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Multi-objective optimization of ring stiffened cylindrical shells using a genetic algorithm

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ABSTRACT

In this paper, the genetic algorithm (GA) method is used for the multi-objective optimization of ring stiffened cylindrical shells. The objective functions seek the maximum fundamental frequency and minimum structural weight of the shell subjected to four constraints including the fundamental frequency, the structural weight, the axial buckling load, and the radial buckling load. The optimization process contains six design variables including the shell thickness, the number of stiffeners, the width and height of stiffeners, the stiffeners eccentricity distribution order, and the stiffeners spacing distribution order. The real coding scheme is used for representing the solution string, while the generation number-based adaptive penalty function is applied for penalizing infeasible solutions. In analytical solution, the Ritz method is applied and the stiffeners are treated as discrete elements. Some examples of simply supported cylindrical shells with nonuniform eccentricity distribution and nonuniform rings spacing distribution are provided to demonstrate the optimality of the solution obtained by the GA technique. The effects of objective weighting coefficients and bounding values of the design variables on the optimum solution are studied for various cases. The results show that the optimal solution can vary with the weighting coefficients significantly. It is also found that extreme reduction and augmentation in turn in the structural weight and fundamental frequency can be simultaneously achieved by selecting suitable stiffeners' geometrical parameters and distributions. Furthermore, the bounding values of the design variables have great effects on the optimum results.

1. Introduction

Ring stiffened cylindrical shells are important configurations widely used in modern structures such as pressure vessels, submarine hulls, aircrafts, and launch vehicles. Structural weight is one of the most important parameters for designers. Most of these shells are required to operate in a dynamical environment. Therefore, it is very critical to investigate the dynamic characteristics of these shells to develop a strategy for controlling their modal vibration on specific operating conditions and determination of their structural integrity and fatigue life. The natural frequencies of vibration are of special interest to aircraft and launch vehicle designers because of increasing use of sensitive electronic instruments, on-board computers, and gyroscopes, which require isolation from the main structure vibrations. Similarly, buckling load is another

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important parameter in designing stiffened shells particularly under compression. Hence, the optimum design of these structures is the objective for designers to achieve the minimum weight either with maximum natural frequencies or with maximum buckling loads.

The optimization of the objective function may be carried out using both gradient-based and non-gradient-based methods. The gradient methods are fast, but have some limitations such as the need for the continuity of the objective function and a large probability of convergence to a local optimum. This means that the computation starts from a single point narrowing the search domain, and the choice of starting point influences the convergence. In contrast, the non-gradient-based optimization techniques do not need derivative evaluations and the optimization is only performed by the objective function. The genetic algorithm (GA) belongs to these methods.

1.1. *Stiffened shells*

In the considerable literature on this subject, there are two main types of analyses depending upon whether the stiffening rings are treated by averaging their properties over the surface of the shell or by considering them as discrete elements. When the ring stiffeners of equal strength are closely and evenly spaced, the stiffened shell can be modeled as an equivalent orthotropic shell, which is called the smearing method. However, as the stiffener spacing increases or the wavelength of vibration becomes smaller than the stiffener spacing, the determination of dynamic characteristics of the stiffened shell becomes inaccurate. Thus, for a more general model, the ring stiffeners have to be treated as discrete elements. As long as modeling in this respect, it is advantageous to utilize the ring stiffeners with different properties such as nonuniform eccentricity, nonuniform spacing, and varying material distributions.

The free vibration and buckling analyses of stiffened cylindrical shells have been investigated since the 1950s by a number of researchers. Hopmann [1] investigated the free vibration of an orthogonally stiffened cylindrical shell with simply supported ends, analytically and experimentally. The smearing method for stiffeners was used in the analytical investigation. Mikulas and McElman [2] examined the free vibration of eccentrically stiffened simply supported cylindrical shells by averaging the stiffeners properties over the surface of the shells. Egle and Sewall [3] and Mustafa and Ali [4] extended this study to treat stiffeners as discrete elements. Wang et al. [5] and Tian et al. [6] employed the Ritz method for solving the free vibration and buckling problems of cylindrical shells with varying ring stiffener distributions. Bagheri and Jafari [7–9] analyzed the free vibration of simply supported ring stiffened cylindrical shells with nonuniform stiffeners distributions analytically, numerically, and experimentally.

1.2. *Genetic algorithm*

GA is a well-known method for global optimization of complex systems. The initiation of GA can be traced back to the 1950s. However, to the authors' best knowledge, the work done by Holland [10] at the University of Michigan led to GA. Then, the method was extended by other researchers, e.g. Goldberg [11] and Gen and Cheng [12].

Since GA is a well-established optimization technique, only a brief description of its theory is given in this paper. The interested readers are referred to the above-mentioned references for many practical and theoretical aspects of GA. DeJong [13] studied the use of GA in a general function optimization. He showed that the ability of GA to learn from the history and exploitation of the environment provides the basis of its effectiveness in the optimization. Recent years have witnessed an exponential growth in the use of GA in a vast variety of sciences and engineering fields. In composite cylindrical shells, the investigations by Callahan and Weeks [14], Nagendra et al. [15], Messager et al. [16], Park et al. [17], Walker and Smith [18], and Adams et al. [19] for the optimization of layers layout to obtain the maximum strength and minimum weight can be mentioned. However, the optimum design of stiffened cylindrical shells by the GA method has not been studied yet. Only a few studies utilizing the gradient-based optimization methods reported by Patnaik and Sankaran [20] and Rao and Reddy [21] are found.

