



Rapid Communications

Enhanced passive targeted energy transfer in strongly nonlinear mechanical oscillators

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ABSTRACT

Single-degree-of-freedom (SDOF) nonlinear energy sinks (NESs) can efficiently mitigate broadband disturbances applied to primary linear systems by means of passive targeted energy transfer (TET), but for a rather limited range of energies. We demonstrate that the TET can be significantly enhanced for broad range of energies by introducing additional internal degrees of freedom to the NES in a highly asymmetric fashion. Numerical simulations demonstrate that the enhanced performance is due to a positive synergistic effect of the internal degrees of freedom of the proposed NES with highly asymmetric stiffnesses.

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1. Introduction

The nonlinear energy sink (NES) has been defined as an essentially nonlinear single-degree-of-freedom (SDOF) structural element with relatively small mass and weak dissipation, attached to a primary structure via essentially nonlinear coupling [1–3]. If the primary structure is excited by a shock whose energy is above a certain critical threshold, the NES can act as broadband passive and adaptive controller by absorbing vibration energy from the primary structure in an almost irreversible manner. This process is referred to as passive targeted energy transfer (TET) [4,5]. TET normally occurs via transient resonance captures, made possible by the essential (nonlinearizable) stiffness nonlinearity of the NES which prevents a preferential resonance frequency. In previous works it has been shown theoretically, numerically and experimentally that the NES can efficiently protect a primary structure against impulsive excitations [6], harmonic (narrowband) loads [7,8], and seismic excitations [9], and it has also been applied to passively suppressing aeroelastic instabilities [10,11] and drill-string instabilities [12]. A detailed discussion of the concepts of NES, TET and related issues is presented in a recent monograph [12].

The efficiency of vibration protection and shock mitigation through the use of SDOF NESs can exceed that of linear tuned mass dampers which are narrowband devices; besides, due to their essential stiffness nonlinearities, the NESs can operate over broad frequency ranges, acting, in essence, as passive adaptive boundary controllers [12]. Still, for the same reason, the SDOF NES can be rather sensitive to the amplitude of the external forcing and the overall energy level of the dynamics. More precisely, high TET efficiency of the SDOF NES is achieved only in relatively narrow ranges of the external forcing amplitudes [12].

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To overcome this deficiency, multiple degree-of-freedom (MDOF) NESs incorporating several degrees of freedom were suggested [13–15]. Enhancement of TET efficiency has been reported in these designs, but the multiple essential stiffnesses employed prevented a systematic analytical study of the governing dynamics and of TET efficiency. In this communication we propose the simplest possible MDOF NES in the form of a two-DOF NES with small total mass which, as we show, is capable of broadening the range of efficiency of a passive TET by an order of magnitude. In accordance with previous works [13–15], the nonlinear stiffnesses of the internal subsystems of the NES are drastically different, introducing intentional strong asymmetry in the nonlinear stiffness distribution of the proposed two-DOF NES design [13,14]. In contrast to previous MDOF NES designs, however, there is direct coupling of the proposed NES with the primary structure through a strongly nonlinear stiffness element. A systematic computational study of TET efficiency of the proposed NES is performed, and compared to the corresponding efficiency of the SDOF NES.

2. Two-DOF NES with strong stiffness asymmetry

We consider a SDOF primary linear oscillator coupled to a two-DOF NES by means of an essentially nonlinear (nonlinearizable) cubic stiffness in parallel with a linear viscous damper. The NES is composed of two equal masses connected by an essentially nonlinear cubic stiffness in parallel to a linear viscous damper; moreover, strong asymmetry of the two nonlinear stiffnesses is assumed, with the nonlinear stiffness connecting the primary oscillator with the NES considered to be stiff, whereas the second nonlinear stiffness coupling the two internal masses of the NES to be soft. The governing equations of motion can be expressed in non-dimensional form as

