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Energy-to-peak control for seismic-excited buildings with actuator faults and parameter uncertainties

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ABSTRACT

This paper deals with the problem of robust reliable energy-to-peak controller design for seismic-excited buildings with actuator faults and parameter uncertainties. It is assumed that uncertainties mainly exist in damping and stiffness of the buildings because they are difficult to be measured precisely. The objective of designing controllers is to guarantee the asymptotic stability of closed-loop systems and attenuate disturbance from earthquake excitation. Energy-to-peak performance is believed to be of great significance when conditions and requirements of active building vibration control are carefully considered. Based on energy-to-peak control theory and linear matrix inequality techniques, a new approach for reliable building vibration control with satisfactory energy-to-peak performance is presented. An n-degree-of-freedom linear building structure under earthquake excitation is analyzed and simulations are employed to validate the effectiveness of the proposed approach in reducing seismic-excited building vibration. Some comparisons are also made between energy-to-peak control systems and H_∞ control systems to further prove the importance of the method raised in this paper.

1. Introduction

Nowadays, vibration control problem of tall buildings has attracted considerable attention because strong earthquakes and tsunamis, such as 2008 Wen Chuan Earthquake and 2007 Indonesia Tsunami, happen frequently. Traditional methods of building vibration control include base isolation [1], many different kinds of dampers [2,3], and various types of bracings [4], but all of them belong to passive or semi-active control methods. However, as buildings become higher and higher, traditional mechanical methods cannot guarantee the structural stability and solidity of seismic-excited and wind-excited buildings. Therefore, active vibration control methods have been heatedly discussed in recent years because of their excellent control performances [5–9].

Realizing the significance of building vibration control and the advantage of active vibration control, many scholars have applied themselves to the research of active vibration control methods in recent years and many control techniques have been utilized. To illustrate, optimal control [10,11], PID control [12,13], classical H_2 and H_∞ control [14–16], sliding model control [12,17], neural networks [18], and fuzzy logic [19,20], have all been developed to attenuate the vibration of seismic-excited or wind-excited buildings. Moreover, some modern approaches have been applied to improve the performance of vibration control systems. For instance, pole placement and mixed H_2/H_∞ control have been used to gain better control effect [21–23], and genetic algorithms have been used to find feasible and optimal solutions to vibration

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control problems [14,19]. With the development of control theory, many practical constraints, such as actuator saturation [3], parameter uncertainties [24,25], time delay in measurements or actions [14,26,27], have been considered in order to make control systems more applicable to real buildings. All these approaches play an important role in building vibration control against earthquakes or wind.

An important problem in active vibration control is that control component faults, such as actuator faults and sensor faults, often occur in the implementation of real control systems. These faults often result in unsatisfactory control performances and even unexpected instability of control systems. Moreover, parameters used by engineers to design controllers cannot always be measured precisely. These uncertainties in parameters often lead to the fact that the performances of designed control systems are not so nice as we expect. To deal with the problems of control component faults and parameter uncertainties, many scholars have discussed the concept and methods of reliable control with uncertainties [28–33]. All these approaches help to ensure the performances of control systems.

All the works mentioned above perform effectively in building vibration attenuation. However, it is worth mentioning that most of the reported approaches are based on the theory of robust H_∞ control, which cares about the energy-to-energy performance of closed-loop systems. For seismic-excited or wind-excited buildings, it is often the excessive peak values of controlled outputs such as displacements or accelerations that make them collapse. Therefore, for building vibration control, a better control performance can be expected if energy-to-peak performance is taken into consideration, which has been studied by many scholars [26,34–40].

Considering the advantage of energy-to-peak control and the necessity of reliable control, we try to design robust reliable energy-to-peak controllers against actuator faults and parameter uncertainties, in order to better attenuate the vibration of seismic-excited buildings. In this paper, we consider a n-degree-of-freedom (n-dof) linear building structure under earthquake excitation. Based on Newton's second law, a state space model of buildings with parameter uncertainties is firstly established. Then, using Lyapunov approach, we consider a method for designing robust reliable controllers against actuator failures to get satisfactory energy-to-peak performance. By using the results in [34,35,38], we further develop the existence conditions for robust reliable controllers with parameter uncertainties in the form of linear matrix inequalities (LMIs). The energy-to-peak controller design problem is transformed into a convex feasibility problem subject to LMI constraints, which can be solved directly by MATLAB LMI Toolbox. If the feasibility problem is solvable, desired controllers can be obtained. We also put forward a method of designing robust reliable H_∞ controllers against actuator faults for further comparisons. Furthermore, we use some illustrative examples to show the effectiveness of our method by comparing displacements and accelerations of buildings with energy-to-peak controllers and those without any controllers under the same earthquake excitation. We finally compare performances between closed-loop systems with energy-to-peak controllers and those with H_∞ controllers to further prove the validity of the method proposed in this paper.

The rest of this paper is organized as follows. In Section 2, a state space model of an n-dof linear building structure under earthquake excitation is established, and the robust reliable energy-to-peak controller design problem is formulated. In Section 3, approaches for designing such controllers are put forward. Section 4 gives simulations to illustrate the effectiveness and applicability of the method proposed in this paper. Finally, conclusions are drawn in Section 5 and proof of the proposed theorems is given in appendix.

Notation: $\mathbb{R}^{\mathbf{n}}$ means the n-dimensional Euclidean space and $\mathbb{R}^{\mathbf{n}\times\mathbf{m}}$ denotes the set of all $n\times m$ real matrices; for a matrix Q, Q^T, Q^{-1} and $Q^{\perp}$ denote its transpose, inverse, and orthogonal complement, respectively; the notation $Q>0(<0)$ is used to denote that Q is real symmetric and positive (negative) definiteness; and $[Q]_F$ denotes $Q+Q^T$. $\|G\|_\infty$ denotes the $H_\infty-$norm of transfer function matrix $G(s)$. I and 0 are used to denote the identity and the zero matrices, respectively, of appropriate dimensions. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and diag{…} stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

In this section, we are going to establish a model of seismic-excited buildings and formulate the reliable energy-to-peak control problem.

2.1. System description

Consider a n-dof linear building structure under earthquake excitation. The building model is shown in Fig. 1, in which $\ddot{\mathbf{x}}_{\mathbf{g}}(t)$ is the ground acceleration; m_i, c_i, k_i, q_i and u_i $(i=1,2,3,\ldots,n)$ are the mass, damping, stiffness, relative drift and control force of each storey, respectively.

According to Newton's second law, the dynamic equation of seismic-excited building motion can be written as

$$\mathbf{M}_0\ddot{\mathbf{q}}(t)+\mathbf{C}_0\dot{\mathbf{q}}(t)+\mathbf{K}_0\mathbf{q}(t)=\mathbf{H}_0\mathbf{u}(t)+\xi_0\ddot{\mathbf{x}}_{\mathbf{g}}(t), \tag{1}$$

where $\mathbf{q}(t)\in\mathbb{R}^{\mathbf{n}}$, $\mathbf{q}(t)=[q_1(t)\ q_2(t)\ q_3(t)\ldots q_n(t)]^T$, $q_1(t)$ is interstorey relative drift between the first floor and ground, and $q_i(t)$ is interstorey relative drift between the ith floor and $(i-1)$th floor, $i=1,2,3,\ldots,n$; $\mathbf{u}(t)$ is the control input, $\mathbf{u}(t)\in\mathbb{R}^{\mathbf{m}}$ gives m control inputs, and here it is supposed that there is one control force embedded at the bottom of each storey, so

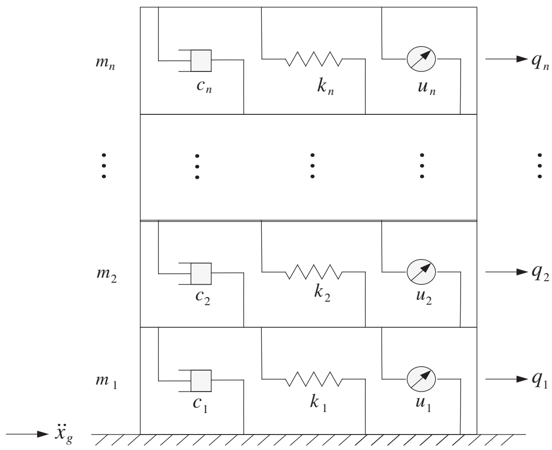

Fig. 1. n-dof building model.

that $\mathbf{u}(t)=[u_1(t)\ u_2(t)\ u_3(t)\ldots u_m(t)]^T$; $\ddot{\mathbf{x}}_\mathbf{g}(t)\in\mathbb{R}^\mathbf{1}$ is the ground acceleration caused by earthquakes, which is defined as the disturbance input $\mathbf{w}(t)\in\mathbb{R}^\mathbf{1}$, that is, $\mathbf{w}(t)=\ddot{\mathbf{x}}_\mathbf{g}(t)$; $\mathbf{M}_0\in\mathbb{R}^{\mathbf{n}\times\mathbf{n}}$, $\mathbf{C}_0\in\mathbb{R}^{\mathbf{n}\times\mathbf{n}}$, and $\mathbf{K}_0\in\mathbb{R}^{\mathbf{n}\times\mathbf{n}}$ are the mass, damping and stiffness matrices, respectively; $\xi_0\in\mathbb{R}^\mathbf{n}$ is a vector representing the influence of earthquake excitation to each floor; $\mathbf{H}_0\in\mathbb{R}^{\mathbf{n}\times\mathbf{m}}$ gives the location of the m controllers.