The present contribution utilizes an analytical method based on GA for the multi-objective optimization of ring stiffened cylindrical shells. In the analytical formulation, the Ritz method is applied and the stiffeners are treated as discrete elements. In the GA technique, the maximization of the fundamental frequency and the minimization of the structural weight are considered as the objective functions and four constraints including fundamental frequency, structural weight, and axial and radial buckling loads are applied. Each chromosome of the population contained six design variables and the real coding scheme is used. Furthermore, the adaptive penalty function is employed for penalizing the infeasible solutions. Some examples of simply supported cylindrical shells with ring stiffeners illustrate the effectiveness of the technique. The effects of the objective weighting coefficients and bounding values of the design variables on the optimum results are investigated.

2. Analytical formulation

Consider a thin uniform cylindrical shell with the uniform thickness h, radius R, length L, mass density ρ, modulus of elasticity E, Poisson's ratio ν, and shear modulus $G=E/2(1+\nu)$, as displayed in Fig. 1. The shell is circumferentially stiffened by N number of the rings, which may be placed either internally or externally. The ith ring stiffener has a rectangular cross

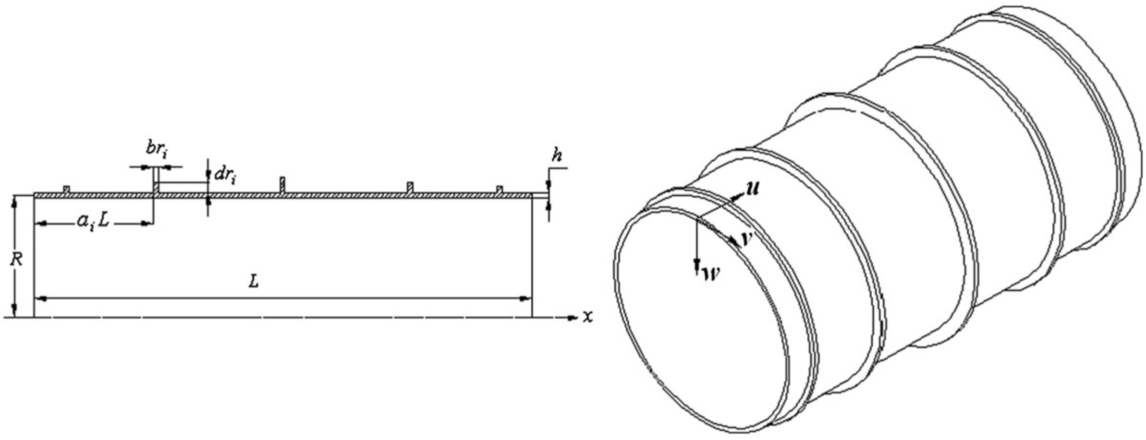

Fig. 1. A ring stiffened cylindrical shell with a nonuniform stiffeners distribution.

section with constant width br_i and depth dr_i, and is located at a distance a_iL from one end of the shell. The spacing and height of the rings may vary along the length of the shell. The material properties of each ring stiffener may differ from those of other ones and also from the parent shell material properties. The ith stiffener's properties are defined as mass density ρr_i, modulus of elasticity Er_i, Poisson's ratio νr_i, and shear modulus Gr_i. The cylindrical shell is subjected to an axis-symmetric lateral pressure P_l and an end uniform axial pressure P_x.

2.1. Shell energy

According to Sander's [22] thin shell theory, the strain energy of stretching and bending of the aforementioned cylindrical shell without stiffeners can be expressed as

$$U=\int_0^L\int_0^{2\pi}\left\{\frac{Eh}{2(1-\upsilon^2)}\left[\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{R^2}\left(\frac{\partial v}{\partial \theta}-w\right)^2+\frac{2\upsilon}{R}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial \theta}-w\right)+\frac{1-\upsilon}{2}\left(\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial u}{\partial \theta}\right)^2\right]\right.$$
$$\left.+\frac{Eh^3}{24(1-\upsilon^2)}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{1}{R^4}\left(\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial v}{\partial \theta}\right)^2+\frac{2\upsilon}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial v}{\partial \theta}\right)+\frac{2(1-\upsilon)}{R^2}\left(\frac{\partial^2 w}{\partial x\partial \theta}+\frac{3}{4}\frac{\partial v}{\partial x}-\frac{1}{4R}\frac{\partial u}{\partial \theta}\right)^2\right]\right\}Rd\theta dx, \quad (1)$$

where u, v, and w stand for the displacements in the longitudinal, tangential, and radial directions, respectively; x and θ stand for the longitudinal and circumferential coordinates, respectively.

Neglecting the effect of rotary inertia (since the shell under discussion is assumed to be thin), the kinetic energy of a cylindrical shell without the stiffeners can be expressed as

$$T=\frac{1}{2}\rho h\int_0^L\int_0^{2\pi}\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right\}Rd\theta\, dx. \quad (2)$$

2.2. Ring stiffener energy

In this research, the geometric characteristics and material properties of each ring may differ from those of other ones. Also, the spacing and the eccentricity of rings may have nonuniform distributions.

