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Discussion

# Comments to paper by Spitas "A continuous piecewise internal friction model of hysteresis for use in dynamical simulations (*Journal of Sound and Vibration* 324: 297–316)"

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ABSTRACT

In this discussion paper, it is shown that the criticism given in the paper mentioned in the title and relating to the so-called modified hysteretic model is not justified.


A significant part and objective of the considered paper consists in a demonstration of an inadequacy connected with the modification presented in [2] of the well-known Reid's model. However, the criticism given in [1] is not justified. The above-mentioned modification is shown in Fig. 1 where the part with parameters $k_1$ and $\eta_1$ corresponds to the hysteretic model (Reid's model) determined by the following equation:

$$F = k_1\left(x_1 + \eta_1 |x_1| \frac{\dot{x}_1}{|\dot{x}_1|}\right) \tag{1}$$

In the case of loading (signs of $x_1$ and $\dot{x}_1$ coincide), the model behaves as a spring with the stiffness $k_1(1+\eta_1)$, and when unloading the equivalent spring has the stiffness $k_1(1-\eta_1)$. The behavior of the hysteretic model for cyclic deforming is shown in Fig. 2. Similarly to the Coulomb friction model, at the first stage of the force decrease (after its increase) the displacement remains constant (the model is locked).

Introducing an additional spring with the stiffness $k_0$ eliminates discontinuities in the force, which are inherent in the model (1) (it is locked at the initial stage of unloading or reloading). The behavior of the system in Fig. 1 corresponds to the hysteretic loop shown in Fig. 3 with the following values of stiffnesses.

The following explanation is taken from [2]: "If a point $(x; F)$ lies at a moment on the line with angle coefficient $k_\alpha$ or $k_\beta$ and the velocity changes its sign passing the zero value, then the point begins to move along the line with angle coefficient $k_0$; and the value $F$ remains continuous. For time intervals in which the velocity does not change its sign, there are linear relations between $\Delta x$ and $\Delta F$ with coefficients $k_\alpha$, $k_\beta$ or $k_0$." In fact, when the hysteretic part of the system in Fig. 1 is in motion (velocity $\dot{x}_1 \neq 0$) then we deal actually with two springs connected serially: the first with the stiffness $k_0$ and the second with the stiffness $k_1(1+\eta_1)$ or $k_1(1-\eta_1)$ accordingly to Eq. (1) and dependently on signs of $x_1$ and $\dot{x}_1$. This explains the expressions for $k_\alpha$ and $k_\beta$. If the velocity changes its sign passing the zero value, then the hysteretic part of the system is

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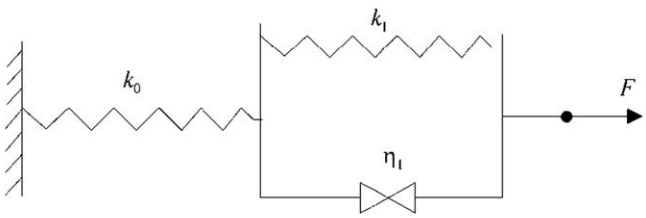


**Fig. 1.** Mechanical system leading to a modified hysteretic model.

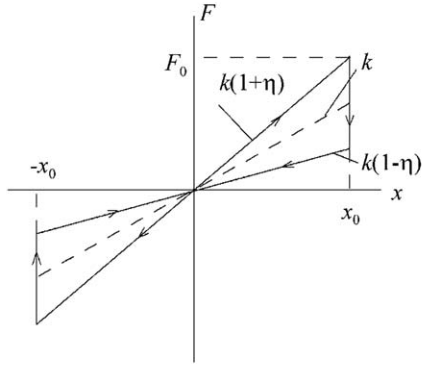


**Fig. 2.** Relation between force and displacement for the hysteretic model.

$$k_\alpha = \left[\frac{1}{k_0} + \frac{1}{k_1(1+\eta_1)}\right]^{-1}, \quad k_\beta = \left[\frac{1}{k_0} + \frac{1}{k_1(1-\eta_1)}\right]^{-1}, \quad k_a = 0.5(k_\alpha + k_\beta).$$

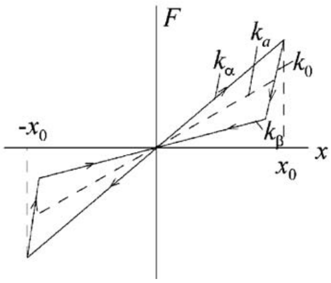


**Fig. 3.** Relation between force and displacement for the modified hysteretic model.

locked initially, the force changes in a time interval without motion in the hysteretic part ($\Delta x_1$=0), and the whole stiffness is represented by the stiffness $k_0$. This explains the lines in Fig. 3 with angle coefficients $k_0$. Thus, Fig. 3 fully corresponds to the model shown in Fig. 1, contrary to the conclusions made in [1]. Knowing the history of deformations, we can construct without difficulties the history of the force using the above description. The main mistake of the paper [1] consists in omitting the possibility for the hysteretic element in Fig. 1 to be locked. It is stated (p. 302 before (17)): "It is easy to observe that all 'springs' of the model are in phase…". However, they are not, when the hysteretic part is locked, and the formulas (18, 20) are correct only for those periods in which the hysteretic part is deforming (the formula (20) simply leads to stiffnesses $k_\alpha$ or $k_\beta$ presented above). One can infer from Fig. 1 at a glance that any force variation necessarily leads to deforming the left spring, and the whole displacement cannot remain constant (contrary to the hysteretic model). The conclusion made in [1] that the both considered models (hysteretic and modified hysteretic) are equivalent, is invalid.

## References

[1] C.H. Spitas, A continuous piecewise internal friction model of hysteresis for use in dynamical simulations, *Journal of Sound and Vibration* 324 (2009) 297–316.
[2] G. Muravskii, On frequency independent damping, *Journal of Sound and Vibration* 274 (2004) 653–668.