By defining the state variables $\mathbf{x}=[\mathbf{q}(t)\ \dot{\mathbf{q}}(t)]^T$, we get that dynamic equation (1) can be expressed in the state space form as below:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B_u}\mathbf{u}(t)+\mathbf{B}_w\boldsymbol{w}(t), \tag{2}$$

where

$$\mathbf{A}=\begin{bmatrix}0 & \mathbf{I}\\ -\mathbf{M}_0^{-1}\mathbf{K}_0 & -\mathbf{M}_0^{-1}\mathbf{C}_0\end{bmatrix},\quad \mathbf{B_u}=\begin{bmatrix}0\\ \mathbf{M}_0^{-1}\mathbf{H}_0\end{bmatrix},\quad \mathbf{B}_w=\begin{bmatrix}0\\ \mathbf{M}_0^{-1}\xi_0\end{bmatrix}.$$

Then the building vibration control system can be described as

$$\begin{aligned}\dot{\mathbf{x}}(t)&=\mathbf{A}\mathbf{x}(t)+\mathbf{B_u}\mathbf{u}(t)+\mathbf{B}_w\boldsymbol{w}(t),\\ \mathbf{z}(t)&=\mathbf{C}\mathbf{x}(t),\end{aligned} \tag{3}$$

where $\mathbf{z}(t)\in\mathbb{R}^\mathbf{p}$ is the controlled output, $\mathbf{p}$ is the number of outputs, and $\mathbf{C}\in\mathbb{R}^{\mathbf{p}\times\mathbf{n}}$ is a constant matrix.

Since the damping and stiffness of buildings cannot be measured easily and precisely, while the mass of buildings can be measured precisely, parameter uncertainties in control systems mainly locate in matrix $\mathbf{A}$. We take $\boldsymbol{\Delta}_\mathbf{A}$ to represent the system uncertainties with the same dimensions as that of $\mathbf{A}$, similar to [41], $\boldsymbol{\Delta}_\mathbf{A}$ is assumed to be norm-bounded, that is

$$\|\boldsymbol{\Delta}_\mathbf{A}\|\le\alpha. \tag{4}$$

Therefore, the building vibration control system with parameter uncertainties can be described as

$$\begin{aligned}\dot{\mathbf{x}}(t)&=(\mathbf{A}+\boldsymbol{\Delta}_\mathbf{A})\mathbf{x}(t)+\mathbf{B_u}\mathbf{u}(t)+\mathbf{B}_w\boldsymbol{w}(t),\\ \mathbf{z}(t)&=\mathbf{C}\mathbf{x}(t).\end{aligned} \tag{5}$$

When considering possible actuator faults in the control system, we introduce a state-feedback controller in the form of

$$\mathbf{u}(t)=\mathbf{K}\mathbf{x}(t)=\mathbf{M_a}\mathbf{K_a}\mathbf{x}(t), \tag{6}$$

where $\mathbf{K_a}$ is the actuator fault-tolerant feedback controller gain to be designed later. Actuator failures are described by fault matrix $\mathbf{M_a}$=diag{m_{a1}, m_{a2}, m_{a3},…,m_{an}}, where m_{ai} (i=1,2,3,…,n) represents possible faults in the actuators of each floor and $0\le m_{ali}\le m_{ai}(t)\le m_{aui}<\infty$, m_{ali} and m_{aui} are known real constraints. If m_{ali}=m_{aui}=0, then $m_{ai}(t)$ =0, which means that the corresponding actuator is completely broken down. To the opposite, if m_{ali}=m_{aui}=1, it is obvious that $m_{ai}(t)$ =1, which indicates that there is no fault in a certain actuator. Otherwise, if $0<m_{ali}<m_{aui}$ and $m_{ai}(t)\neq 1$, there exists partial fault in the corresponding actuator.

According to (5) and (6), the closed-loop system can be written as

$$\begin{aligned}\dot{\mathbf{x}}(t)&=(\mathbf{A}+\boldsymbol{\Delta}_\mathbf{A})\mathbf{x}(t)+\mathbf{B_u}\mathbf{M_a}\mathbf{K_a}\mathbf{x}(t)+\mathbf{B_w}\mathbf{w}(t)\\ &=(\mathbf{A}+\boldsymbol{\Delta}_\mathbf{A}+\mathbf{B_u}\mathbf{M_a}\mathbf{K_a})\mathbf{x}(t)+\mathbf{B_w}\mathbf{w}(t),\\ &=\overline{\mathbf{A}}\mathbf{x}(t)+\mathbf{B}_w\boldsymbol{w}(t),\\ \mathbf{z}(t)&=\mathbf{C}\mathbf{x}(t).\end{aligned} \tag{7}$$

2.2. Formulation of energy-to-peak control problem

In this paper, the disturbance signal $\mathbf{w}(t)$ is assumed to be bounded with finite energy, that is,

$$\|\mathbf{w}(t)\|_2 = \sqrt{\int_{t=0}^{\infty} |\mathbf{w}(t)|^2 \, \mathrm{d}t} < \infty. \tag{8}$$

Here, (8) is a reasonable assumption, which can be satisfied by most earthquakes. It is also assumed that the peak response quantity of the controlled output is bounded, that is,

$$\|\mathbf{z}(t)\|_\infty = \sup_{t\in[0,\infty)} \sqrt{\mathbf{z}(t)^\mathbf{T}\mathbf{z}(t)} < \infty. \tag{9}$$

Based on the analysis above, the problem to be studied in this paper can be illustrated as follows. Considering the uncertain seismic-excited building model in (5), we aim at putting forward a feasible method for designing a robust reliable energy-to-peak controller (6) and seeking the corresponding controller gain $\mathbf{K}$, such that the closed-loop system is asymptotically stable, and for a given performance index $\gamma > 0$, the energy-to-peak performance

$$\|\mathbf{z}(t)\|_\infty < \gamma \|\mathbf{w}(t)\|_2 \tag{10}$$

is satisfied for any $\mathbf{w}(t)$ given by (8).

3. Controller design

In this section, the robust reliable energy-to-peak controller design problem is analyzed. A new method is brought forward for designing a feasible controller such that the closed-loop system is asymptotically stable and the energy-to-peak performance in (10) is satisfied. At the same time, a method for designing a robust reliable H_∞ controller is also put forward for further comparison.

3.1. Robust reliable energy-to-peak controller design

First, some lemmas that are useful for later controller design are given below. Their proofs and applications can be found in [34,35,42].

