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Vibration suppression of rotating beams using time-varying internal tensile force

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ABSTRACT

A new strategy for vibration suppression of a rotating beam using a time-increasing internal tensile force is proposed in this paper. Nonlinear coupled longitudinal and bending equations of motion are derived in non-dimensional form using the Hamilton principle. The first-order analytical solution of the equations of motion is obtained using the Galerkin technique combined with the multiple scales method (MSM). Numerical simulations are then performed for various increasing rates of the internal tensile force and performance of the vibration suppression strategy is studied. A very close agreement between the simulation results obtained by the numerical integration and the first-order analytical solution is achieved. Forced vibrations of the system for input excitations of either a sinusoidal or a random function with white noise time history are considered. The simulation results and dynamic performance of the suppressed system for an externally excited rotating beam show an interesting phenomenon of the form of remarkable effectiveness of the proposed vibration reduction strategy.

0. Introduction

Vibration suppression of rotating beams has recently received much attention due to many different engineering applications. Rotating turbine and helicopter blades, robot arms, vehicular propulsion systems, flexible rotating space booms, automotive cooling systems and recently rotating-beam gyroscopes and rotary micro-electro-mechanical systems (MEMS) are a few to mention. The research trend has recently been directed toward development of various control systems to damp out the structural vibration of rotating beams. The interest mainly arises from the engineering desire to reduce the noise and vibration levels and, consequently, improve performance and fatigue life of rotating systems. The control systems designed for vibration suppression of beams can be mainly divided into three different categories, namely passive, semi-active and active control systems.

In passive control systems, the conventional constrained viscoelastic damping layers are used to damp out vibration of a rotating beam [1]. Systems with semi-active controllers employ the technique of activated damping in which an electro-rheological (ER) damper [2] or a piezoelectric element [3] is attached to the rotating beams. In the active control strategy, an external actuator is employed and its control force is designed based on one of the existing control theories, e.g., feedback control system [4–8], modal control [9], sliding-mode control [10], optimal control [11,12] and fuzzy-based control system [13].

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Nomenclature

a	cross-sectional area
A	function of T
A_0	hole area
b	hub radius
c	increasing rate of the tensile force
C	damping coefficient
e	strain
f	external force
F_{NC}	non-conservative force due to damping
F	non-dimensional force
I	area moment of inertia of the beam
K	kinetic energy
L	beam length
m	mass per unit length of the beam
M_{NC}	non-conservative bending moment due to damping
M	bending moment
p_0	internal pressure
P	internal compressive force
q	modal coefficient
r	position vector
s	non-dimensional length
t	time
T	tension force
$\hat{T}$	$=\varepsilon\tau$
u	longitudinal displacement
U	potential energy
V	velocity
w	transversal displacement
W	work
x	longitudinal coordinate
y	transverse coordinate
β	hub ratio
ε	coefficient of proportionality
η	damping factor
ρ	material density
λ	eigen-value
σ	stress
τ	non-dimensional time
φ	variable scale
ψ	mode shape function
ω	rotational speed
Ω	non-dimensional rotational speed
$(\,)'$	$=\partial/\partial s$
$\dot{(\,)}$	$=\partial/\partial\tau$

Due to importance of the subject, there are many research articles recently published in the literature, mainly covering the areas of dynamic modeling, active vibration control and stability analysis of rotating beams. Use of piezoelectric stacks in vibration control of helicopter blades was studied by Straub et al. [1]. An electro-rheological (ER) sandwiched structure was experimentally analyzed for the vibration control of a rotating flexible beam with variable speed and acceleration [2]. Vibration suppression of rotating beams using an active constrained layer consisting of a viscoelastic damping layer sandwiched between two piezoelectric layers was studied by Baz and Ro [3]. A combined proportional and derivative control for vibration suppression of a rotating beam provided with a pair of piezoelectric sensors and actuator layers was investigated by Lin [4]. Suppression of transverse vibrations of a rotating beam was simulated by Yang et al. [5] using a control system designed by combining positive position feedback and the momentum exchange feedback control laws.

An optimal control theory was employed by Choi et al. [6] to design a flow source vibration controller and a conventional PD controller for vibration suppression of a rotating beam. A velocity feedback control system was proposed by Na et al. [7,8] for vibration control of a rotating beam. An analytical design of an active control scheme for vibration suppression in an elastic rotating beam was presented by Khulief [9] based on an optimal modal control employing a set of significant modes. An integral sliding-mode control approach was proposed by Xue and Tang [10] for the vibration control of a rotating system with nonlinear coupling effect between hub rotation and beam transverse vibration. An optimal control system was presented by Cai et al. [11] to damp out transverse vibration of a rotating cantilever beam.

An optimal control strategy for a rotating, composite, pre-twisted and single-celled box beam was proposed by Shete et al. [12], for which an optimum pre-twist leading to the lowest response, power and settling time, was obtained. Marghitu et al. [13] presented a control system for vibration suppression of a parametrically excited rotating beam using a fuzzy-logic controller. Controllability and observability of a rotating beam system was studied by Kuo and Lin [14] and a controller system based on the pole assignment criterion was proposed.

It is a well-known fact that a tensile force can increase flexural rigidity and, consequently, natural frequency of a rotating beam. However, it should be noted that for a known initial condition, more rigidity can only lead to larger natural frequencies and not to any vibration suppression in a natural vibration. The new concept presented in this paper is time-varying tensile force. It is proved that a time increasing tensile force behaves as virtual damping in an un-damped rotating system. Literature review on the subject indicates that very few research studies have been carried out in the area of passive control systems for vibration suppression of rotating beams. This is because of various difficulties due to the rotation of such systems. In other words, conventional passive control systems like tuned-mass-damper (TMD) systems practically do not have the potential to be utilized in a rotating system.

Vibration suppression of a rotating beam using time-increasing internal tensile force is investigated in this paper. Coupled nonlinear longitudinal and bending vibrations of a beam are considered and the governing equations of motion for a rotating beam with time-varying internal tensile force using the Hamilton principle are derived. These equations are then converted to a non-dimensional form in order to generalize the study and its analytical solution. This procedure has the

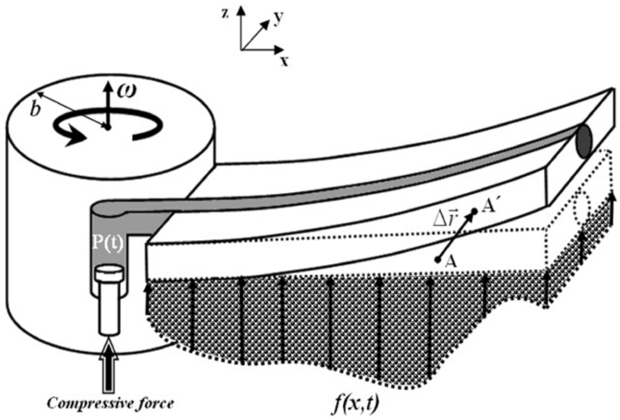

Fig. 1. A rotating viscoelastic beam subjected to time varying tensile force.

