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Influence of the random dynamic parameters of the human body on the dynamic characteristics of the coupled system of structure–crowd

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ABSTRACT

The presence of human occupants may change the dynamic behaviour of structures considerably. While this effect is considered in mechanical engineering (e.g. interaction between driver seat and driver) and biomechanics (potentially damaging effects of vibrations) by using equivalent mass–spring–damper systems for the human body, the design practice in civil engineering still often clings to the so-called mass-only model, i.e. the occupants are considered only as additional masses when analysing the dynamic behaviour of floor slabs and stand structures. Recent research efforts aim to improve this situation by recommending averaged models for the human body. This approach seems to be reasonable for large crowds; however, for smaller groups, the question arises whether the random scatter in the dynamic characteristics of the human body leads to random scatter in the effective natural frequency and the effective damping of the coupled structure–crowd system. Based on a probabilistic model for the dynamic characteristics of the human body, an extensive study is presented in this paper. The key variables are the natural frequency of the bare structure, the ratio of the crowd's mass to the structure mass and the group size. The scatter in the effective dynamic characteristics of the coupled system is revealed by the 90%-confidence interval. Furthermore, the maximum span of the respective bounds is used to identify cases where the averaged model fails to predict the real behaviour of the coupled system.

1. Introduction

The presence of human occupants may change the dynamic behaviour of structures considerably. This can be seen in Fig. 1 which, based on the peak-picking method for the Fourier transforms of measured accelerations for time windows of 13.653 s, displays the change of the natural frequency of a simply supported reinforced slab during the period of filling a stand in a stadium. During this phase, the size of the crowd increases, leading to a more or less constant decrease of the effective natural frequency until the slab is completely occupied.

It is important to note that during the filling process, usually some people leave shortly after their arrival, e.g. to get food and drinks for themselves and others. Furthermore, although the stand has permanent seats, the audience is neither permanently seated during the filling process nor during the match. This unrest of the crowd, i.e. the change of postures

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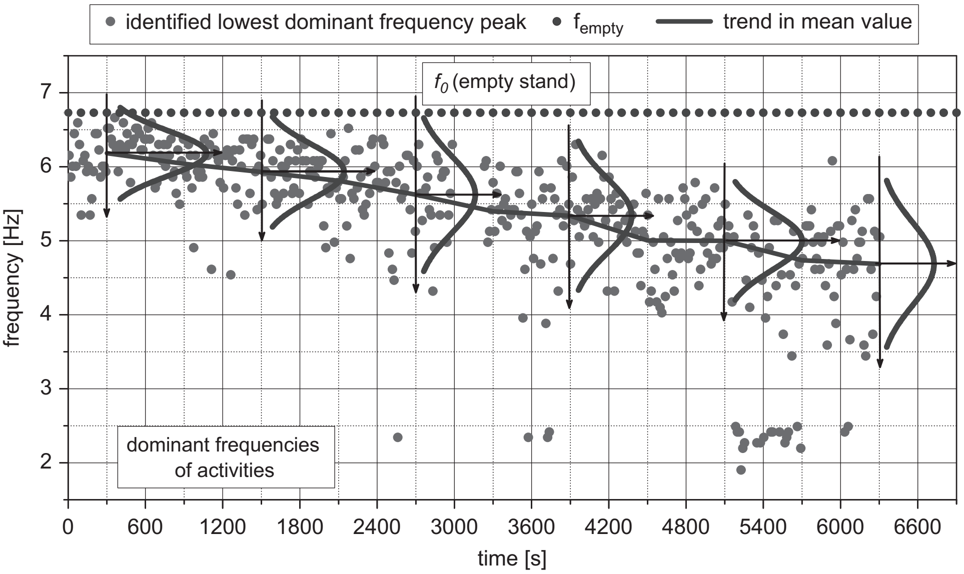

Fig. 1. Change and scatter of the natural frequency during filling of a stand in a stadium.

and activities, leads to a large scatter of the effective natural frequency which is reflected in the change of the probability densities of the identified dominant frequencies sampled over sub-periods of 10 min.

All dominant frequencies in Fig. 1 which are below 3 Hz indicate rhythmic activities of the crowd. These frequencies have not been considered in the basic trend of the natural frequency and the corresponding probability densities. Altogether, the slab with a mass of 15.72 tons provides 63 seats. Considering only the additional mass of the audience with an average weight of 80 kg per person, the frequency is predicted to drop from 6.73 to 5.86 Hz. However, the observed drop of the natural frequency in Fig. 1 is much larger. The drop of the natural frequency below 5 Hz points out the need for an appropriate method to describe the dynamic behaviour of the coupled structure–crowd system for all possible stages during the filling process and during the match. Realistic values of the effective natural frequency are required for both the Serviceability and Ultimate Limit State, since the dynamic loads and the human sensation of accelerations strongly depend on this frequency.

The vibration amplitudes are influenced by the effective damping of the coupled structure–crowd system. It is important to note that the presence of human occupants also influences the damping capacity. This has been documented for civil engineering applications already in the 1960s, e.g. by Lenzen [1], who found that the presence of occupants leads to a decrease in the natural frequency and an increase of the damping. The influence of occupants can be modelled by mass–spring–damper representations of the human body. The first models have been developed since the 1950s [2]; however, the main application field was not civil engineering but mechanical engineering (interaction between driver seat and driver) and biomechanics (potentially damaging effects of vibrations).

Based on measurements of the impedance of the human body, Dieckmann published in 1957 a first approach [3] using a linear single-degree-of-freedom model (sdof) which was able to reproduce approximately the dynamic behaviour of the human body under sinusoidal vertical excitation up to frequencies of 10 Hz. The influence of the second natural frequency around 12 Hz was considered later in a two-degree-of-freedom model (2dof) [4]. The results of the following decades of intensive international research were summarised and condensed in ISO 5982 [5], presenting a deterministic two-degree-of-freedom model for standing and sitting posture. In 1987, ISO 7962 [6] published a four-degree-of-freedom model, specifying also minimum and maximum values of the impedance curves. However, application of dynamic models of the human body in civil engineering remained scarce with only a few exceptions.

For the dynamic analysis of floor responses, in Ref. [7] a sdof-system was used and in Ref. [8] a 2dof and additionally an 11dof system were used to model the increased damping due to human occupants. In regard to the natural frequency, it was generally believed that for the prediction of the natural frequency, the mass-only model was appropriate [9,10], i.e. occupants were considered only as additional masses.

A first more consistent approach was developed by Ellis and Ji [11] who recommended to model passive persons as spring–mass–damper systems and to consider active persons (walking or jumping) as external loads. Strictly speaking, this concept requires at least one further degree of freedom for each passive individual in the crowd, assuming that the dynamic characteristics of the human body are, to some extent, random variables. If the random scatter of the dynamic characteristics of the human body can be neglected and the dynamic behaviour of the structure is modelled by an equivalent modal system a two-degree-of-freedom system can be achieved. Sachse et al. [12] tried to identify an equivalent sdof-system for the dynamic influence of up to five sitting persons based on measurements on a pre-stressed concrete slab with a weight of 15 tons and a lowest natural frequency of 4.51 Hz when empty. Sim adopted this idea [13] and obtained equivalent 2dof-systems for the coupled system for standing or sitting posture of the audience. The dynamic behaviour of the crowd was modelled as the average apparent mass, considering the raw data of the dynamic characteristics of the human body of 60 sitting [14] or 12 standing persons [15]. For seated posture, Sim specified separate models for men, women and children, while for standing posture, only a model for men was given. With the averaged models for only male persons, the change of the lowest natural frequency and the effective damping was determined for a range of crowd to structural mass ratios varying from 5% to 40%.