$$\begin{aligned} \ddot{u} + \varepsilon\lambda_1\dot{u} + u + \varepsilon\lambda(\dot{u} - \dot{v}_1) + \frac{4\varepsilon}{3}(u - v_1)^3 &= 0 \\ \varepsilon\ddot{v}_1 - \varepsilon\lambda(\dot{u} - \dot{v}_1) - \frac{4\varepsilon}{3}(u - v_1)^3 + \varepsilon\lambda(\dot{v}_1 - \dot{v}_2) + \frac{4\varepsilon k}{3}(v_1 - v_2)^3 &= 0 \\ \varepsilon\ddot{v}_2 - \varepsilon\lambda(\dot{v}_1 - \dot{v}_2) - \frac{4\varepsilon k}{3}(v_1 - v_2)^3 &= 0 \end{aligned} \quad (1)$$

where $0 < \varepsilon \ll 1$ scales each of the small masses of the NES, the weak viscous damping elements, and the essentially nonlinear stiffness coefficients. In addition, u denotes the displacement of the primary oscillator, v_i , $i=1,2$ the displacements of two internal masses of the MDOF NES, $\varepsilon\lambda$ the viscous damping coefficient in both elements of the NES, $\varepsilon\lambda_1$ is the damping at the primary mass and k the coefficient characterizing the stiffness asymmetry of the NES. It follows that system (1) possesses four non-dimensional parameters, namely ε , λ , λ_1 and k ; in addition, except for the small parameter ε , all parameters are assumed to be $O(1)$ quantities. We will examine the response of system (1) when an impulse of magnitude A is applied to the linear oscillator, with the system being initially at rest. Scheme of the system under consideration is presented in Fig. 1.

3. Numerical results and discussion

In order to assess the efficiency of the two-DOF NES design, we will set in the beginning $\lambda_1=0$ and compare its performance to the performance of the corresponding SDOF NES with normalized mass equal to 2ε , i.e., equal to the total normalized mass of the proposed two-DOF NES, so that no mass effect enters into the comparison of performance. This standard [12] system attached to the same primary linear oscillator is described by the equations

$$\begin{aligned} \ddot{u} + u + 2\varepsilon\lambda(\dot{u} - \dot{v}) + \frac{8\varepsilon}{3}(u - v)^3 &= 0 \\ 2\varepsilon\ddot{v} + 2\varepsilon\lambda(\dot{v} - \dot{u}) + \frac{8\varepsilon}{3}(v - u)^3 &= 0 \end{aligned} \quad (2)$$

where v is the displacement of the NES. Note that both damping and stiffness terms have been modified as well, since they are assumed to scale linearly with the NES mass.

We will study passive mitigation of the impulsive responses of systems (1) and (2) subject to an impulsive excitation $F(t)=A\delta(t)$ applied to the linear primary oscillator with the system being initially at rest. This is equivalent to initiating the dynamics of (1) and (2) with initial condition $\dot{u}(0^+)=A$, and all other initial conditions zero.

For optimal mitigation of the impulsive responses of these systems by means of passive targeted energy transfer (TET) to their respective NESs it will be necessary to dissipate the overall energy induced by the applied impulse as effectively and as fast as possible. Hence, we will be comparing the rate of overall energy dissipation in systems (1) and (2) subject to

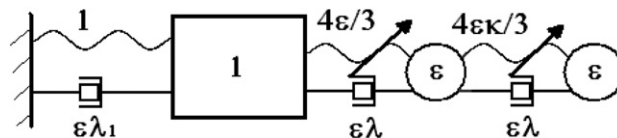


Fig. 1. Scheme of the system under consideration with two-DOF NES.