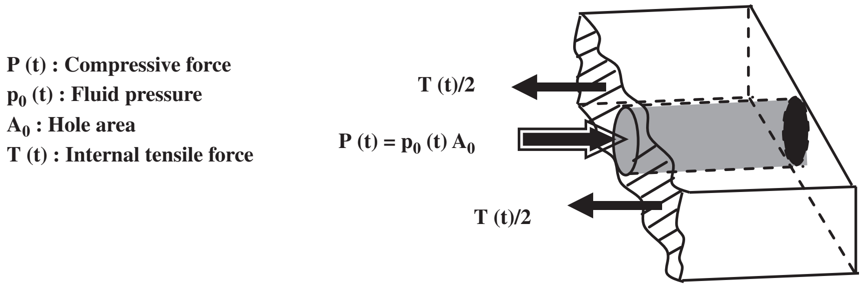

Fig. 2. A mechanism for supplying the internal tensile force.

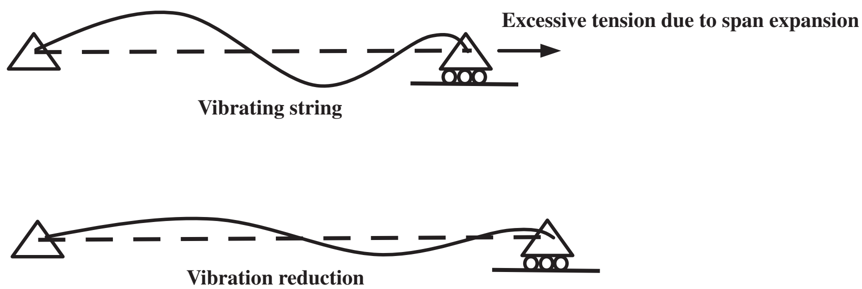

Fig. 3. Vibration reduction concept employing an increase of tensile force.

advantage of having outcomes to be in the general form and not dependent on a specific case study. The first-order analytical solution of the nonlinear partial differential equation of motion is obtained using a combinational procedure employing the Galerkin technique and the multiple scales method (MSM).

Analytical solutions can always provide a much better sense for the engineering design of complex systems. Numerical simulations are carried out for various increasing rates of the internal tensile force and a very close agreement between the simulation results obtained by numerical integration and the first-order analytical solution is observed. The rotating beams are always subjected to a variety of external excitations due to the vortex shedding phenomenon. For the case of forced vibration, the sources of external excitation are assumed to be a sinusoidal function or random excitation with white-noise time history, and dynamic performance of the passive control system is investigated.

1. Mathematical modeling

A vibrating viscoelastic rotating beam subjected to an external force and a time-varying internal tensile force is shown schematically in Fig. 1. Because of the rotating nature of the rotary beam, there is no feasible solution in which any external tensile force can be applied. The only solution to provide such a desired tensile force is by an internal system. The concept here is to provide a pressure internally exerted on the end wall of the beam via a hole located inside it. Such a pressure can be supplied by a fluid injection system, as illustrated in Fig. 1, or by a compression rod inserted into the rotating beam. Considering the cross section of the rotating beam (Fig. 2) the compressive force $P(t)$ can be obtained by $p_0(t)A_0$, in which $p_0(t)$ and A_0 denote the fluid pressure and area of the internal hole, respectively. Consequently, the reaction internal tensile force $T(t)$ is supposed to be proportional to the fluid pressure and the compressive force. The main concept of the vibration suppression strategy is illustrated in Fig. 3. In a stretched vibrating string if any excessive tensile force is exerted via expansion of string span, amplitude of the vibration remarkably decreases quite matched with physical sense.

As modeling assumptions, the beam is assumed to be inextensible and a classical linear viscoelastic model, i.e. a Kelvin–Voigt model, is considered. It is assumed that the beam satisfies the Euler–Bernoulli beam theory, where shear deformation and rotary inertia terms are negligible. The beam is also assumed to possess uniform cross-sectional area and mass distribution along its length. Position vector of each element of the beam is expressed by

$$\vec{r} = (b+x+u(x,t))\,\hat{i} + w(x,t)\,\hat{j} \tag{1}$$

and the components of velocity vector for a beam element positioned at x are

$$V_x = \frac{\partial u}{\partial t}; \quad V_y = (b+x+u)\omega; \quad V_z = \frac{\partial w}{\partial t} \tag{2}$$

The total kinetic energy of the beam can now be constructed by [21]

$$K = \int_0^L \frac{1}{2} mV^2\,dx = \int_0^L \frac{1}{2} m(V_x^2+V_y^2+V_z^2)dx \tag{3}$$

Substituting Eq. (2) in (3) gives

$$K = \int_0^L \frac{m}{2}\left[\left(\frac{\partial u}{\partial t}\right)^2 + (b+x+u)^2\omega^2 + \left(\frac{\partial w}{\partial t}\right)\right]^2 dx \tag{4}$$

in which m denotes the mass per unit length of the beam.

One can express the stress–strain relationship for a Kelvin–Voigt standard viscoelastic material in the form

$$\sigma = Ee + C\dot{e} \tag{5}$$

in which E and C are Young's modulus of elasticity and the damping factor, respectively, and e represents the strain value. Using the kinematics of large deformation one can reach the following relation for curvature at any given point along the beam [17]:

$$\rho = \frac{w''(1+u') - w'u''}{\sqrt{[(1+u')^2 + w'^2]^3}} \tag{6}$$

The bending moment at each section of the beam is then obtained by [15]

$$M = \int_a \sigma y\,da = EI\rho \tag{7}$$

in which the parameter σ is the stress value and parameters I and a represent the area second moment of inertia and the cross-sectional area of the beam, respectively. Combining Eqs. (6) and (7) would lead to [15]

$$M = EI\frac{w''(1+u') - w'u''}{\sqrt{[(1+u')^2 + w'^2]^3}} \tag{8}$$

The potential energy due to bending of the beam can be obtained by [15,17]

$$U = \frac{1}{2}\left(\int_0^L M\rho dx + \int_0^L T(t)e\,dx\right) \tag{9}$$

in which the strain value is given by [20]

$$e = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{10}$$

The above definition for strain value and Eqs. (6) and (8) lead to

$$U = \frac{1}{2}\left(\int_0^L EI\left[\frac{(w''(1+u') - w'u'')}{[\sqrt{(1+u')^2 + w'^2}]^3}\right]^2 dx + \int_0^L T(t)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)dx\right) \tag{11}$$

