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Energy flow model considering near field energy for predictions of acoustic energy in low damping medium

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ABSTRACT

The Acoustic Energy Flow Boundary Element Method (AEFBEM) is developed to predict the acoustic energy density and intensity of an engineering system. Up to now, the acoustic energy flow model has been used only for analysis of high frequencies or radiation noise because of plane wave and far-field assumptions. In this research, a new energy flow governing equation that can consider the near field acoustic energy term and spherical wave characteristics is derived successfully to predict the acoustic energy density and intensity of a system in the medium-to-high frequency range. A near field term of acoustic energy in spherical coordinate is added to the relationship between energy density and energy flow. But with the far-field assumption, this term vanishes, so the relationship between energy density and energy flow becomes the same as that of the plane wave. By considering the near field energy term without far-field assumption, the energy density at medium frequencies can be estimated. However, the governing equation has to be numerically manipulated for use in the analysis of complex structures; therefore, the Boundary Element Method (BEM) is implemented. AEFBEM is a numerical analysis method formulated by applying the boundary element method to an acoustic energy flow governing equation. It is very powerful in predicting the acoustic energy density and intensity of complex structures in medium-to-high frequency ranges, and can analyze interior noise and radiating sound. To verify its validity, several numerical results are provided. BEM and AEFBEM were compared with respect to energy density, and the results from both methods were similar.

1. Introduction

Engineers in industries such as automotive, shipbuilding and aerospace have tried to improve the vibro-acoustic comfort and reduce the noise and vibration of structures. Because of much interest in the problems of noise and vibration, the need for their exact prediction is increasing. For engineers, computational analysis is the most popular method used to predict the noise level of structures during the design process because it saves experimental costs and can be used to analyze large or dangerous structures. In addition, the designs of structures can be conveniently changed and re-analyzed by computer simulation. Therefore, the ability of an analysis method to calculate the noise level of a mechanical system and its spectral distribution at a certain receiver point is very powerful and important in mechanical design practice.

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Noise prediction of complex structures can be carried out in three ranges of low, medium and high frequencies. The noise level of a mechanical system is analyzed in many ways. Generally at low frequencies, the conventional Finite Element Method (FEM) and Boundary Element Method (BEM) are widely used as analytical noise and vibration simulation tools. However, these methods have critical disadvantages in medium-to-high frequency ranges since unreasonably fine mesh size or higher order shape functions are required as the frequency increases. For a boundary element analysis, at least six elements should be included in one wavelength to represent waves similar to sine wave. Analogous to BEM, the FE-element has to be made very small at high frequencies because the shape function is frequency-independent. Such an excessive number of elements induces longer operation time and high cost as well as considerable numerical errors due to so many repeated computations. For this reason, FEM and BEM are considered inadequate methods to use in medium-to-high frequency ranges, especially for a large engineering system. Researchers have tried to develop a new method which can analyze noise and vibration of a system in medium-to-high frequency ranges without increase of numerical costs. Statistical Energy Analysis (SEA), which provides a single averaged energy density with respect to time and space in a subsystem, in contrast to the traditional methods like FEM and BEM that deal with physical quantities at a particular instant and coordinates, has been developed for analysis at high frequencies by Lyon and Dejong [1]. Energy methods like the SEA are not bound to any constraints of the number of elements in medium-to-high frequency ranges. Because of modeling convenience and the advantage of reduced computational time, SEA has been used in various industrial fields for the analysis of noise and vibration of a complex system at high frequencies. However, SEA cannot provide any information about phase and energy flow in the domain of interest because it is based on the restrictive assumption of diffuse vibrational fields. Besides, SEA is a method for high frequency analysis not for medium frequency analysis.

To overcome these weaknesses of the FEM, BEM and SEA for analysis in the medium-to-high frequency ranges, an alternative method has been developed for analysis in the medium-to-high frequency ranges. Energy Flow Analysis (EFA), which is based on an energy equation analogous to the steady-state heat conduction equation, is representative alternative method. EFA, which was first introduced by Belov et al. [2], can be applied to numerical manipulations such as the finite element method and boundary element method, and can be used to obtain information about the spatial distribution of acoustic or vibrational energy densities and energy flows of subsystems in medium-to-high frequency ranges. Nefske and Sung [3] applied a vibrational energy governing equation to the FEM to predict the vibrational response of an Euler–Bernoulli beam, and Wohlever and Bernhard [4] studied additionally about rods and the Euler–Bernoulli beam. Bouthier and Bernhard [5,6] expanded the studies of Wohlever to analyze a membrane, Kirchhoff plate and acoustic cavity. Smith [7] suggested a hybrid method for predicting vibrational response of point loaded plate which could calculate direct field and reverberant field, respectively. Le Bot [8] developed a vibro-acoustic model for analysis of multidimensional systems like plates or acoustic cavities. Park et al. [9] studied the energy flow models of the in-planes waves in isotropic thin plates and the flexural waves in orthotropic thin plates. And an energy flow model of reinforced beam–plate coupled structures was suggested by Seo et al. [10], who developed the software PFADS based on the Energy Flow Finite Element Method (EFFEM). Recently, Park and Hong [11,12] newly derived the energy governing equations of the Timoshenko beam and the Mindlin plate. On the other hand, Lee et al. [13] applied an EFA energy governing equation to the boundary element technique. The Energy Flow Boundary Element Method (EFBEM) is efficient way to analyze noise such as radiation or scattering because BEM offers advantages in the analysis of acoustic fields over FEM.

But the method suggested by Lee et al. has some disadvantages in the analysis of radiation sound or interior noise for low-to-medium frequency ranges because it uses an energy governing equation derived based on the assumption that acoustic waves are plane waves in diffused field. Therefore, it is more effective for analyzing interior noise in the high-frequency range. Thus Kwon [14] researched a new acoustic energy governing equation by considering spherical wave characteristics and by assuming the far-field condition, and implemented an indirect boundary element method to the governing equation. And Wang et al. also suggest Energy Boundary Element Analysis (EBEA) formulation to calculate sound radiation at high frequency from a radiator [15]. Up to now, it has been used mainly for analysis of radiation noise, which makes the far-field assumption be reasonable.

In this paper, the acoustic energy governing equation of Kwon et al. is expanded by including a near field energy term. The derived energy governing equation without assumptions of diffused field and with weak far-field condition is very useful for predicting noise level not only at high frequencies but at medium frequencies. The indirect boundary element method is implemented on the new acoustic energy flow model for the prediction of the energy density of complex

Table 1
values of $(2k^2r^2+1)/2k^2r^2$.

	200 Hz	300 Hz	400 Hz	500 Hz	600 Hz	700 Hz	800 Hz	900 Hz	1000 Hz
0.5 m	1.1490	1.0662	1.0372	1.0238	1.0165	1.0121	1.0093	1.0073	1.0059
1 m	1.0372	1.0165	1.0093	1.0059	1.0041	1.0030	1.0023	1.0018	1.0014
2 m	1.0093	1.0041	1.0023	1.0014	1.0010	1.0007	1.0005	1.0004	1.0003
3 m	1.0041	1.0018	1.0010	1.0006	1.0004	1.0003	1.0002	1.0002	1.0001
4 m	1.0023	1.0010	1.0005	1.0003	1.0002	1.0001	1.0001	1.0001	1.0000
5 m	1.0019	1.0006	1.0003	1.0002	1.0001	1.0001	1.0000	1.0000	1.0000

engineering systems. Finally, to validate the developed acoustic energy governing equation, the results of the developed acoustic energy flow model and conventional BEM are compared for several frequencies.