Lemma 1 (*Grigoriadis and Watson [34], Du and Lam [35]*). *Consider a linear system*

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B_u}\mathbf{u}(t) + \mathbf{B_w}\mathbf{w}(t),$$

$$\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t). \tag{11}$$

Suppose $(\mathbf{A},\mathbf{B_u},\mathbf{B_w},\mathbf{C}) \in \mathbb{R}$ *is arbitrary but fixed and let* $\gamma > 0$ *be given. Then closed-loop system* (7) *is asymptotically stable and the energy-to-peak performance in* (10) *is satisfied for any* $\mathbf{w}(t)$ *given by* (8) *if and only if there exists a symmetric matrix* $\mathbf{X} > 0$ *satisfying*

$$\overline{\mathbf{A}}\mathbf{X} + \mathbf{X}\overline{\mathbf{A}}^\mathbf{T} + \mathbf{B_w}\mathbf{B_w^T} < 0, \tag{12}$$

and

$$\mathbf{C}\mathbf{X}\mathbf{C}^\mathbf{T} < \gamma^2 \mathbf{I}. \tag{13}$$

Lemma 2 (*Yao et al. [42]*). *Let* $\mathbf{E}$,$\mathbf{F}$ *and* $\mathbf{\Sigma}$ are *matrices of appropriate dimensions with* $\|\mathbf{\Sigma}\| \le 1$. *Then, for any scalar* $\varepsilon > 0$,

$$\mathbf{E\Sigma F} + \mathbf{F^T\Sigma^T E^T} \le \varepsilon^{-1}\mathbf{EE^T} + \varepsilon\mathbf{F^T F}. \tag{14}$$

Lemma 3 (*Yao et al. [42]*). *For a time-varying diagonal matrix* $\mathbf{\Phi}(t) = diag\{\phi_1(t), \phi_2(t), \ldots, \phi_m(t)\}$ *and two matrices* $\mathbf{R}$ *and* $\mathbf{S}$, *if* $|\mathbf{\Phi}(t)| \le \mathbf{V}$, *where* $|\mathbf{\Phi}(t)| = diag\{|\phi_1(t)|, |\phi_2(t)|, \ldots, |\phi_m(t)|\}$ *and* $\mathbf{V} > 0$ *is a known diagonal matrix then for any scalar* $\varepsilon > 0$,

$$\mathbf{R\Phi S} + \mathbf{S^T\Phi^T R^T} \le \varepsilon^{-1}\mathbf{S^T V S} + \varepsilon\mathbf{RVR^T}. \tag{15}$$

Next, we introduce the following matrices that will be used in our later development.

$$\mathbf{M_{a0}} = \mathrm{diag}\{m_{a01}, m_{a02}, m_{a03}, \ldots, m_{a0n}\},$$

$$\mathbf{L_a} = \mathrm{diag}\{l_{a1}, l_{a2}, l_{a3}, \ldots, l_{an}\},$$

$$\mathbf{J_a} = \mathrm{diag}\{j_{a1}, j_{a2}, j_{a3}, \ldots, j_{an}\},$$

where $m_{a0i}=(m_{ali}+m_{aui})/2$, $l_{ai}=[m_{ai}(t)-m_{a0i}]/m_{a0i}$, and $j_{ai}=(m_{aui}-m_{ali})/(m_{aui}+m_{ali})$ with $i=1,2,3,\ldots,n$. According to the definition, we have

$$\mathbf{M_a}=\mathbf{M_{a0}}(\mathbf{I}+\mathbf{L_a}), \tag{16}$$

$$\mathbf{L_a^T L_a} \le \mathbf{J_a^T J_a} \le \mathbf{I}. \tag{17}$$

Then, we put forward the following theorem to solve the reliable controller design problem for seismic-excited buildings.

Theorem 1. *Consider building vibration control system* (5) *with robust reliable energy-to-peak state-feedback controller* (6), *closed-loop system* (7) *is asymptotically stable and the energy-to-peak performance in* (10) *is satisfied for a given performance index* $\gamma>0$ *and for all* $\mathbf{w}(t)$ *given by* (8), *if there exist positive symmetric matrix* $\mathbf{X}$, *matrix* $\mathbf{S}$ *and positive scalars* $\varepsilon_1,\varepsilon_2$ *satisfying*

$$\begin{bmatrix} \boldsymbol{\Omega} & \mathbf{X} & \mathbf{S^T} & \mathbf{B_w} \\ * & -\varepsilon_1\mathbf{I} & 0 & 0 \\ * & * & -\varepsilon_2\mathbf{J_a^{-1}} & 0 \\ * & * & * & -\mathbf{I} \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} \mathbf{X} & \mathbf{XC^T} \\ * & \gamma^2\mathbf{I} \end{bmatrix} > 0, \tag{19}$$

where

$$\boldsymbol{\Omega} \triangleq \mathbf{AX}+\mathbf{XA^T}+\mathbf{B_u S S^T B_u^T}+\varepsilon_1\alpha^2\mathbf{I}+\varepsilon_2\mathbf{B_u J_a^T B_u^T}.$$

The desired control law is given by

$$\mathbf{K_{al}}=\mathbf{M_{a0}^{-1}}\mathbf{S}\mathbf{X}^{-1}. \tag{20}$$

The proof of this theorem can be seen in the appendix.

Remark 1. Theorem 1 provides us a process to design robust reliable energy-to-peak controllers with actuator faults and parameter uncertainties. Closed-loop system (7) will be asymptotically stable and the energy-to-peak performance in (10) will be satisfied if Lemma 1 holds. However, Lemma 1 cannot be directly used to design controllers due to the parameter uncertainties in matrix **A** and some possible actuator faults. Therefore, it is necessary to make some transformations before Lemma 1 is used. Firstly, in order to solve the problem of parameter uncertainties, we use Lemma 2 to find the upper bound of the uncertainty and replace the uncertain part with its upper bound. Furthermore, a proper form is chosen to describe the possible actuator faults. Then Lemmas 2 and 3 are used to substitute the time-variant part with its time-invariant upper bound. Through out the process mentioned above, the initial problem of designing controllers under uncertain and time-variant conditions, which are almost impossible to be solved, is transformed into the final problem of designing controllers under certain and time-invariant conditions, which can be handled by the available approaches. Finally, we transform requirements (31) into two LMIs (18) and (19), which can be easily solved with the help of MATLAB LMI Toolbox.

Remark 2. The energy-to-peak performance index γ should be specified before designing robust reliable energy-to-peak controllers with Theorem 1. When a certain γ is given, our work is to find positive symmetric matrix **X**, matrix **S** and positive scalars ε_1, ε_2 satisfying LMIs (18) and (19). If feasible solutions are found, we can get the desired controller gain $\mathbf{K_{al}}$ by (20) and thus the entire problem can be solved.

Till now, we have proposed a theorem to solve the robust reliable energy-to-peak controller design problem. We can obtain desirable controllers by solving the LMI conditions in Theorem 1 so that the corresponding energy-to-peak performance can be satisfied. In order to testify the advantages of our controllers with satisfactory energy-to-peak performance, we will present a possible way to design robust reliable H_∞ state-feedback controllers under the same conditions, and it will be used in future comparision.

3.2. Robust reliable H_∞ controller design

In this subsection, our attention is focused on designing robust reliable H_∞ controllers such that the asymptotic stability of closed-loop system (7) is guaranteed and the H_∞ norm of the closed-loop transfer function $\mathbf{G}(s)$ satisfies

$$\|\mathbf{G}(s)\|_\infty=\sup_{\omega}\sigma_{\max}(\mathbf{G}(j\omega))=\frac{\|\mathbf{z}(t)\|_2}{\|\mathbf{w}(t)\|_2}<\eta, \tag{21}$$

where η is a given positive scalar. For seismic-excited building vibration control systems, in terms of the design objective, we aim at minimizing $\|\mathbf{G}(s)\|_\infty$. The following lemma will be used in the design of robust reliable H_∞ controllers.