For an inextensible uniform beam, one can assume [16]

$$(1+u')^2 + w'^2 \approx 1 \tag{12}$$

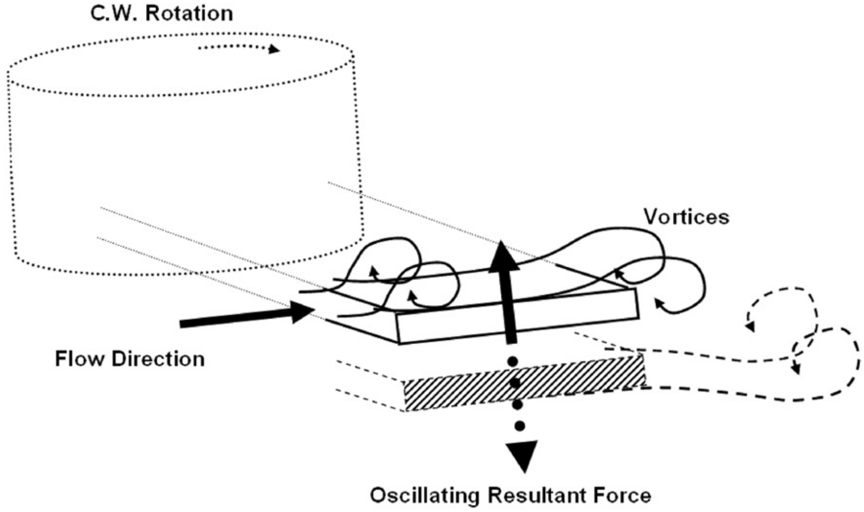

Fig. 4. Loading mechanism exerted by vortex shedding around a rectangular cross section.

and consequently

$$u' \approx -\frac{1}{2}w'^2 \quad \text{and} \quad u'' \approx -w'w'' \tag{13}$$

Using the above approximation, the potential energy due to bending is given by

$$U = \frac{1}{2}\int_0^L EI\left[w'' + \frac{1}{2}(w''w'^2)\right]^2 dx \tag{14}$$

The work done by an external force and a non-conservative force due to damping F_{NC} can be obtained by

$$W_{NC} = \int_0^L (f(x,t)w\,dx - \int_0^L F_{NC}w\,dx \tag{15}$$

$f(x, t)$ denotes the external distributed load, practically representing air flow induced forces exerted on the beam surface. Vortex-shedding induced force, which can have a random form or harmonic nature dependence on the flow regime, is an example of such a distributed load. The vortex shedding is caused when the fluid flow past the beam creates alternating low-pressure vortices on the downstream side of the object and consequently the object will tend to move toward the low-pressure zone. As shown in Fig. 4, these alternating low pressure zones cause the beam to move in an oscillatory manner perpendicular to the direction of the flow.

Similar to Eq. (8), the non-conservative moment due to structural damping is given by [17]

$$M_{NC} = CI\frac{d}{dt}\frac{w''(1+u')-w'u''}{\sqrt{[(1+u')^2+w'^2]^3}}) \tag{16}$$

The correlated non-conservative force is then given by [17].

$$F_{NC} = \int_0^L \frac{\partial^2 M_{NC}}{\partial x^2}dx \tag{17}$$

Using Eqs. (15)–(17) and the approximations given by Eq. (13), the total non-conservative virtual work done by structural damping and the external force on the system is given obtained as [18,19]

$$\delta W_{NC} = \int_0^L \left(f(x,t) - CI\left[\dot{w}'' + \frac{1}{2}(\dot{w}''w'^2) + w''w'\dot{w}'\right]''\right)\delta w\,dx \tag{18}$$

Using the Hamilton principle given by

$$\int_0^t (\delta K - \delta U + \delta W_{NC})dt = 0 \tag{19}$$

and well-known theorems for calculus of variations, the governing nonlinear differential equation of motion is then obtained as

$$m\ddot{w} + CI\dot{w}^{(4)} + EIw^{(4)} - \omega^2\left[(a(L-x) + \frac{(L^2-x^2)}{2})w'' - (a+x)w'\right] - T(t)w''$$
$$= -3T(t)w''w'^2 - \frac{m}{2}\left[w'\int_L^x\int_0^x(\dot{w}'^2 + w'\ddot{w}')dx\,dx\right]' - EI\left[w'(w'w'')'\right]' - CI\left[\frac{1}{2}\dot{w}''w'^2 + w''w'\dot{w}'\right]'' + f(x,t) \tag{20}$$

with the conventional boundary conditions for either ends of the beam in the form

$$BC\begin{cases} EI[w''' + w'''w'^2 + w'w''^2]\delta w|_0^L = 0 \\ EI[w'' + w''w'^2]\delta w'|_0^L = 0 \end{cases} \tag{21}$$

In order to make the solution method and the results as general as possible, the equation of motion is now converted to a non-dimensional form using the following non-dimensional parameters:

$$s = \frac{x}{L};\ \tau = t\sqrt{\frac{EI}{mL^4}};\ ()' = \frac{\partial}{\partial s};\ \mathring{()} = \frac{\partial}{\partial \tau};\ \Omega = \omega\sqrt{\frac{mL^4}{EI}};\ \beta = \frac{b}{L};\ \eta = C\sqrt{\frac{I}{EmL^4}};\ T* = T\frac{L^2}{EI};\ F = f\frac{L^3}{EI} \tag{22}$$

Therefore the governing equation of motion in its non-dimensional form is

$$\begin{aligned} &\ddot{w} + \eta\dot{w}^{(4)} + w^{(4)} - \Omega^2\left[\left(\beta(1-s) + \frac{(1-s^2)}{2}\right)w'' - (\beta+s)w'\right] - T*(\tau)w'' \\ &= 3T*(\tau)w''w'^2 - \frac{1}{2}\left[w'\int_1^s\int_0^s(\dot{w}'^2 + w'\ddot{w}')\,ds\,ds\right]' - [w'(w'w'')']' \\ &\quad -\eta\left[\frac{1}{2}\dot{w}''w'^2 + w''w'\dot{w}'\right]'' + F(s,\tau) \end{aligned} \tag{23}$$

and similarly for the boundary conditions one can write

$$BC\begin{cases} [w''' + w'''w'^2 + w'w''^2]\delta w|_0^1 = 0 \\ [w'' + w''w'^2]\delta w'|_0^1 = 0 \end{cases} \tag{24}$$

