a pair of functional recursive equations governing the evolution of the pdf of response conditioned on measurements can be derived based on the application of the Chapman Kolmogorov equation and Bayes' theorem [34]. These equations, however, are not amenable for closed form solutions nor are they amenable for evaluation via classical quadrature formulae. Nevertheless, the equations can still be tackled using Monte Carlo simulation strategies and this has resulted in methods known variously as Monte Carlo filters, Bayesian filters, population Monte Carlo algorithms, or particle filters [34–39]. The application of these tools in structural system identification has been explored in recent years by a few authors [40–48].

In the context of inverse problems involving VSI-s, the particle filtering methods, to the best of authors' knowledge, have not been explored so far in the existing literature. The present study, accordingly, aims to investigate this possibility.

This is achieved by combining the finite element method for modeling VSI problems with the particle filtering method as proposed by Gordon et al. [35]. A modification to the filtering algorithm is proposed that takes into account the fact that the system parameters being estimated are time invariant in nature and the measurement data is time varying often obtained by sampling at high rates. The work aims to make a contribution at a conceptual level and the illustrative examples are limited to measurement data simulated numerically from FE codes. Illustrative examples, that demonstrate the performance of the method, include studies on a parametrically and externally driven SDOF oscillator and on a beam-moving oscillator system. Questions on the influence of guideway unevenness and estimation of parameters associated with both the moving oscillator and beam parameters are addressed.

2. Finite element model for vehicle structure interaction

The beam-moving oscillator system shown in Fig. 1 is an archetypal system that has been widely used to conceptualize problems of bridge–vehicle interaction problems. The bridge is modeled as an Euler–Bernoulli beam and the vehicle as a SDOF oscillator. Fig. 1 also introduces the notations used to represent the beam-oscillator characteristics. The beam is taken to be supported on rotary springs to account for partial fixity conditions that may prevail at the bearings. The guideway is assumed to be uneven and the surface roughness is represented by the function $r(x)$. The oscillator is taken to enter the beam at t=0, travel with an uniform velocity v, and exit the bridge at t_f. The governing equations for the beam-oscillator system valid for the time interval $0<t\le t_f$ can be shown to be given by [1,2]

$$m_1\ddot{y}+c_1\left[\dot{y}-\frac{D}{Dt}\{w(vt,t)+r(vt)\}\right]+k_1\{y_1-w(vt,t)-r(vt)\}=0$$

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}\right]+m\frac{\partial^2 w}{\partial t^2}+c\frac{\partial w}{\partial t}=f(x,t)\delta(x-vt)$$

$$f(x,t)=(m_1+m_2)g-m_2\frac{D^2}{Dt^2}\{w(x,t)+r(x)\}+c_1\left[\dot{y}-\frac{D}{Dt}\{w(x,t)+r(x)\}\right]+k_1\{y_1-w(x,t)-r(x)\} \tag{1}$$

In the above equation $\delta(\,)$ denotes the Dirac delta function, g is the acceleration due to gravity, D/Dt denotes the total derivative (with $Dw/Dt=(\partial w/\partial x)(\partial x/\partial t)+\partial w/\partial t$) that allows for Coriolis forces that arise due to rolling of oscillator on the deflected profile of the beam and $f(x,t)$ denotes the interaction force. The relevant boundary conditions are given by

$$w(0,t)=0,\quad \frac{\partial}{\partial x}\left[EI(0)\frac{\partial^2 w(0,t)}{\partial x^2}\right]+k_{\theta 1}\frac{\partial w(0,t)}{\partial x}=0$$

$$w(L,t)=0,\quad \frac{\partial}{\partial x}\left[EI(L)\frac{\partial^2 w(L,t)}{\partial x^2}\right]+k_{\theta 2}\frac{\partial w(L,t)}{\partial x}=0 \tag{2}$$

In order to develop finite element approximation to the above set of differential equations, we represent the beam displacement as

$$w(x,t)=\mathbf{N}(x)\mathbf{d}(t) \tag{3}$$

Here $\mathbf{N}(x)$ is the matrix of interpolation functions (assumed to be cubic polynomials in the present study) and $\mathbf{d}(t)$ is the vector of nodal dof-s of the beam. Following the steps outlined by Filho [2], the governing equations, in semi-discretized

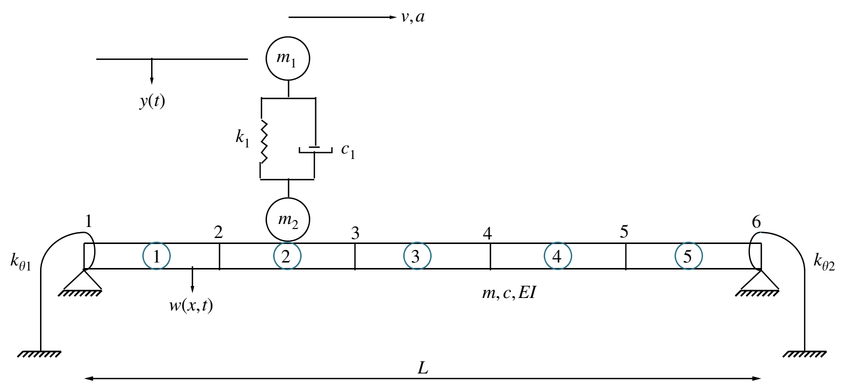

Fig. 1. A SDOF oscillator moving on a flexibly supported single span beam; details of FE discretization are also shown; numbers within the circles indicate element numbers and numbers on the top of the beam indicate node numbers.

form, valid for the interval $0 < t \le t_f$, can be shown to be given by

$$\begin{bmatrix} \mathbf{M}+\mathbf{m}^* & \mathbf{0} \\ 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{d}} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} \mathbf{C}+\mathbf{c}^* & -c_1\mathbf{N}^{\mathrm{T}} \\ -c_1\mathbf{N} & c_1 \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{d}} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}+\mathbf{k}^* & -k_1\mathbf{N}^{\mathrm{T}} \\ -c_1 v\mathbf{N}_x - k_1\mathbf{N} & k_1 \end{bmatrix} \begin{Bmatrix} \mathbf{d} \\ y \end{Bmatrix} = \begin{Bmatrix} \mathbf{N}^{\mathrm{T}}(m_1+m_2)g \\ 0 \end{Bmatrix} \quad (4a)$$

where

$$\mathbf{m}^* = m_2\mathbf{N}\mathbf{N}^{\mathrm{T}}; \quad \mathbf{c}^* = 2m_2 v\mathbf{N}\mathbf{N}_x + c_1\mathbf{N}^{\mathrm{T}}\mathbf{N}; \quad \mathbf{k}^* = m_2 v^2\mathbf{N}^{\mathrm{T}}\mathbf{N}_{xx} + c_1 v\mathbf{N}^{\mathrm{T}}\mathbf{N}_x + k_1\mathbf{N}^{\mathrm{T}}\mathbf{N} \quad (4b)$$

Here **K**, **M** and **C** are the beam stiffness, mass and damping matrices respectively; the subscript x denotes differentiation with respect to x with $x=vt$. The stiffness matrix here not only takes into account the beam flexural rigidities but also the rotary springs k_{θ_1} and k_{θ_2}, which are used to model the bearing flexibility. After the vehicle leaves the bridge, that is, for $t > t_f$, the ensuing free vibration of the beam is governed by

$$\mathbf{M}\ddot{\mathbf{d}}(t) + \mathbf{C}\dot{\mathbf{d}}(t) + \mathbf{K}\mathbf{d}(t) = 0 \quad (5)$$

with the *initial* conditions $\mathbf{d}(t_f)$ and $\dot{\mathbf{d}}(t_f)$ to be obtained by considering solution of Eq. (4) at $t=t_f$. It may be observed that Eq. (4) represents a time varying linear system with the structural matrices being asymmetric and functions of time. The time varying nature of the structural matrices rules out the possibility of uncoupling the equations of motion through modal coordinate transformation.

3. Structural system identification

For the purpose of system identification we take that the elemental flexural rigidity, mass per unit length, damping properties of the beam structure and the rotary springs at the supports to constitute the vector of parameters to be identified. This vector could also include parameters associated with the moving oscillator, namely the unsprung and sprung masses, spring constant and damper. We parameterize the damping characteristics of the beam in its uncoupled state by assuming that, at the element level, the damping matrix is proportional to the element mass matrix and take the proportionality constants as parameters to be identified. That is, we take $\mathbf{C} = \sum_{e=1}^{N_e} \alpha_e \mathbf{M}_e$, where N_e is the number of elements into which the beam (in its uncoupled state) is discretized, M_e=mass matrix of the e-th beam element and $\{\alpha_e\}_{e=1}^{N_e}$ are constants to be identified. This model of damping does not guarantee that the beam would be classically damped. We represent the parameters to be identified as $\boldsymbol{\Theta} = (\theta_i)_{i=1}^q$. The velocity and acceleration with which the oscillator traverses the beam are taken to be known; these could also be treated as unknown to be identified but this option is not considered in the present study. The objective of the present study is to develop a procedure based on dynamic state estimation methods to estimate the parameters $\boldsymbol{\Theta} = (\theta_i)_{i=1}^q$. A first step in achieving this is to formulate the relevant process and measurement equations. Since these parameters $\boldsymbol{\Theta} = (\theta_i)_{i=1}^q$ are time invariant, it is clear that

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = 0; \quad \theta_i(0) = \theta_{i0}; \quad i = 1, 2, \ldots, q \quad (6)$$

In the present study, we consider these equations to constitute the process equation. In a discretized form the above equation can be written as $\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k$; $k = 0, 1, 2, \ldots, N_k$ with $\boldsymbol{\Theta}_0$ specified. Here k refers to the discretized time dimension and N_k is the number of time instants considered. For the sake of computational expediency, and, to facilitate the development of dynamic state estimation based identification procedure, we write this equation as

$$\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k + \mathbf{w}_k; \quad k = 0, 1, 2, \ldots, N_k \quad (7)$$

Here w_k denotes a $q \times 1$ vector of random variables and the sequence of these random vectors for k=0, 1, 2, … are taken to be identical and independently distributed with $\mathbf{w}_k \sim p(\mathbf{w}_k)$; ('$\sim$' denotes 'distributed as'). We also interpret Θ_0 as being random with $\boldsymbol{\Theta}_0 \sim p(\boldsymbol{\Theta}_0)$. It should be noted that the idea of treating time invariant parameters as artificial dynamic variable with artificial evolution noise has been used earlier by other researchers: see for example Refs. [49,50] and the citations therein.