The strain energy of the ith ring stiffener with the effects of stretching, biaxial bending and wrapping is given by

$$Ur_i=\int_0^{2\pi}\left\{\frac{Er_iIzr_i}{2}\frac{1}{R+er_i}\left(\frac{\partial wr_i}{\partial x}+\frac{1}{R+er_i}\frac{\partial^2 ur_i}{\partial \theta^2}\right)^2+\frac{Er_iIxr_i}{2}\frac{1}{(R+er_i)^3}\left(wr_i+\frac{1}{R+er_i}\frac{\partial^2 wr_i}{\partial \theta^2}\right)^2\right.$$
$$\left.+\frac{Er_iAr_i}{2}\frac{1}{R+er_i}\left(\frac{\partial vr_i}{\partial \theta}-wr_i\right)+\frac{Gr_iJr_i}{2}\frac{1}{R+er_i}\left(-\frac{\partial^2 wr_i}{\partial x\partial \theta}+\frac{1}{R+er_i}\frac{\partial ur_i}{\partial \theta}\right)^2\right\}d\theta. \quad (3)$$

The kinetic energy of the ith ring stiffener with the effects of triaxial translational inertia and rotary inertia about x and z axes is given by

$$Tr_i=\frac{1}{2}\rho r_i\int_0^{2\pi}\left\{Ar_i\left[\left(\frac{\partial ur_i}{\partial t}\right)^2+\left(\frac{\partial vr_i}{\partial t}\right)^2+\left(\frac{\partial wr_i}{\partial t}\right)^2\right]+(Ixr_i+Izr_i)\left(\frac{\partial^2 wr_i}{\partial t\partial x}\right)^2\right\}(R+er_i)d\theta, \quad (4)$$

where the second moments of areas Izr_i, Ixr_i, the cross sectional area Ar_i, and the torsional rigidity Jr_i are given by

$$Izr_i = \frac{br_i{}^3 dr_i}{12}; \quad Ixr_i = \frac{br_i dr_i{}^3}{12}, \quad Ar_i = br_i dr_i, \quad Jr_i = \frac{1}{3}\left[1 - \frac{192 br_i}{\pi^5 dr_i}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^5}\tanh\frac{n\pi dr_i}{2br_i}\right] br_i{}^3 dr_i, \tag{5}$$

and the eccentricity of the ring stiffener can be defined as

$$er_i = \pm\frac{h+dr_i}{2}, \tag{6}$$

where the signs (+) and (−) represent the external and internal stiffening, respectively.

From geometrical considerations, the relationships between the displacements (ur_i, vr_i, wr_i) of the ith stiffener and the displacements (u,v,w) of the shell at the position of the stiffener are given by

$$\begin{aligned} ur_i &= u + er_i\frac{\partial w}{\partial x} \\ vr_i &= v\left(1+\frac{er_i}{R}\right) + \frac{er_i}{R}\frac{\partial w}{\partial \theta} \\ wr_i &= w. \end{aligned} \tag{7}$$

Substituting Eqs. (5–7) into Eqs. (3,4), the ring stiffener energy can be expressed in terms of the middle surface displacements of the shell.

2.3. External pressure energy

The potential energies of the axis-symmetric radial pressure P_r and the end uniform axial pressure P_x are given by

$$V = -\int_0^L\int_0^{2\pi}\frac{P_r}{2}\left\{\left[\frac{\partial^2 w}{\partial\theta^2}+w\right]w\right\}d\theta\, dx - \frac{\varepsilon P_x}{4}\int_0^L\int_0^{2\pi}\left(\frac{\partial w}{\partial x}\right)^2 R^2 d\theta\, dx, \tag{8}$$

where ε is a scalar indicator, which takes the value of 1 or 0 depending on whether there is an end axial pressure or not, respectively.

Therefore, the energy functional of the ring stiffened cylindrical shell can be written as

$$F = U - T + \sum_{i=1}^{N}(Ur_i - Tr_i) + \mathbf{V}. \tag{9}$$

The following functions are adopted to separate the spatial variables x, θ and the time variable t

$$\begin{aligned} u(x,\theta,t) &= u(x)\sin(n\theta+\omega t) \\ v(x,\theta,t) &= v(x)\cos(n\theta+\omega t) \\ w(x,\theta,t) &= w(x)\sin(n\theta+\omega t), \end{aligned} \tag{10}$$

where n indicates the number of circumferential waves and ω is the circular frequency of vibration.

For generality and convenience, the following nondimensional variables are defined as follows:

$$\overline{F} = \frac{2(1-\upsilon^2)}{\pi hRLE}F, \quad \Omega^2 = \frac{(1-\upsilon^2)\rho R^2}{E}\omega^2, \quad \lambda = \frac{P_x R(1-\upsilon^2)}{Eh} \tag{11}$$

2.4. Geometric boundary conditions

For simply supported cylindrical shells, four kinds of boundary conditions can be designated:

$$S_1: \overline{w} = \overline{v} = 0, \quad S_2: \overline{w} = 0, \quad S_3: \overline{w} = \overline{u} = 0, \quad S_4: \overline{w} = \overline{v} = \overline{u} = 0. \tag{12}$$

2.5. Ritz functions

In view of satisfying the foregoing geometric boundary conditions, the proposed Ritz functions for approximating the displacements are as follows:

$$\begin{aligned} \overline{u} &= \left(\sum_{j=1}^{NS} p_j \overline{x}^{j-1}\right)(\overline{x})^{\Gamma_u^0}(1-\overline{x})^{\Gamma_u^1} = \sum_{j=1}^{NS} p_j \overline{u}_j \\ \overline{v} &= \left(\sum_{j=1}^{NS} q_j \overline{x}^{j-1}\right)(\overline{x})^{\Gamma_v^0}(1-\overline{x})^{\Gamma_v^1} = \sum_{j=1}^{NS} q_j \overline{v}_j \\ \overline{w} &= \left(\sum_{j=1}^{NS} r_j \overline{x}^{j-1}\right)(\overline{x})^{\Gamma_w^0}(1-\overline{x})^{\Gamma_w^1} = \sum_{j=1}^{NS} r_j \overline{w}_j, \end{aligned} \tag{13}$$

Table 1
Powers of Γfor the Ritz functions.