2. Method of solution

The Galerkin mode summation approach is employed in this research to discretize the nonlinear partial differential equation of motion. The solution is then expressed as

$$w(s,\tau) = \sum_i \psi_i(s)q_i(\tau) \tag{25}$$

in which $q_i(\tau)$ are the modal coefficients. The mode shape functions $\psi_i(s)$ are presented in the following form:

$$\psi_i(s) = [\sinh(\lambda_i s) - \sin(\lambda_i s)] - \frac{[\sinh(\lambda_i) + \sin(\lambda_i)]}{[\cosh(\lambda_i) + \cos(\lambda_i)]}[\cosh(\lambda_i s) - \cos(\lambda_i s)] \tag{26}$$

where λ_i values satisfy the frequency characteristic equation

$$1 + \cosh(\lambda_i)\cos(\lambda_i) = 0 \tag{27}$$

Using a first-order approximation, in conjunction with the orthogonality principle of the mode shapes, one can arrive at

$$\begin{aligned} &\ddot{q} + \frac{1}{(1+4.265q^2)}\Big[(12.362 + \Omega^2(1.193 + 1.987\beta) + 5.01T*(\tau))q \\ &+ 12.362\eta\dot{q} + (75.043 + 23.994T*(\tau))q^3 + 4.265q\dot{q}^2 - 17.963\eta q\dot{q}\Big] = 0.575F(\tau) \end{aligned} \tag{28}$$

Using the Taylor series expansion one would obtain

$$\begin{aligned} &\ddot{q} + [12.362 + \Omega^2(1.193 + 1.987\beta) + 5.01T*(\tau)]q + 12.362\eta\dot{q} + (22.319 - 5.088\Omega^2)q^3 \\ &+ 4.265q\dot{q}^2 - 17.963\eta q\dot{q} - 320.058q^5 - 52.724\eta q^2\dot{q} - 18.19q^3\dot{q}^2 \\ &+ 76.61\eta q^3\dot{q} + 2.626T*(\tau)q^3 - 102.334T*(\tau)q^5 = 0.575F(\tau)(1 - 4.265q^2) \end{aligned} \tag{29}$$

Recognizing the linear and nonlinear terms and splitting them into two parts give

$$\ddot{q} + [12.362 + \Omega^2(1.193 + 1.987\beta) + 5.01T*(\tau)]q = \varepsilon g(\tau,q,\dot{q}) \tag{30}$$

in which

$$\begin{aligned} g(\tau,q,\dot{q}) = &-[12.362\eta\dot{q} + (22.319 - 5.088\Omega^2)q^3 \quad + 4.265q\dot{q}^2 - 17.963\eta q\dot{q} - 320.058q^5 - 52.724\eta q^2\dot{q} - 18.19q^3\dot{q}^2 \\ &+ 76.61\eta q^3\dot{q} + 2.626T*(\tau)q^3 - 102.334T*(\tau)q^5] + 0.575F(\tau)(1 - 4.265q^2) \end{aligned} \tag{31}$$

For non-stationary free vibration, in which T^* varies with time, multiple scales method can be implemented [20,21] so that Eq. (31) will be replaced by

$$\ddot{q} + [12.362 + \Omega^2(1.193 + 1.987\beta) + 5.01T*(\hat{T})]q = \varepsilon f(T,q,\dot{q}) \tag{32}$$

where $\hat{T} = \varepsilon\tau$.

The general equation of non-stationary motion indicates that the case of harmonic variations in angular velocity results in a Mathieu–Duffing type oscillator [22]. Now, one should seek an expansion for the solution in terms of two scales τ and

φ so that

$$\hat{\Omega}(\hat{T}) = \left[12.362 + \Omega^2(1.193 + 1.987\,\beta) + 5.01T*(\hat{T})\right] = \frac{d\varphi}{d\tau} \tag{33}$$

The time derivatives will then become

$$\frac{d}{d\tau} = \hat{\Omega}\frac{\partial}{\partial\varphi} + \varepsilon\frac{\partial}{\partial\hat{T}} \tag{34}$$

$$\frac{d^2}{d\tau^2} = \hat{\Omega}^2\frac{\partial^2}{\partial\varphi^2} + \varepsilon\left[2\hat{\Omega}\frac{\partial^2}{\partial\varphi\partial\hat{T}} + \hat{\Omega}'\frac{\partial}{\partial\varphi}\right] + \varepsilon^2\frac{\partial^2}{\partial\hat{T}^2} \tag{35}$$

Hence Eq. (32) yields

$$\hat{\Omega}^2\frac{\partial^2 q}{\partial\varphi^2} + \varepsilon\left[2\hat{\Omega}\frac{\partial^2 q}{\partial\varphi\partial\hat{T}} + \hat{\Omega}'\frac{\partial q}{\partial\varphi}\right] + \varepsilon^2\frac{\partial^2 q}{\partial\hat{T}^2} + \hat{\Omega}^2 q = \varepsilon g\left[\hat{T}, q, \hat{\Omega}\frac{\partial q}{\partial\varphi} + \varepsilon\frac{\partial q}{\partial\hat{T}}\right] \tag{36}$$

Now, expanding q in the series form

$$q = q_0(\varphi_0, \hat{T}) + \varepsilon q_1(\varphi_0, \hat{T}) + \cdots \tag{37}$$

substituting Eq. (37) in (36) and equating the coefficients of the same power of ε, one can arrive at

$$\hat{\Omega}^2\left(\frac{\partial^2 q_0}{\partial\varphi^2} + q_0\right) = 0 \tag{38}$$

$$\hat{\Omega}^2\left(\frac{\partial^2 q_1}{\partial\varphi^2} + q_1\right) = -2\hat{\Omega}\frac{\partial^2 q_0}{\partial\varphi\partial\hat{T}} - \hat{\Omega}'\frac{\partial q_0}{\partial\varphi} + g\left[\hat{T}, q, \hat{\Omega}\frac{\partial q}{\partial\varphi} + \varepsilon\frac{\partial q}{\partial\hat{T}}\right] \tag{39}$$

The general solution of Eq. (39) can be obtained in the complex form of

$$q_0 = A(\hat{T})\exp(i\phi) + cc \tag{40}$$

where *cc* denotes the complex conjugate term. Hence equation Eq. (39) becomes

$$\hat{\Omega}^2\left(\frac{\partial^2 q_1}{\partial\varphi^2} + q_1\right) = -i\left[2\hat{\Omega}A' + \hat{\Omega}'A\right]\exp(i\varphi) + i\left[2\hat{\Omega}\overline{A'} + \hat{\Omega}'A\right]\exp(-i\varphi) \quad + g\left[\hat{T}, A\exp(i\varphi) + cc, i\hat{\Omega}(A\exp(i\varphi) + cc\right] \tag{41}$$