We assume that a set of measurements on beam displacements, strains and (or) accelerations have been made and we denote these measurements at the kth time instant by an $n_y \times 1$ vector $\mathbf{y}_k$ and assume that this vector is related to $\boldsymbol{\Theta}_k$ through the relation

$$\mathbf{y}_k = \mathbf{h}_k(\boldsymbol{\Theta}_k) + \xi_k; \quad k = 1, 2, \ldots, N_k \quad (8)$$

Here $\mathbf{h}_k(\boldsymbol{\Theta}_k)$ represents the nonlinear relationship that exists between the measured quantities y_k and the system parameters $\boldsymbol{\Theta}_k$ and, in the present study, this relation is based on the finite element model as in Eqs. (4) and (5). In the above equation ξ_k represents a $n_y \times 1$ vector of random variables with $\xi_k \sim p(\xi_k)$. It is assumed that, for k=1, 2, … the vectors ξ_k constitute a sequence of independent and identically distributed random vectors. Furthermore, in the present study, we take $\mathbf{w}_k$, $\boldsymbol{\Theta}_0$ and ξ_k to be mutually independent. The system identification problem on hand can now be perceived as being equivalent to the determination of the posterior pdf $p(\boldsymbol{\Theta}_k|\mathbf{y}_{1:k})$, also known as the filtering density, where $\mathbf{y}_{1:k} = \{\mathbf{y}_1 \ \mathbf{y}_2 \ \ldots \ \mathbf{y}_k\}^{\mathrm{T}}$. This also leads to the determination of the mean and covariance of $\boldsymbol{\Theta}_k$ conditioned on $\mathbf{y}_{1:k}$

given respectively by

$$\mathbf{a}_{k|k} = \int \mathbf{\Theta}_k p(\mathbf{\Theta}_k|\mathbf{y}_{1:k})\,\mathrm{d}\mathbf{\Theta}_k$$

$$\mathbf{\Sigma}_{k|k} = \int (\mathbf{\Theta}_k - \mathbf{a}_{k|k})^t (\mathbf{\Theta}_k - \mathbf{a}_{k|k}) p(\mathbf{\Theta}_k|\mathbf{y}_{1:k})\,\mathrm{d}\mathbf{\Theta}_k \tag{9}$$

It may be remarked that the process Eq. (7) here is linear but the measurement Eq. (8) is nonlinear in $\mathbf{\Theta}_k$. Thus, even if we take $\mathbf{w}_k$ and ξ_k to be Gaussian distributed, the problem of determination of $p(\mathbf{\Theta}_k|\mathbf{y}_{1:k})$ is not amenable for an exact solution via the Kalman filter. One alternative to proceed further would be to linearize the measurement Eq. (8) in $\mathbf{\Theta}_k$ and apply Kalman filter on the linearized equations; another alternative is to employ Monte Carlo simulation based strategy to approximately solve the problem. We adopt the latter strategy in the present work details of which are provided in the next section.

In summary, it may be emphasized that the problem of system parameter identification has been posed as a problem in nonlinear filtering. It is also important to note that the system states contains only the parameters to be identified and the response of beam and vehicle are contained only in the measurement model. The formulation thus does not require any special step to be implemented to take into account spatial incompleteness of measurements. The present approach thus differs from the existing system identification methods that employ dynamic state estimation methods (as in, for example, [49,50]) in which the problem of system parameter estimation is embedded into a dual problem of combined state and parameter estimation.

4. Bootstrap filtering

We assume that the random vectors $\mathbf{w}_k$ and ξ_k appearing in Eqs. (7) and (8) are Gaussian with zero mean and specified covariance matrix. It follows from Eq. (7) that the process $\mathbf{\Theta}_k$ has Markovian property, that is, $p(\mathbf{\Theta}_k|\mathbf{\Theta}_{k-1}, \mathbf{\Theta}_{k-2}, \ldots, \mathbf{\Theta}_0) = p(\mathbf{\Theta}_k|\mathbf{\Theta}_{k-1})$. Based on this result, and, also, by using Bayes' theorem, it can be shown that the time evolution of $p(\mathbf{\Theta}_k|\mathbf{y}_{1:k})$ is governed by the following set of functional recursive equations [35]:

$$p(\mathbf{\Theta}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{\Theta}_k|\mathbf{\Theta}_{k-1}) p(\mathbf{\Theta}_{k-1}|\mathbf{y}_{1:k-1})\,\mathrm{d}\mathbf{\Theta}_{k-1}$$

$$p(\mathbf{\Theta}_k|\mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_k|\mathbf{\Theta}_k) p(\mathbf{\Theta}_k|\mathbf{y}_{1:k-1})}{\int p(\mathbf{y}_k|\mathbf{\Theta}_k) p(\mathbf{\Theta}_k|\mathbf{y}_{1:k-1})\,\mathrm{d}\mathbf{\Theta}_k} \tag{10a,b}$$

A solution to the dynamic state estimation problem by using numerical quadrature on these equations is generally infeasible and Monte Carlo simulation strategies offer viable alternatives to deal with this problem. In the present study, we use the bootstrap particle filtering algorithm as developed by Gordon et al. [35] to tackle the filtering problem. The theoretical foundations of this method are based on an earlier result by Smith and Gelfand [51]. This result can be stated as follows: suppose that samples $\{x_i^*(i) : i = 1, 2, \ldots, N\}$ are available from a continuous density function $G(x)$ and that samples are required from the pdf proportional to $L(x)G(x)$, where $L(x)$ is a known function. The theorem states that a sample drawn from the discrete distribution over $\{x_i^*(i) : i = 1, 2, \ldots, N\}$ with probability mass function $L(x_k^*(i))/\sum_{j=1}^{N} L(x_k^*(j))$ on $x_k^*(i)$, tends in distribution to the required density as $N \to \infty$. The algorithm for implementing the filter (with reference to Eqs. (3) and (4)) using N particles is as follows:

1. Set k=0. Generate N samples $\{\mathbf{\Theta}_{i,0}^*\}_{i=1}^N$ from the initial pdf $p(\mathbf{\Theta}_0^*)$ and $\{\mathbf{w}_{i,0}\}_{i=1}^N$ from $p(\mathbf{w})$.
2. Run the ordinary differential equation (ODE) solvers to integrate Eqs. (4) and (5) and generate $\mathbf{h}_k[\mathbf{\Theta}_{i,k}^*]_{i=1}^N$; evaluate $p(\mathbf{y}_k|\mathbf{\Theta}_{i,k}^*)$.
3. Consider the kth measurement $\mathbf{y}_k$. Define
$$q_i = \frac{p(\mathbf{y}_k|\mathbf{h}_k[\mathbf{\Theta}_{i,k}^*])}{\sum_{j=1}^{N} p(\mathbf{y}_k|\mathbf{h}_k[\mathbf{\Theta}_{j,k}^*])}$$
4. Define the probability mass function $P[\mathbf{\Theta}_k(j) = \mathbf{\Theta}_{i,k}^*] = q_i$; generate N samples $\{\mathbf{\Theta}_{i,k}\}_{i=1}^N$ from this discrete distribution.
5. Evaluate
$$\mathbf{a}_{k|k} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{\Theta}_{i,k} \quad \text{and} \quad \mathbf{\Sigma}_{k|k} = \frac{1}{N-1}\sum_{i=1}^{N} (\mathbf{\Theta}_{i,k} - \mathbf{a}_{k|k})^t (\mathbf{\Theta}_{i,k} - \mathbf{a}_{k|k}).$$
6. Set k=k+1 and go to step 2 if $k < N_k$; otherwise, stop.

The central contention in the formulation of this algorithm is that the samples $\{\mathbf{\Theta}_{i,k}\}_{i=1}^N$ drawn in step 4 above are approximately distributed as per the required pdf $p(\mathbf{\Theta}_k|\mathbf{y}_{1:k})$. The justification for this is provided by considering the updation Eq. (10b) in conjunction with the result due to Smith and Gelfand with $G(\mathbf{\Theta})$ identified with $p(\mathbf{\Theta}_k|\mathbf{y}_{1:k-1})$ and $L(\theta)$ with $p(\mathbf{\Theta}_k|\mathbf{y}_k)$. This filtering procedure is applicable to nonlinear process and measurement equations, and, non-Gaussian noise models. If the process ξ_k (Eq. (8)) is taken to be Gaussian with zero mean and covariance $\mathbf{\Sigma}_\xi$, it follows that in step 2 above we use $p(\mathbf{y}_k|\mathbf{\Theta}^*_{i,k}) \sim N[\mathbf{h}_k(\mathbf{\Theta}^*_{i,k}), \mathbf{\Sigma}_\xi]$. The number of calls to the ODE solvers for Eqs. (4) and (5) in step 2 depends upon the number of observation points N_k and the major computational effort in implementing the algorithm is spent in this step. Bearing in mind that the problem of state estimation here essentially consists of estimating system parameters that are intrinsically time invariant, and, also, with a view to lessen the number of calls to the ODE solvers, the procedure listed above could be simplified as follows: We group the N_k discrete values of k into R groups with N_i number of discrete values of k in the ith group with $\sum_{i=1}^{R} N_i = N_k$. We assume that $\mathbf{\Theta}_i$ remains constant for all values of k lying within the ith group. Also, the values of the probability measures q_i (in steps 3 and 4 above) are averaged over all values of k within the ith group. Accordingly, the filtering steps, using N particles, as per this modified procedure, are as follows:

1. Set r=0. Generate N samples $\{\mathbf{\Theta}^*_{i,0}\}_{i=1}^N$ from the initial pdf $p(\mathbf{\Theta}^*_0)$ and $\{\mathbf{w}_{i,0}\}_{i=1}^N$ from $p(\mathbf{w})$.
2. Evaluate $\{\mathbf{\Theta}^*_{i,r+1}\}_{i=1}^N$ using $\mathbf{\Theta}_{r+1} = \mathbf{\Theta}_r + \mathbf{w}_r$. Set r=r+1.
3. Run ODE solvers to solve Eqs. (4) and (5) and generate $h_k[\mathbf{\Theta}^*_{i,r+1}]$; evaluate $p(\mathbf{y}_k|\mathbf{\Theta}^*_{i,r})$ for $(r-1)n_k \le k \le rn_k$ where $n_k = N_k/R$.
4. Consider the kth measurement y_k. Define

$$q^*_{k,j} = \frac{p(\mathbf{y}_k|\mathbf{h}_k[\mathbf{\Theta}^*_{j,r}])}{\sum_{i=1}^{N} p(\mathbf{y}_k|\mathbf{h}_k[\mathbf{\Theta}^*_{j,r}])}$$

5. Evaluate

$$q_i = \frac{1}{n_k} \sum_{k=(r-1)n_k}^{rn_k} q^*_{k,i}.$$

6. Define the probability mass function $P[\mathbf{\Theta}_r(j) = \mathbf{\Theta}^*_{i,r}] = q_i$. Generate N samples $(\mathbf{\Theta}_{i,r})_{i=1}^N$ from this discrete distribution.
7. Evaluate

$$\mathbf{a}_{r|r} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{\Theta}_{i,r} \quad \text{and} \quad \mathbf{\Sigma}_{r|r} = \frac{1}{N-1}\sum_{i=1}^{N} (\mathbf{\Theta}_{i,r} - \mathbf{a}_{r|r})^t (\mathbf{\Theta}_{i,r} - \mathbf{a}_{r|r}).$$

8. Set r=r+1 and go to step 2 if $r < R$; otherwise, stop.

Remarks.