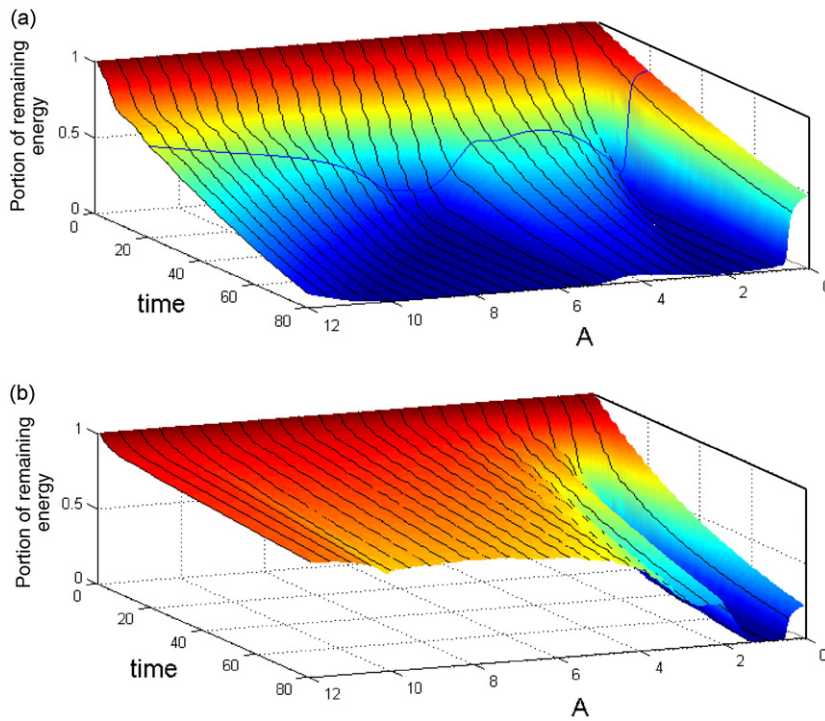


Fig. 2. Portion of energy remaining in a system versus time elapsed after the application of an impulse of magnitude A at the linear oscillator, for parameters $\varepsilon=0.05$, $\lambda=0.2$, $k=0.01$, $\lambda_1=0$ and varying values of A : (a) system (1) with two-DOF NES and (b) system (2) with SDOF NES.