Eliminating the terms in Eq. (41) that produce secular terms in q_1 yields

$$-i\left[2\hat{\Omega}A' + \hat{\Omega}'A\right] = \frac{1}{2\pi}\int_0^{2\pi} g(A, \overline{A}, \varphi, \hat{T})\exp(-i\varphi)d\varphi \tag{42}$$

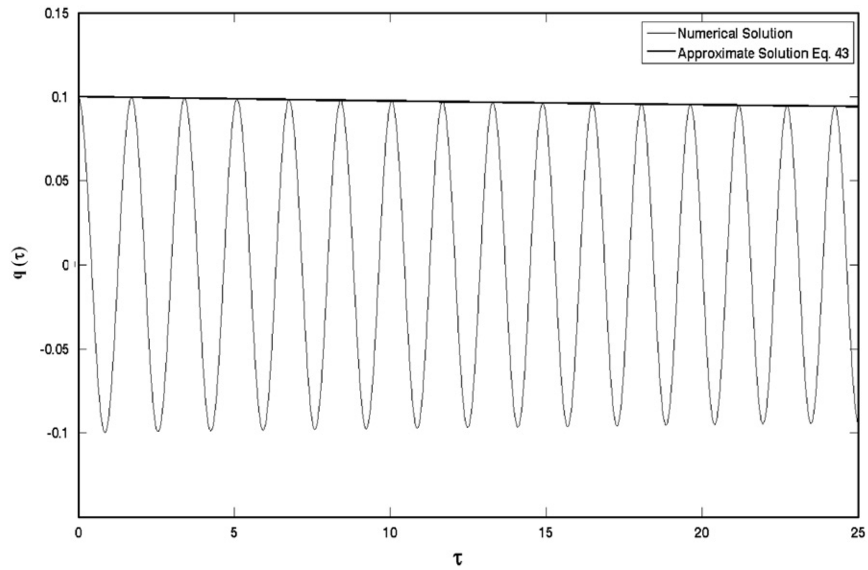

Fig. 5. Effect of tensile force increasing rate on time history of vibration of an un-damped beam ($\eta = 0.0$; $\Omega = 1.0$; $\beta = 0.1$), Case 1: c=0.1.

An exact solution of Eq. (42) exists for any arbitrary $T*$ varying with time. The solution can be obtained as

$$A(\tau)=A(0)\sqrt{\frac{\exp(-12.36\eta\tau)\sqrt{\frac{12.362+\Omega^2(1.193+1.987\beta)+5.01T*(0)}{12.362+\Omega^2(1.193+1.987\beta)+5.01T*(\tau)}}}{1-131.81\eta A(0)^2\int_0^\tau \exp(-12.36\eta x)\sqrt{\frac{12.362+\Omega^2(1.193+1.987\beta)+5.01T*(0)}{12.362+\Omega^2(1.193+1.987\beta)+5.01T*(x)}}}dx} \quad (43)$$

3. Numerical results

Two approaches, namely the time integration method and the first-order multiple scales method, are employed here as solution methods in this study. A computer program is developed using the MATLAB©–MAPLE© software link for numerical

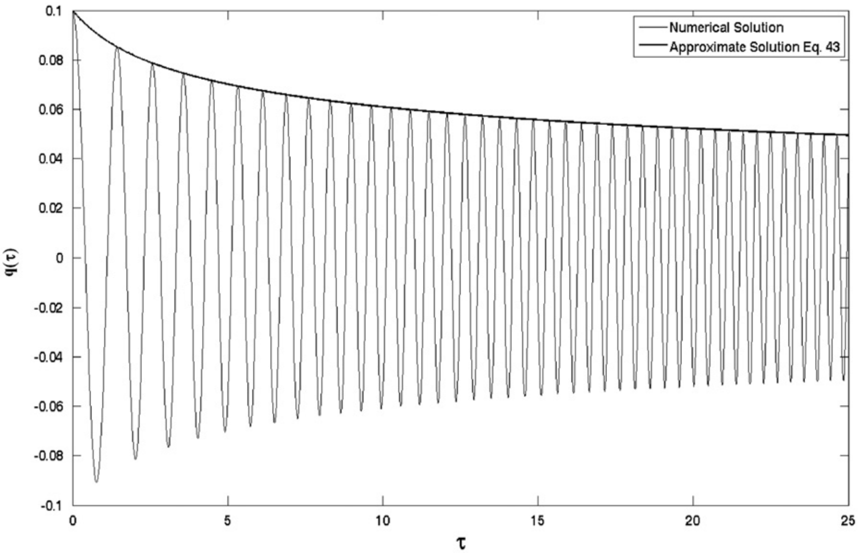

Fig. 6. Effect of tensile force increasing rate on time history of vibration of an un-damped beam ($\eta=0.0$; $\Omega=1.0$; $\beta=0.1$), Case 2: c=10.

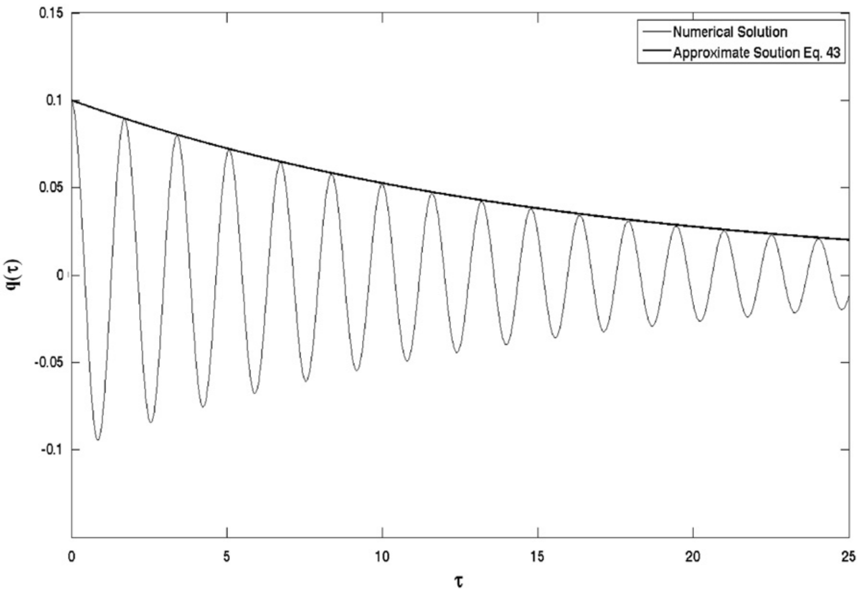

Fig. 7. Effect of tensile force increasing rate on time history of vibration of a damped beam ($\eta=0.01$; $\Omega=1.0$; $\beta=0.1$), Case 1: c=0.1.