(a) The presence of the artificial noise $\mathbf{w}_k$ in Eq. (6) ensures that the samples of $\mathbf{\Theta}_k$ would not get frozen at their initial values drawn from $p(\mathbf{\Theta}_0)$ and would evolve as k increases. The standard deviation of the noise $\mathbf{w}_k$ needs to be non-zero and, as long as it remains small its magnitude is not expected to affect the performance of the filter. During the course of this study it was empirically found that noise levels with standard deviations less than about 0.5% of nominal values of $\mathbf{\Theta}$ generally lead to satisfactory performance of the algorithm. The present study does not theoretically investigate the performance of the estimation procedure *vis-à-vis* the noise levels.

(b) The initiation of the filtering algorithm requires an assumption to be made on the initial form of the pdf $p(\mathbf{\Theta}^*_0)$. In this study it is assumed that $\mathbf{\Theta}_0$ is uniformly distributed over a plausible range of values of its components and these components were taken to be independent. It was also found advantageous to draw samples using optimal space filling Latin hypercube sampling strategy [52] so that the initial guess distributes the sample points as uniformly distributed as possible within the chosen domain. It may be noted that the Matlab platform has readily available subroutines to implement this.

(c) To enhance the performance of the identification procedure, an additional step involving a global iteration is also employed in the present study. Here, the guess on pdf of $\mathbf{\Theta}_0$ is updated at the end of a given cycle of filtering by the final $p(\mathbf{\Theta}_{N_k}|\mathbf{y}_{1:N_k})$ and is used as the starting guess for the next cycle of filtering. This global iteration loop is repeated till a satisfactory convergence on the expected value of $\mathbf{\Theta}$ conditioned on measurements is obtained.

(d) The performance of the identification procedure outlined above depends upon several factors: number of particles used, details of averaging of weights (choice of parameter R), characteristics of noise process $\mathbf{w}_k$, initial guess made on $p(\mathbf{\Theta}_0)$ and number of global iterations used. Some of the steps such as the averaging of weights and global iterations are employed in the present study based mainly on intuitive arguments. A systematic investigation into assessment of each of these factors on convergence of the filtering algorithm leads to several questions on the nature of the statistical estimator for the system parameter. While these questions are worthy of serious research, we have not made such efforts in this study to address these questions. Moreover, we have limited our attention to the application of only one class of particle filtering, namely the bootstrap filter. There exist several other alternative strategies such as, for example, sequential importance sampling filtering [37–39],and the Markov chain Monte Carlo methods [53], which could possibly be applied to tackle the state estimation problem considered in this paper.

(e) Furthermore, the formulation developed in this study has been with reference to the simple example of a one-span Euler Bernoulli beam traversed by a SDOF moving oscillator. The application of the tools developed to practical problems requires careful modeling of not only the bridge structure but also the moving vehicle with multiple axles. A time domain sub-structuring scheme involving finite element models for the vehicle and the supporting structure, in their uncoupled states, could be developed and imbedded into the identification procedure outlined in this paper. This requires considerable modeling and software development tasks. We believe that the study reported in this paper serves as the starting point to initiate such an effort.

5. Numerical examples

We illustrate the procedure developed in the previous section by considering synthetically generated measurements. We consider two sets of examples: the first refers to a SDOF parametrically excited system and the second deals with the beam-oscillator system (Fig. 1). The first example is considered to illustrate a few algorithmic aspects of the identification procedure. As a part of the second example, the following cases are considered: (a) identification of beam parameters

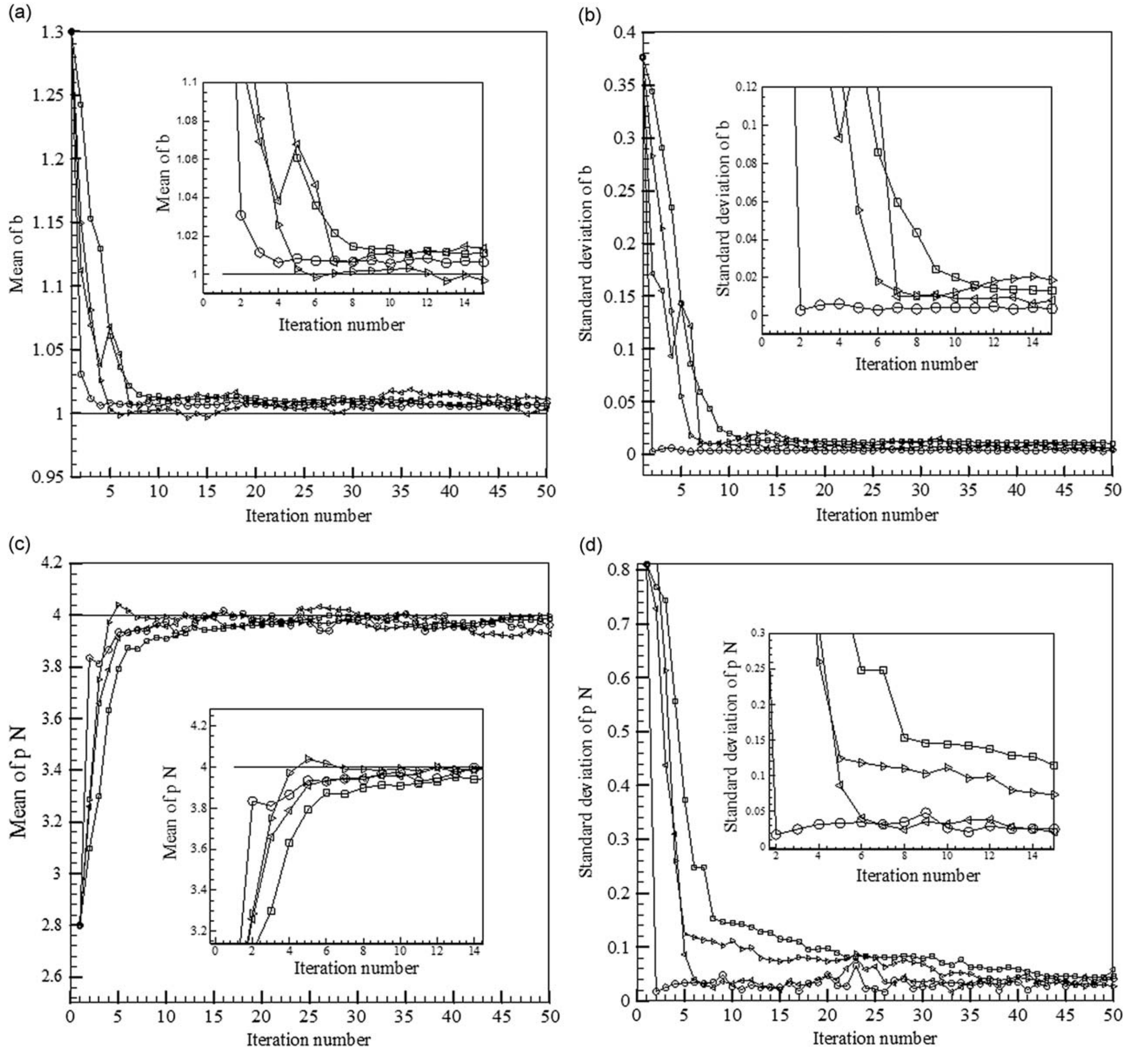

Fig. 2. Example in Section 5.1; Evolution of posterior mean and standard deviation of system parameters as a function of the global iteration number: (a) expected value of b; (b) standard deviation of b; (c) expected value of p; (d) standard deviation of p; ——— reference, —□— R=1, —▷— R=5, —◁— R=25, —○— R=125.

assuming vehicle properties are known, (b) presence of guideway unevenness and (c) identification of beam as well as vehicle parameters. In all these cases the velocity and acceleration with which the vehicle traverses the beam are taken to be known. In all the examples considered, the elements of noise vector ξ_k and w_k are taken to have zero mean, and to be independent and Gaussian distributed. The standard deviations of elements of ξ_k are taken to be 5% of the maximum value of the associated noise-free measured quantity. Similarly, the standard deviations of elements of w_k were taken to be 0.5% of the nominal values of the respective system parameters.

5.1. Preparatory example

We begin by considering the problem of identification of parameters of a SDOF linear time variant system. This example serves to clarify the influence of choices made on algorithmic parameters in implementing the identification procedure outlined in the preceding sections. The system under consideration is taken to be governed by the equation

$$\ddot{x}+2\eta\omega[1+a\cos(\lambda_1 t)]\dot{x}+\omega^2[1+b\cos(\lambda_2 t)]x=p\cos(\mu t);\quad x(0)=0;\quad \dot{x}(0)=0 \tag{11}$$