A further refinement of the basic approach by Ellis and Ji has been published recently in Ref. [16]. Active persons are modelled as a self-generated dynamic action within a dynamic system representing the active human body. This leads to additional damping effects also for the active persons. The model has been calibrated to the activity bobbing, which is characterised by permanent contact of the active person to the ground. For jumping, however, there is a distinct flight phase, i.e. the human body (or the corresponding equivalent dynamic model) is not in contact with the structure. So far, there is no indication that the body of a jumping person contributes to the effective damping. Hence, for a conservative approach, the model proposed by Ellis and Ji is recommended for jumping crowds. Dougill's model has been adopted in the recommendations of the Joint IStructE working group [17]. The document also recommends deterministic values for a single-degree-of-freedom model for passive persons.

The approach by Sim and the refined approach by Dougill both suffer from the basic shortcoming that only averaged input values are used, i.e. the basic dynamic characteristics of the human body are the same for all individuals and, in the case of Dougill's approach, the loads generated by each individual are the same. Especially for smaller groups of passive persons, the large scatter in the individual dynamic characteristics of the human body may lead to scatter in the position of the resulting lowest frequency (effective natural frequency) and the amplitude of the effective damping.

The refined analysis requires a probabilistic model of the basic dynamic characteristics of the human body for the two postures seating and standing. In Section 2, the basic features of the probabilistic model are explained. As input data, the results by Griffin [14,15] are used. In the further analysis, each individual is considered with two degrees of freedom. The random characteristics of each individual are obtained by Monte-Carlo simulation. Finally, the dynamic behaviour of a specific deterministic structure coupled with the random crowd is analysed in terms of the dynamic amplification function. The randomness in the dynamic behaviour of the coupled system is evaluated based on 10,000 simulations for each combination of the basic variables which are the natural frequency of the empty structure, the crowd size, the mass ratio (defined as the ratio of the crowd's total mass to the mass of the empty structure) and the posture. While in Sim's study only a variation of the mass ratio from 0 to 0.4 has been considered, the actual study analyses the range from 0 to 1, thus covering also pre-stressed structures.

2. Dynamic characteristics of the human body

An efficient and simple description of the dynamic characteristics of the human body has been obtained by Griffin, modelling the human body as a two-degree-of-freedom system for sitting [14] and standing [15] persons. In Ref. [14] the test subjects were exposed to random vertical vibrations of 1.0 m/s^2 rms in the frequency range between 0.25 and 20 Hz, whereas in Ref. [15] the frequency range was from 0.5 to 30 Hz and the subjects were exposed to five random vertical vibration amplitudes from 0.125 to 2.0 m/s^2 rms. Griffin offered alternative models for each posture; Fig. 2 shows the two corresponding models which were selected for the further analysis since they deemed most applicable for this investigation. Individual values are provided for the stiffness parameters k_1 and k_2, for the damping parameters c_1 and c_2,

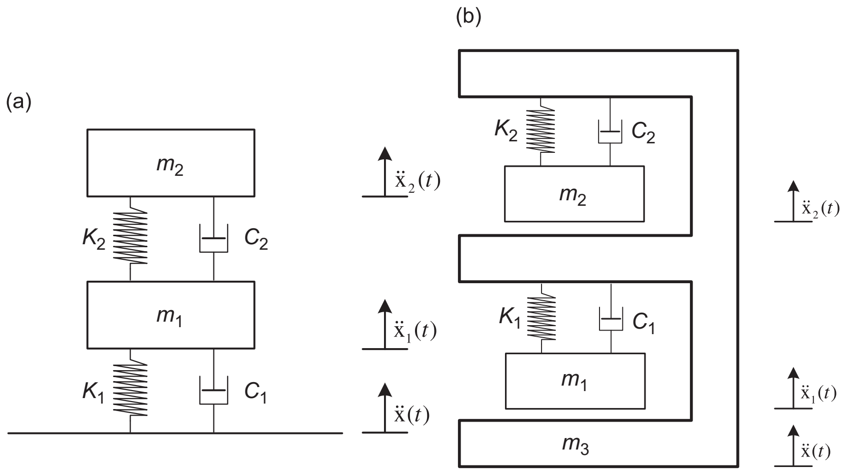

Fig. 2. Basic dynamic systems used for the human body: (a) standing posture and (b) sitting posture.

and the corresponding masses. While the model for sitting persons has a rigid support with an additional mass m, the model for standing persons corresponds to a simple two-degree-of-freedom system.

In the following, only adult persons are considered, thus reducing the original ensemble for sitting from 60 to 48 persons. It is important to note that the raw data by Griffin do not allow modelling the full variation range of the body weight. In fact, the ensemble for standing persons, who are all male, leads to a mean value of only 74 kg and a variation coefficient of only 10%, while observations in Germany suggest a mean value of 82.6 kg for males with a corresponding variation coefficient of 15.4% [18]. This problem is solved by transferring the body masses from the Griffin study into dimensionless mass ratios. Furthermore, the corresponding parameters for stiffness k_i and c_i are transferred to frequencies f_i and damping ratios D_i using the following relations:

$$f_i = \frac{1}{2\pi}\sqrt{\frac{k_i}{m_i}}$$
$$D_i = \frac{c_i}{4\pi f_i m_i} \tag{1}$$

In the further analysis, for the body weights of female and male persons, a probabilistic model is used which is based on a survey of the Robert Koch-Institute RKI [18]. In Fig. 3, the observed probability densities of the body weight for German adult persons are shown in comparison to the normal distribution. Clearly, body weights are not normally distributed.

It is important to note that introducing an appropriate probabilistic model for the body masses with the full range of variation is going to lead to larger variation coefficients for k_i and c_i than those that are obtained from Griffin's raw data. The final value of the damping parameter c_1 for instance is influenced by the scatter in D_1, f_1, m_1/m_{tot} and m_{tot}. While the raw data lead to a variation coefficient of 33% for c_1, the final probabilistic model leads to a variation coefficient of 42%. In Table 1, the average values for the basic parameters are summarised.

The random dynamic characteristics of the human body are analysed based on order statistics to identify the probability distributions. The observed values are sorted in ascending order. For each position in the sorted list, the non-exceedance probability is estimated by dividing the rank r by N+1, where r is the position in the list and N is the ensemble size. For biomechanical reasons it seems reasonable to assume that the probability distributions are limited at both ends, i.e. there should be an upper value which cannot be exceeded and all values should be at least positive. The Beta-Distribution offers these basic features, i.e. it has a lower bound a and an upper bound b. Its probability density is given with the following expression:

$$f(x) = \frac{(x-a)^{r-1}(b-x)^{t-1}}{(b-a)^{r+t-1}B(r,t)} \quad \text{for} \quad a \le x \le b \tag{2}$$

where $B(r,t) = (\Gamma(r)\Gamma(t))/(\Gamma(r+t))$ and Γ is the Gamma function

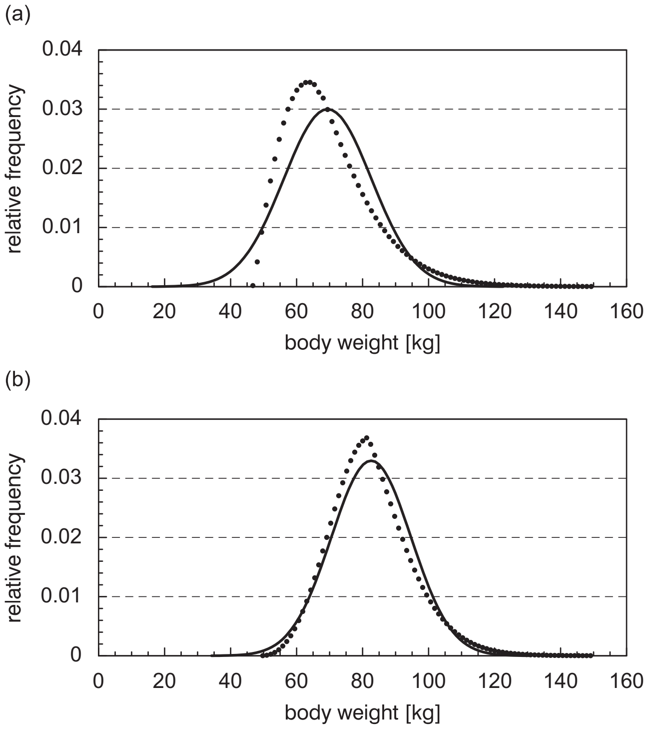

Fig. 3. Probability density of the body weight of female and male persons: (a) female and (b) male (● observed values, — fitted normal distribution).