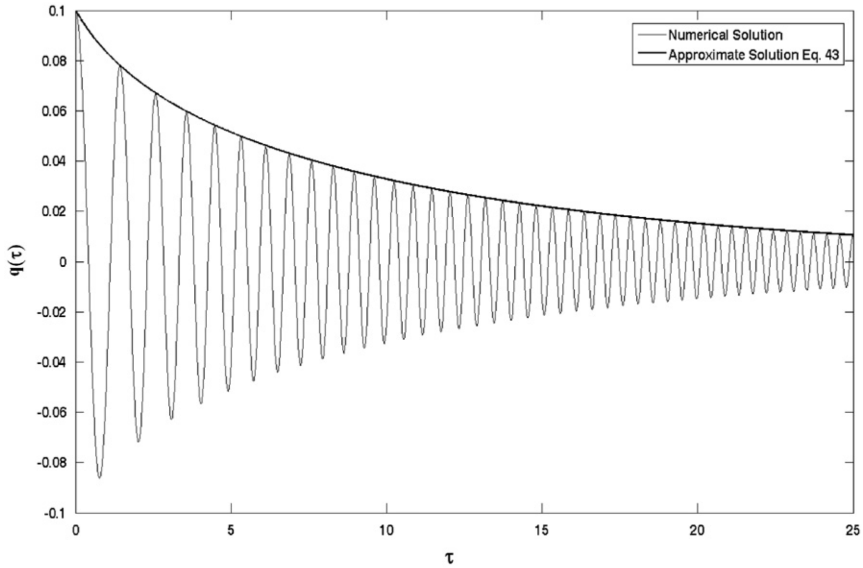

Fig. 8. Effect of tensile force increasing rate on time history of vibration of a damped beam ($\eta = 0.01$; $\Omega = 1.0$; $\beta = 0.1$), Case 2: c=10.

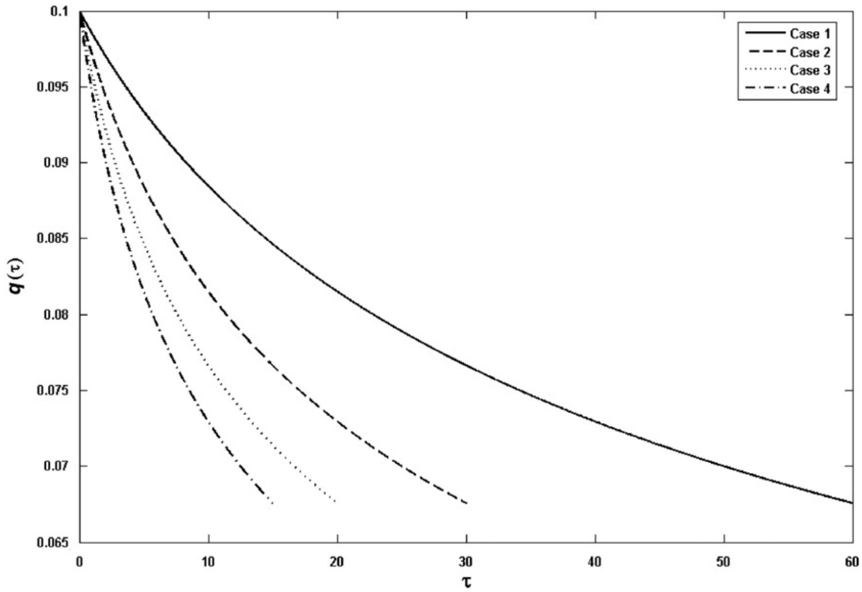

Fig. 9. Effect of tensile force increasing rate on vibration amplitude of an un-damped beam when the final value of tensile force is fixed to P^*=60. Case 1: c=1.0; Case 2: c=2.0; Case 3: c=4.0; Case 4: c=5.0.

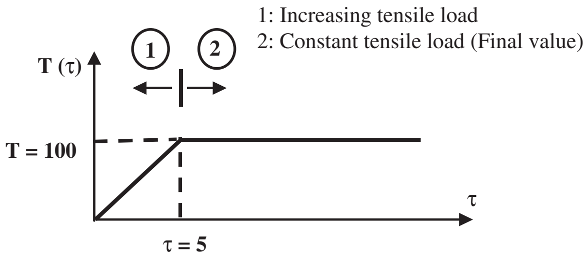

Fig. 10. Time history of tensile load.

simulation, and a parametric sensitivity study is carried out to evaluate the effects of different parameters on time history of natural responses.

The effects of time increasing rates of the internal tensile force on time responses are illustrated in Figs. 5 and 6 for an un-damped beam. The internal tensile force is assumed to be $T*(\tau)=c\tau$. A very close agreement between the numerical solution and the first-order MSM approximation is observed. It is seen that the internal tensile force causes virtual damping and reduces the vibration level with an increase in time. Larger values of internal tensile force increasing time rates are correlated with larger virtual damping values. It should be noted that for a constant internal tensile force, even for large values, attenuation phenomenon is not achievable. Large values of tensile force can only increase the flexural rigidity of the beam and consequently its natural frequency but cannot damp out the beam vibration; however its positive time increasing rate can do that.

The effects of time rate of the internal tensile force on time responses are illustrated in Figs. 7 and 8 for a damped beam. As seen, time increasing rate of the internal tensile force contributes to reduce the vibration level in the presence of the

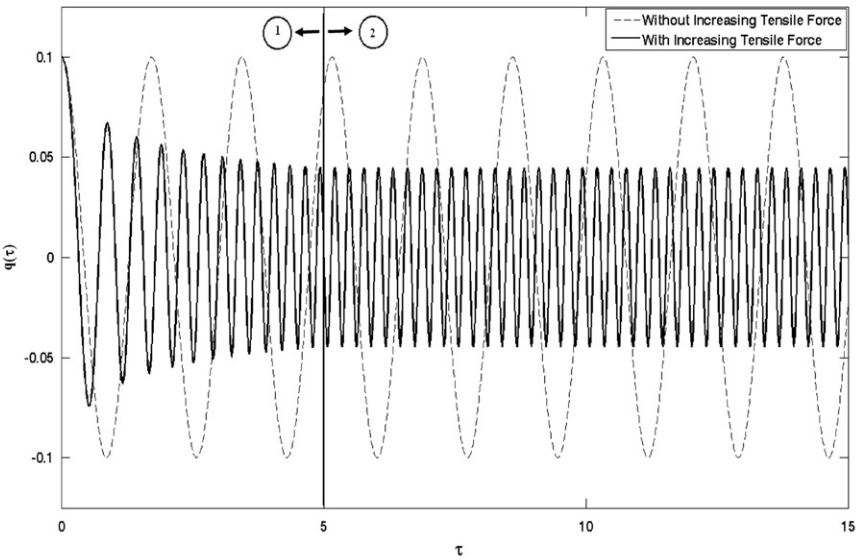

Fig. 11. Vibration of un-damped beam before and after reaching the final value of tensile load ($\eta=0$; $\Omega=1.0$; $\beta=0.1$), Region 1: increasing tensile force, Region 2: constant tensile force (final value).