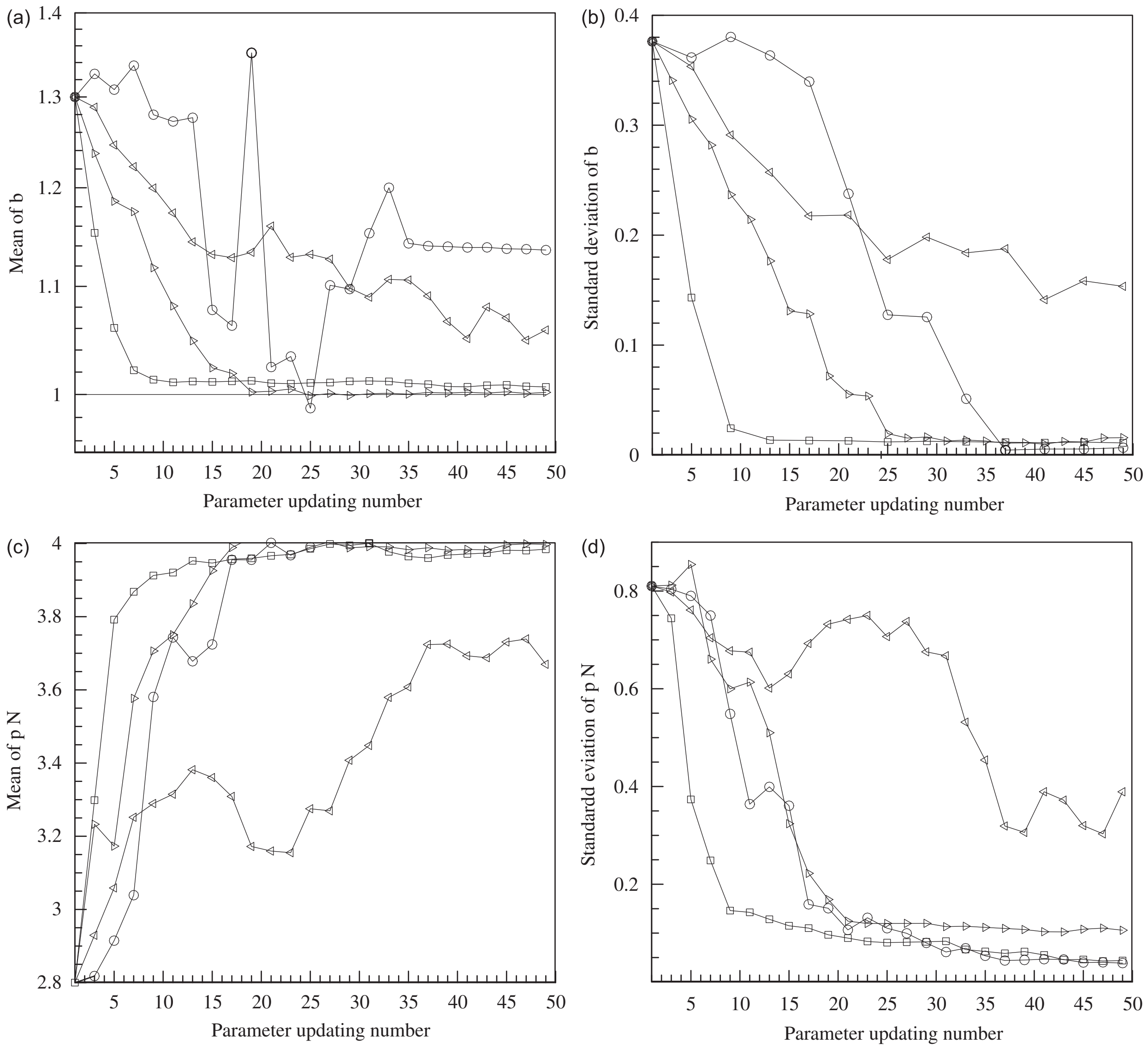

Fig. 3. Example in Section 5.1; Evolution of posterior mean and standard deviation of system parameters as a function of the number of calls to the ODE solver for Eq. (11): (a) expected value of b; (b) standard deviation of b; (c) expected value of p; (d) standard deviation of p; ——— reference, —□— R=1, —▷— R=5, —◁— R=25, —○— R=125.

This model is fashioned after the well known Mathieu–Hill system, which is encountered in a wide variety of applications (see, for example, the monograph by Bologtin [54]). The problem on hand consists of estimating the parameters b and p based on measurements made on $x(t)$ and $\dot{x}(t)$. Synthetic measurements are generated by solving the above equation using the Wilson-θ method with η=0.04, ω=4π rad/s, a=2, λ=0.8ω rad/s and μ=2λ_1; the step size used in integration was Δt=(2π/25μ)s. The reference values of b and p were taken to be b=1 and p=4 N. Initially, the parameters b and p were taken to be independent and uniformly distributed in the limits [0.65, 1.95] and [1.4, 4.2] N, respectively. The number of data points N_k was taken to be 125 and the number of global iterations was fixed to be 50. The filtering was carried out using 200 samples. The procedure outlined in section 4.0 is applied for R=1, 5, 25 and 125. The number of calls to the ODE solver of Eq. (11) (that is, the number of updations) for these four cases thus turns out to be 50, 250, 1250 and 6250, respectively.

Fig. 2(a–d) shows the evolution of the estimates of mean and standard deviation conditioned on measurements of parameters b and p as functions of the iteration number for different values of parameter R. It may be observed that the results for R=1, 5, 25 and 125 all show satisfactory convergence as iterations are increased with the case of R=125 showing earlier convergence than the rest. It is to be noted that the computational effort needed for the cases of R=1, 5, 25 and 125 are quite different. Thus, for R=1, in any single iteration, we need to call the ODE solver of Eq. (11) only once while for R=125, the solver is called 125 times. Thus the better convergence for the case of R=125 in Fig. 2 is achieved at the expense of higher computational effort. To compare the computational efforts involved in cases with R=1, 5, 25 and 125, the results

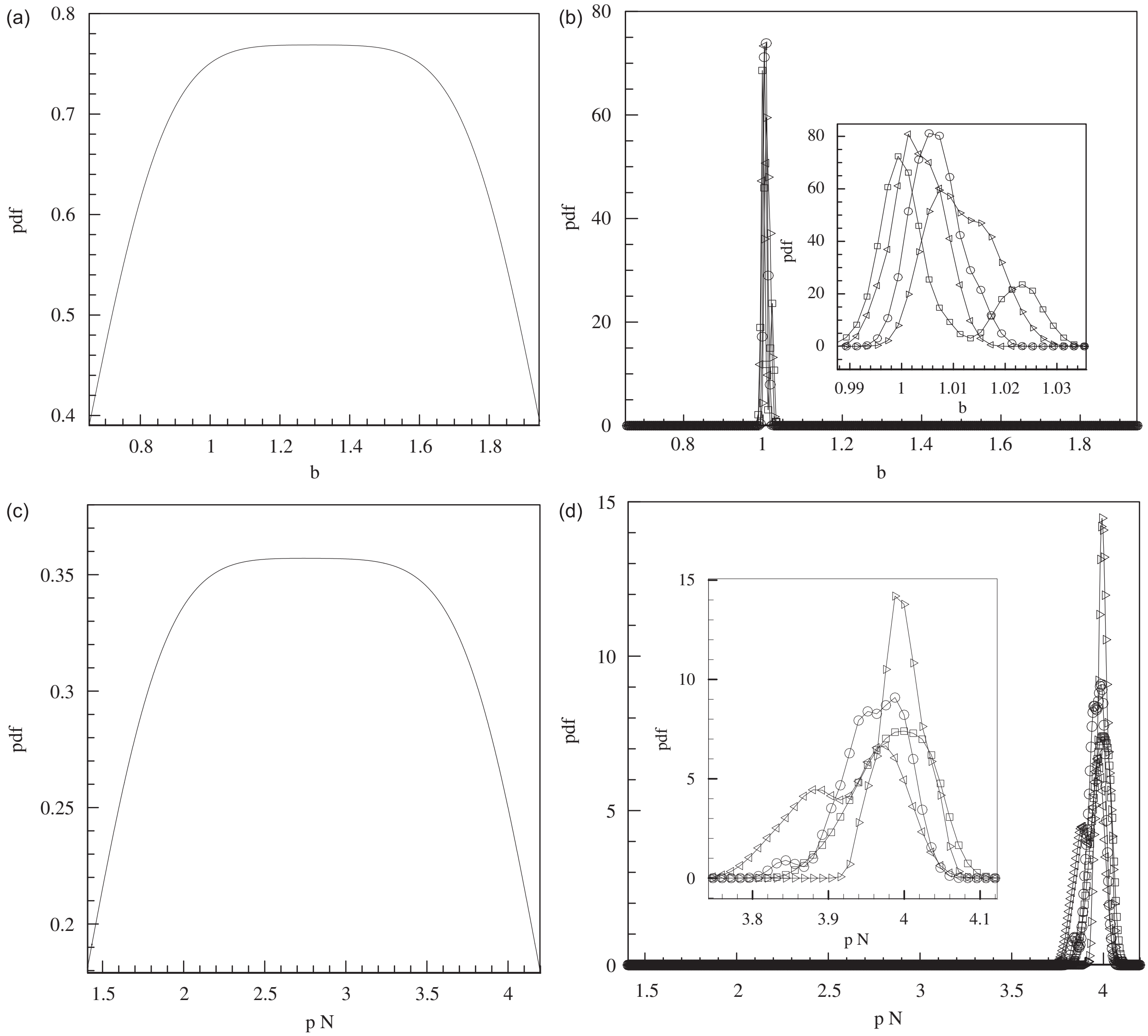

Fig. 4. Example in Section 5.1: (a) pdf of the initial guess on b; (b) pdf of b at the end of identificaiton proceudre; (c) pdf of the initial guess on p; (d) pdf of p at the end of identificaiton procedure; ——— reference, —□— R=1, —▷— R=5, —◁— R=25, —○— R=125.

of estimation of b and p are shown in Fig. 3(a–d) as a function of number of calls to the ODE solver. It is to be noted here that the number of global iterations for R=1, 5, 25 and 125 varies; thus, for R=1, 50 global iterations are involved, for R=5, 10 iterations are involved, etc. It may be observed from Fig. 3 that the case of R=1 provides superior performance when compared with other values of R. Fig. 4 illustrates the initial pdf-s of the parameters b and p and the final pdf-s, at the end of identification procedure, are also shown. It is to be noted that the final pdf-s peak sharply and these peaks are not necessarily near the expected value of the initial guess. The measured displacement and velocity are compared with the respective reference values and the estimates (using the identified system parameters) in Fig. 5(a–c). Fig. 5(c) shows the results on acceleration response which was not used in the identification procedure. The reference values of system response are simulated from the known system model which has been used for already for generating the synthetic measurement data. Based on the results presented, it may be concluded that the proposed method performs satisfactorily and the choice of R=1 provides acceptable results with attendant benefits of lower computational effort. It was observed during numerical investigations that the effect of increasing the standard deviation of noise $\mathbf{w}_k$ was felt more pronouncedly on the estimate of the standard deviation of the system parameters than on their expected values.

5.2. Studies on the beam-oscillator system

The beam-oscillator system under study (Fig. 1) is taken to have the following properties: span, L=30 m; beam mass per unit length, m=38277.0 kg/m; Young's modulus, $E=2.0\times10^{11}\,\mathrm{N/m^2}$; area moment of inertia of the beam cross section, $I=0.503\,\mathrm{m^4}$; area of cross section of the beam, $A=4.876\,\mathrm{m^2}$; coefficient of viscous damping for the beam, c=317000.0 Ns/m; rotary spring constants at the supports, $k_{\theta_1}=2.8168\times10^9\,\mathrm{Nm/rad}$ and $k_{\theta_2}=4.2252\mathrm{E}{+}09\,\mathrm{Nm/rad}$; sprung mass of the vehicle, m_1=114830.0 kg; unsprung mass of the vehicle m_2=459320.0 kg; damping ratio for the vehicle, η=0.05; vehicle