2. Energy governing equation

When a mechanical system or fluid medium is steady state, incoming energy flow into a subsystem is equal to the sum of internal energy dissipation and outflow of energy through its boundary. This energy conservation concept is derived from the general control volume approach. The energy balancing equation derived by Bouthier and Bernhard [16] is as follows:

$$\nabla \cdot \mathbf{I} + \Pi_{\mathrm{diss}} = \Pi_{\mathrm{in}}, \tag{1}$$

where Π_{diss} is the dissipated power due to the internal damping of the subsystem and Π_{in} is external input power to the subsystem. **I** is the intensity vector which is coming or leaving through the boundaries. For a system which has an internal damping loss factor, the dissipated power is proportional to the time averaged energy and can be expressed as

$$\Pi_{\mathrm{diss}} = \eta\omega\langle e\rangle, \tag{2}$$

where $\langle\,\rangle$ means a time-averaged value and ω is the excitation angular frequency and η is the damping loss factor [17]. Therefore Eq. (1) can be re-written as

$$\nabla \cdot I + \eta\omega\langle e\rangle = \Pi_{\mathrm{in}}. \tag{3}$$

In an acoustic problem, intensity can be expressed in terms of time-averaged energy. The acoustic wave equation is transformed to a Helmholtz equation below with the assumption that it has a harmonic solution:

$$\nabla^2 p + \tilde{k}^2 p = 0, \tag{4}$$

where p is the acoustic pressure and $\tilde{k}$ means the complex acoustic wavenumber $k(1-j\eta/2)$, which includes the damping loss factor η [18]. The general solution in spherical coordinate of the Helmholtz equation for free space or unbounded medium is

$$p(r) = \frac{A}{r}\exp(-jkr). \tag{5}$$

And Eq. (5) is the pressure at distance r from the center of the simple source [19]. The pressure amplitude is proportional to $1/r$ from the source. Using Euler equation, which defines the relation between the acoustic pressure and the acoustic particle velocity, the acoustic particle velocity can be expressed as below [19]:

$$\mathbf{u}(r) = \frac{j}{\rho\omega}\frac{\partial p(r)}{\partial r} = -j\frac{A}{\omega\rho}\frac{1+jkr}{r^2}\exp(-jkr). \tag{6}$$

The particle velocity is radial, and it is in-phase with the pressure at distance r, which is much larger than the wavelength, and it is out of phase near the source. Since the average values of energy density and intensity are more interesting than instantaneous values, the energy density and intensity are time-averaged by averaging over a period [20].

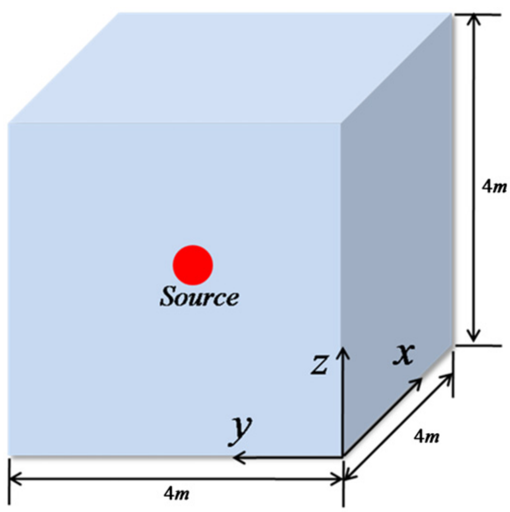

Fig. 1. Three dimensional acoustic cavity model.

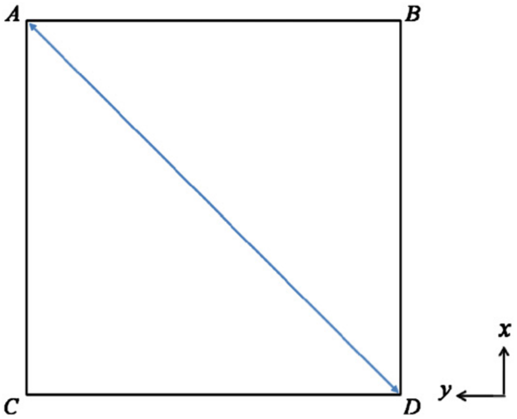

Fig. 2. Centerline of field points (the points of which $X=Y$).

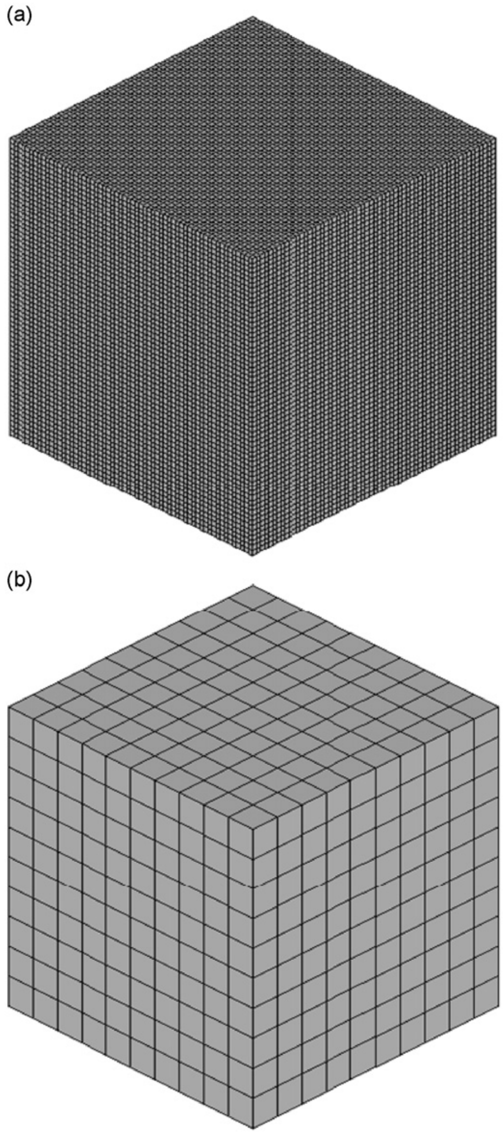

Fig. 3. The cubic acoustic cavity, $L_x \times L_y \times L_z$=4 m × 4 m × 4 m: (a) BE-model of 21,600 elements and (b) AEFBE-model of 600 elements.