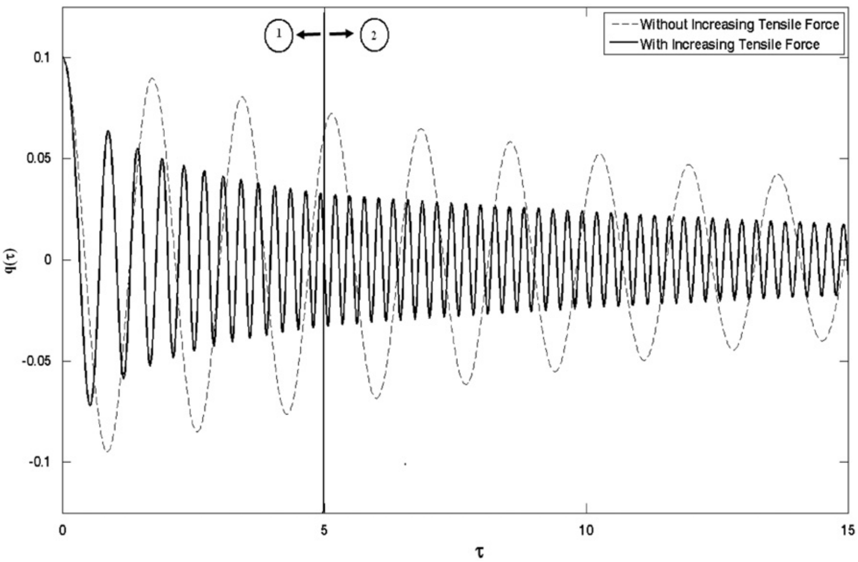

Fig. 12. Vibration of damped beam before and after reaching the final value of tensile load ($\eta=0.01$; $\Omega=1.0$; $\beta=0.1$), Region 1: increasing tensile force, Region 2: constant tensile force (final value).

structural damping. In other words, it can play the same role with the same significance as that of structural damping. The effect of tensile force increasing rate on vibration amplitude of an un-damped beam is illustrated in Fig. 9, where the final value of tensile force is assumed to be fixed. The objective here is to show that the reduction rate is dependent on the time rate of increasing tensile force but the reduction factor (final displacement/initial displacement) is dependent just on the final value of tensile force.

In order to evaluate the system behavior when the final value of the tensile force is attained, some case studies based on the tensile force profile given in Fig. 10 are simulated. As seen for an un-damped beam (Fig. 11), once the final tensile value is attained, the beam continues its oscillation at its new natural frequency with the amplitude remarkably smaller than that of the uncontrolled beam (40% in this case). The end condition of the first part of the tensile loading profile acts as the initial condition of free vibration of the beam with final value of tensile force (100 in this case). In the case of damped beam, as illustrated in Fig. 12, after reaching the final value of the tensile load, the beam behaves as a free damped system

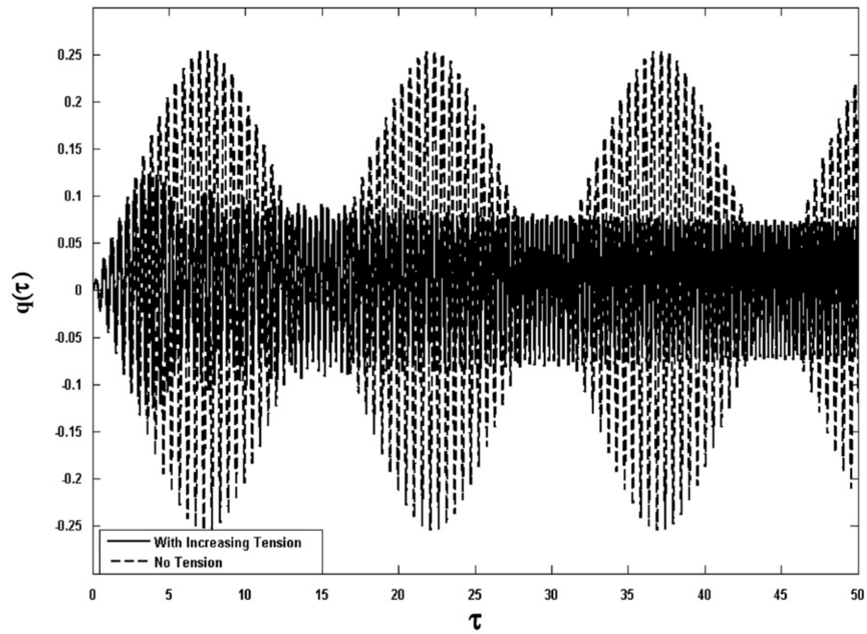

Fig. 13. Performance of the suppression system for an un-damped beam subjected to harmonic excitation ($\eta=0.0$; $\Omega=10$; $\beta=0.1$).

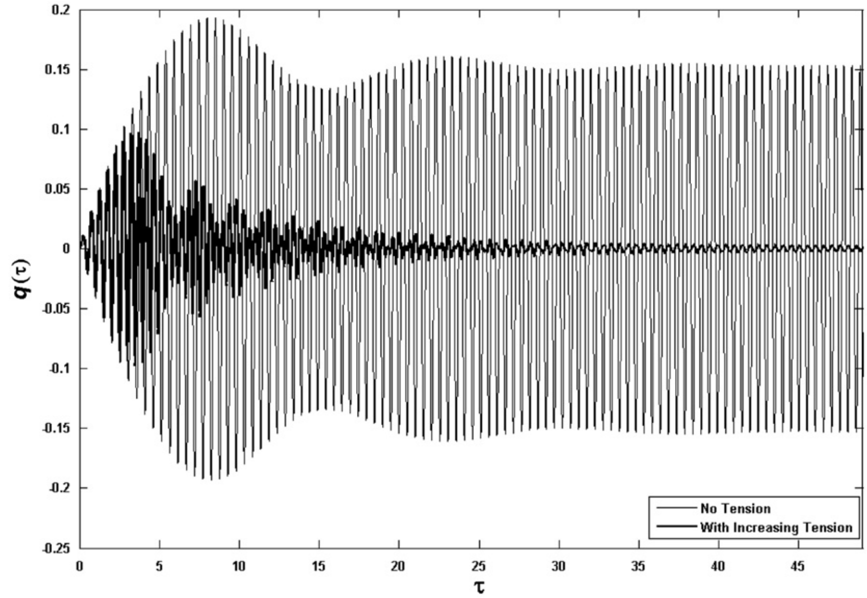

Fig. 14. Performance of the suppression system for a damped beam subjected to harmonic excitation ($\eta=0.01$; $\Omega=10$; $\beta=0.1$).