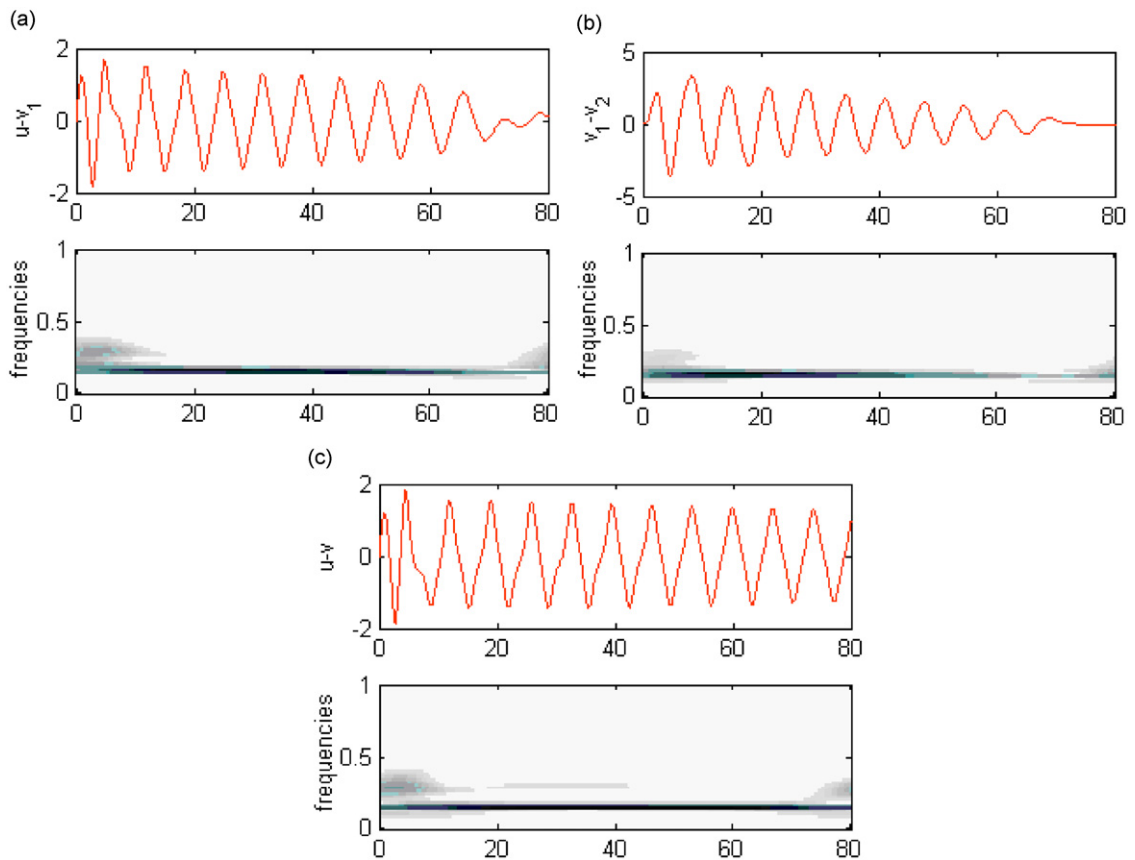


Fig. 3. Transient responses and corresponding wavelet transform spectra for parameters $\varepsilon=0.05$, $\lambda=0.2$, $k=0.01$, $\lambda_1=0$ and $A=2.0$: (a) $u-v_1$ for system (1), (b) v_1-v_2 for system (1), and (c) $u-v$ for system (2).

identical applied impulses. Such a study of NES efficiency differs from alternative measures employed in previous works [12], where the portion of total energy eventually dissipated only by the NES was evaluated, but no consideration of the rate (i.e., the time scale) of energy dissipation was made. Clearly, such a measure would be irrelevant for systems (1) and (2), since no damping is assumed for the primary system. Besides, from an engineering viewpoint it is important to rapidly dampen out the overall energy induced by the applied shock; indeed, when only a small portion of the initial energy remains in the system, its distribution among the substructures is nearly irrelevant.

In Figs. 2a and b we present the portion of total energy remaining in the systems (1) and (2), respectively, versus time and magnitudes of applied impulses, keeping the asymmetry parameter fixed at $k=0.01$, $\varepsilon=0.05$, $\lambda=0.2$, $\lambda_1=0$. We note that, whereas for very low applied impulses the SDOF NES proves to be marginally more effective, for higher impulses the two-DOF NES proves to be significantly more effective in rapidly dissipating the energy of the system and for a broad range of applied impulses. Indeed, for the two-DOF NES the range of effective energy dissipation proves to be robust for the range of applied impulses $0.6 < A < 7$, contrary to the SDOF NES which is effective only for a narrow range of applied impulses close to $A \approx 0.6$. Hence, we note an essential improvement in the robustness of passive mitigation when using the proposed highly asymmetric two-DOF NES. These results indicate that by splitting the (relatively small) mass of the SDOF NES and introducing an additional soft nonlinear spring in the NES design, one can obtain a much more effective and robust shock mitigation for rather broad range of the initial excitation amplitudes.

It is of importance to study in more detail the frequency content of the transient dynamic responses of the two systems (1) and (2) in order to get more insight into the dynamics that govern energy dissipation in the two cases. In addition, such an analysis should provide us with the reasons for the more effective TET in the case of the two-DOF NES. To this end we will employ wavelet transform spectra [16], which will provide us with the temporal evolution of the basic harmonic components of the simulated transient nonlinear responses (in contrast to the classical Fourier transform which provides only a ‘static’ description of the harmonic content of the time series). As discussed and demonstrated in numerous applications in [12] the wavelet transform is a powerful signal processing tool for analyzing the transient dynamics of nonlinear systems.

In Fig. 3—we provide the transient responses and the corresponding wavelet spectra of systems (1) and (2) possessing the two different NES configurations, for different magnitudes of the applied impulses. Considering the numerical simulations depicted in Fig. 3 for impulse magnitude equal to $A=2.0$ (i.e., close to the threshold $A^* \approx 0.6$), we note that the