Lemma 4 (*Grigoriadis and Watson [34]*). *Suppose* $(\mathbf{A},\mathbf{B_u},\mathbf{B_w},\mathbf{C}) \in \mathbb{R}$ *in* (11) *is arbitrary but fixed and let* $\eta > 0$ *be given. Closed-loop system* (7) *is asymptotically stable and* $\|\mathbf{G}(s)\|_\infty < \eta$ *is satisfied for any* $\mathbf{w}(t)$ *given by* (8), *if and only if there exists a symmetric matrix* $\mathbf{W} > 0$ *satisfying*

$$\begin{bmatrix} \overline{\mathbf{A}}^\mathbf{T}\mathbf{W}+\mathbf{W}\overline{\mathbf{A}} & \mathbf{W}\mathbf{B_w} & \mathbf{C}^\mathbf{T} \\ * & -\mathbf{I} & 0 \\ * & * & -\eta^2\mathbf{I} \end{bmatrix} < 0. \tag{22}$$

Theorem 2. *Consider a building vibration control system with robust reliable* H_∞ *state-feedback controller* (6), *closed-loop system* (7) *is asymptotically stable and* $\|\mathbf{G}(s)\|_\infty < \eta$ *is satisfied, if there exist positive symmetric matrix* $\mathbf{X}$, *matrix* $\mathbf{Y}$ *and positive scalars* ε_3, ε_4 *satisfying*

$$\begin{bmatrix} \Gamma & \mathbf{X} & \mathbf{Y} & \mathbf{X}\mathbf{B_w} & \mathbf{C}^\mathbf{T} \\ * & -\varepsilon_3\mathbf{I} & 0 & 0 & 0 \\ * & * & -\varepsilon_4\mathbf{J_a}^{-1} & 0 & 0 \\ * & * & * & -\mathbf{I} & 0 \\ * & * & * & * & -\eta^2\mathbf{I} \end{bmatrix} < 0, \tag{23}$$

where

$$\Gamma \triangleq \mathbf{A}\mathbf{X}+\mathbf{X}\mathbf{A}^\mathbf{T}+\mathbf{B_u}\mathbf{Y}+\mathbf{Y}^\mathbf{T}\mathbf{B_u^T}+\varepsilon_3\alpha^2\mathbf{I}+\varepsilon_4\mathbf{B_u}\mathbf{J_a^T}\mathbf{B_u^T}.$$

Moreover, the desired control law is given by

$$\mathbf{K_{ah}} = \mathbf{M_{a0}^{-1}}\mathbf{Y}\mathbf{X}^{-1}. \tag{24}$$

The proof of this theorem can be seen in the appendix.

Different from the utilization of Theorem 1, we can solve the problem of designing robust reliable H_∞ controllers in two aspects. One aspect is designing suboptimal H_∞ controllers: we aim at seeking state-feedback controllers for a given H_∞ performance index η, so that the corresponding closed-loop system (7) is asymptotically stable and (21) is satisfied. The other is designing optimal H_∞ controllers: our goal is to look for state-feedback controllers in order to minimize $\|\mathbf{G}(s)\|_\infty$. Since optimal controllers are usually better in attenuating disturbances, we choose to look for optimal robust reliable H_∞ controllers in illustrative examples.

We use the following remark to summarize this section and make some expections for the next section.

Remark 3. For building vibration control systems, all the state variables are supposed to be measurable. Under such assumption, we bring forward two theorems to design robust reliable state-feedback controllers. By solving the feasibility problem of LMIs shown in Theorems 1 and 2, we can eventually obtain the robust reliable energy-to-peak controllers and the robust reliable H_∞ controllers. We believe that the robust reliable energy-to-peak controllers will have satisfactory performance in vibration attenuation because they focus on decreasing the peak value of the controlled outputs such as displacements and accelerations, which will be more helpful to protect buildings in strong earthquakes. We will give several illustrative examples to prove the effectiveness of our ideas in the next section. First, feasible robust reliable energy-to-peak controllers for a given energy-to-peak performance index γ will be designed under some different fault conditions and we will make comparisons between performances of closed-loop systems with energy-to-peak controllers and those of open-loop systems. The simulations contain two cases with different controlled outputs, which indicate the wide use of reliable energy-to-peak controllers. Second, optimal robust reliable H_∞ controllers will be designed under the same fault conditions. If control performances of the feasible robust reliable energy-to-peak controllers are better than those of the optimal robust reliable H_∞ controllers, the effectiveness of our method will be further proved.

4. Illustrative example

In this section, we are going to employ simulations to show the effectiveness of the robust reliable energy-to-peak controllers in building vibration attenuation. The data of El Centro 1940 earthquake are used as the excitation signal (see Fig. 2). We design controllers by using the approaches presented in Section 3.

In the simulations, a three-storey shear-beam building model considered in [35] is studied. The active bracing systems (ABSs) are installed on each floor, respectively. Here, it is assumed that the masses, damping and stiffness coefficients are identical for each storey unit and given as m_i=1 t, $c_i = 1.407$ kN s/m and k_i=980 kN/m, where i=1, 2, 3. It is also assumed the initial state $\mathbf{x}(0)$=[0,0,0,0,0,0]$^\mathbf{T}$. Similar to (1), the dynamic equation of the three-storey shear-beam building model is

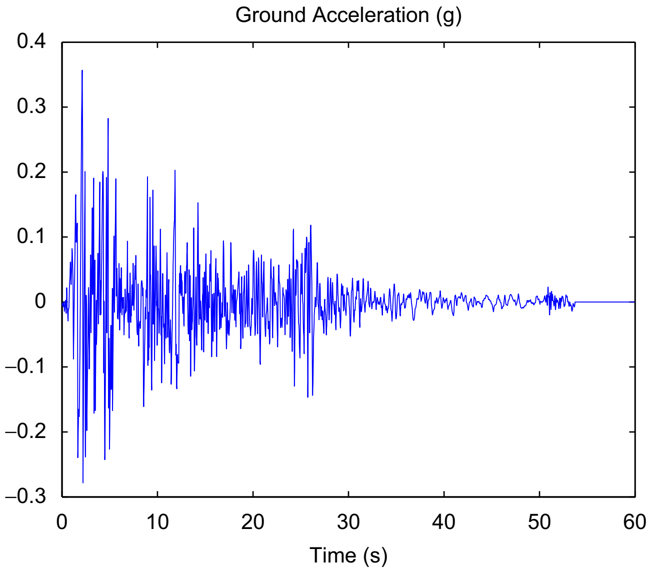

Fig. 2. El Centro 1940 earthquake.

obtained as follows:

$$\mathbf{M}_0=\begin{bmatrix} m_1 & 0 & 0 \\ m_2 & m_2 & 0 \\ m_3 & m_3 & m_3 \end{bmatrix},\quad \mathbf{C}_0=\begin{bmatrix} c_1 & -c_2 & 0 \\ 0 & c_2 & -c_3 \\ 0 & 0 & c_3 \end{bmatrix},\quad \mathbf{K}_0=\begin{bmatrix} k_1 & -k_2 & 0 \\ 0 & k_2 & -k_3 \\ 0 & 0 & k_3 \end{bmatrix},$$

$$\xi_0=\begin{bmatrix} -m_1 \\ -m_2 \\ -m_3 \end{bmatrix},\quad \mathbf{H}_0=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Other parameters are listed as follows: considering parameter uncertainties in the form of (4), it is supposed that $\alpha=0.01$; for the description of actuator faults, it is supposed that m_{ali}=0.2, m_{aui}=1.2, where i=1, 2, 3, and then we get $\mathbf{M_{a0}}$=diag{0.7, 0.7, 0.7}.

To simulate the actuator fault conditions, it is assumed that faults occur periodically, and the percentage scalar $\Delta(t)$ of the signal loss is defined as

$$\Delta(t)=\begin{cases} \delta, & kT<t\leq kT+f_t, \\ 0 \text{ percent}, & kT+f_t<t\leq(k+1)T, \end{cases}\quad k=0,1,2,3\ldots, \tag{25}$$

where T is a known time period, f_t is a section of T $(0\leq f_t<T)$, which informs how much time there exists faults in a period, and δ is the percentage of the signal loss when faults exist. According to the definition above, the fault condition can be described as follows: during the first part of each period $(kT<t\leq kT+f_t)$, faults exist and the percentage of signal loss is δ; in the second part of each period $(kT+f_t<t\leq(k+1)T)$, no faults exist and the control system works normally. As is mentioned above, if f_t is longer or δ is larger, faults in the control system are more serious.

In the following, two cases with different controlled outputs will be shown to illustrate the effectiveness of energy-to-peak control of building vibration, and some comparisons will be made between closed-loop systems with energy-to-peak controllers and those with H_∞ controllers to further prove the method proposed in this paper. For the sake of comparison, the robust reliable energy-to-peak controllers are denoted as controller L, and the robust reliable H_∞ controllers are denoted as controller H. Time response curves of the interstorey drifts and accelerations for each case are plotted. The differences of control performances can be compared by the peak values of interstorey drifts and accelerations, as well as the duration of building vibration, which will be shown in the figures of the following part. For both of the cases, it is supposed that all the state variables are measurable. In Case A and B, we will first make comparisons between the performances of closed-loop systems with energy-to-peak controllers and those of open-loop systems.