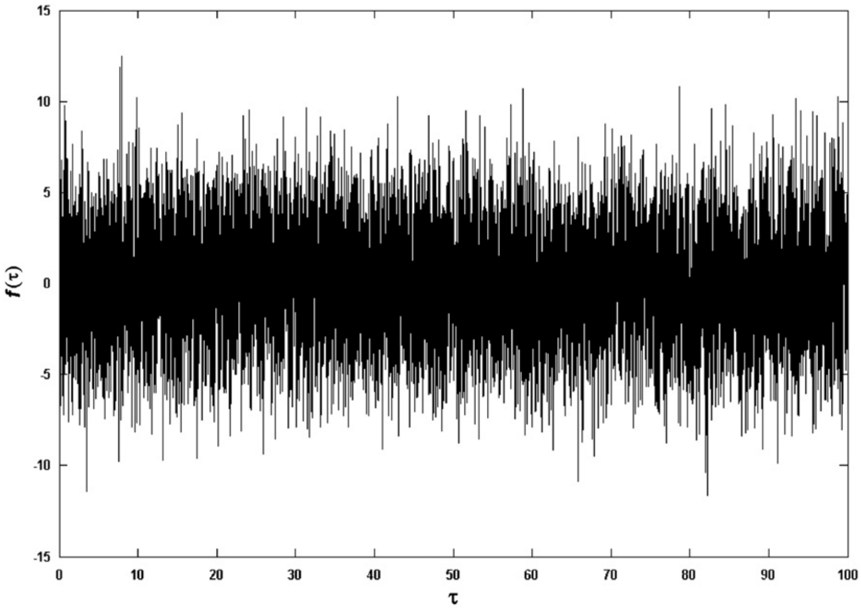

Fig. 15. White noise random excitation force exerted on the rotating beam.

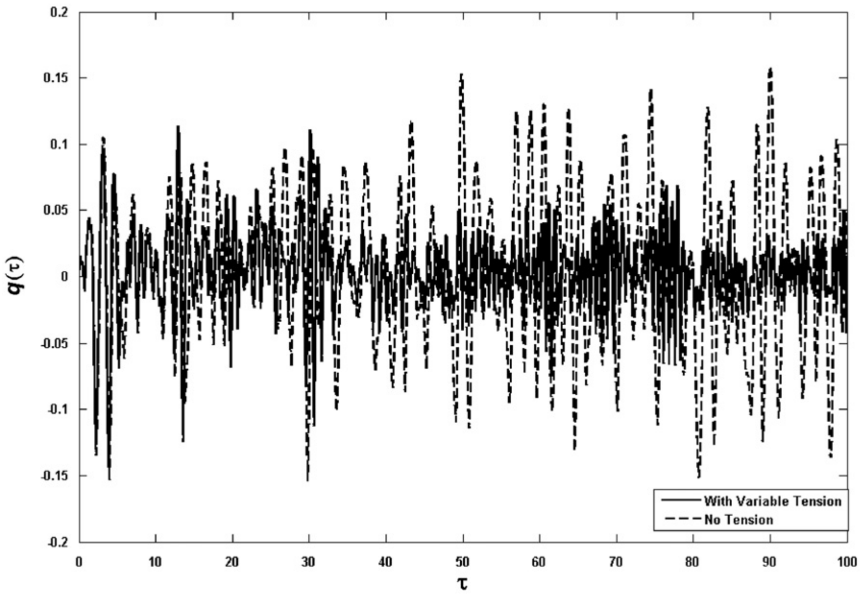

Fig. 16. Performance of the suppression system for an un-damped beam subjected to random excitation ($\eta = 0.0$; $\Omega = 10$; $\beta = 0.1$).

and its amplitude is attenuated due to structural damping. As seen, the amplitude of vibration is quite smaller than the amplitude of uncontrolled beam for both parts of the loading history.

The performances of the suppressed system, for a rotating beam subjected to external excitations, are presented in Figs. 13–16. In the case of a harmonic external force with the excitation frequency equal to the first natural frequency of the linearized system (i.e. 12.31 in our case) results of the numerical simulation are illustrated in Fig. 13 for an un-damped beam. As seen, the tensile force can suppress beatings in the time responses and interestingly leads to a steady state response for the un-damped beam. The time responses for a damped rotating beam subjected to harmonic excitation are illustrated in Fig. 14. It is seen that vibration is damped out when an increasing tensile force is exerted on the system. For a typical random excitation having the white noise nature shown in Fig. 15, numerical results are illustrated in Fig. 16. Existence of the increasing tensile force reduces the root mean square (RMS) of the responses up to 40% from 0.0509 to 0.0309.

4. Conclusion

Vibration suppression of a rotating beam using time-increasing internal tensile force was studied in this paper. The governing nonlinear differential equation of motion, with coupled longitudinal and bending vibrations of the beam, was derived using the Hamilton principle. The first-order approximate solution of the equation of motion was obtained using the Galerkin method in conjunction with the multiple scales method (MSM). Numerical simulations were carried out for various increasing rates for the internal tensile force and performance of the proposed vibration suppression system was investigated. A very close agreement between the simulation results obtained by the numerical integration and the first-order analytical solution was observed. Forced vibrations due to sinusoidal and random excitation with white noise time history were simulated and performance of the suppression system was investigated for externally excited rotating beams. It was found that internal tensile force causes virtual damping in the rotating system and reduces the vibration level with an increase in time. Constant tensile forces have no effect on vibration reduction even if they have large values. Larger values of internal tensile force increasing rates are correlated with larger virtual damping values. The increasing tensile force retains its reduction effect even in the presence of structural damping. The vibration reduction rate is dependent on time rate of increasing tensile force but the reduction factor (final displacement/initial displacement) is dependent on the final value of increasing tensile force. In the case of harmonic external force with the excitation frequency equal to the first natural frequency of the linearized system, an increasing tensile force can suppress beatings in time responses and even lead to a steady state response for the un-damped beam. It can also comprehensively damp out vibrations for a damped rotating beam subjected to harmonic excitation. It was also shown that the suppression system can effectively reduce vibration when it is subjected to a broad-band random excitation.

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