From Eqs. (5) and (6), the time averaged acoustic energy and intensity can be obtained as

$$e(r) = \frac{1}{4}\left(\rho \mathbf{u}_r \mathbf{u}_r^* + \frac{1}{\rho {c_g}^2} pp^*\right) = \frac{|A|^2}{2\rho {c_g}^2 r^2} e^{-\eta k r}\left[1 + \frac{1}{2(kr)^2} + \frac{\eta}{2kr} + \frac{\eta^2}{8}\right], \tag{7}$$

and

$$\mathbf{I}_r(r) = \frac{1}{2}\mathrm{Re}(p\mathbf{u}_r^*) = \frac{|A|^2}{2\rho c_g r^2} e^{-\eta k r}\overrightarrow{\mathbf{r}}, \tag{8}$$

where * means the conjugate of a complex number, ρ is the density of an acoustic medium, c_g is the group velocity and $\overrightarrow{\mathbf{r}}$ indicate direction vector, the subscript r in intensity and particle velocity vector denotes direction vector component of intensity and particle velocity, respectively [19]. If the damping loss factor of the acoustic medium is very low ($\eta \ll 1$), the last two terms in Eq. (7) can be vanished and the equation can be simplified as follows:

$$e(r) = \frac{|A|^2}{2\rho c_g^2 r^2} e^{-\eta k r}\left[1 + \frac{1}{2(kr)^2}\right]. \tag{9}$$

When distance r from the source is larger than the wavelength, the relationship between energy and intensity becomes a simple one, like the one of a plane wave (i.e. $\mathbf{I}_r = c_g e$). The sound field in this region is called the far field. In the near field, in region $r \ll \lambda$, the velocity has a large value additionally, which is out of phase with the pressure. And it produces additional energy term, as shown in Eq. (9), that contains the near field term $\frac{1}{2}(kr)^2$. Considering the effect of the near field term, the relationship between the acoustic energy and intensity also include the near field term and it can be

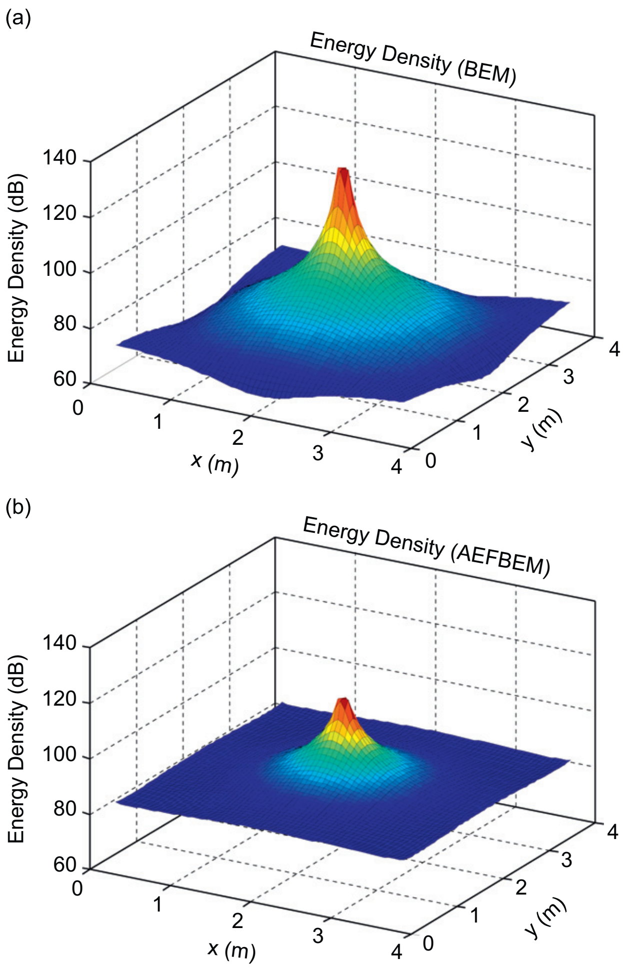

Fig. 4. The energy density level distribution of acoustic cavity when f=63 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: (a) BEM and (b) AEFBEM.

expressed as

$$\mathbf{I}_r = c_g\left(\frac{2k^2r^2}{2k^2r^2+1}\right)\langle e\rangle\,\vec{\mathbf{r}}. \tag{10}$$

Eq. (10) is called the energy transmission relationship between the acoustic energy density and intensity. As mentioned above, without near-field consideration, the energy transmission relationship is the same as that of a plane wave, so the energy governing equation using that relationship cannot express spherical wave characteristics well. Therefore, Eq. (10) is important for formulating the energy governing equation of a spherical wave. To formulate the energy governing equation of an acoustic space, the energy balance equation of Eq. (1) and the energy loss equation of Eq. (2) may be used. By substitution of Eqs. (2) and (10) into Eq. (1), the energy governing equation representing the properties of a spherical wave can be obtained as

$$c_g\nabla\cdot\left\{\left(\frac{2k^2r^2}{2k^2r^2+1}\right)\langle e\rangle\right\}+\eta\omega\langle e\rangle=\Pi_{\text{in}}. \tag{11}$$

Eq. (11) can be expressed in spherical coordinates and rearranged as below:

$$\frac{c_g}{r^2}\left\{\frac{\partial}{\partial r}\left(\left(\frac{2k^2r^2}{2k^2r^2+1}\right)\langle r^2e\rangle+\frac{\eta\omega}{c_g}\langle r^2e\rangle\right)\right\}=\Pi_{\text{in}}, \tag{12}$$

$$\frac{c_g}{r^2}\left\{\left(\frac{2k^2r^2}{2k^2r^2+1}\right)\frac{\partial}{\partial r}\langle r^2e\rangle+\left(\frac{\eta\omega}{c_g}+\frac{4k^2r}{2k^2r^2+1}-\frac{8k^4r^3}{(2k^2r^2+1)^2}\right)\langle r^2e\rangle\right\}=\delta(r), \tag{13}$$

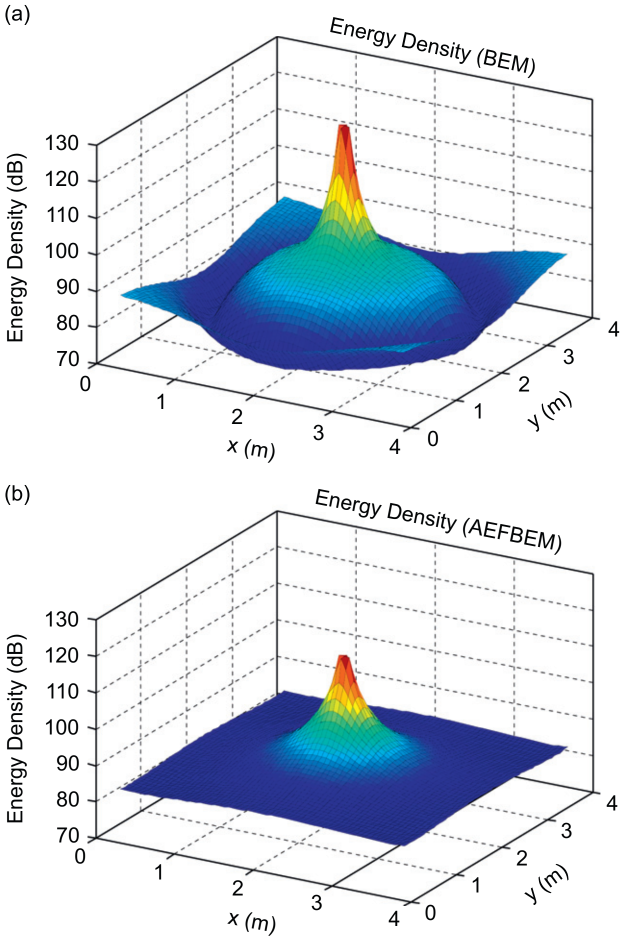

Fig. 5. The energy density level distribution of acoustic cavity when f=125 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: (a) BEM and (b) AEFBEM.