4.1. Case A: Full interstorey drift output

In this case, we choose the interstorey drift of each floor as the controlled outputs, that is, $\mathbf{z}=[q_1\ q_2\ q_3]^{\mathbf{T}}$.

We first designate the energy-to-peak performance index $\gamma = 0.02$. After solving the linear matrix inequalities in (18) and (19), the gain matrix of Controller L is obtained

$$\mathbf{K_{al}} = 10^5 \times \begin{bmatrix} -3.7914 & -0.5410 & -0.4729 & -0.6203 & -0.0942 & -0.0405 \\ -3.1740 & -4.6198 & -0.6493 & -0.7234 & -0.6697 & -0.0989 \\ -2.8948 & -4.2988 & -4.6304 & -0.7694 & -0.7289 & -0.6301 \end{bmatrix}.$$

After obtaining Controller L, we make comparisons between closed-loop systems equipped with Controller L and open-loop systems to illustrate the effectiveness of robust reliable energy-to-peak control through some simulations. Firstly, we draw the frequency response of energy-to-peak control system and open-loop system in Fig. 3.

Then, we begin some simulations in time domain. It is supposed that $T=5$ s. Two different fault conditions will be discussed below:

$$\text{Case A1}: f_t = 1\,\text{s}, \delta = 20 \text{ percent (20 percent loss of the actuator thrust)};$$

$$\text{Case A2}: f_t = 3\,\text{s}, \delta = 100 \text{ percent (100 percent loss of the actuator thrust)}.$$

We want to emphasize that in most cases, even if actuator faults happen, the actuator will just loss part of its thrust, but it will seldom work in reverse to the command control sign. So we choose Case A2, in which actuator loss all its thrust, and we believe that if the energy-to-peak control system is proved to be effective in this case, the proposed method can be implemented into practical building vibration control systems. Time responses of the interstorey drifts and accelerations of the three floors under the two subcases are described in Figs. 4–11.

Figs. 4–11 depict the time responses of the interstorey drifts, relative accelerations, and control forces of the three floors under El Centro 1940 earthquake excitation in Case A, where "open-loop" refers to the case of no control force being applied to the building structure, and "L_2–L_∞ control" indicates the use of robust reliable energy-to-peak control approaches presented above. Figs. 4, 5, 8, 9 clearly demonstrate that Controller L results in less peak value and shorter duration of vibration among the two drift curves of each floor. The relative accelerations of the three floors are depicted in Figs. 6 and 10, and Controller L also holds less peak value and shorter duration of vibration. We also plot the control forces in Figs. 7 and 11. In a word, although there exist actuator faults and parameter uncertainties in the closed-loop systems, it can be seen from Figs. 4–11 that improved responses are obtained for the interstorey drifts and accelerations of the three floors under reliable energy-to-peak control compared with the open-loop system. It is clear that actuator completely losses its thrust in Case 2, but the controlled outputs are still very satisfying, which indicates that robust reliable energy-to-peak controllers can be successfully used in different cases of actuator faults.

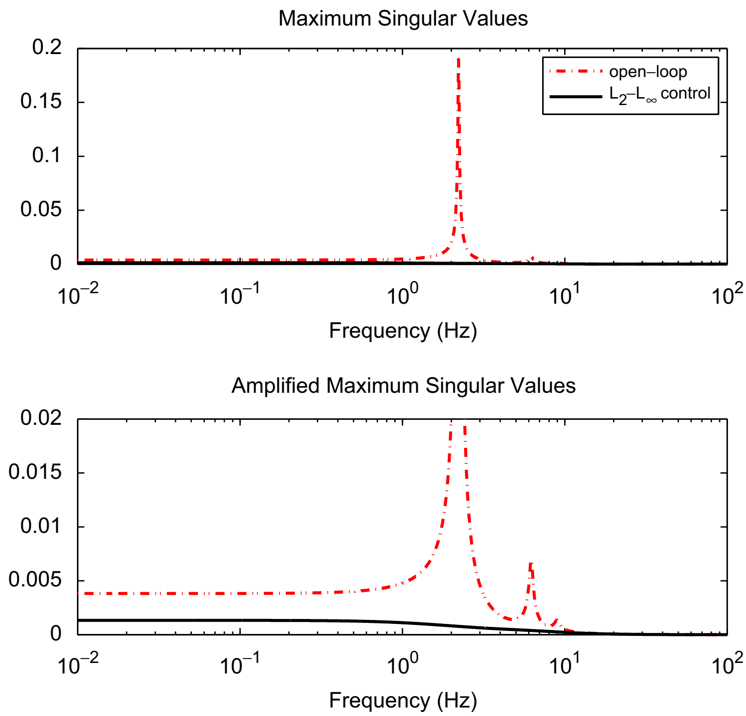

Fig. 3. The frequency response in Case A.

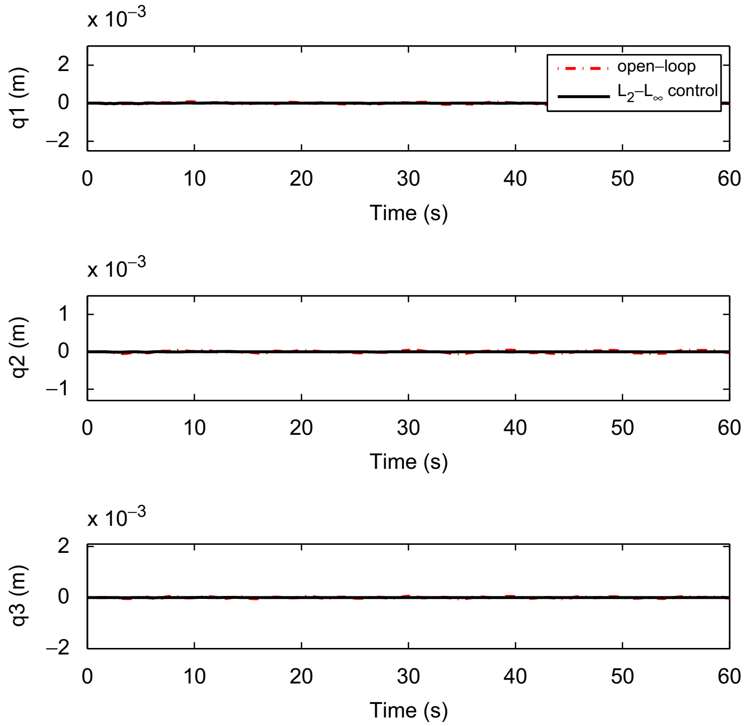

Fig. 4. Interstorey drifts of the three floors in Case A1.

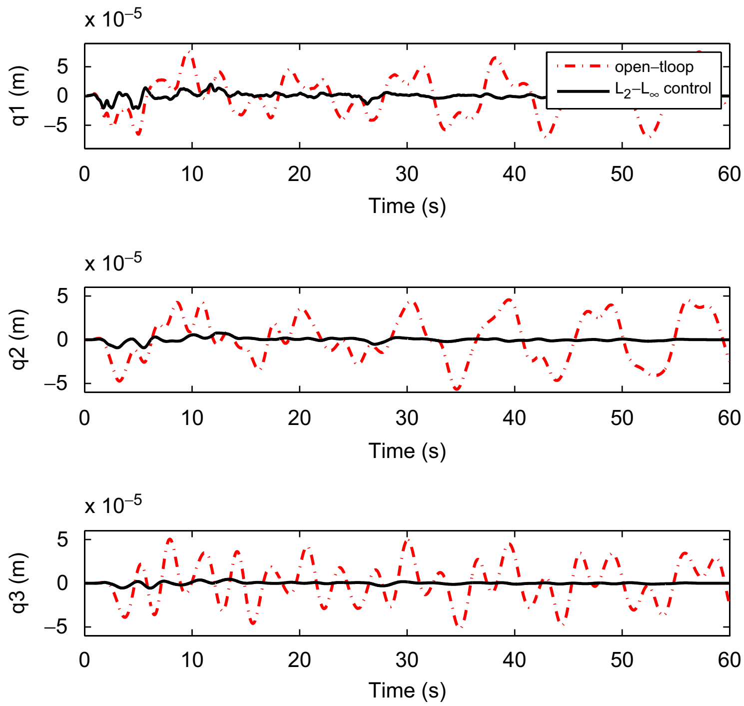

Fig. 5. Amplified interstorey drifts of the three floors in Case A1.