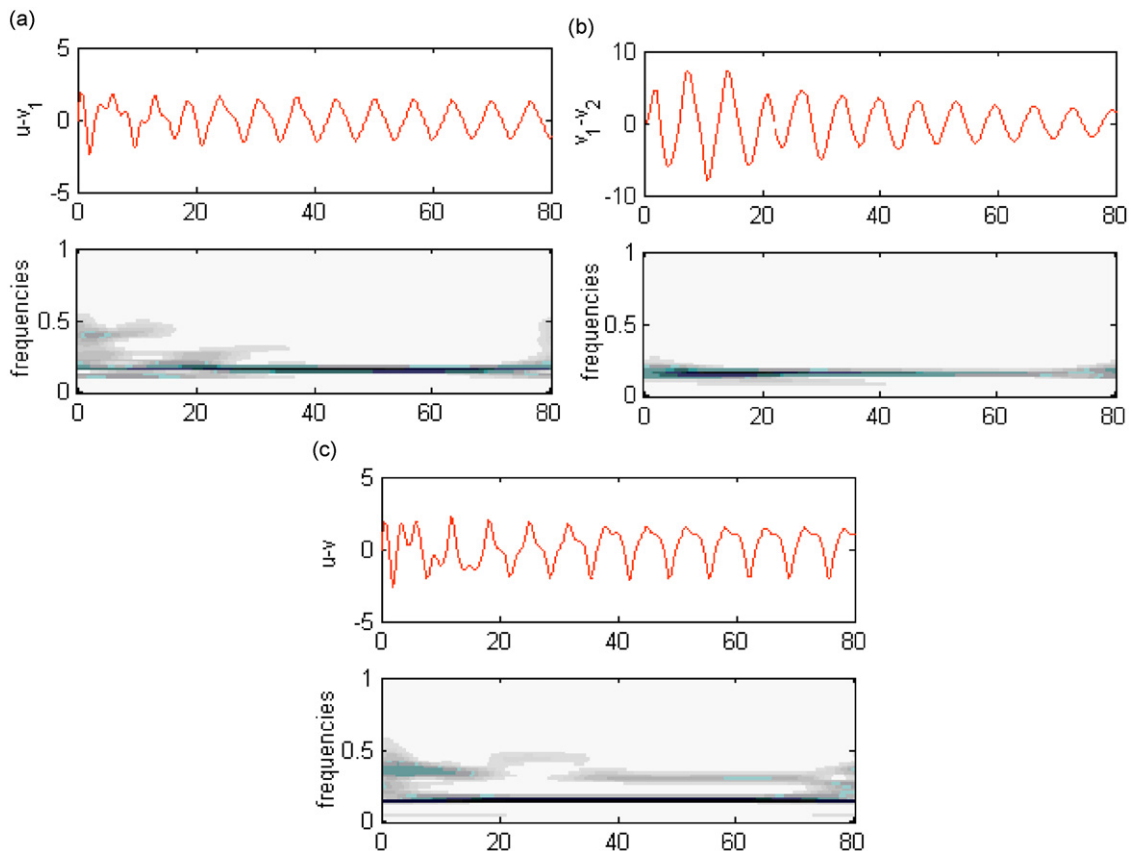


Fig. 4. Transient responses and corresponding wavelet transform spectra for parameters $\varepsilon=0.05$, $\lambda=0.2$, $k=0.01$, $\lambda_1=0$ and $A=4.0$: (a) $u-v_1$ for system (1), (b) v_1-v_2 for system (1), and (c) $u-v$ for system (2).

dynamics in both systems is governed by 1:1 resonance. This is evidenced by the appearance of a single dominant harmonic in the wavelet spectra of the relative responses of both systems, at frequency close to the natural frequency of the linear primary oscillator. It follows that the transient dynamics of both the SDOF and two-DOF NES 'lock' into 1:1 transient resonance capture with the dynamics of the linear oscillator, so the transient dynamics of both systems is qualitatively similar.