where $\delta(r)$ is the Dirac-delta function. At the non-singular region, Eq. (13) can be simplified as

$$\frac{\partial}{\partial r}\langle r^2 e\rangle + \left(\frac{\eta\omega}{c_g} + \frac{4k^2 r}{(2k^2r^2+1)^2}\right)\left(\frac{2k^2r^2+1}{2k^2r^2}\right)\langle r^2 e\rangle = 0. \tag{14}$$

Assuming the medium-to-high frequency range condition $k^2r^2 \gg 1$, $(2k^2r^2+1)/(2k^2r^2) \approx 1$ can be reasonable. As shown in Table 1, the values of $(2k^2r^2+1)/(2k^2r^2)$ are nearly unity at the mid-frequencies; therefore, $(2k^2r^2+1)/(2k^2r^2)$ can be neglected in multiplication. If the distance r is large enough, that approximation satisfies even in the low frequency range and it can be thought weaker condition than far-field or high-frequency assumption $kr \gg 1$. However, $4k^2r/(2k^2r^2+1)^2$ is not vanished, although it is very small, because the damping loss factor η is so small that $4k^2r/(2k^2r^2+1)^2$ is not negligible compared with $\eta\omega/c_g$. Finally, the acoustic energy governing equation is as below:

$$\frac{\partial}{\partial r}\langle r^2 e\rangle + \left(\frac{\eta\omega}{c_g} + \frac{4k^2 r}{(2k^2r^2+1)^2}\right)\langle r^2 e\rangle = \delta(r). \tag{15}$$

$4k^2r/(2k^2r^2+1)^2$ is the term which is newly obtained by considering the additional near field energy, and it belongs to the damping part of the differential equation.

3. AEFBEM

3.1. Green's function

Eq. (15) is the differential equation in the form

$$\frac{\partial}{\partial r}\langle r^2 e\rangle + q(r)\langle r^2 e\rangle = 0, \tag{16}$$

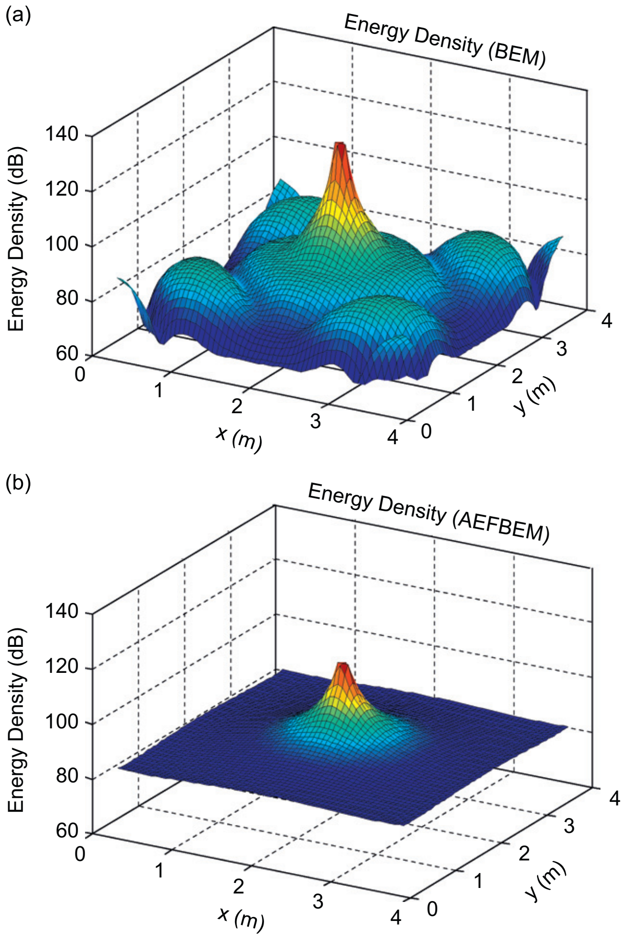

Fig. 6. The energy density level distribution of acoustic cavity when f=250 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: (a) BEM and (b) AEFBEM.

$$q(r)=\frac{\eta\omega}{c_g}+\frac{4k^2r}{(2k^2r^2+1)^2}. \tag{17}$$

This kind of differential equation has a formal solution as below:

$$\langle r^2e\rangle = Ce^{-\int q(r)\,\mathrm{d}r}. \tag{18}$$

Therefore, the energy density equation and intensity equation, which satisfy Eq. (15), can be obtained as follows:

$$\langle e\rangle = \frac{C}{r^2}e^{-(\eta\omega/c_g)r+(1/(1+2k^2r^2))}, \tag{19}$$

$$\langle \mathbf{I}_r\rangle = c_g\frac{C}{r^2}e^{-(\eta\omega/c_g)r+(1/(1+2k^2r^2))}\vec{\mathbf{r}}. \tag{20}$$

To evaluate the constant C, the relationship between intensity and input power can be applied. The input power is defined as integral of intensity over a vanishingly small volume surrounding the origin of the source. This can be expressed as

$$\Pi_{\text{in}}=\lim_{r\to 0}\int_S \mathbf{I}_r(r)\,\mathrm{d}S=\lim_{r\to 0}4\pi r^2\mathbf{I}_r(r), \tag{21}$$

where S indicates the boundary of the source.

From Eq. (21), the constant C is $e^{-1}/(4\pi c_g)$ when the amplitude of the input power Π_{in} is unity. Therefore, free-field Green's function $G(r)$ for acoustic energy is

$$G(r)=\frac{1}{4\pi c_g r^2}e^{-\eta kr-(2k^2r^2/(1+2k^2r^2))}, \tag{22}$$

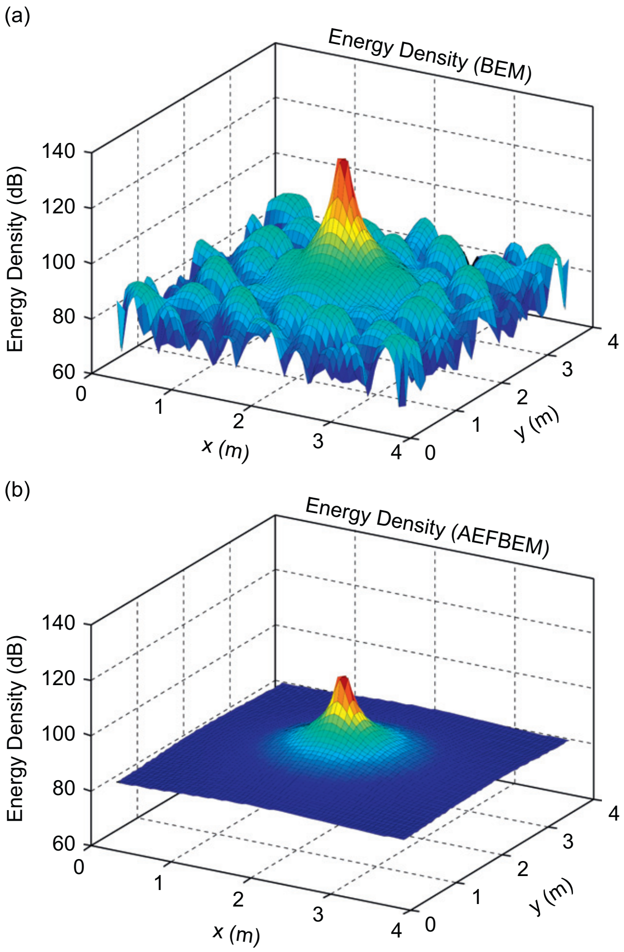

Fig. 7. The energy density level distribution of acoustic cavity when f=500 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: (a) BEM and (b) AEFBEM.

and free field Green's function $H(r)$ for intensity is

$$H_r(r)=\frac{1}{4\pi r^2}e^{-\eta kr-(2k^2r^2/(1+2k^2r^2))}. \tag{23}$$

Here $-2k^2r^2/(1+2k^2r^2)$ is additional term compared with conventional EFBEM Green's function $1/4\pi r^2\exp(-\eta kr)$. These Green's functions will be used in the development of the indirect energy flow boundary element method.