Boundary condition	$S1$	$S2$	$S3$	$S4$
Γ_u	0	0	1	1
Γ_v	1	0	0	1
Γ_w	1	1	1	1

where the powers of Γ are listed in Table 1. The superscripts of Γ, i.e. 0 and 1, denote the cylindrical shell ends at $\bar{x}=0$ and $\bar{x}=1$, respectively.

These forms of the Ritz functions allow easy, exact differentiation and integration. In addition, the more the increase in the number of polynomial sentences NS, the better the convergence to the exact solution can be achieved.

2.6. Equations of motion

Applying the Ritz method (the minimization of the nondimensional energy functional with respect to the Ritz functions coefficients), equations of motion are derived as follows:

$$\left.\begin{aligned} \frac{\partial \overline{F}}{\partial p_j} &= 0 \\ \frac{\partial \overline{F}}{\partial q_j} &= 0 \\ \frac{\partial \overline{F}}{\partial r_j} &= 0 \end{aligned}\right\} \quad j=1,2,\ldots,NS. \tag{14}$$

Substituting Eq. (13) into Eq. (9) and then into Eq. (14) yields the following eigenvalue equation:

$$\left[[\mathbf{K}]+\sum_{i=1}^{N}[\mathbf{Kr_i}]-\lambda[\mathbf{G}]-\Omega^2\left([\mathbf{M}]+\sum_{i=1}^{N}[\mathbf{Mr_i}]\right)\right]\{\mathbf{C}\}=\{\mathbf{0}\}, \tag{15}$$

where $[\mathbf{K}]$ and $[\mathbf{M}]$ are the stiffness and mass matrices of the cylindrical shell, $[\mathbf{Kr}_i]$ and $[\mathbf{Mr}_i]$ are corresponding matrices of the ith ring stiffener, and $[\mathbf{G}]$ is related to the external pressure loading. Also, $\{\mathbf{C}\}=\{p_1, \ldots, p_{NS}, q_1, \ldots, q_{NS}, r_1, \ldots, r_{NS}\}^T$ is the column vector of the Ritz coefficients, and Ω and λ are the nondimensional frequency parameter and the external pressure parameter, respectively.

3. Optimization based on GA

GA is a search and optimization method, which mimics the processing of chromosomes in natural genetics. The algorithm starts with an initial random configuration, which called a population with a fixed initial size or a number of individuals. Each individual in that population is a string or a chromosome, and is defined by optimization variables. The chromosome represents a possible solution to the optimization problem, and its length is dictated by the number of optimization variables and their required precision. Also, each optimization variable has to be bounded by a minimum and a maximum value.

Major components of GA are encoding scheme, fitness evaluation, parent selection, crossover, and mutation operators. Moreover, elitism is a supplementary component in the GA procedure.

The encoding scheme is the first step to transform points from the parameter space into the bit string representations. In this study, the real-coded scheme is used and individuals are coded as vectors of real numbers corresponding to the design variables. There are no encoding and decoding operations involved.

In the second step, the fitness should be evaluated for each design in every generation since it depends on the objective function value.

After evaluating the fitness of each member of the current population, a selection process for individuals to participate in the creation of the next generation is in order. In this study, the tournament selection method is used, where a few members of the population are selected randomly and their fitness values are compared. A number with higher fitness advances to the next generation.

The crossover operator allows the genetic information contained in the best individuals to be combined to form offspring. In this paper, a one-point crossover is applied.

After that the crossover process is performed, mutation process takes place. This step diversifies the population as different areas of the parameters space that can be explored, and also prevents the solution from premature convergence. Nonetheless, this would not occur very often, because GA will perform a random search in the real world.

After selection, crossover and mutation are applied to the initial population, a new population will be formed and the generational counter is increased by one. In this new population, there is a chance that the best solution will be lost due to these genetic operations. Elitism is the method that copies the best chromosome (or a few of the best chromosomes) to a new population and can raise the performance of GA rapidly, because it prevents losing the best found solution.

GA has been well suited to unconstrained optimizations, but most real-world engineering design problems involve constrained optimizations. Traditionally, external penalty functions have been used to convert a constrained optimization problem into an unconstrained problem for GA-based optimizations. This approach, somewhat, requires an arbitrary selection of penalty draw-down coefficient. The performance of the approach largely depends on the penalty parameters employed. In this respect, the adaptive penalty functions are preferred. The goal of an adaptive penalty function is to change the value of the draw-down coefficient during the search, allowing exploration of infeasible regions to find the optimal building blocks, while preserving the feasibility of the final solution.