Proceeding now to the results depicted in Figs. 4 and 5 that correspond to stronger applied impulses, we note a qualitative difference in the dynamics of systems (1) and (2), which explains the radically enhanced targeted energy transfer in the two-DOF NES. Specifically, sustained 3:1 transient resonance capture is noted in the dynamics of the SDOF NES, which settles into a slowly decaying oscillation containing two harmonics: a harmonic at the natural frequency of the linear oscillator and a higher harmonic equal to three times that frequency. This is evidenced by the wavelet spectra depicted in Figs. 4c and 5c. A similar early 3:1 transient resonance capture is similarly noted in the dynamics of the stiffer subsystem of the two-DOF NES (i.e., in the part of the NES that is directly coupled to the linear oscillator—cf. Figs. 4a and 5a), but this is followed by a transition to 1:1 transient resonance capture as time increases. The most important feature, however, that clearly differentiates the dynamics of the two NESs concerns the dynamics of the softer subsystem of the two-DOF NES (i.e., the part of the NES that is further from the linear oscillator—cf. Figs. 4b and 5b) which executes large-amplitude oscillations with a frequency close to the natural frequency of the linear primary oscillator. Hence, the softer subsystem of the two-DOF NES engages in 1:1 transient resonance capture with the stiffer subsystem and the linear oscillator right from the beginning of the motion, which results in high-amplitude oscillations. It is these large-amplitude oscillations under condition of 1:1 transient resonance capture that cause the rapid and efficient dissipation of shock energy by the viscous damper of the softer subsystem of the two-DOF NES, and radically enhances TET in this system. It follows that the asymmetric (stiff-soft) design of the two-DOF NES permits the simultaneous excitation of both 3:1 and 1:1 transient resonance captures in its stiff and soft components which, in turn, broadens the range of effective TET for a wider range of applied impulses compared to the SDOF NES.

Finally, in Fig. 6 we present the energy remaining in both systems after time equal to $1/\varepsilon$ for different values of impulse magnitude A , asymmetry coefficient k and damping coefficient λ . This plot provides a measure of the time scale of energy dissipation by the two NESs, and clearly demonstrates the enhanced effectiveness of the two-DOF NES.