Table 1
Mean values of the dynamic characteristics of the human body.

Posture	f_1 (Hz)	f_2 (Hz)	D_1	D_2	m_1/m_2
Sitting [N=48]	5.222	9.220	0.345	0.318	3.884
Standing [N=12]	6.025	13.463	0.379	0.337	1.664

For larger values of the exponents r and t, the Gamma function values, which are required for the calculation of $B(r, t)$, may become very large, thus leading to numerical problems. These problems can be solved by using series expansion considering that

$$\Gamma(x+1)=x\Gamma(x) \tag{3}$$

The number of factors is the same in the numerator and the denominator, leading to

$$B(r,t)=\prod_{i=1}^{n_r-1}\frac{(r-i)}{(r+t-i)}\frac{\Gamma(r-n_r+1)}{(r+t-n_r+1)}\prod_{j=1}^{n_t-1}\frac{(t-j)}{(r+t-n_r-j)}\frac{\Gamma(t-n_t+1)}{\Gamma(r+t-n_r-n_t+1)} \tag{4}$$

where n_r is the next integer number larger than r, n_t the next integer number larger than t.

There is no general closed form solution for the cumulative probability distribution, i.e. the cumulative probability distribution has to be obtained by numerical integration

$$F(x)=\int_a^x \frac{(u-a)^{r-1}(b-u)^{t-1}}{(b-a)^{r+t-1}B(r,t)}\,\mathrm{d}u \tag{5}$$

The mean value and the standard deviation of the Beta-Distribution are given as

$$m=a+(b-a)\frac{r}{r+t}, \quad \sigma=\frac{b-a}{r+t}\sqrt{\frac{rt}{r+t+1}} \tag{6}$$

With the two limits a and b known, the application of the method of moments, i.e. using the ensemble mean value and standard deviation, leads to a closed form solution for the exponents r and t. If, however, the limiting values are unknown, only trial-and-error in combination with an appropriate fitting method leads to the identification of the four parameters. For the present study, a least-squares-error fitting is used.

In Fig. 4, the observed traces of the non-exceedance probability for the parameters f_1, f_2, D_1 and D_2 are compared to the identified Beta-Distributions using the data for sitting persons. Generally, the identified distributions form a fair basis of the probabilistic analysis. Further parameters of the complete model are the mass ratios m_1/m_2 and m_3/m_{tot}, with m_{tot} being the total body mass.

3. Influence of the random body characteristics on the natural frequency and the effective logarithmic damping decrement

It is reasonable to assume that for very large groups of passive persons the average characteristics of the human body may be used to describe the influence of occupants on the dynamic behaviour of the coupled system. However, there are a lot of practical design cases where this scenario does not apply. For assembly halls, only wide-span floors meet the demand of a large influencing area which suppresses the individual influences. If, on the other hand, floor systems have smaller influence areas for the load bearing structures (e.g. ribbed ceiling), the influence of the individuals may become important. In case of stand structures, the following three groups of structural systems can be distinguished:

1. The first group covers all systems where the slabs are much stiffer than the supporting trusses. In this case, the number of persons forming the crowd is large, and using average characteristics of the human body seems to be a reasonable approach to estimate the influence of the passive audience.
2. The second group covers multiple-row supporting slabs, which may have smaller frequencies than the supporting trusses. The crowd size depends on the number of rows per element, the span, and the basic function of the stand, i.e. stand with or without permanent seats. Depending on the total number of passive persons, in this case, average- or random characteristics of the human body may used.
3. The third group are single-row supporting slabs. This type of system is only able to accommodate a small group of persons, limited by the dimensions of the slab and therefore random dynamic characteristics of the human body must be used.

It is important to note that not the total number of persons determines the effects on the dynamic behaviour of the coupled structure–crowd system but the total number of passive persons. Hence, for different activity rates, the effective damping of a crowd varies considerably. The higher the activity rate becomes, the larger will be the randomness in the effective damping and in the position of the effective natural frequency.