4. Results and discussion

4.1. Multi-objective optimization problem

In this study, the multi-objective optimization of ring stiffened cylindrical shells is implemented. The fundamental frequency and the structural weight are the objective functions and four constraints including the fundamental frequency, the structural weight, and the axial and radial critical buckling loads are considered. At first, a cylindrical shell without stiffeners, with thickness h_0, radius R and length L with simply supported boundary conditions (S_1-S_1) is chosen. The physical dimensions and properties of the shell model are given in Table 2.

The weight, the fundamental frequency, and the axial and radial buckling loads of the shell are W_0, ω_0, $P_{0\mathrm{axial}}$, and $P_{0\mathrm{rad}}$, respectively. The shell is stiffened with nonuniformly spaced ring stiffeners. The new weight, the new fundamental frequency, and the new axial and radial buckling loads are W, Ω, P_{axial}, and P_{rad}, respectively. The objectives are maximizing the fundamental frequency and minimizing the structural weight without reduction in the fundamental frequency and critical buckling load and an increase in the structural weight, with respect to the original unstiffened shell. As a result, the fitness, the objectives, the constraints, and the adaptive penalty functions namely f, φ, G_κ, and P_κ are defined as follows:

$$f(C)=Max\left\{\varphi(C)-\delta_p\sum_{\kappa=1}^{4}P_\kappa(C)\right\}, \tag{16}$$

$$\varphi(C)=\delta_\omega\varphi_\omega+\delta_\mathrm{w}\varphi_w, \tag{17}$$

$$\varphi_\omega=\frac{\Omega}{\omega_0},\quad \varphi_w=\left(\frac{W}{W_0}\right)^{-1}, \tag{18}$$

$$\delta_\omega+\delta_w=1, \tag{19}$$

$$G(C)=\begin{cases} G_1(C)\rightarrow\left(\dfrac{\Omega}{\omega_0}-1\right)\geq 0\\ G_2(C)\rightarrow\left(1-\dfrac{W}{W_0}\right)\geq 0\\ G_3(C)\rightarrow\left(\dfrac{P_{rad}}{P_{0rad}}-1\right)\geq 0\\ G_4(C)\rightarrow\left(\dfrac{P_{axial}}{P_{0axial}}-1\right)\geq 0,\end{cases} \tag{20}$$

$$P_\kappa(C)=\begin{cases}0 & \text{if } G_\kappa(C)\geq 0\\ \left[\mathrm{gn}\left(\dfrac{1}{1+G_\kappa(C)}\right)\right]^2 & \text{else,}\end{cases}\qquad \kappa=1,\ldots,4 \tag{21}$$

Table 2
Geometrical and material properties of the cylindrical shell.

Characteristics	values
Shell radius R(mm)	82.5
Shell thickness h_0(mm)	2.5
Shell length L(mm)	247.5
Modulus of elasticity E (GPa)	200
Mass density ρ (kg/m^3)	7823
Poisson's ratio ν	0.29

where δ_p stands for a constant denoting the penalty weighting coefficient, and δ_ω and δ_w are the objectives weighting coefficients representing the relative importance of the objective functions, respectively. Besides, gn is the generation number and C is the vector of the design variables, where $C=\{h,N,b,d_{\max},\gamma,\beta\}$. Moreover, γ and β are in turn the eccentricity and spacing distribution orders of the ring stiffeners, respectively.

The location and the depth of the rings corresponding to γ and β values are derived from Eq. (22). Some cases of the nonuniform stiffeners distribution are shown in Fig. 2.

$$\left.\begin{aligned}\bar{x}r_i &= \frac{1}{2}\left(\frac{2i}{N+1}\right)^{\beta}\\ dr_i &= d_{\max}\times(2\bar{x}r_i)^{\gamma}\end{aligned}\right\} \quad \text{for } i=1\ldots N/2. \tag{22}$$

4.2. Case studies

In this section, the effects of the objective functions weighting coefficients and the bounding values of the design variables on the optimum solutions are discussed. Each design variable is bounded by the minimum and maximum values. To demonstrate the effects of bounding values of the design variables on the optimum solutions, five case studies are performed, namely Cases I–V. The cases have different bounding values of design variables, which are presented in Table 4.

A flowchart of the general procedure, which is used for GA, is outlined in Fig. 3. For this flowchart, a computer code is developed in MATLAB software. In this study, the tournament selection method is used and a few members of the population are selected randomly and their fitness values are compared. A number with higher fitness advances to the next generation. The advantage of this method is owing to having less computational effort in comparison to other methods.

The GA control parameters contain the population size (Ps=40), the probability of the crossover (Pc=0.7) with the one-point crossover, and the probability of the mutation (Pm=0.1) with the linear deterministic rule decrement. The probability of the mutation is usually less than 0.01. However, the great value selection for Pm with the linear deterministic decrement allows exploring and exploiting of searching space to find the optimum solution and preventing convergence to local optimum. In this study, uniform and creep mutations are employed. For some initial generations, only uniform mutation can take place, but for the remaining generations, if the maximum fitness value is not changed for the long periods of iterations, both uniform and creep mutations will be applied.

Generally, the population size (Ps) ranges from 10 to 100 and the probability of the crossover (Pc) ranges from 0.6 to 0.9 relative to the optimization problem and the number of variables. Selecting the lower values for Ps may lead to convergence to the local optimum. However, selecting the high values causes the lower rate of convergence of the optimization process. In contrast, selecting the lower values for Pc causes the lower rate of convergence of the optimization process. However, selecting the high values may yield convergence to the local optimum.