3.2. Indirect AEFBEM

An indirect approach of the acoustic energy flow boundary element method (AEFBEM) will be formulated for noise analysis. The basic concept of the indirect boundary element method is that a real system is embedded in an infinite acoustic field and the fictitious source $\phi(\vec{\xi})$ is distributed on the boundary of the real system. In indirect AEFBEM, another assumption is additionally applied; a propagating wave does not interfere with the propagation of the other waves. Namely, the acoustic energy quantity at a point is merely the summation of the energy quantities of each propagating field, and there is no cancelation of energy (linear superposition). Therefore, the energy density and intensity can be represented as

$$e(\vec{\mathbf{x}})=\int_S G(|\vec{\mathbf{x}}-\vec{\xi}|)\phi(\vec{\xi})\,\mathrm{d}S(\vec{\xi})+\int_V G(|\vec{\mathbf{x}}-\vec{\mathbf{z}}|)\Pi_{\mathrm{in}}(\vec{\mathbf{z}})\,\mathrm{d}V(\vec{\mathbf{z}}), \tag{24}$$

and

$$\mathbf{I}(\vec{\mathbf{x}})=\int_S H(|\vec{\mathbf{x}}-\vec{\xi}|)\phi(\vec{\xi})\,\mathrm{d}S(\vec{\xi})+\int_V H(|\vec{\mathbf{x}}-\vec{\mathbf{z}}|)\Pi_{\mathrm{in}}(\vec{\mathbf{z}})\,\mathrm{d}V(\vec{\mathbf{z}}), \tag{25}$$

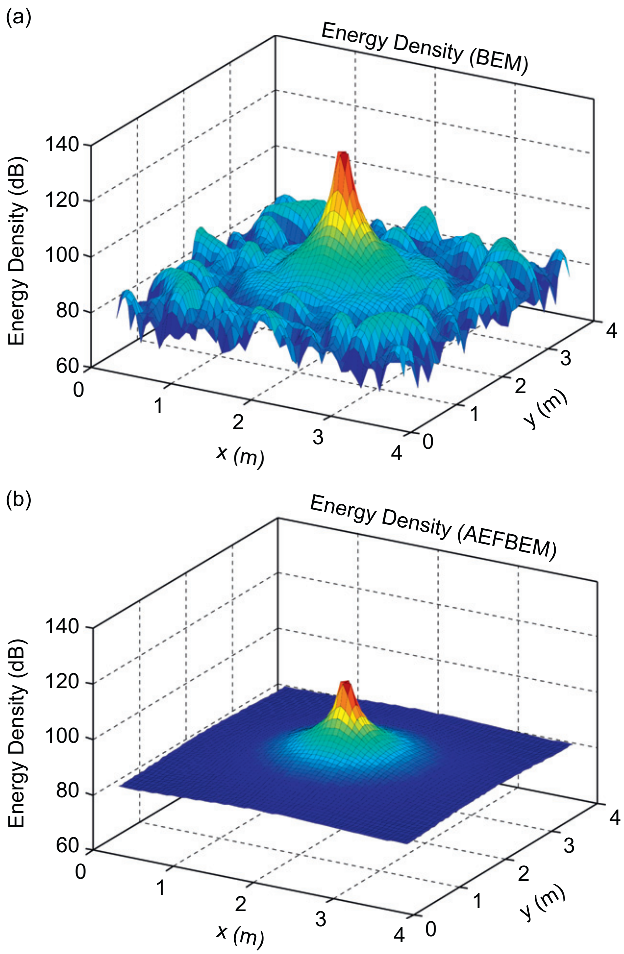

Fig. 8. The energy density level distribution of acoustic cavity when f=650 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: (a) BEM and (b) AEFBEM.

where free-field Green's functions for energy and intensity are Eqs. (22) and (23), respectively. V means the domain and S denotes the boundary. $\vec{\mathbf{x}}$ is a field point in the domain, $\vec{\xi}$ is the location of the fictitious source on the boundary, $\vec{\mathbf{z}}$ is the position of the input power, and $\phi(\vec{\xi})$ is the strength of fictitious source on the boundary [21]. Using Eqs. (24) and (25), the acoustic energy density and intensity can be calculated at the frequencies of interest. For computational analysis, the domain and boundary in Eqs. (24) and (25) should be discretized. With the discretization, Eqs. (24) and (25) are restated as

$$e(\vec{\mathbf{x}}^i) = \sum_{j=1}^{N} \phi(\vec{\xi}^j) \int_{\Delta S} G(\vec{\mathbf{x}}^i, \vec{\xi}^j)\,\mathrm{d}S^j + \sum_{k=1}^{M} \Pi_{\mathrm{in}}(\vec{\mathbf{z}}^k) \int_{\Delta V} G(\vec{\mathbf{x}}^i, \vec{\mathbf{z}}^k)\,\mathrm{d}V^k, \tag{26}$$

and

$$\mathbf{I}(\vec{\mathbf{x}}^i) = \sum_{j=1}^{N} \phi(\vec{\xi}^j) \int_{\Delta S} H(\vec{\mathbf{x}}^i, \vec{\xi}^j)\,\mathrm{d}S^j + \sum_{k=1}^{M} \Pi_{\mathrm{in}}(\vec{\mathbf{z}}^k) \int_{\Delta V} H(\vec{\mathbf{x}}^i, \vec{\mathbf{z}}^k)\,\mathrm{d}V^k. \tag{27}$$

Here, the boundary S and domain V are discretized into N boundary elements and M internal cells, respectively.

The fictitious source strengths are determined by specifying one of three boundary conditions at all boundary elements. The first boundary condition is energy density. By transforming Eq. (26) into a matrix index, the energy density at point i is re-written as

$$e_i = G_{ij}\varphi_j + G_{ik}\Pi_{\mathrm{in},k} \tag{28}$$

Eq. (28) means the energy density superposition of the field produced by discrete fictitious sources ϕ_j on the boundary and the field created by discrete real point sources of strengths $\Pi_{\mathrm{in},k}$. G_{ij} and G_{ik} are row vectors of Green's functions relating the fictitious source strengths located at point j and the real source strengths located at point k to the energy density at point i on the boundary. ϕ_j and $\Pi_{\mathrm{in},k}$ are column vectors of fictitious sources and real sources, respectively. The intensity boundary condition at point i of Eq. (27) can be restated as a matrix index by the superposition of the normal intensity. The normal intensity is due to the fields created by the fictitious sources ϕ_j on the boundary and the real sources of strengths $\Pi_{\mathrm{in},k}$ as

$$\mathbf{I}_{n,i} = \mathbf{I}_i\cdot\mathbf{n}_i = [H_{ij}(\mathbf{r}_{ij}\cdot\mathbf{n}_i)]\phi_j + [H_{ik}(\mathbf{r}_{ik}\cdot\mathbf{n}_i)]\Pi_{\mathrm{in},k} \tag{29}$$

where H_{ij} and H_{ik} are row vectors of intensity Green's functions relating the fictitious and the real sources to the intensity boundary condition. The third condition is the absorption boundary condition,

$$\mathbf{I}_{n,i} = \mathbf{I}_i\cdot\mathbf{n}_i = \frac{1}{4}c_g\alpha_i e_i \tag{30}$$

where α_i is the Sabine absorption coefficient and $\mathbf{n}_i$ is the normal vector at point i on the discrete boundary.