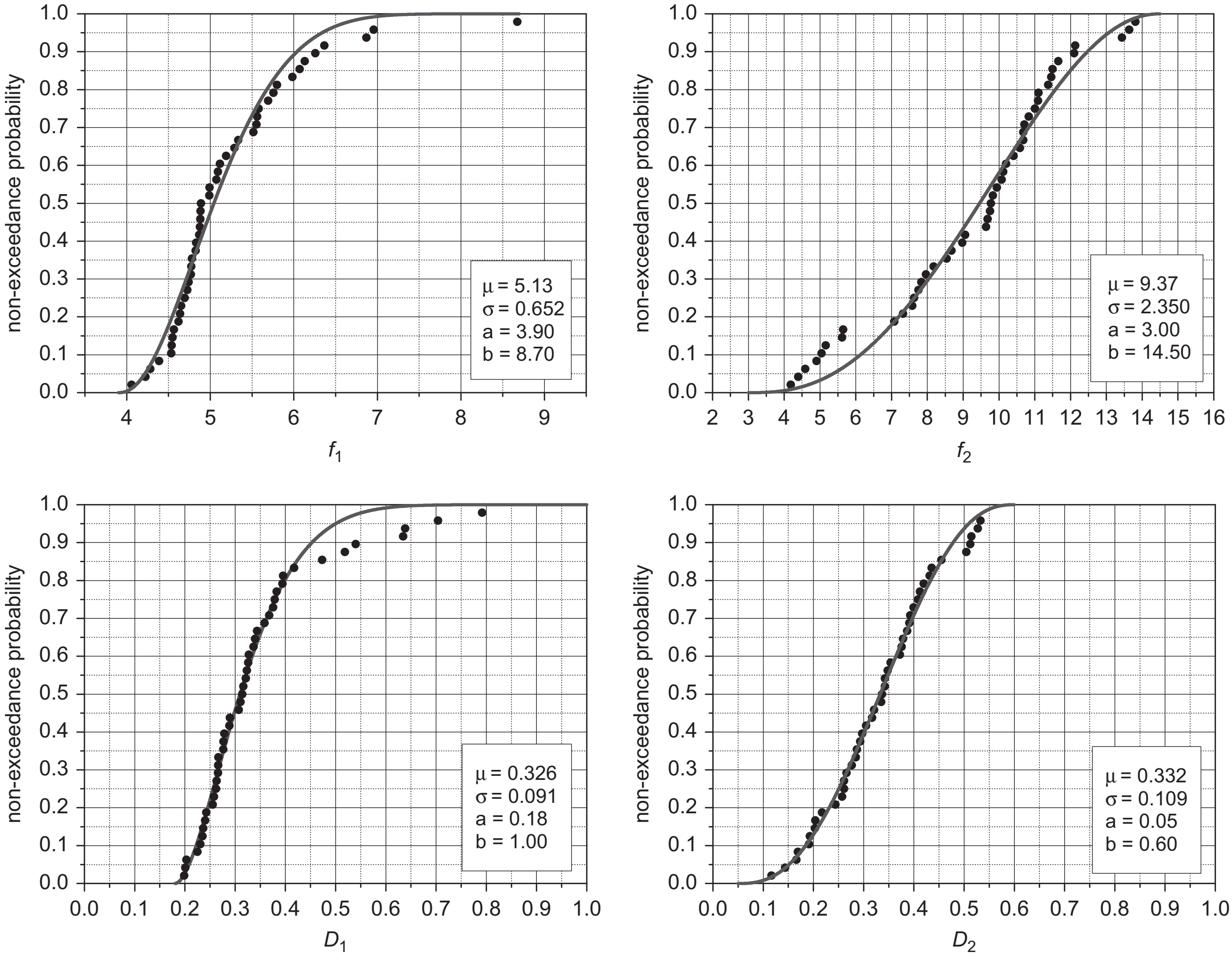

Fig. 4. Comparison of the observed non-exceedance probabilities to fitted Beta-Distributions for different random dynamic characteristics of the human body.

In the further analysis, the structure is considered as a single-degree-of-freedom system with a corresponding mode shape Φ. For each person, two additional degrees of freedom are introduced. Based on the position of the person, a weighting coefficient transfers the dynamic parameters k_i, c_i and m_i into effective dynamic parameters k_i^*, c_i^* and m_i^* as follows:

$$\begin{aligned} k_i^* &= \Phi(\xi_i)^2 k_i \\ c_i^* &= \Phi(\xi_{\mathrm{i}})^2 c_i \\ m_i^* &= \Phi(\xi_{\mathrm{i}})^2 m_i \end{aligned} \tag{7}$$

where ξ_i is the normalised control variable$=x_i/L$, L is the length of the slab, x_i the position of person i along the slab.

In Fig. 5 the equivalent model of the coupled system structure–crowd is shown. The differential equation of motion is solved in the frequency domain, leading to the dynamic amplification function for stationary harmonic excitation. Additionally, for each simulation, the position of the largest amplitude is sampled as the effective natural frequency of the coupled system. The maximum amplitude of the dynamic amplification function is translated into the effective logarithmic damping decrement

$$\delta_{\mathrm{eff}} = \frac{\pi}{V_{\max}} \tag{8}$$

δ_{eff} is the effective logarithmic damping decrement, $V_{\max}$ the maximum amplitude of the dynamic amplification function.

The dynamic behaviour of the coupled system also depends on the structural damping capacity. To date, an analytical method to predict the effective damping for a structure under design does not exist. Therefore, codes usually recommend lower limits for the effective damping which are generally believed to be exceeded in the real structure. In the scope of the present study, the damping ratio D is set to 0.016, corresponding to the recommended value for reinforced concrete in the German codes [19].

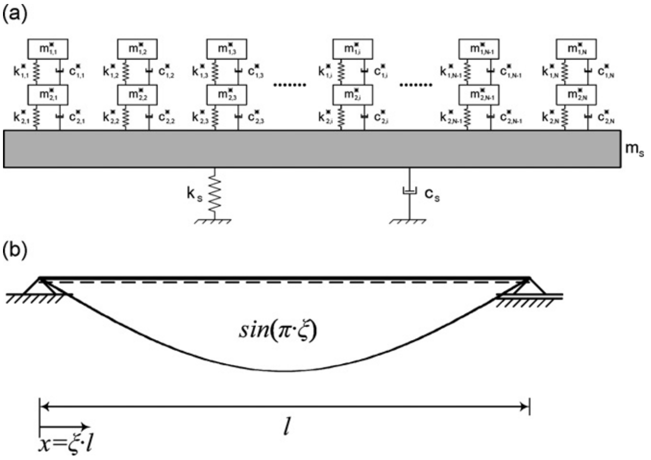

Fig. 5. Equivalent coupled system structure–crowd (a) and example of mode shape of the structure (b).

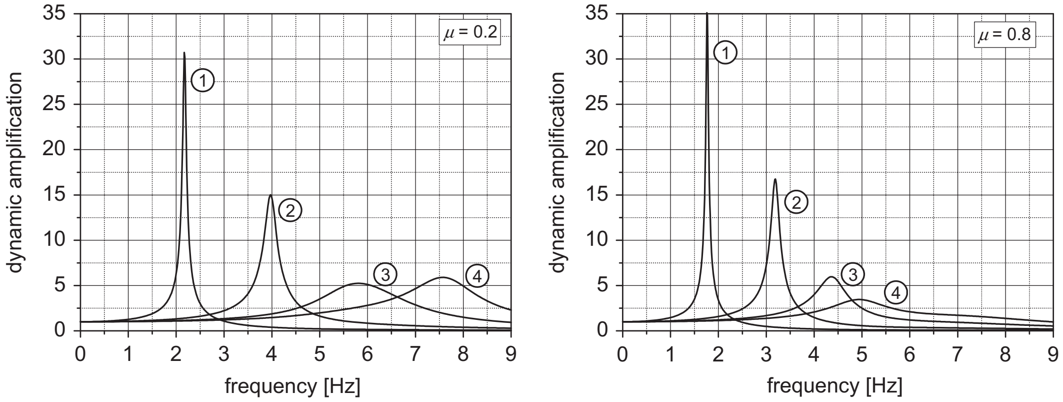

Fig. 6. Examples of the dynamic amplification function for the coupled system structure–crowd: ①- f_{empty}=2.4 Hz; ②- f_{empty}=4.5 Hz; ③- f_{empty}=6.6 Hz and ④- f_{empty}=8.0 Hz.

In the complete study, the mass ratio μ is varied from 0.1 to 1.0 in 0.1 steps and the natural frequency f_0 of the empty slab from 2.4 to 8.0 Hz in 0.1 steps. For each basic configuration (sitting or standing posture, group size, mass ratio, natural frequency of the empty structure), 10,000 runs were performed. For each basic value on the frequency axis, the further statistical analysis leads to the 5% and 95% fractile values of the random dynamic amplification. Similar statistics are performed for the effective natural frequency of the coupled system and the effective damping.

Before a detailed discussion of the randomness in the dynamic behaviour is presented, first, the basic influence of a crowd on structures with different natural frequencies is studied. Only for this step, the average values of the dynamic characteristics of the human body are used. In Fig. 6, the dynamic amplification functions are shown for four exemplary frequencies of the empty structure (2.4, 4.5, 6.6 and 8.0 Hz) for two mass ratios (μ=0.2 and 0.8). If the natural frequency of the empty structure f_{empty} is well below the first natural frequency of the human body, the crowd mainly acts as an additional mass, leading to a decrease in the effective natural frequency. Considerable damping and a decrease of the effective natural frequency are obtained if the natural frequency of the empty structures approaches the first natural frequency of the human body. A further increase of f_{empty} leads to very large effective damping values; however, the shift of the effective natural frequency is reduced. This basic behaviour is more or less independent of the mass ratio.

A more detailed analysis is shown in Fig. 7. The mass-only model predicts an increasing frequency drop for increasing mass ratio with $(1/1+\mu)^{1/2}$. The more realistic model shows that the frequency drop also depends on the frequency of the empty structure. The frequency drop increases with increasing frequency for mass ratios larger than 0.4 in the complete analysed frequency range and for mass ratios smaller than 0.4 up to a natural frequency of the empty structure of 6 Hz. An increase of the effective damping is obtained only for structures with a natural frequency larger than 3 Hz when empty. Beyond 4 Hz, the increase of effective damping exceeds 50%, the effective damping is doubled beyond 4.5 and reaches more than 300% of the structural damping for frequencies beyond 5 Hz.