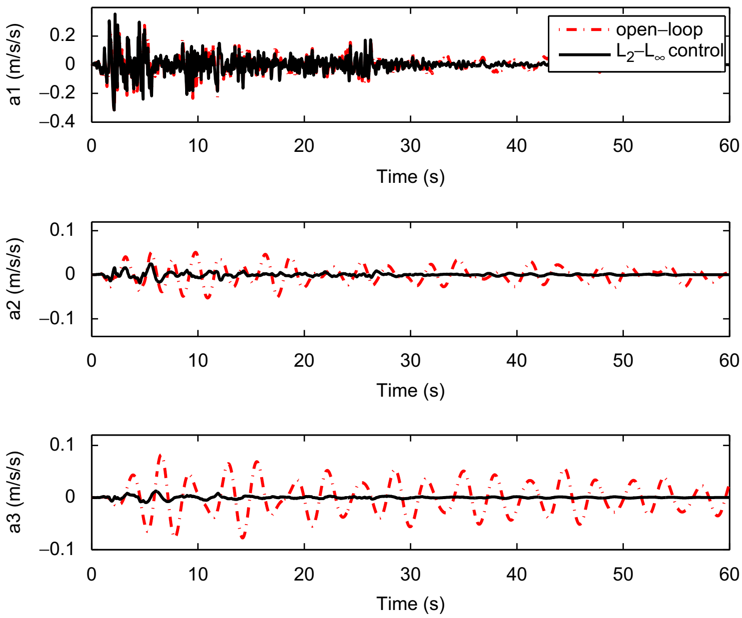

Fig. 6. Accelerations of the three floors in Case A1.

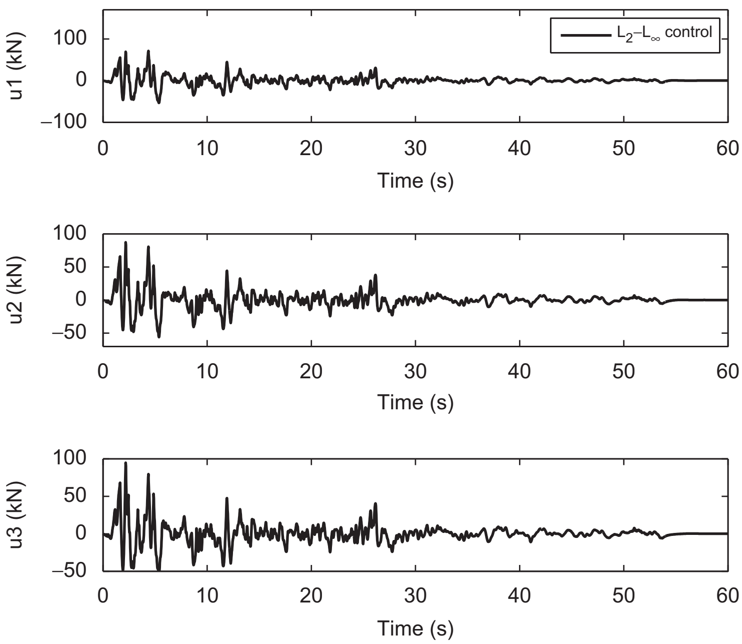

Fig. 7. Control forces of the three floors in Case A1.

4.2. *Case B: Full relative velocity output*

In this case, we choose the relative velocity of each floor as the controlled outputs, that is, $\mathbf{z} = [\dot{q}_1\ \dot{q}_2\ \dot{q}_3]^{\mathbf{T}}$.

We first designate the energy-to-peak performance index $\gamma = 0.3$. After solving the linear matrix inequalities in (18) and (19), the gain matrix of Controller L is

$$\mathbf{K_{bl}} = 10^7 \times \begin{bmatrix} -1.2362 & 2.2028 & -1.2727 & -0.0336 & 0.0586 & -0.0213 \\ 1.0760 & -2.5882 & 2.2361 & 0.0164 & -0.0635 & 0.0560 \\ -0.2380 & 0.9839 & -1.2353 & -0.0039 & 0.0174 & -0.0386 \end{bmatrix}.$$

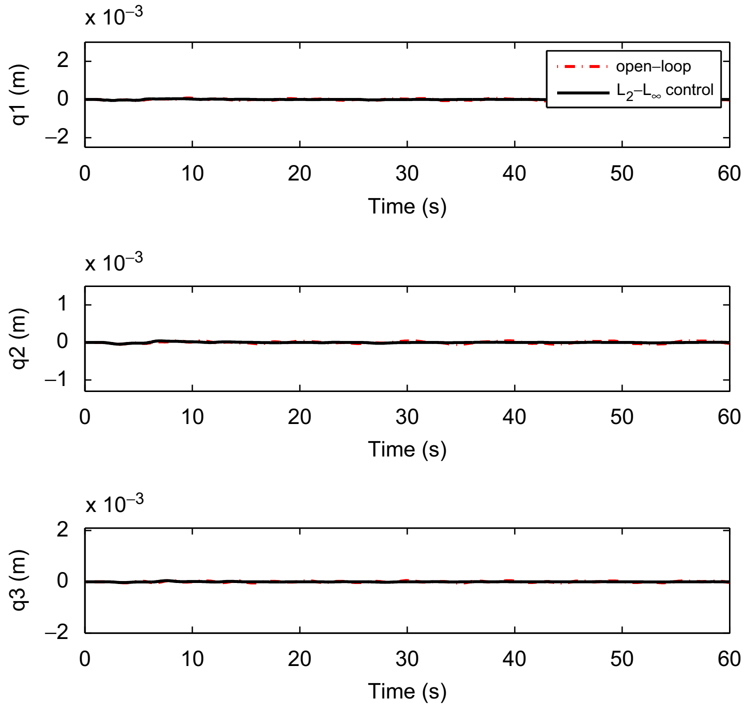

Fig. 8. Interstorey drifts of the three floors in Case A2.

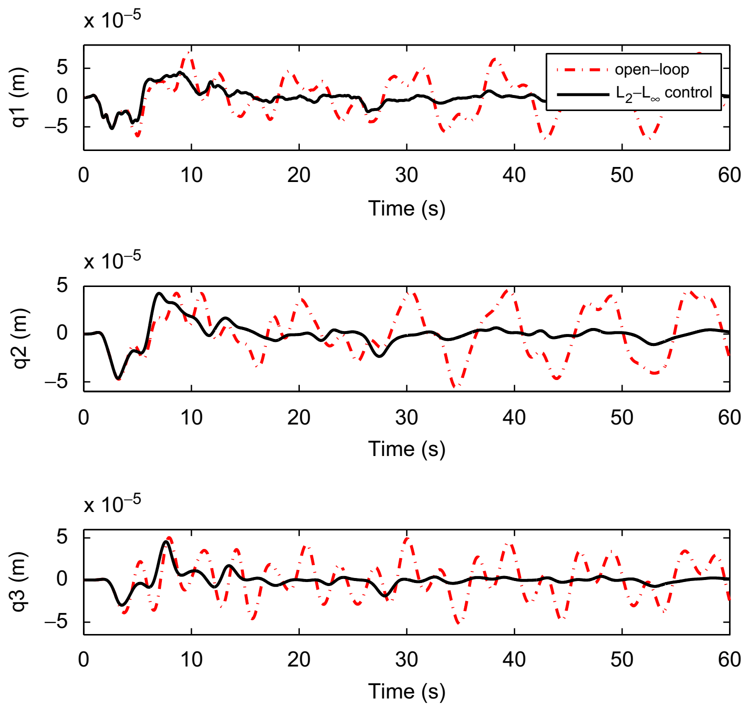

Fig. 9. Amplified interstorey drifts of the three floors in Case A2.

After obtaining Controller L, we also make comparison between closed-loop systems equipped with Controller L and open-loop systems to illustrate the effectiveness of robust reliable energy-to-peak control. First, we draw the frequency response of energy-to-peak control system and open-loop system in Fig. 12.