The set of linear matrix equation from the relationships of Eqs. (28), (29), and (30) is constructed to solve the fictitious source strength ϕ_j. Applying one of these three prescribed boundary conditions at each boundary element will yield a set of equations which can be written in matrix form as

$$K\mathbf{\Phi} = \mathbf{F}, \tag{31}$$

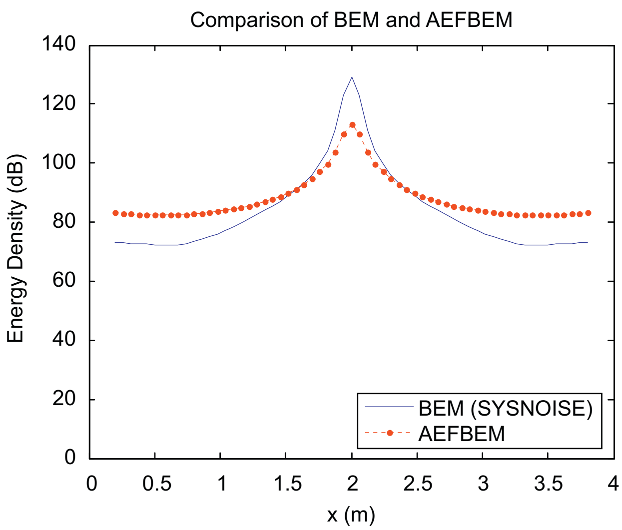

Fig. 9. The comparison of the energy density distribution along the centerline of field points when f=63 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, AEFBEM; - -●- -, BEM.

where rows of $\mathbf{K}$ can either be equal to G_{ij}, $H_{ij}(\mathbf{r}_{ij}\cdot\mathbf{n}_i)$, or $H_{ij}(\mathbf{r}_{ij}\cdot\mathbf{n}_i)-\frac{1}{4}c_g\alpha_i e_i$, and the forcing matrix $\mathbf{F}$ can either be equal to $e_i-G_{ij}\Pi_{\text{in},k}$, $\mathbf{I}_{n,i}-H_{i,k}(\mathbf{r}_{ik}\cdot\mathbf{n}_i)\Pi_{\text{in},k}$, or $\{\frac{1}{4}c_g\alpha_i G_{ik}-H_{i,k}(\mathbf{r}_{ik}\cdot\mathbf{n}_i)\}\Pi_{\text{in},k}$. When the fictitious source strengths are calculated, the relationships Eq. (28) or Eq. (29) are used to get the energy density or intensity at field points in the acoustic domain.

The point source input used in this paper can be obtained after some manipulations by using relationship between acoustic pressure and acoustic power [18]. The pressure field generated in free field by the uniform, radial, harmonic pulsation of a sphere of equilibrium radius a at frequency ω can be expressed as

$$p(r,t)=\frac{1}{1+jka}\frac{j\omega\rho_0\tilde{Q}}{4\pi r}\exp[j(\omega t-k(r-a))] \tag{32}$$

where k is acoustic wavenumber, a is radius of the pulsating sphere and $\tilde{Q}$ is the complex amplitude of volume velocity of the source. In the case of point source, or $ka \ll 1$ the pressure field equation can be restated as

$$p(r,t)=\frac{j\omega\rho_0\tilde{Q}}{4\pi r}\exp[j(\omega t-kr)]. \tag{33}$$

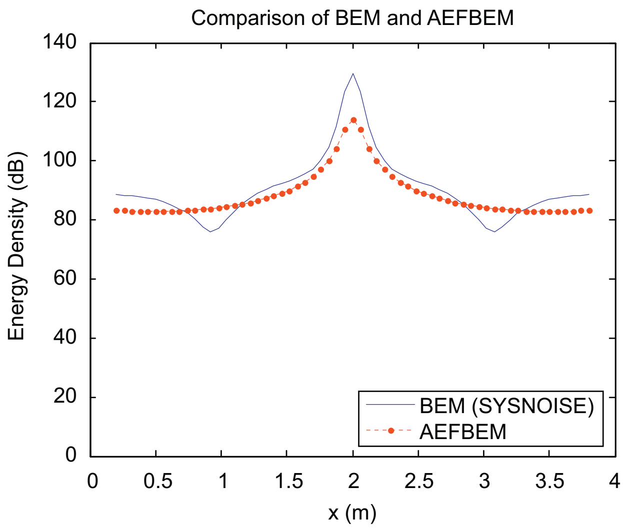

Fig. 10. The comparison of the energy density distribution along the centerline of field points when f=125 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, AEFBEM; - -●- -, BEM.

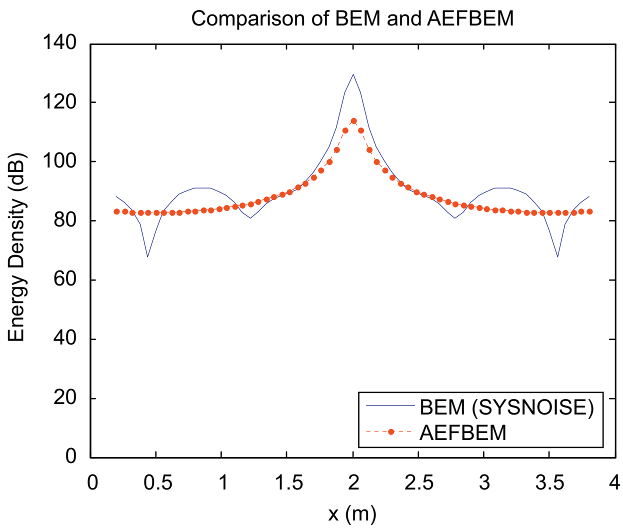

Fig. 11. The comparison of the energy density distribution along the centerline of field points when f=250 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, AEFBEM; - -●- -, BEM.

Using Euler equation of Eq. (6) the time averaged intensity and acoustic power can be derived as

$$\mathbf{I} = \frac{1}{2}\mathrm{Re}\{p\mathbf{u}_r^*\} = \frac{\rho_0\omega^2\tilde{Q}^2}{32\pi^2 c r^2}, \tag{34}$$

$$\Pi = 4\pi r^2 \mathbf{I} = \frac{\rho_0\omega^2\tilde{Q}^2}{8\pi c}. \tag{35}$$

Therefore, the source strength which would be applied to AEFBEM formulations can be calculated by using the volume velocity of the source and the above relationship between the volume velocity and the acoustic power of the source makes it possible to compare the results of BEM and AEFBEM with the same amplitude of the point source.