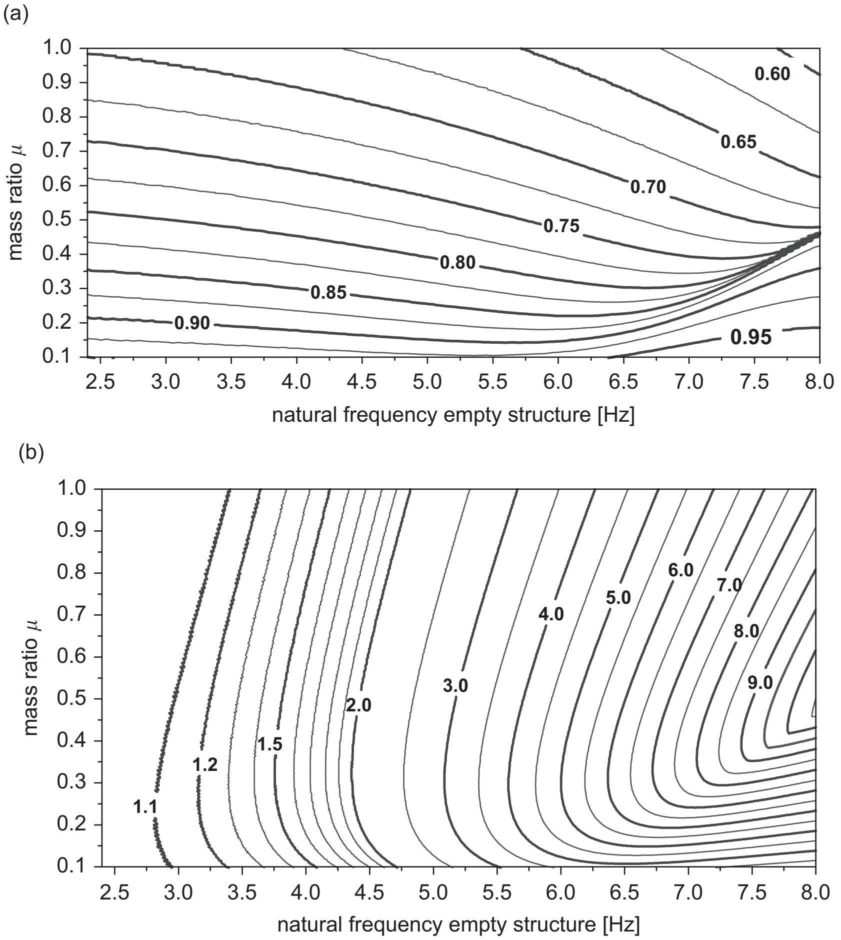

Fig. 7. Relative change of the effective natural frequency due to the presence of crowd and influence on the damping: (a) $f_{\mathrm{eff}}/f_{\mathrm{empty}}$ and (b) $D_{\mathrm{eff}}/D_{\mathrm{empty}}$.

In Fig. 8 the ratio of the 95%-fractile to the 5%-fractile is shown for the effective natural frequency and the effective damping. For most combinations of f_{empty} and μ the scatter of the effective natural frequency remains negligibly small. However, there is a zone where the scatter becomes large. This zone can be marked by a triangle spanned by the coordinates (6.5, 0.2), (8.0, 0.4) and (8.0, 0.65). In this zone, the 3-D structure resembles the form of a shark fin. Along the edge, the differences rapidly increase and exceed values of 40% for larger frequencies. The effective damping shows considerable influences of the scatter in the dynamic characteristics of the human body for almost the whole analysed range of frequencies and mass ratios. Major influence is obtained from the frequency of the empty structure; the mass ratio is of smaller importance. For frequencies between 5 and 6.5 Hz the differences exceed 30%. Beyond this range, the differences become smaller again and the mass ratio gains increasing importance, leading to a modulated 3-D structure. Altogether, in this range the differences are larger than 20%.

Fig. 9 shows the ratio of the predicted frequency obtained with the mass-only model to the more realistic effective frequency. Except for a small range of large natural frequencies of the empty structure and small mass ratios, the mass-only model overestimates the effective natural frequency. For mass ratios larger than 0.55, the overestimation increases with increasing natural frequency f_{empty}. For mass ratios below 0.55, the mismatch depends on both f_{empty} and μ.

In Fig. 10, the randomness in the dynamic amplification function is analysed for a slab which provides 20 seats. The basic static system is a simple beam, i.e. the mode shape follows the half-sine wave. The mass ratio is set to μ=0.2, the natural frequency of the empty slab is assumed to be 4.5 Hz. The group size of 20 persons is not large enough to suppress random influences. Even for the total weight of the group, there is considerable scatter to be expected. For the centred 90% confidence interval, the total weight of 20 persons ranges from 1440 to 1770 kg. The scatter increases for the effective weight, which is the sum of the individual weights multiplied by the respective weighting coefficient corresponding to the position. The corresponding 90% confidence interval predicts the effective weight in the range from 715 to 895 kg. The different graphs show the situation for different ratios of passive to active persons, namely 20/0, 15/5, 10/10 and 5/15. For all situations with less than 20 passive persons, the position of the passive person has been randomized as well. Beside the curve for the empty slab, all graphs show a band which corresponds to the 95% and 5% fractile value of the respective

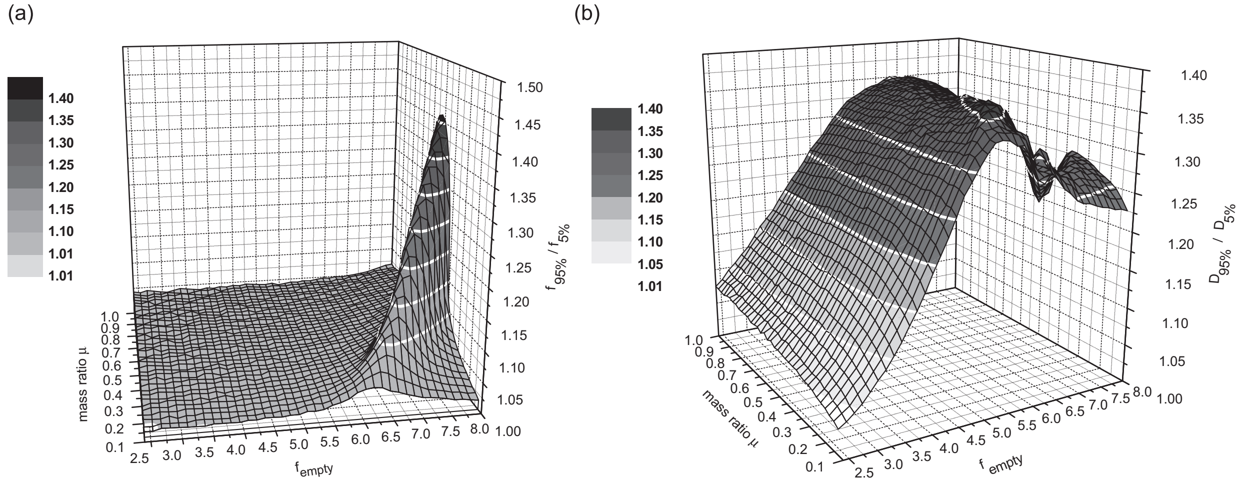

Fig. 8. Ratio of the 95%-fractile to the 5%-fractile for the natural frequency and the effective damping: (a) ratio of the natural frequency and (b) ratio of the effective damping.