Moreover, the elitism and adaptive penalty function are used for better performance of GA. It should be noted that the adaptive penalty function utilizes the generation number-based strategy, which causes the value of the draw-down coefficient to rise with a successive generation. All of the genetic operations are repeated about 200 iterations to obtain a converged solution.

The variations of the objectives values (φ_ω, φ_w) for the different weighting coefficients (δ_ω, δ_w) for Case I are shown in Table 3. Moreover, plots of the obtained optimal objectives are illustrated in Fig. 4. For $\delta_\omega \gg \delta_w$, the increase of the

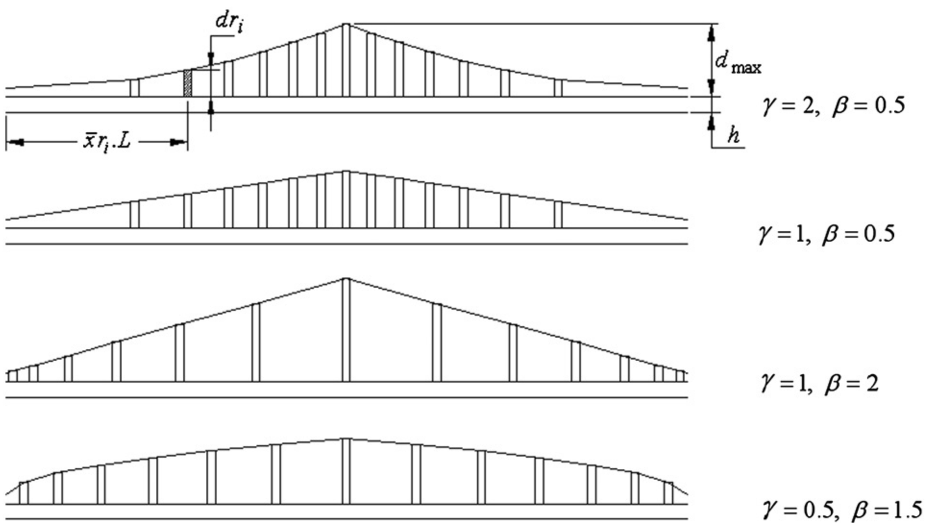

Fig. 2. The nonuniform distributions of the rings spacing and eccentricity.

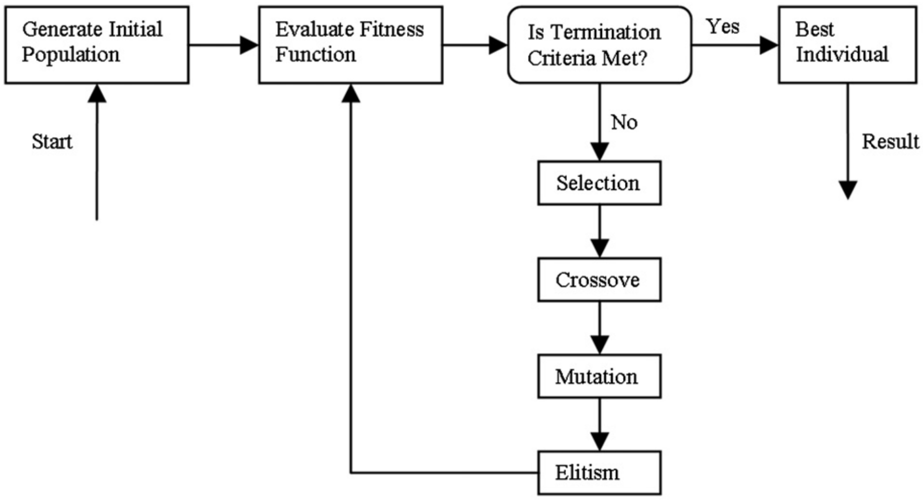

Fig. 3. Flowchart of the proposed GA.

Table 3
The optimal objective values versus the weighting coefficients.

Weighting coefficients		Objective values	
δ_ω	δ_w	φ_ω	φ_w
0.00	1.00	1.1290	1.3460
0.05	0.95	1.1299	1.3452
0.10	0.90	1.1784	1.3426
0.15	0.85	1.1919	1.3372
0.20	0.80	1.1922	1.3369
0.25	0.75	1.1968	1.3359
0.30	0.70	1.2039	1.3327
0.35	0.65	1.2071	1.3306
0.40	0.60	1.2349	1.2976
0.45	0.55	1.2407	1.2908
0.50	0.50	1.2480	1.2811
0.55	0.45	1.3603	1.1172
0.60	0.40	1.3792	1.0879
0.65	0.35	1.3922	1.0670
0.70	0.30	1.4258	1.0148
0.75	0.25	1.4308	1.0072
0.80	0.20	1.4317	1.0060
0.85	0.15	1.4326	1.0049
0.90	0.10	1.4332	1.0032
0.95	0.05	1.4337	1.0023
1.00	0.00	1.4339	1.0002

fundamental frequency is more important than the decrease of the structural weight. Therefore, the significant increment of the fundamental frequency can be seen without a significant decrement in the structural weight. Conversely, for $\delta_\omega \ll \delta_w$, reducing the structural weight is more important than augmenting the fundamental frequency. Therefore, a remarkable drop in the structural weight can be observed with a few increments in the fundamental frequency. However, for the values of the weighting coefficients ranging from 0.4 to 0.6, severe changes in the objectives values can be viewed.