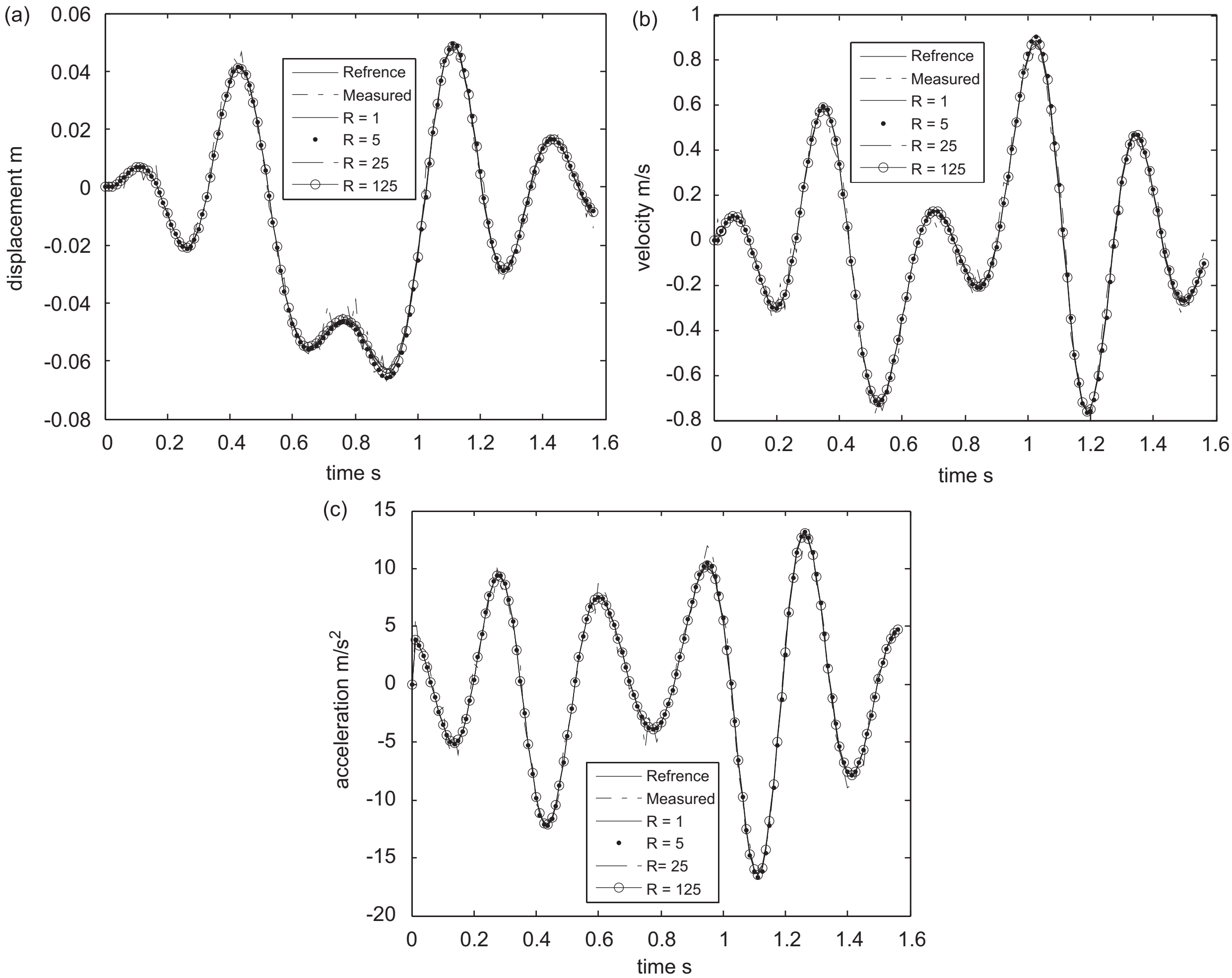

Fig. 5. Example in Section 5.1; Comparison of predictions on system response from identified and initial models with corresponding measurements: (a) $x(t)$; (b) $\dot{x}(t)$; (c) $\ddot{x}(t)$.

spring stiffness, k_1=1.568E+10 N/m; longitudinal velocity of the vehicle, v=20.0 m/s and longitudinal acceleration of the vehicle, a=0 m/s^2. A finite element model for the beam with 12 dof-s is constructed with five Euler–Bernoulli beam elements with consistent mass matrix formulation (details of discretization as in Fig. 1). The first five natural frequencies of the beam (when it is uncoupled from the vehicle) were found to be 18.22, 72.99, 165.26, 298.19 and 505.49 rad/s; similarly, the vehicle's natural frequency was found to be 171.61 rad/s. In order to generate the measurement data, the governing equations for the beam-oscillatory system were integrated using Wilson-θ method with a step size of 2.0×10^{-4} s. The oscillator was taken to enter the beam at t=0 and the beam and vehicle were taken to be at rest at this time instant. The governing equations of motion were integrated for a length of 1.5t_f, where t_f=time taken by the oscillator to traverse the beam span. Fig. 6(a and b) shows the beam mid-span displacement and the time history of the oscillator force on the

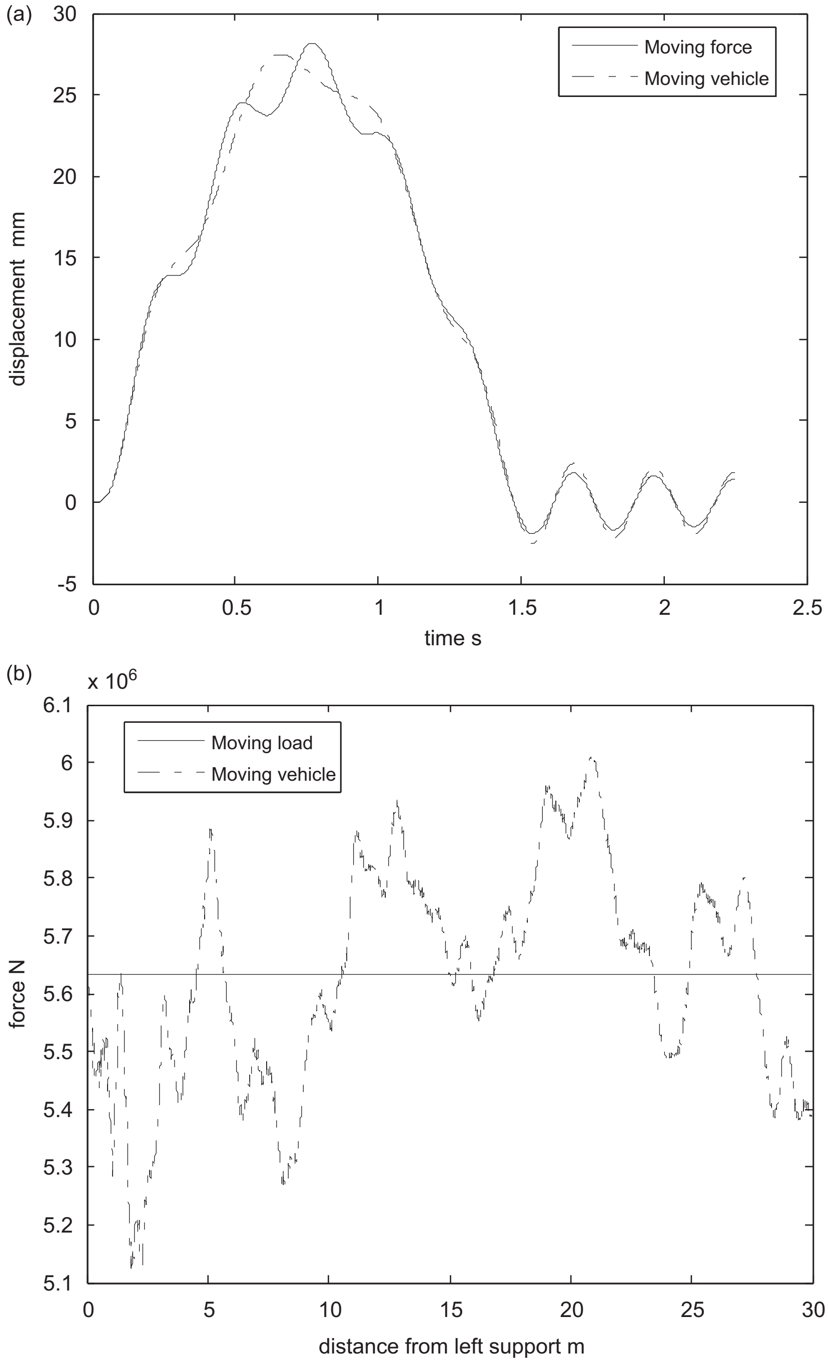

Fig. 6. Response of beam oscillator system considered in Section 5.2 (Fig. 1): (a) mid-span translation; (b) interaction force.

Table 1
Summary of estimated system parameters for studies 1–3 in Sections 5.2.1 and 5.2.2; in columns 5–13, the numbers within the bracket [ ] are the standard deviations; the numbers within parentheses ( ) are % errors.

Element	Reference value of the parameters			Study 1			Study 2			Study 3		
	Stiffness	Damping, Ns/m	Density, kg/m	Stiffness	Damping, Ns/m	Density, kg/m	Stiffness	Damping, Ns/m	Density, kg/m	Stiffness	Damping, Ns/m	Density, kg/m
1	8.134×10^{10} Nm^2	23458.0	38277.0	7.939×10^{10} [4.796×10^{8}] Nm^2 (2.39)	23556.0 [146.0] (0.42)	37158.0 [166.0] (2.92)	7.030×10^{10} [4.870×10^{8}] Nm^2 (13.57)	20353.0 [148.0] (13.24)	39115.0 [168.0] (2.19)	7.945×10^{10} [4.657×10^{8}] Nm^2 (2.31)	22967.0 [141.0] (2.09)	38145.0 [170.0] (0.34)
2	1.004×10^{11} Nm^2	26945.0	33683.0	9.731×10^{10} [4.788×10^{8}] Nm^2 (3.02)	26317.0 [141.0] (2.33)	31949.0 [177.0] (5.15)	1.026×10^{11} [4.861×10^{8}] Nm^2 (2.21)	25715.0 [143.0] (4.57)	30353.0 [180.0] (9.89)	1.007×10^{11} [4.830×10^{8}] Nm^2 (0.32)	27838.0 [139.0] (3.31)	34698.0 [168.0] (3.01)
3	$8.87310^{8} \times 10^{10}$ Nm^2	25994.0	26028.0	8.915×10^{10} [4.834×10^{8}] Nm^2 (0.47)	26328.0 [146.0] (1.28)	26861.0 [174.0] (3.20)	9.854×10^{10} [4.912×10^{8} Nm^2 (11.06)	27816.0 [148.0] (7.01)	31313.0 [177.0] (20.31	9.169×10^{10} [4.674×10^{8}] Nm^2 (3.34)	26966.0 [140.0] (3.74)	26951.0 [173.0] (3.55)
4	9.507×10^{10} Nm^2	29481.0	30621.0	9.621×10^{10} [4.627×10^{8}] Nm^2 (1.20)	30163.0 [140.0] (2.31)	30226.0 [176.0] (1.29)	1.002×10^{11} [4.683×10^{8}] Nm^2 (5.41)	32181.0 [142.0] (9.16)	33051.0 [179.0] (7.93)	9.461×10^{10} [4.739×10^{8}] Nm^2 (0.48)	30194.0 [141.0] (2.42)	30209.0 [176.0] (1.35)
5	8.450×10^{10} Nm^2	30749.0	28707.0	8.304×10^{10} [4.781×10^{8}] Nm^2 (1.73)	30343.0 [143.0] (1.32)	28851.0 [174.0] (0.50)	1.104×10^{11} [4.853×10^{8}] Nm^2 (30.64)	32551.0 [145.0] (5.86)	32443.0 [176.0] (13.01)	8.605×10^{10} [4.825×10^{8}] Nm^2 (1.83)	30815.0 [144.0] (0.21)	29922.0 [175.0] (4.23)
$k_{\theta 1}$	2.817×10^{9} Nm/rad	–	–	2.847×10^{9} [4.606×10^{7}] Nm/ rad (1.07)	–	–	3.101×10^{9} [4.659×10^{6}] Nm/ rad (10.10)	–	–	2.863×10^{9} [4.771×10^{6}] Nm/ rad (1.65)	–	–
$k_{\theta 2}$	4.225×10^{9} Nm/rad	–	–	4.316×10^{9} [4.747×10^{7}] Nm/ rad (2.16)	–	–	3.357×10^{9} [4.815×10^{6}] Nm/ rad (20.56)	–	–	4.084×10^{9} [4.653×10^{6}] Nm/ rad (3.34)	–	–

beam for the cases when oscillator-beam dynamic interactions are included and when oscillator is modeled simply as a moving force. The influence of vehicle structure interactions can clearly be discerned from these figures. In all the examples to follow, the particle filter was implemented with 100 numbers of particles, with a maximum of 50 global iterations and with $R=1$.