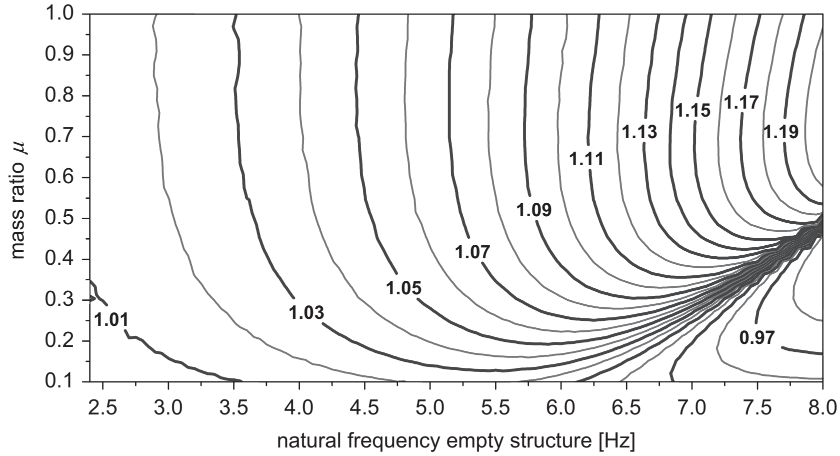

Fig. 9. Ratio of effective natural frequency for the mass-only model and the realistic approach.

amplitudes of the dynamic amplification function, i.e. the band marks the centred 90%-confidence interval of the dynamic amplification function under crowd influence. The differences are not negligible, i.e. the randomness in the individual parameters of the dynamic characteristics of the human body leads also to a randomness in the effective dynamic behaviour of the coupled system structure–crowd. As measure of the randomness, the largest ratio of the 95% to the 5% fractile may be used. For the combinations '20 passive persons and no active person' and '15 passive and 5 active persons', respectively, this ratio is almost the same with a value of 1.37. A further decrease of the passive persons to 10 and 5 leads to an increase in the differences between the 95% and 5% fractile value with ratios of 1.49 and 1.64.

Although the IStructE-model [17] is, strictly speaking, only applicable for crowds of more than 50 people, it is interesting to compare the predictions of this deterministic model to the results for the situation with 20 passive persons. It is important to note that compared to Griffin's results, the IStructE-model specifies a fairly large value for the damping capacity of the human body which is well above the observed mean value. For a group size of 20 passive persons, the IStructE-model is still in the range of the natural scatter, i.e. the IStructE-model corresponds to the 5%-fractile. For increasing group size, the range of the natural scatter becomes smaller. Consequently, the IStructE-model becomes more and more unrealistic. For 40 passive persons, the deterministic approach of the IStructE-model corresponds to the 1%-fractile value, for 80 persons it corresponds to the 0.1%-fractile value. It is important to note that the overestimation of the damping capacity leads to an underestimation of the structural responses. A better approach for large groups can be obtained with a reduced damping value of say 0.35 instead of 0.4.

The question arises from which group size on the individual influences may be neglected. A general answer to this question requires a huge number of simulations for all combinations of f_{empty} and μ which may become important in design situations for a range of group sizes from 10 to say 300 persons. This is beyond the scope of this paper. Therefore, in

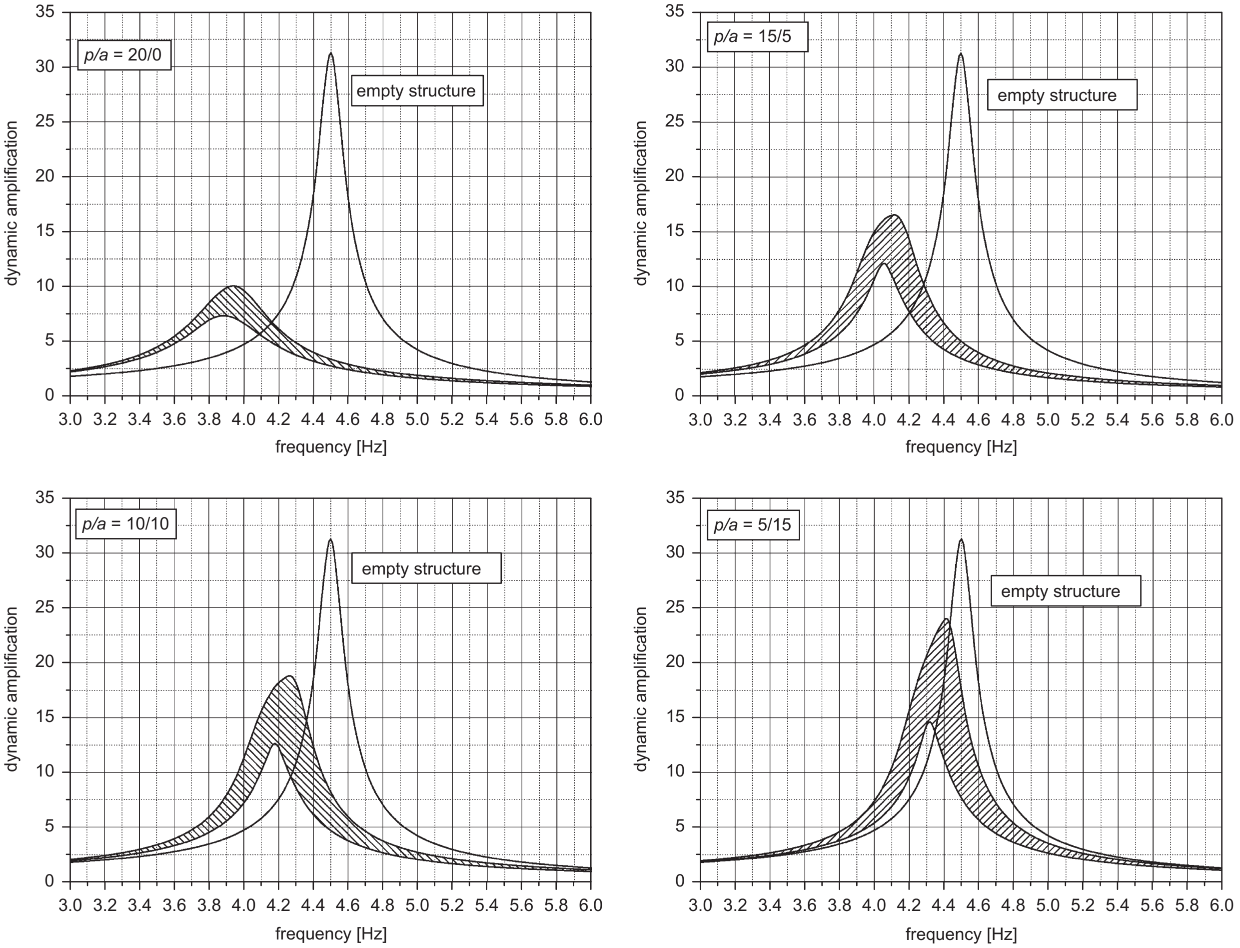

Fig. 10. 90%-confidence interval of the dynamic amplification factor for different number of passive persons (p) and active persons (a) for mass ratio μ=0.2 and f_{empty}=4.5 Hz.

Table 2
Scatter in the effective damping and the natural frequency for different group sizes for a structure with D_0=0.016, μ=0.2 and f_{empty}=4.5 Hz.