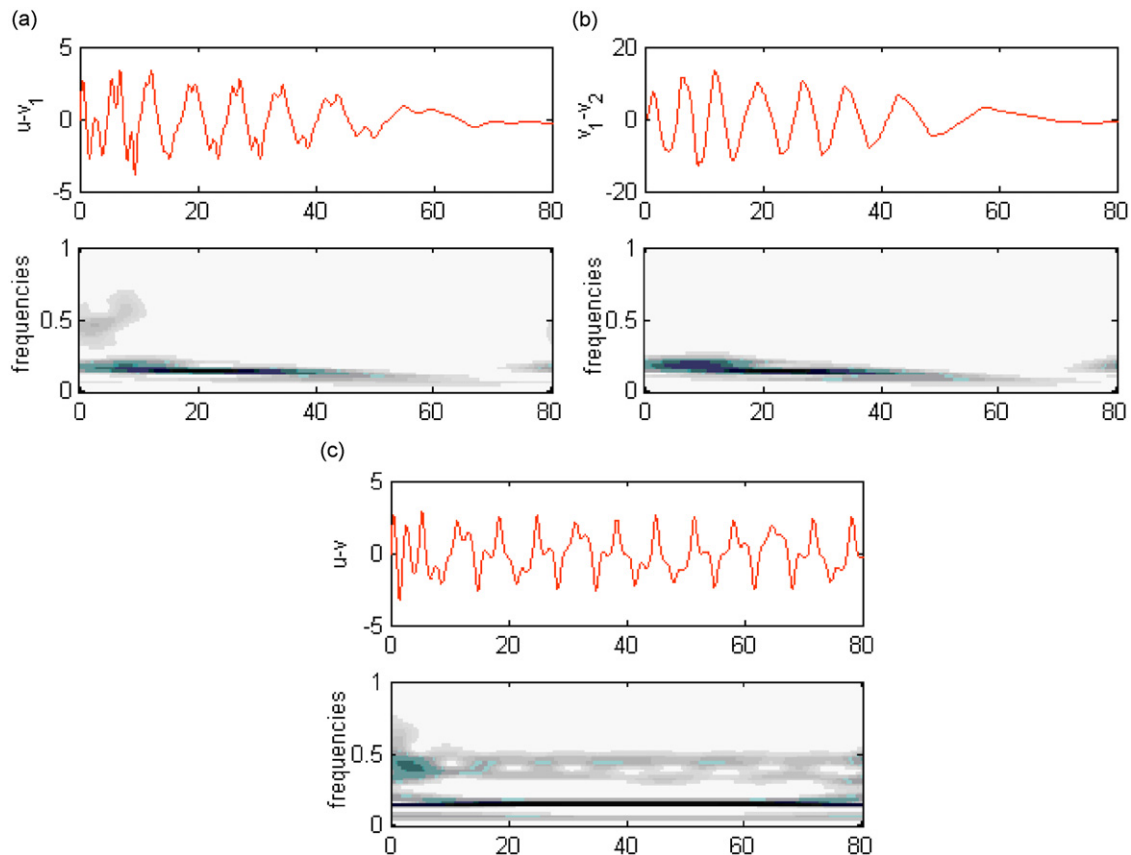


Fig. 5. Transient responses and corresponding wavelet transform spectra for parameters $\varepsilon=0.05$, $\lambda=0.2$, $k=0.01$, $\lambda_1=0$ and $A=7.0$: (a) $u-v_1$ for system (1), (b) v_1-v_2 for system (1), and (c) $u-v$ for system (2).