The convergence of the fitness function for Cases I–V corresponding to $\delta_w/\delta_\omega=1$ is shown in Fig. 5(a, b). The optimum values of the design variables obtained by the GA technique are presented in Table 4. Moreover, the corresponding objectives and constraints functions values are given in Table 5.

It should be noted that significant weight reductions of about 21.9%, 17.2%, 13.6%, 12.4%, and 7.5% are obtained for Cases I–V, respectively. In addition, the fundamental frequency increments about 24.8%, 17.6%, 13.2%, 13.4%, and 33.5% for these cases can be observed. Also, Fig. 6(a,b) illustrates a comparison of the natural frequencies of the ring stiffened cylindrical shell with those of the unstiffened shell for the foregoing cases for $m=1$.

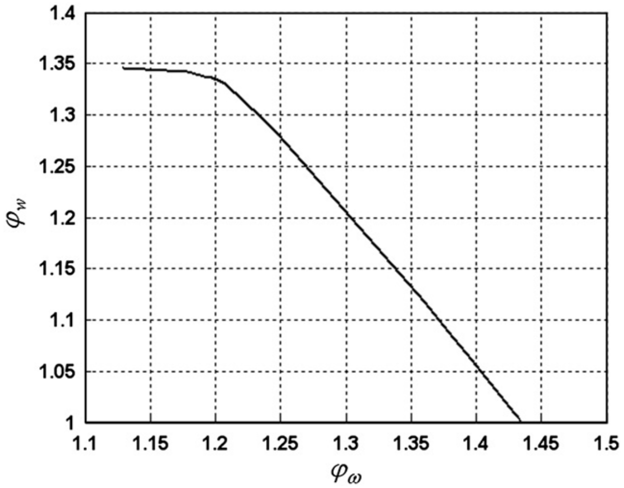

Fig. 4. The effects of the weighting coefficients ratio on the optimum results for Case I.

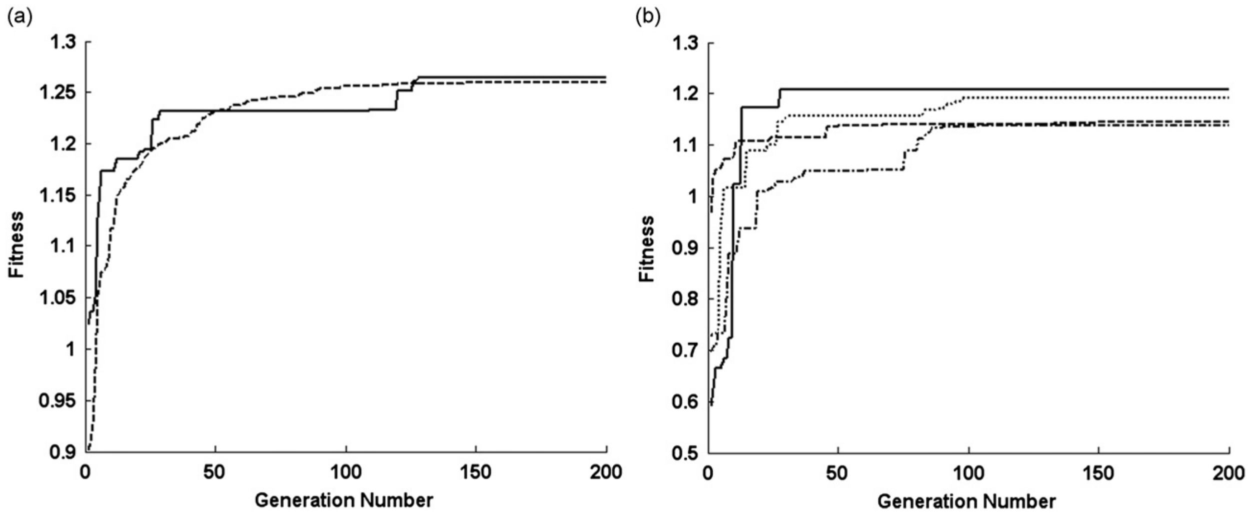

Fig. 5. Convergence of the fitness function for Cases I–V, δ_w/δ_ω=1. (a) Case I (— the best run, - - - - average 20 run). (b) — Case V, … Case II, - - - - Case III, - - - Case IV.

Table 4
The bounding and optimum values of the design variables for Cases I–V.

Case no.	Variable		h	N	b (mm)	d_{max} (mm)	γ	β
I	Bounding values	Min	$h_0/2$	1	2	2	0	0.1
		Max	h_0	30	10	10	2	2
	Optimum values		1.5657	6	2	7.5391	0	1.7670
II	Bounding values	Min	$h_0/2$	1	4	4	0	0.1
		Max	h_0	30	8	8	2	2
	Optimum values		1.4831	6	4	5.7931	0.0003	1.7422
III	Bounding values	Min	$h_0/2$	4	2	2	0	0.1
		Max	h_0	40	5	5	2	2
	Optimum values		1.5486	6	4.8486	5	0	1.5064
IV	Bounding values	Min	$h_0/2$	8	3	3	0	0.1
		Max	h_0	40	5	5	2	2
	Optimum values		1.6848	8	3	5	0	1.4144
V	Bounding values	Min	$h_0/2$	10	2	2	0	0.1
		Max	h_0	40	10	10	2	2
	Optimum values		1.9171	10	2	8.6297	0.5095	2

Table 5
The objective and constraints functions values for Cases I–V ($\delta_\omega=\delta_w=0.5$).