	Group size N						
	10	20	40	80	160	240	320
$D_{95\%}/D_{5\%}$	1.40	1.27	1.19	1.13	1.09	1.08	1.06
$f_{95\%}/f_{5\%}$	1.03	1.02	1.01	1.01	1.01	1.01	1.01

a first step, the influence of the group size is studied in this paper only for the combination f_{empty}=4.5 Hz and μ=0.2. The corresponding results are summarised in Table 2. For this combination, there is only small influence of the scatter in the dynamic characteristics of the human body on the effective natural frequency. However, the influence on the effective damping is large. For groups of 10 persons, the ratio of the 95–5%-fractile is 1.4, slightly dropping to 1.27, 1.19 and 1.13 for increasing group size of 20, 40 and 80 persons, respectively. Only for groups larger than 150 persons, the influence becomes smaller than 10%.

In a second step, the study is extended to four distinct frequencies (2.4, 4.5, 6.6 and 8 Hz) and four distinct group sizes (10, 20, 40, 80) with varying mass ratios from 0.1 to 1.0. As a basic measure of the influence of the randomness, the ratios are used between the 95%- and the 5%-fractile for the effective damping and the effective natural frequency. The analysed posture in all cases is standing. The corresponding results are shown in Fig. 11. Generally, the differences between the 95%- and the 5%-fractile become smaller with increasing group size. This is true for the effective damping and also for the natural frequency. In most cases, the influence on the effective damping is larger than on the effective natural frequency of

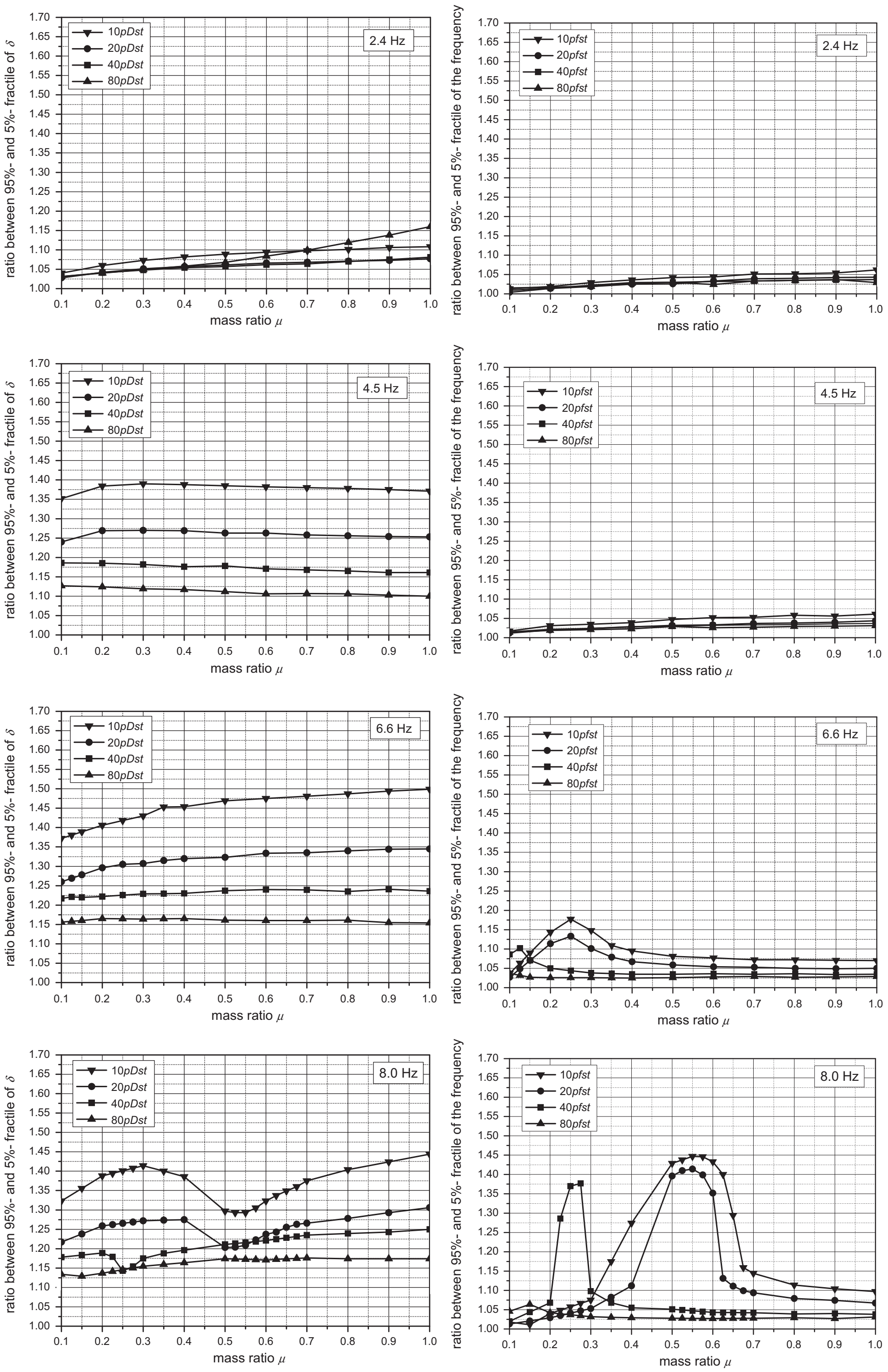

Fig. 11. Random scatter in the natural frequency and effective damping for different group sizes and mass ratios for a standing audience for a natural frequency of the empty structure of 2.4, 4.5, 6.6 and 8.0 Hz.

the coupled structure–crowd system. Particularly large influences on the effective damping are found when the natural frequency of the empty structure is in the range of the first natural frequency of the human body (examples f_0=4.5 and 6.6 Hz). For 10 persons, the ratio of the 95–5% fractile value varies from 1.35 to 1.5, it decreases to about 1.3 for 20 persons, to about 1.2 for 40 persons and finally to about 1.15 for 80 persons. The influence on the natural frequency of the coupled system remains small for an empty frequency of 2.4 and 4.5 Hz. It becomes large for some mass ratios for an empty frequency of 6.6 and 8 Hz.