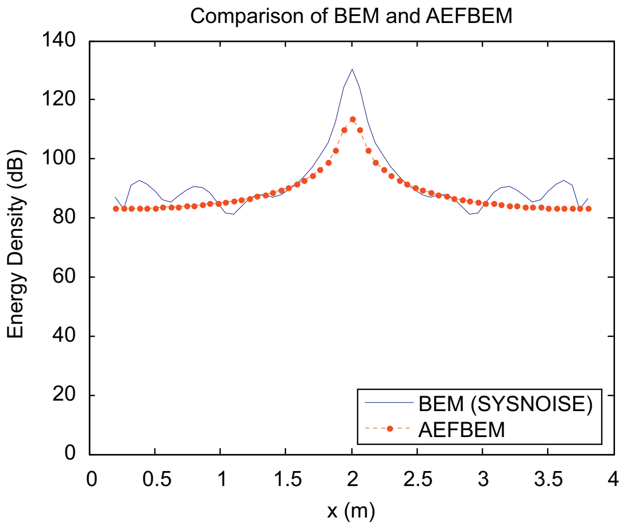

Fig. 12. The comparison of the energy density distribution along the centerline of field points when f=500 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, AEFBEM; - -●- -, BEM.

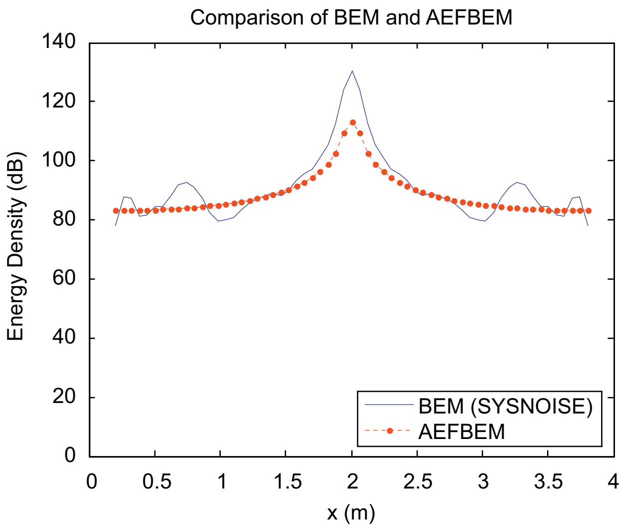

Fig. 13. The comparison of the energy density distribution along the centerline of field points when f=650 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, AEFBEM; - -●- -, BEM.

4. Numerical application of AEFBEM

4.1. Numerical verification

To verify the derived energy flow model for acoustic problems, numerical analysis was performed for a cubic room. The analysis model is a cube, as shown in Fig. 1, where the dimension of the cube is $L_x \times L_y \times L_z$=4 m × 4 m × 4 m, and the fluid properties are the same as those of air (ρ=1.21 kg/m^3, c_g=343 m/s). The point source is located at the center of the room (2 m, 2 m, 2 m), the boundary is assumed to be a rigid wall and the intensity boundary condition is used. Field points, which are the points of interest, are defined on the XY plane where z=2 m. The energy density distribution along the centerline between BEM and AEFBEM is compared and $\overline{AD}$ is the centerline in Fig. 2. When the energy density is calculated by using AEFBEM, Eqs. (26) and (27) are used. In the case of the BEM analysis, the indirect approach is used as AEFBEM. Once the pressure and velocity values of the field points are obtained, then the energy density can be calculated by Eq. (7). Therefore, the energy density calculated by using the results of BEM and the energy density of AEFBEM can be compared. Figs. 4–8 show the spatial distribution of the energy density obtained by each solution for the frequencies of 63, 125, 250, 500, 650 Hz and (a) of each figure corresponds to the BEM result and (b) to the AEFBEM result. The comparison of the

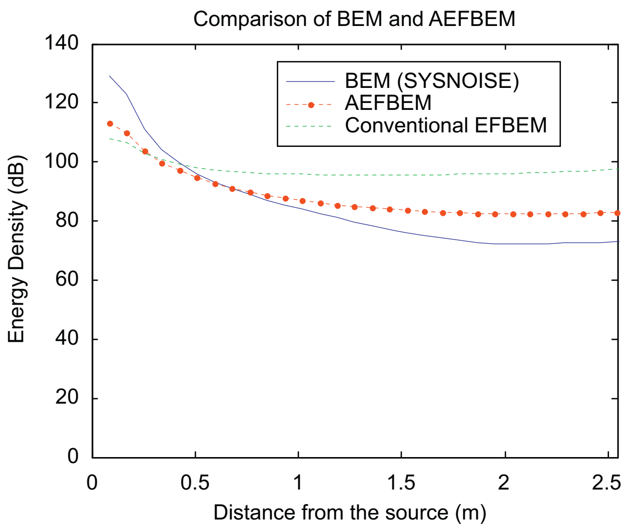

Fig. 14. The comparison of the energy density distribution along the centerline of field points when f=63 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, BEM; --●--, AEFBEM; – – – -, conventional EFBEM.

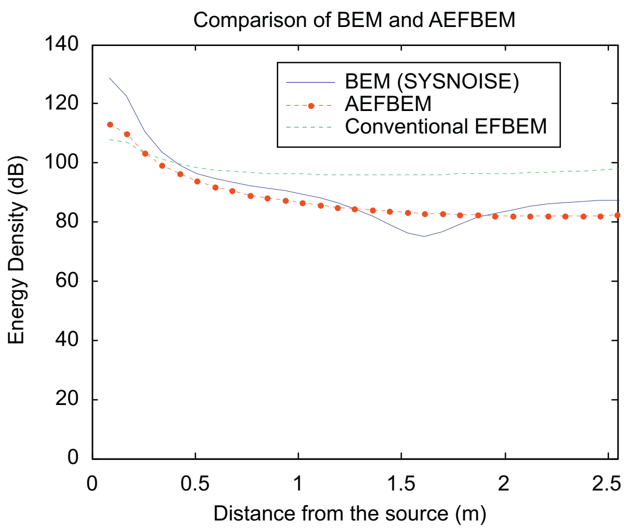

Fig. 15. The comparison of the energy density distribution along the centerline of field points when f=125 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, BEM; --●--, AEFBEM; – – – -, conventional EFBEM.

energy density along the centerline is shown in Figs. 9–13, where solid line represents the conventional BEM result and the dotted line indicates the result of AEFBEM. In Figs. 4–8, the developed EFA model considering the near field term represents well the global variation of the boundary element method. These results can be clearly observed in Figs. 9–13. In the results of both analysis methods, as the frequency increases, the spatial distributions of the energy densities predicted by the BEM and AEFBEM solutions become more similar and it can be said that AEFBEM gives a good averaged energy density of the acoustic field. The reason is that the acoustic energy flow model is made under the assumption of medium-to-high frequency range condition $k^2r^2 \gg 1$, according to this assumption, if the acoustic cavity is large enough, the energy flow model developed in this paper can be applied even at low frequencies, of course except at very low frequencies, at which the modes are very important. To discuss the validity of this developed method more clearly, comparisons among the BEM, conventional EFBEM and developed AEFBEM are presented in Figs. 14–18.