5.2.1. Studies 1 and 2

Here we make the following assumptions: (a) the guideway is smooth, (b) the vehicle properties are known and (c) measurements are made on beam transverse displacements and bending strains at midpoint of the elements 1, 2, 3 and 5, and rotations at the two ends. Moreover, separate studies were carried out by including the dynamic effects of VSI in the identification model (Study 1) and by excluding this effect in the model (Study 2). It is emphasized that in both these cases, the measurements were made by including the effects of VSI. The results of these studies are summarized in Table 1 (together with results from Study 3 to be reported in the next section). It may be noted that the numbers within the brackets [ ] in columns 5–13 in this table are the estimated standard deviations and the numbers within the parenthesis ( ) are % errors computed as $(|\theta_i^{ref}-\theta_i^{estimated}|/\theta_i^{ref})\times 100$. It may be observed that the estimation of parameters in Study 1 is

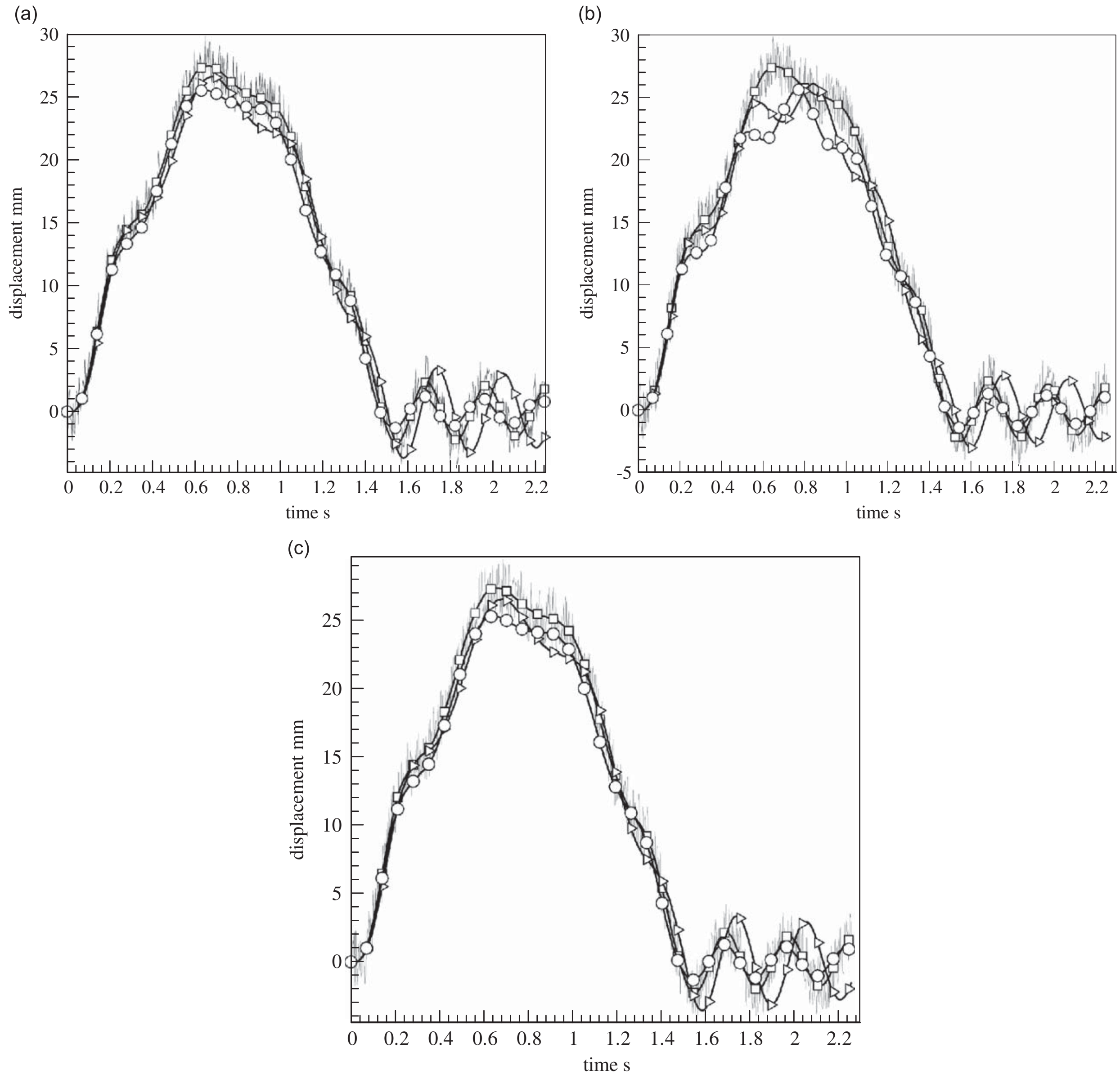

Fig. 7. Response of beam oscillator system considered in Section 5.2 (Fig. 1): (a) mid-span translation (a) study1; (b) study 2; (c) study 3; –··–··–·· measurement, ——□—— reference, ——▷—— initial model, ——○—— final model.

achieved with a maximum error of 3.02% in estimating stiffness parameters, 2.33% in estimating damping parameters and 5.15% in estimating mass parameters. In Study 2, however, the maximum error is about 30.64% in stiffness parameters, 13.24% in damping parameters and 20.31% is mass parameters. The unacceptably large errors here are to be expected since the identification model used here ignores the vehicle structure interactions. Fig. 7(a and b) compares the estimated mid-span response from the identified system along with the reference trajectory, response from the initial model and the noisy

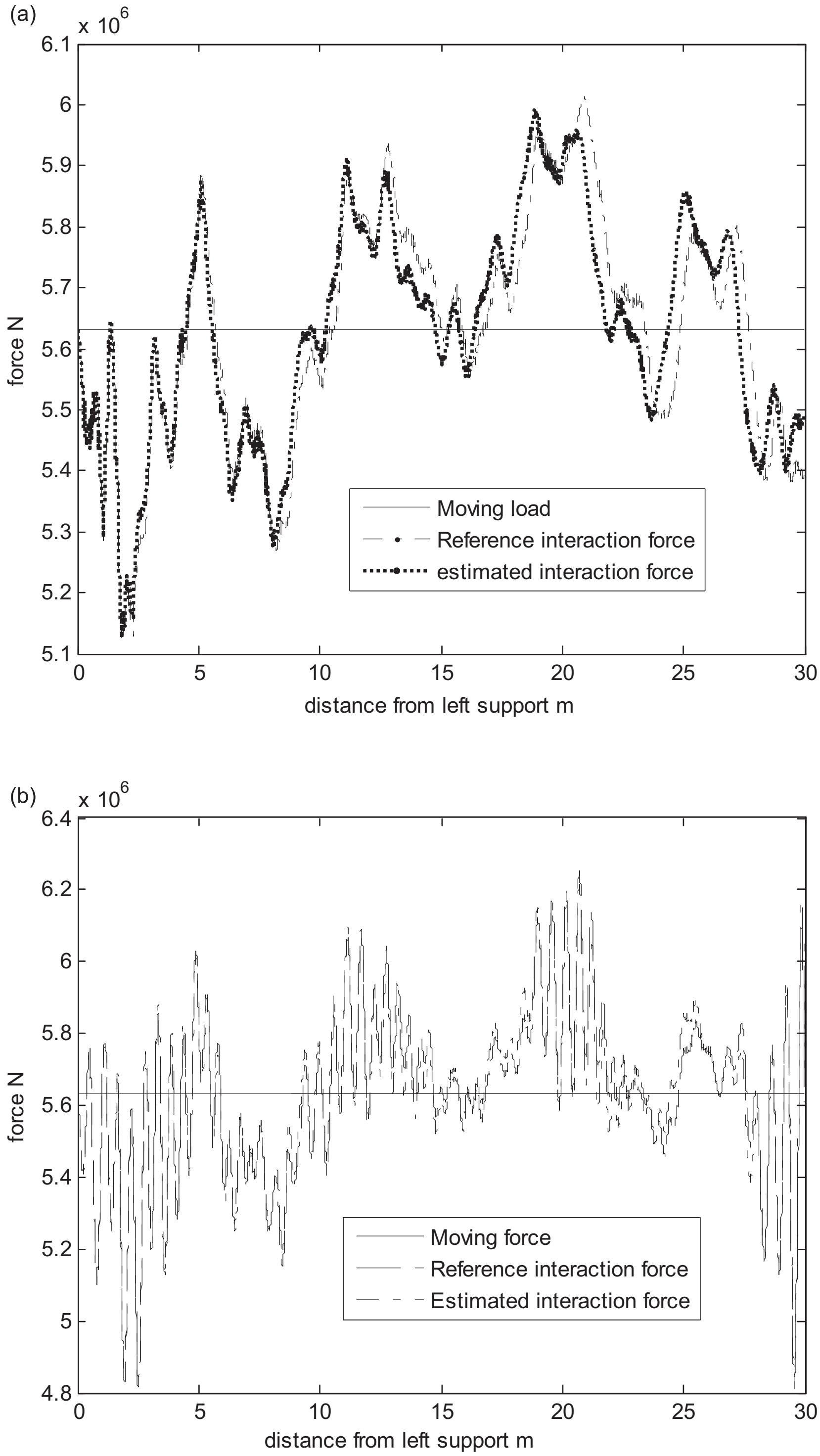

Fig. 8. Response of interaction force for the beam oscillator system considered in Section 5.2 (Fig. 1): (a) study1; (b) study3.