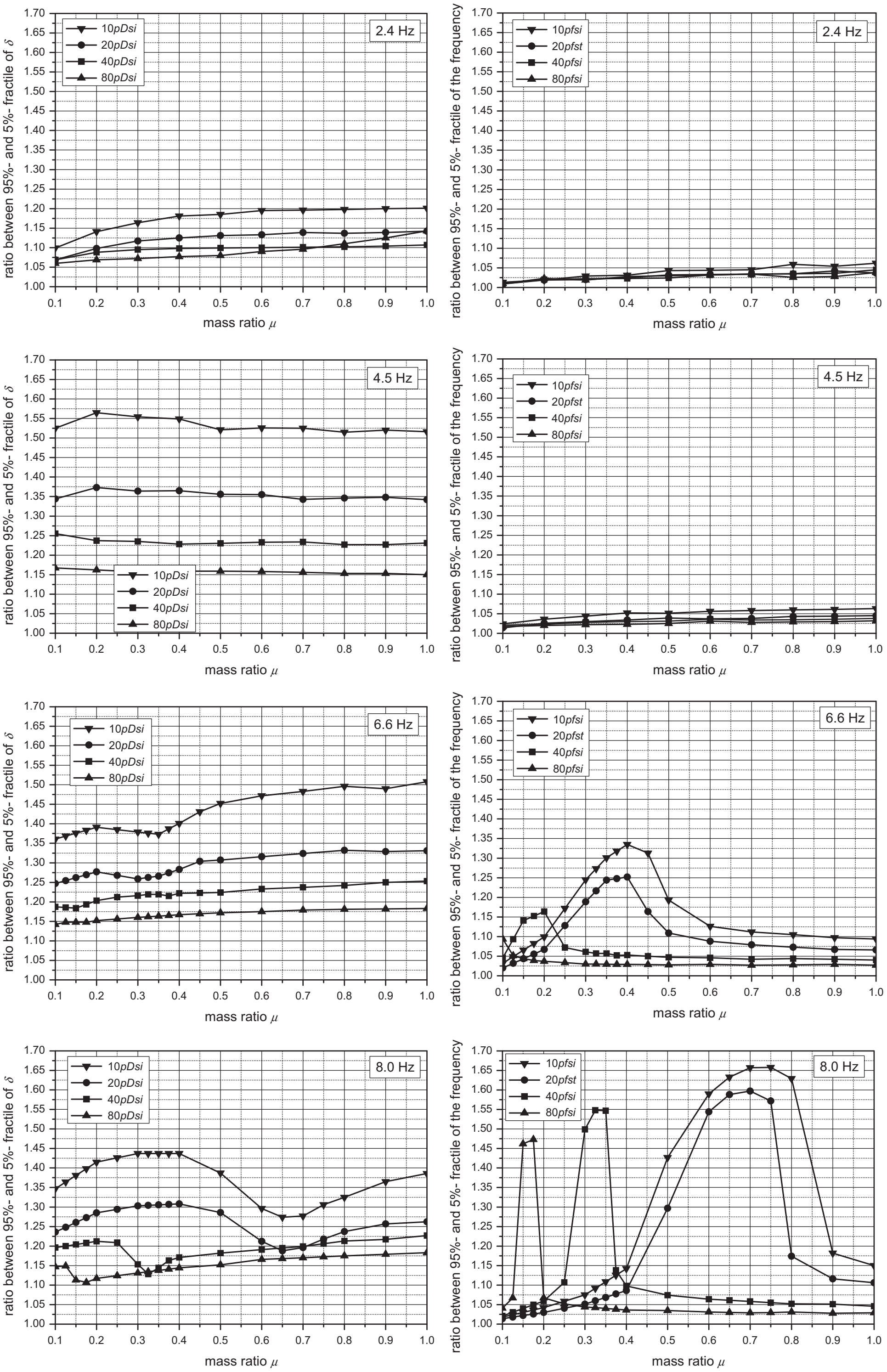

Fig. 12. Random scatter in the natural frequency and effective damping for different group sizes and mass ratios for a sitting audience for a natural frequency of the empty structure of 2.4, 4.5, 6.6 and 8.0 Hz.

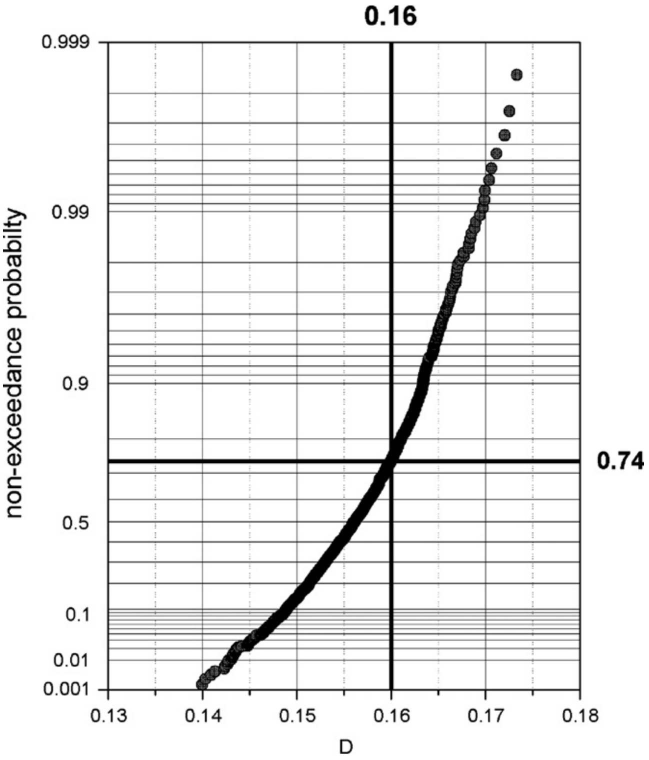

Fig. 13. Probability distribution for the effective damping using the dynamic characteristic of the structure from in-situ measurements published in Ref. [20].

For natural frequencies smaller than 3 Hz, the influence of the individual scatter can be neglected even for small group sizes. For natural frequencies beyond the first natural frequency of the human body, there is a strong influence on the mass ratio. Strong individual influences at least occur up to a group size of 40 persons.

A similar behaviour is observed for the sitting posture (Fig. 12). For natural frequencies of the empty structure below the natural frequency of the human body, generally, the influence of the individual scatter is larger for the sitting posture than for the standing posture. However, for f_{empty} below 3 Hz, the influence can be neglected. Very large influences on the natural frequency of the coupled system are found for $f_{\text{empty}}=8$ Hz and mass ratios from 0.6 to 0.85. The ratio of the 95–5% fractile values exceeds 1.6 for smaller group size, i.e. the range of possible resonance frequencies is very large. Even for a large group of 80 persons, considerable influences may be obtained. For the mass ratio in the range from 0.125 to 0.175, the ratio of the 95–5% fractile values may become as large as 1.4.

As a first step of validating the random model of the coupled structure–crowd system, in-situ measurements published in Ref. [20] can be used. It is reported that during the dynamic test on two 9 m × 6 m corner bays occupied with 64 people, the damping value of the coupled system increased to 16% as all the people stood still on the bay. The basic layout of the tests and the dynamic characteristics of the corner bays were given in Ref. [21]. This 'singular' result can be compared to the estimated probability distribution for the effective damping using a sufficient large number of simulations corresponding to this specific problem. The respective results are shown in Fig. 13. The observed value of 16% effective damping lies well in the range of the random effective damping values and corresponds to the 74%-fractile value.

4. Summary and conclusions

The paper studies the effect of the human body on the dynamic characteristics of the coupled structure–crowd system with special emphasis on the randomness in the effective dynamic characteristics. A probabilistic model of the random dynamic characteristics of passive persons is developed. Based on this probabilistic model extensive Monte-Carlo simulations are performed for different group sizes, postures (standing/sitting), frequencies of the empty structure, and mass ratios (mass of the empty structure to the mass of the crowd).

Generally, the smaller the group of persons on the structure will be, the larger the scatter in the dynamic characteristics of the coupled system will become. Depending on the mass ratio and the natural frequency of the empty structure, even for larger groups the scatter in the dynamic characteristics may be large. Due to this fact, it is not appropriate to use the mean values of the dynamic characteristics of the human body in all possible cases.

The deterministic sdof-model as recommended in the IStructE-guide is overestimating the damping effects of the crowd. Hence, applying this model to the dynamic analysis of structures may lead to estimations on the unsafe side. The mismatch in the effective damping is larger for smaller frequencies and becomes smaller for increasing frequencies.

It is important to note that the large damping capacity of passive persons may mask vibration problems. In case of low structural damping, even a small number of passive persons may induce very large values of effective damping. Therefore, observations of the vibration amplitudes during phases with passive persons may mask dynamic problems, which only are revealed if the activity rate approaches or reaches the 100% level. Then, the effective damping 'suddenly' drops back to the low structural damping value and the resonance response becomes large. This seems to be true at least for a jumping audience. For the activity bobbing, some researchers [16] assume that there is a contribution to the damping even by active persons, which would mean that the differences between say 90% and 100% activity rate might be less onerous.

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