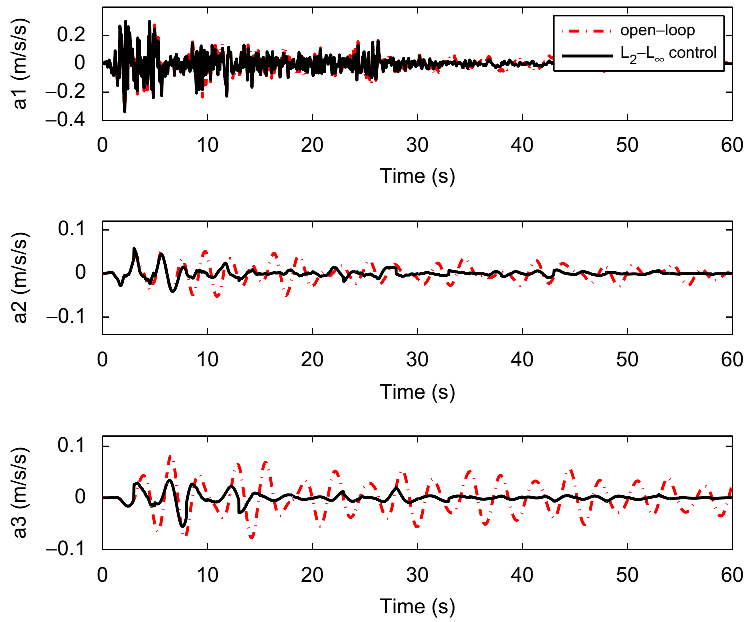

Fig. 10. Accelerations of the three floors in Case A2.

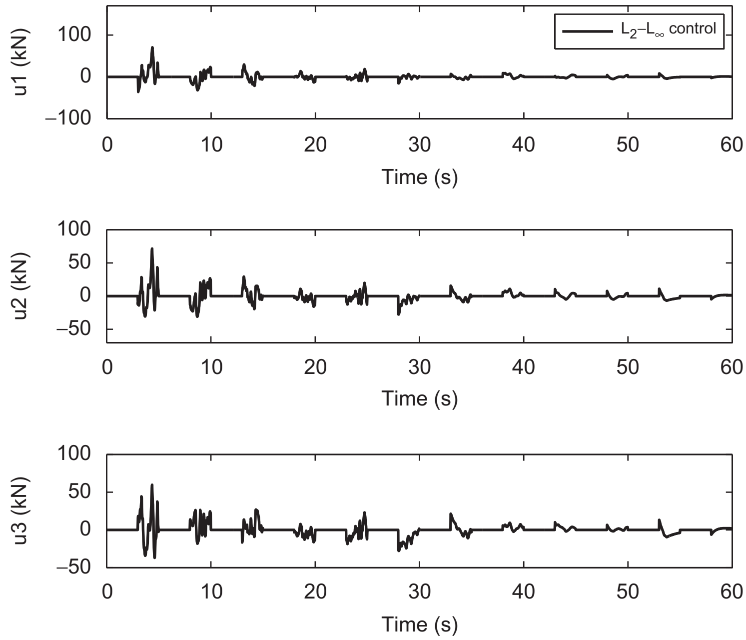

Fig. 11. Control forces of the three floors in Case A2.

Then, two different fault conditions will be discussed below. It is also supposed that $T=5$ s. Two fault conditions, will be discussed below:

$$\text{Case B1}: f_t = 1\,\text{s}, \quad \delta = 20 \text{ percent (20 percent loss of the actuator thrust);}$$

$$\text{Case B2}: f_t = 3\,\text{s}, \quad \delta = 80 \text{ percent (80 percent loss of the actuator thrust).}$$

Time responses of the interstorey drifts and accelerations of the three floors under the two cases are described in Figs. 13–18.

Similar to Case A, Figs. 13 and 16 depict the drift curves of each floor, and the relative accelerations of the three floors are depicted in Figs. 14 and 17. Control forces are also plotted in Figs. 15 and 18. Although the performances of

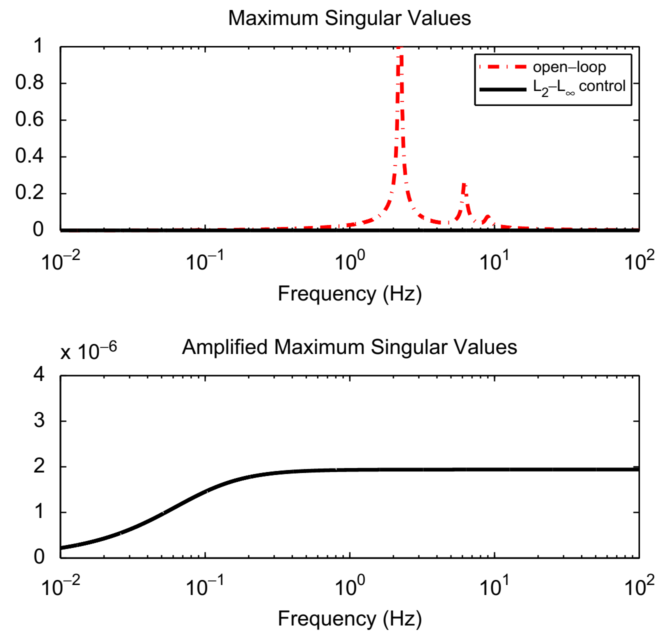

Fig. 12. The frequency response in Case B.

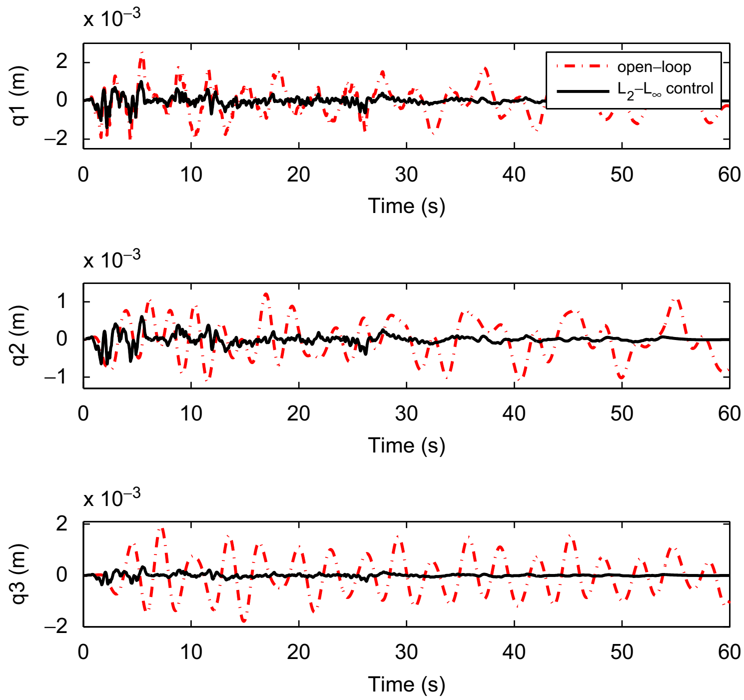

Fig. 13. Interstorey drifts of the three floors in Case B1.

energy-to-peak controllers are not as good as those in Case A, the significance of using such controllers can still be found because of their help in keeping closed-loop systems stable, reducing the peak value of drifts and accelerations, and shortening the duration of vibration.

4.3. Discussion about Case A and Case B

Based on the figures of interstorey drifts and accelerations, Cases A and B have shown the effectiveness of robust reliable energy-to-peak control systems in building vibration control in comparison with open-loop systems. By the

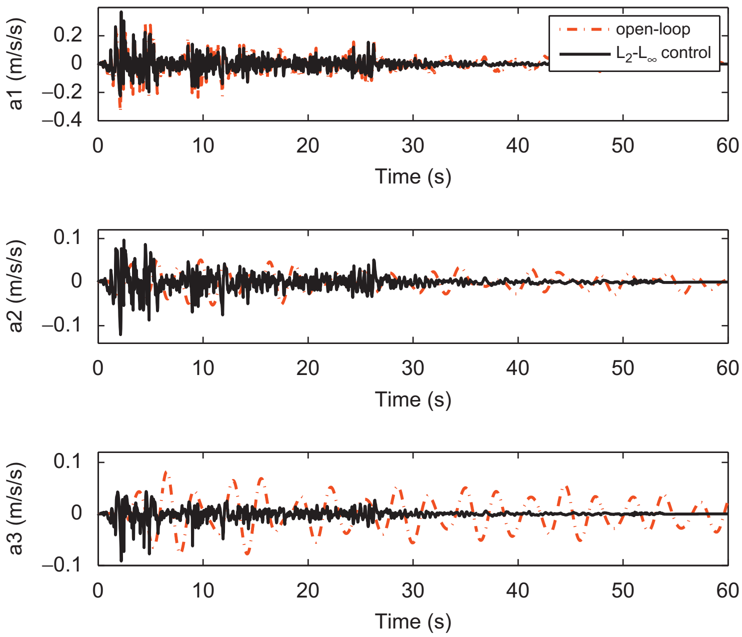

Fig. 14. Accelerations of the three floors in Case B1.