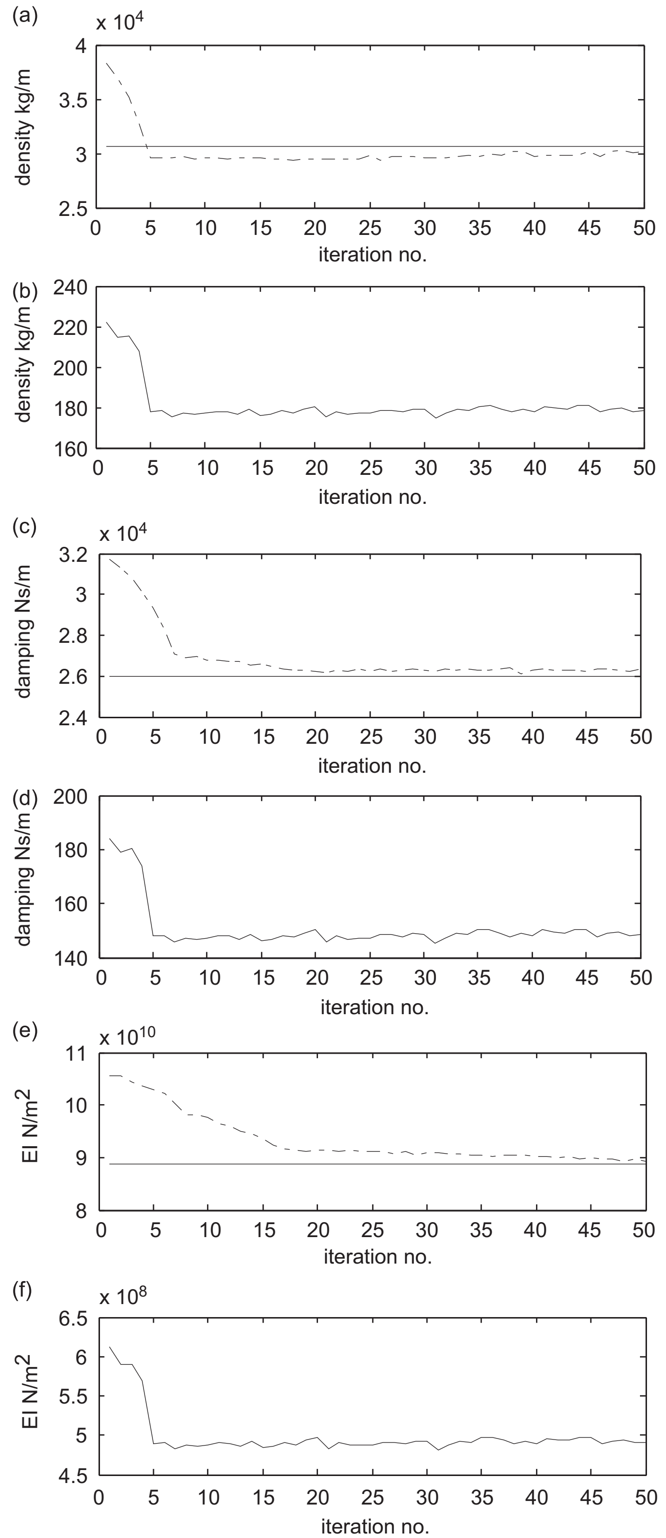

Fig. 9. Example in Section 5.2; Study 1; Estimation of flexural rigidity, damping and mass per unit length of element 4: (a) expected value of the m_4; (b) standard deviation of m_4; (c) expected value of c_4; (d) standard deviation c_4; (e) expected value of EI_3; (f) standard deviation of EI_3.

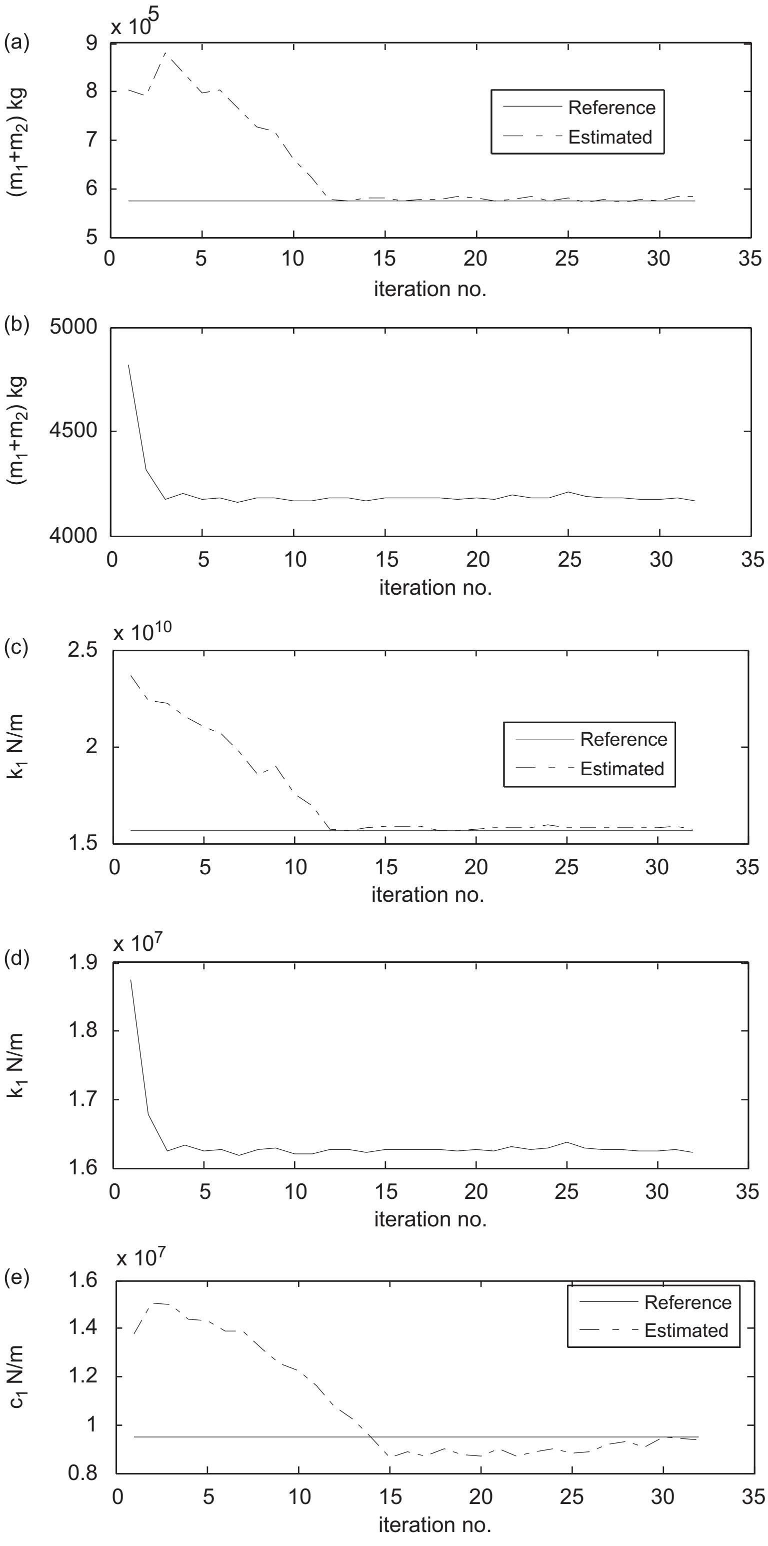

Fig. 10. Example in Section 5.2.3; Estimation of $(m_1+m_2), k_1, c_1, k_{\theta 1}$ and $k_{\theta 2}$: (a) expected value of the (m_1+m_2); (b) standard deviation of (m_1+m_2); (c) expected value of k_1; (d) standard deviation k_1; (e) expected value of c_1; (f) standard deviation of c_1; (g) expected value of $k_{\theta 1}$; (h) standard deviation of $k_{\theta 1}$; (i) expected value of $k_{\theta 2}$; (j) standard deviation of $k_{\theta 2}$.

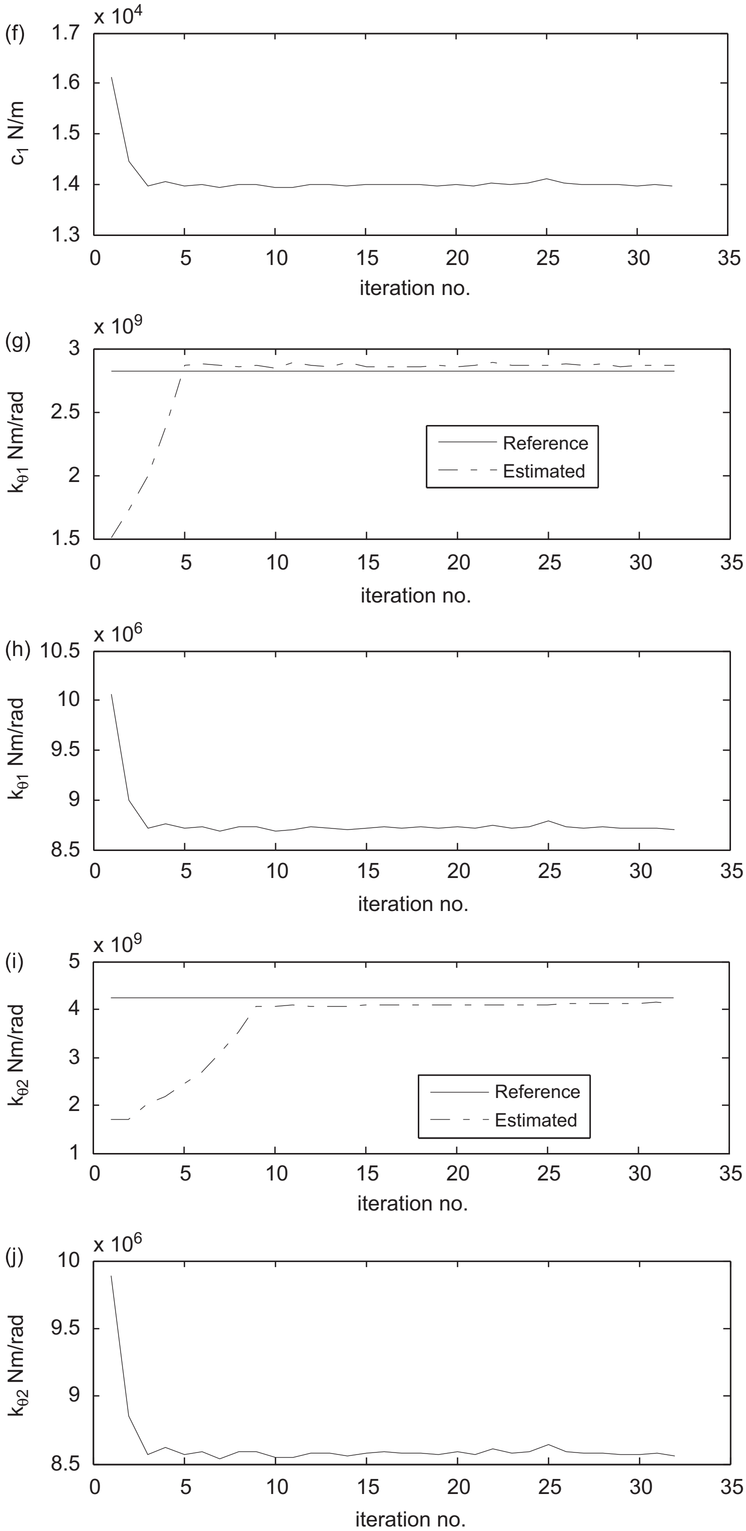

Fig. 10. (*Continued*)

measurement. Similar plot for the force transmitted by the oscillator to the beam are shown in Fig. 8(a). Clearly, results from Study 1 (Figs. 6(a) and 8(a)) show acceptable comparison with the reference trajectory (that is, the response time history obtained from known system model from which the synthetic measurements are numerically simulated). The evolution of mean and standard deviation of properties of beam element 4 as a function of global iteration number is shown in Fig. 9 and the estimates are observed to converge in about 25 iterations.