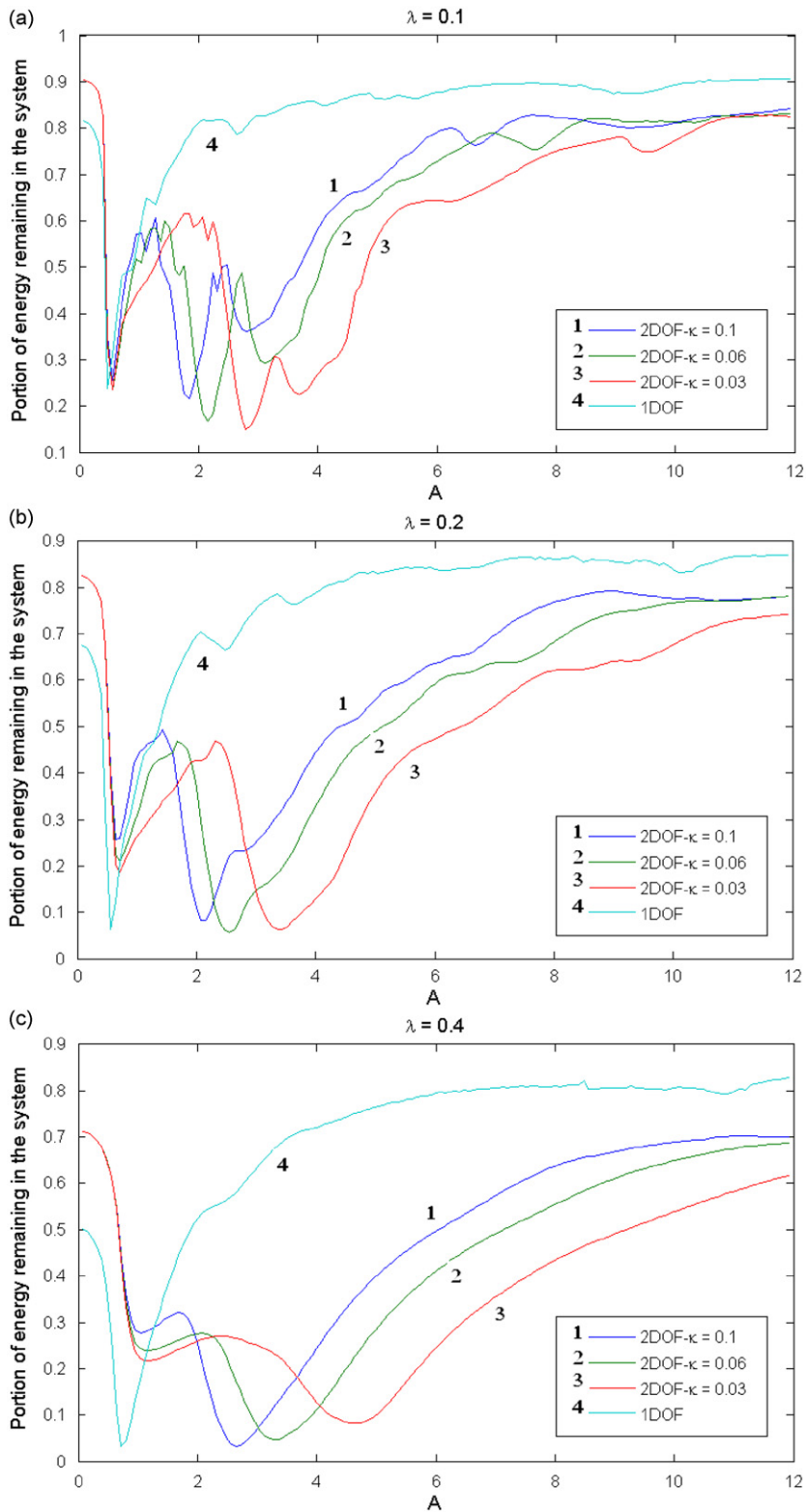


Fig. 6. Portion of energy remaining in the system after time $1/\varepsilon$ for systems (1) and (2) and for different values of damping (a)–(c) and asymmetry parameter k over varying magnitude of the impulse A , $\varepsilon=0.05$, $\lambda_1=0$.

It turns out that the choice of the appropriate values for the asymmetry parameter k for the NES is crucial for optimizing the NES performance in terms of enhanced TET. It is well known [12] that the ‘activation’ of a regular SDOF NES occurs when the amplitude of the impulse A is close to a certain critical value [12]. If for the SDOF NES with pure cubic stiffness nonlinearity u^3 this critical impulse threshold is denoted by A^* , then one can speculate that for a cubic stiffness ku^3 one should get a similar critical threshold equal to $A^*/k^{1/2}$. So, on a qualitative level, if two SDOF NESs with pure cubic nonlinearities u^3 and ku^3 are attached to a primary SDOF linear oscillator, one should expect that the first NES will be optimal at the level of impulse $A \approx A^*$, whereas the second at $A \approx A^*/k^{1/2}$. However, if a two-DOF NES is considered instead, combining both of these SDOF NESs in series, one might expect that with positive synergy between these two internal degrees of freedom the MDOF NES can be effective for a range of intensities of the applied impulses; intuitively one would expect this range to be close to $(A^*, A^*/k^{1/2})$. Indeed, we will show that by employing such a two-DOF NES one should be able to extend radically the range of effective TET to an entire range of applied impulses which matches the above range which was derived following simple scaling arguments.

We note that the composite two-DOF NES can overcome one of the major shortcomings of the traditional SDOF NES: namely, that its TET efficiency is confined to only a narrow range of energies. Of course, the above qualitative treatment based on scaling arguments is supported by numeric simulations but is not rigorous and further detailed analysis is desirable.

It should be mentioned that in all numeric experiments presented above the value of λ_1 was set to zero. Qualitatively, similar results are obtained when this coefficient is varied in the diapason $\lambda_1 \in [0, 0.5]$.

4. Concluding remarks

We have demonstrated that it is possible to radically enhance the passive TET performance of an NES by adding internal degrees of freedom to it, and introducing strong asymmetry in its strongly nonlinear components. It is interesting to note that this enhanced performance occurs without the need of adding mass compared to the SDOF NES. Quite obviously, this enhancement does not come for free—one has to allow ample oscillations of the NES; besides, the reliable design of the nonlinear springs with highly asymmetric stiffnesses may pose technical challenges. It should be mentioned that no optimization was performed besides variation of the asymmetry parameter k , and even in this case, the possibility of certain tradeoffs in performance was detected. There are other possibilities for TET enhancement, particularly, adjustment of the damping characteristics of the components of the proposed two-DOF NES.

Numerical simulations confirm our qualitative estimations of the range of efficiency of the two-DOF NES based on simple scaling arguments. Still, there is the need for theoretical investigation of the dynamics of the proposed two-DOF NES in order to study the complex dynamics of this highly degenerate system (its linearization possesses a highly degenerate structure indicating the existence of high co-dimension bifurcations), and study the different possible mechanisms for broadband targeted energy transfer through the excitation of simultaneous transient resonance captures in its stiff and soft components. This should provide us with the capacity for predictive design of these systems leading to an optimal and robust TET.

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