Conventional energy flow models have given better results in the high frequency range, in which reverberant field can be formed or spherical waves act like plane waves. In general room acoustics the frequency range for large room condition is proposed as Eq. (36) [18] where V is volume of the room and T is the reverberation time in seconds

$$f > 2000\sqrt{\frac{T}{V}} \tag{36}$$

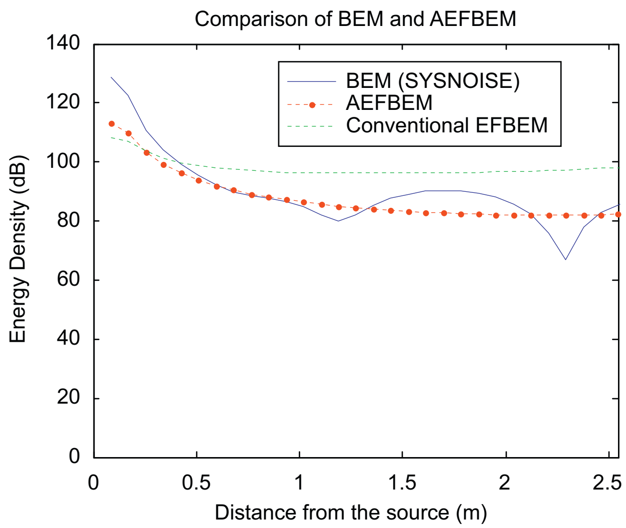

Fig. 16. The comparison of the energy density distribution along the centerline of field points when f=250 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, BEM; --●--, AEFBEM; – – – -, conventional EFBEM.

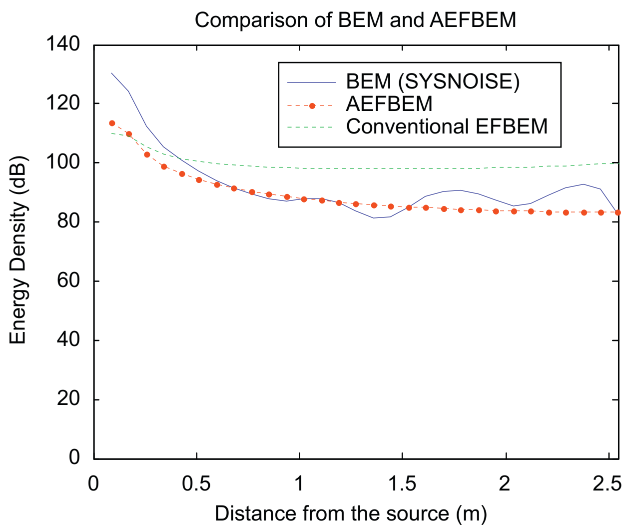

Fig. 17. The comparison of the energy density distribution along the centerline of field points when f=500 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, BEM; --●--, AEFBEM; – – – -, conventional EFBEM.

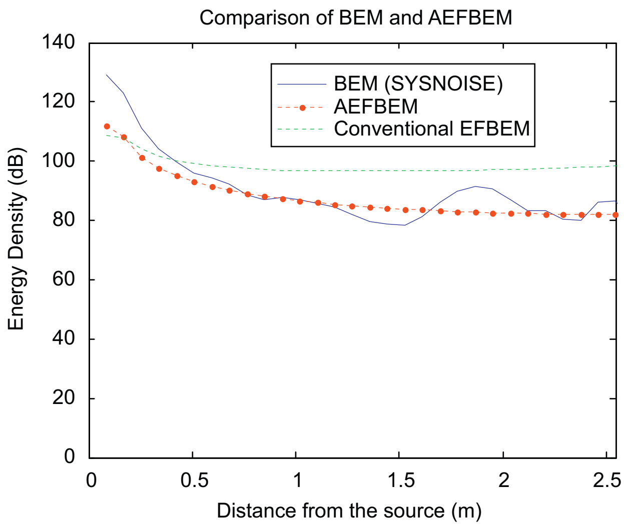

Fig. 18. The comparison of the energy density distribution along the centerline of field points when f=650 Hz. The reference energy density is 10^{-12} J/m^3 and η=0.0001: ——, BEM; - -●- -, AEFBEM; – – – -, conventional EFBEM.

The energy flow boundary element model of Lee et al. [13] is used when the pressure field is a perfectly reverberant field. The other model which was developed by Kwon [14] is used in the far-field case, which deals with the underwater radiation noise problem in the sea. Therefore, these models are not adequate for interior noise problems at mid-high frequencies. This work provides an alternative method for the conventional energy flow model for noise analysis.

The boundary element model (BE-model) and acoustic energy flow boundary element model (AEFBE-model) used in this analysis are shown in Fig. 3(a) and (b), respectively. The BE-model has 21,600 elements and 21,602 nodes. As mentioned, the elements of BE-model are fine and frequency-dependent, but those of the AEFBE-model are frequency-independent. As shown in Fig. 3(b), the AEFBE-model is very sparse compared with the BE-model. It has only 600 elements and 602 nodes. Although the BE-model has much more elements than the AEFBE-model, it can be used only up to 850 Hz due to the rule of 6 elements per wavelength. On the other hand, the AEFBE-model shows sufficient convergence with only hundreds of elements. Using BEM scheme, in matrix equation **Ax**=**B** matrix **A** is a full matrix(not a band matrix or sparse matrix). We can use Gauss–Jordan elimination to compare computational time between BEM and AEFBEM. The difference of computational time basically comes from the number of elements between two methods. Generally Gauss–Jordan elimination as an algorithm has a time complexity of $O(n^3)$. The BE-model used in this paper has 21,600 elements and 21,602 nodes while APFBE-model has only 600 elements and 602 nodes, or BEM needs about 36 times more elements than AEFBEM to calculate 4 m $\times$ 4 m $\times$ 4 m room with maximum frequency 850 Hz. Therefore, the BEM has much computational time than AEFBEM as 36^3 times (i.e. 46,668 times) to analyze same size room. Therefore, it is reasonable to conclude that the newly developed acoustic energy flow model is a good choice with respect to accuracy and efficiency when analyzing noise level at medium-to-high frequency ranges.

5. Conclusion

In this paper, an energy flow model for an acoustic problem with low damping was newly derived, and the boundary element technique was applied to the developed governing equation. By using medium-to-high frequency assumptions and considering the near field term of acoustic energy density, this acoustic energy flow model was well suited for the medium-to-high frequency domain. The governing energy equation can be used easily when analyzing an arbitrary form of acoustic cavities by applying indirect boundary element method to this model. For this reason, AEFBEM can be a powerful method for predicting acoustic energy distributions.

Several numerical results were provided, and the results of AEFBEM and BEM simulating the acoustic energy field of a cubic room were compared to verify the validity of AEFBEM. As expected, the developed energy flow solutions agreed well with the global variation of the conventional boundary element solutions. These results suggest that the method presented in this research would be very useful for the predictions of noise level at the medium-to-high frequency ranges in the first stage of design of a mechanical system like automobiles and ships. Especially in the case of large systems, such as ships, submarines and airplanes, the efficient modeling capability of AEFBEM will be a great advantage, and its efficiency and accuracy make AEFBEM a promising method for noise analysis in these industries.

In conclusion, this developed acoustic energy flow model considering the near field term is expected to save computational costs and improve the convenience and accuracy of noise analysis of engineering systems.

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