5.2.2. Studies 3 and 4

The assumptions made here are as in the previous section except that we now introduce a guideway unevenness assumed to be of the form $r(x)=r_0[1-\cos\lambda x]$. In the numerical work we assume $\lambda=10\pi/3$ and select r_0 such that if the vehicle were to run on a rough (rigid) road with profile given by $r(x)$ with a velocity of 30 m/s, it would experience a peak

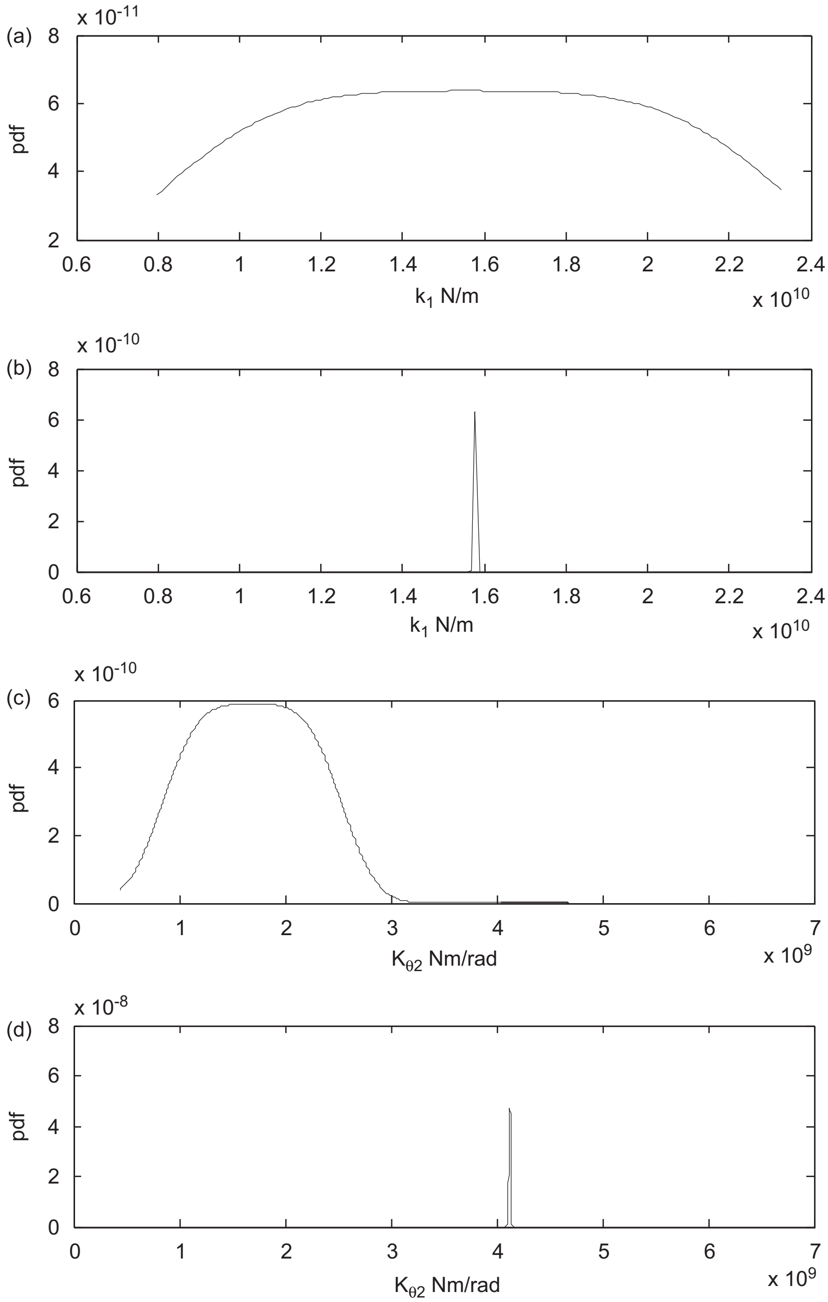

Fig. 11. Example in Section 5.2.3: (a) pdf of the initial guess on k_1; (b) pdf of k_1 at the end of identificaiton proceudre; (c) pdf of the initial guess on $k_{\theta 2}$; (d) pdf of $k_{\theta 2}$ at the end of identificaiton proceudre.

support acceleration of about 0.8 m/s^2. We perform system identification using a model that includes VSI (Study 3) and a model that ignores the interaction (Study 4). The results from Study-3 are shown in Table 1 and it can be seen that the maximum error of 3.34% in stiffness parameter estimation, 3.74% for damping parameters and 4.23% for mass parameters. Fig. 7(c) shows a comparison of the estimated mid-span deflection, using identified system parameters, with reference and measured trajectories. Similar results on interaction force are shown in Fig. 8(b). Based on these results, it may be concluded that the identification procedure has performed satisfactorily in Study3. The identification procedure in Study 4 yielded very poor results with errors in estimating system parameters being as high as 52.3%. Again, this is to be expected, since, a moving force model, not only ignores effects of VSI, but also, cannot take into account effect of guideway unevenness. The details of these results are not reported herein.

5.2.3. Study 5

Here we consider the parameters $(m_1+m_2), k_1, c_1, k_{\theta 1}$ and $k_{\theta 2}$ to be the quantities to be estimated with all other system characteristics taken to be known. Thus, this example represents a case in which parameters of both the moving oscillator and beam structure are identified. Measurements are assumed to be made on beam transverse displacements and bending strains at midpoint of the elements 1, 2, 3 and 5, and rotations at the two ends. The guideway assumed to be uneven with $r(x)$ as specified in the previous section. Fig. 10 shows the evolution of posterior mean and standard deviation of system parameters as a function of global iteration number. The estimates are observed to converge in about 15–20 iterations. A comparison of initial and final pdf-s of two of the system parameters (k_1 and k_{θ_2}) is depicted in Fig. 11. It may be observed that the estimated pdf for k_{θ_2} is dramatically different from the initial guess. Table 2 summarizes the results of system identification and it is observed that the maximum error of identification is about 2.4% which occurs in the estimation of the parameter k_{θ_2}. A comparison of reference values of oscillator weight and the force transferred to the beam

Table 2
Example in Section 5.2.3; summary of results of system identification.

Parameter	Mean of the initial guess	Reference value	Estimated mean	Estimated standard deviation	% Error
(m_1+m_2), kg	8.038×10^5	5.742×10^5	5.825×10^5	4.168×10^3	1.44
k_1, N/m	2.367×10^{10}	1.568×10^{10}	1.574×10^{10}	1.622×10^7	0.38
c_1, Ns/m	1.379×10^7	9.488×10^6	9.345×10^6	1.396×10^4	1.50
$k_{\theta 1}$, Nm/rad	1.627×10^9	2.817×10^9	2.861×10^9	8.700×10^6	1.58
$k_{\theta 2}$, Nm/rad	1.627×10^9	4.225×10^9	4.124×10^9	8.558×10^6	2.39

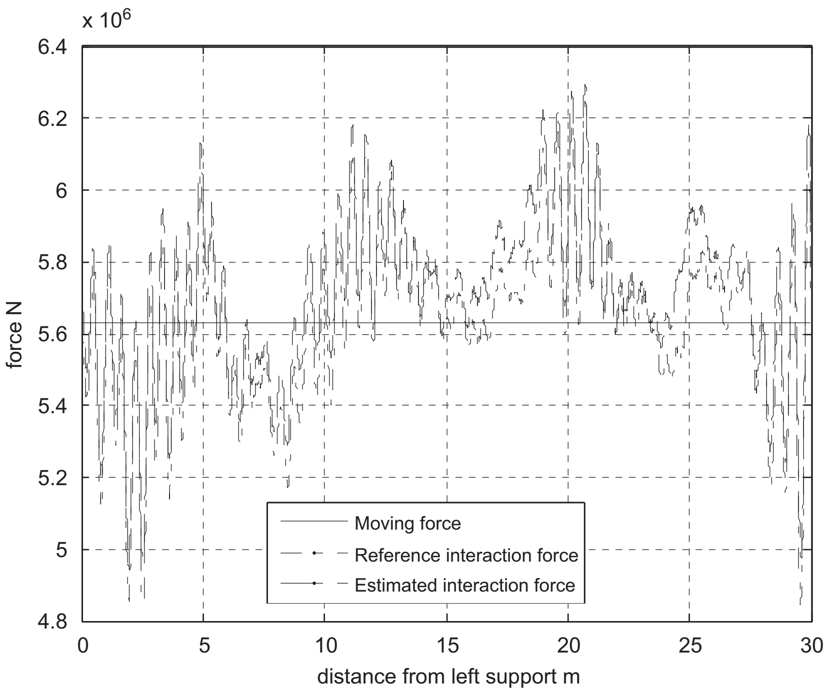

Fig. 12. Example in Section 5.2.3; Prediction on interaction force for the beam oscillator system.

with the corresponding estimates from the identified system is shown in Fig. 12. Here again the identification procedure may be deemed to have performed satisfactorily.

6. Closing remarks

A strategy based on the application of dynamic state estimation using particle filters has been developed for solving problems of system identification in vehicle–structure interaction problems. The method has capability to identify parameters of both the vehicle and supporting structure characteristics and has potential for application in structural health monitoring bridges and also in problems of load identification in bridge engineering problems. These methods have sound mathematical moorings in the theory of Markov processes and Bayes' theorem. The main result in the theory of dynamic state estimation is the pair of functional recursion relations that the posterior pdf of the system states satisfy. These equations are exact in nature and they form the basis on which approximate solutions can be obtained based on Monte Carlo simulation strategies. The proposed method has wide ranging capabilities that includes ability to handle imperfections in mathematical models, measurement noise and guideway unevenness, application FE models with time varying structural matrices and treatment of spatially incomplete measurements. Illustrative examples are presented with respect to an archetypal system made up of flexibly supported, single span, Euler–Bernoulli beam that is traversed by a SDOF oscillator. The response measurements are assumed to be limited to beam displacements and strains and parameters to be identified includes properties of both the beam and vehicle systems. For well formulated mathematical models, the errors in identification of system parameters are seen to be less than about 5%.

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