Cases	Objective functions		Constraints			
	$\left(\frac{\Omega}{\omega_0}\right)$	$\left(\frac{W}{W_0}\right)^{-1}$	$\left(\frac{\Omega}{\omega_0}-1\right)$	$\left(1-\frac{W}{W_0}\right)$	$\left(\frac{P_{axial}}{P_{0axial}}-1\right)$	$\left(\frac{P_{rad}}{P_{0rad}}-1\right)$
I	1.2480	1.2811	0.2480	0.2194	0.0009	0.8949
II	1.1765	1.2081	0.1765	0.1723	0.0007	0.7351
III	1.1323	1.1576	0.1323	0.1362	0.0001	0.6811
IV	1.1341	1.1419	0.1341	0.1243	0.0009	0.6712
V	1.3354	1.0816	0.3354	0.0755	0.0010	1.4443

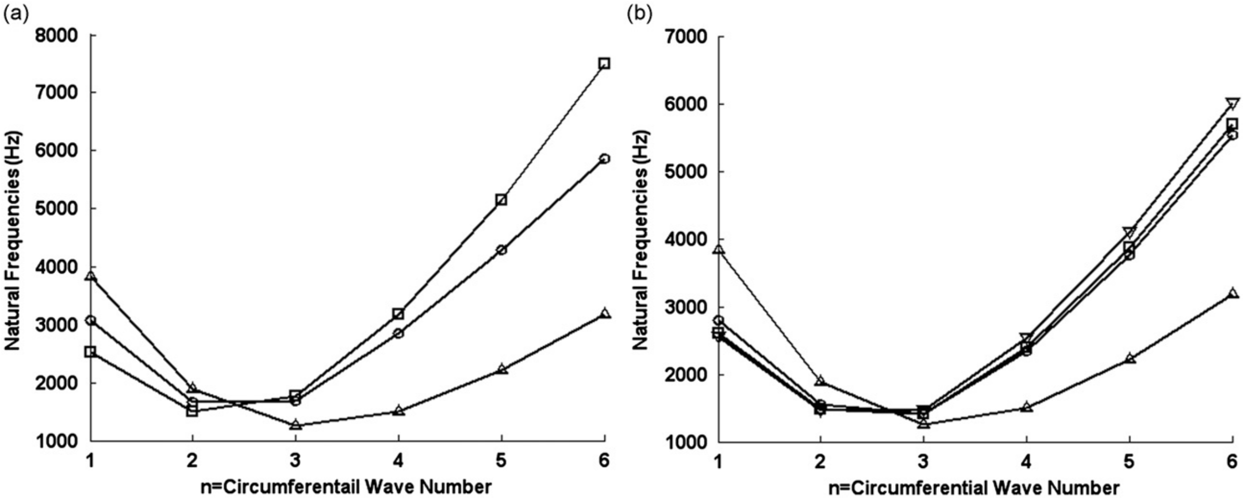

Fig. 6. The effect of the ring stiffeners on the natural frequencies for $m=1$. (a) —□—, Case 1, —○— Case V, —△— un-stiffened; (b) —▽— Case II, —□— Case III, —○— Case IV, —△— un-stiffened.

For Cases I and V, the fundamental frequency occurs at $n=2$ and $m=1$. However, for Cases II, III and IV, it takes place at $n=3$ and $m=1$. Note that integer m refers to the number of longitudinal half waves and integer n refers to the circumferential wavenumber. It should be noted that the fundamental frequency of the unstiffened shell occurs at $n=3$ and $m=1$. Although the maximization of the fundamental frequency and the minimization of the structural weight are the main objectives of the present study, fortunately, the significant increase in the radial buckling loads can be viewed (see Table 5). Moreover, a small augmentation in the axial buckling load is observable. From Fig. 6(a,b), it is seen that the natural frequencies corresponding to $m=1$ and $n=4$–6 are also increased about 55% to 135%, respectively. On the other hand, small decrements in the natural frequencies for $n=1$ and 2 are observed.

5. Conclusions

The GA method is applied to the multi-objective optimization problem of ring stiffened cylindrical shells. The optimum design is analytically obtained by employing the maximum fundamental frequency and the minimum structural weight as the objective functions. Moreover, four constraints including the fundamental frequency, the structural weight, and the axial and radial general buckling loads are considered. The real coding scheme is used for representing the solution string, and the generation number-based adaptive penalty function is applied for penalizing the infeasible solutions. In the analytical solution, the Ritz method is applied and the stiffeners are treated as discrete elements. The effects of the weighting coefficients of the objective functions and the bounding values of the design variables on the optimum solution are studied for various cases.

The results demonstrate that changing the weighting coefficients of the objectives lead to different optimum solutions. Moreover, the bounding values of the design variables have considerable effects on the optimum results. Some variables are sensitive to the minimum bounding values while the others are sensitive to the maximum bounding values. It can be concluded that stiffening a cylindrical shell yields lower structural weight, and the higher natural frequencies and buckling loads. In addition, it is found that the distribution of the stiffeners plays a key role on the magnitudes of the natural frequencies and buckling loads. For instance, the improvement in the natural frequencies and buckling loads for the uniform stiffeners distribution is not as significant as that for the nonuniform distributions.

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