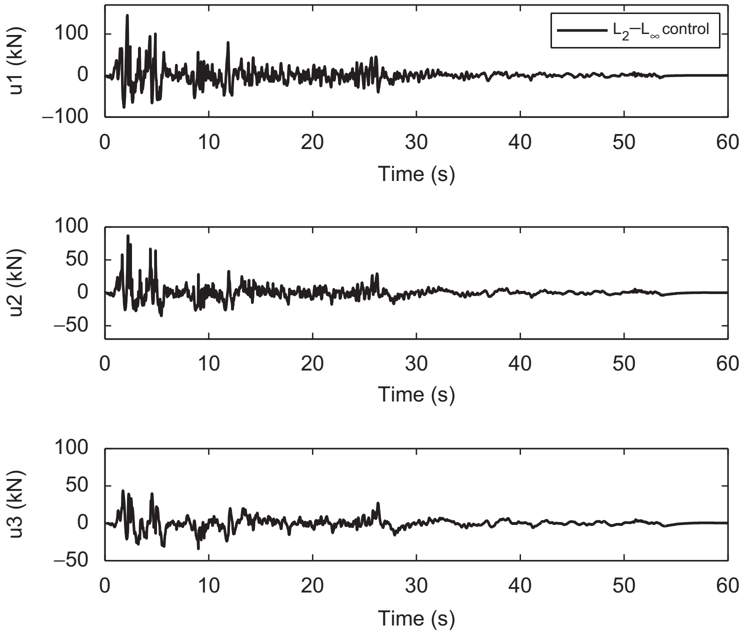

Fig. 15. Control forces of the three floors in Case B1.

comparison, we can get that the performances of closed-loop systems are much better than those of open-loop systems in different fault conditions. Different controlled outputs are chosen to justify that the energy-to-peak controllers can be trustworthy according to the conditions and requirements of real building vibration control systems. We can therefore ensure the effectiveness of the method raised in this paper. Moreover, with fast progress of computers' processing speed, this energy-to-peak control approach in the form of LMIs is applicable to real control systems.

4.4. Comparison between energy-to-peak control and H_∞ control

Here in particular, we would like to make comparisons between robust reliable energy-to-peak control and widely used robust reliable H_∞ control to further verify the good performances of the former. Let us take the same parameters as Case

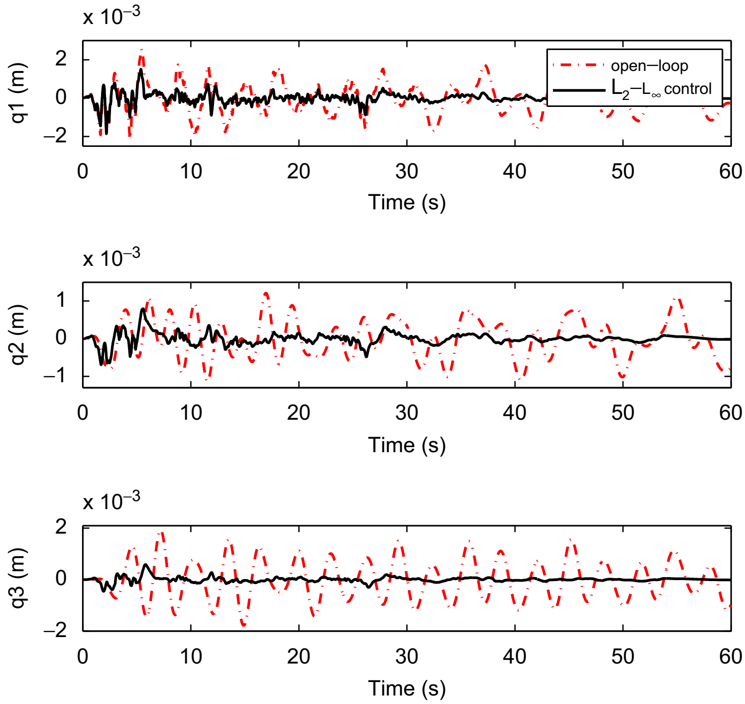

Fig. 16. Interstorey drifts of the three floors in Case B2.

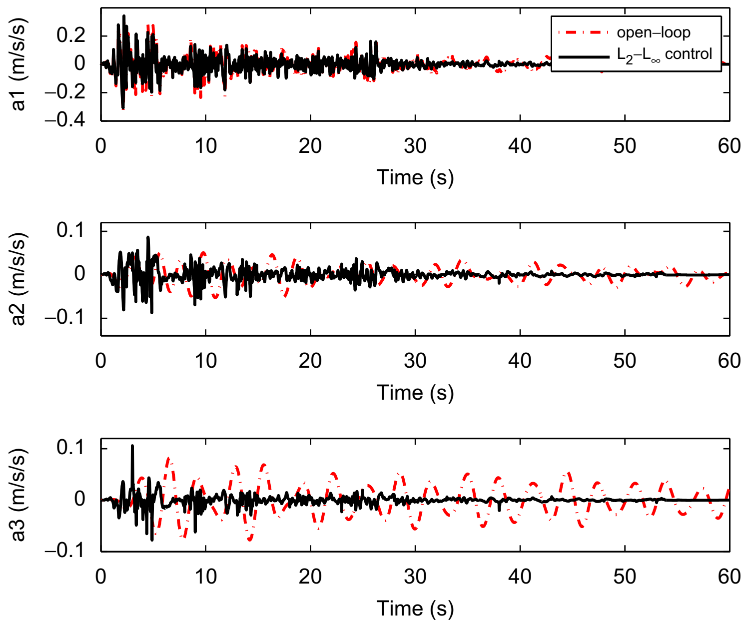

Fig. 17. Accelerations of the three floors in Case B2.

A, and the gain matrix of Controller L has been obtained in Case A

$$\mathbf{K}_{\mathbf{al}} = 10^5 \times \begin{bmatrix} -3.7914 & -0.5410 & -0.4729 & -0.6203 & -0.0942 & -0.0405 \\ -3.1740 & -4.6198 & -0.6493 & -0.7234 & -0.6697 & -0.0989 \\ -2.8948 & -4.2988 & -4.6304 & -0.7694 & -0.7289 & -0.6301 \end{bmatrix}.$$

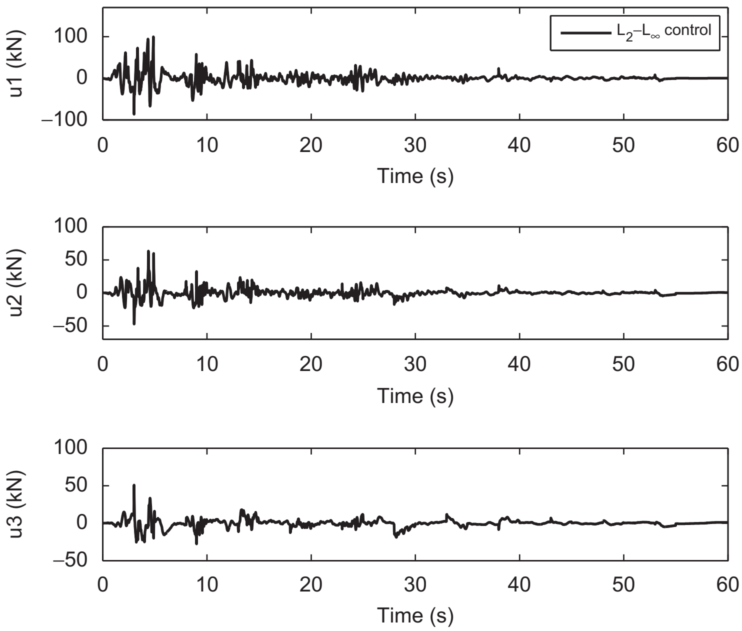

Fig. 18. Control forces of the three floors in Case B2.

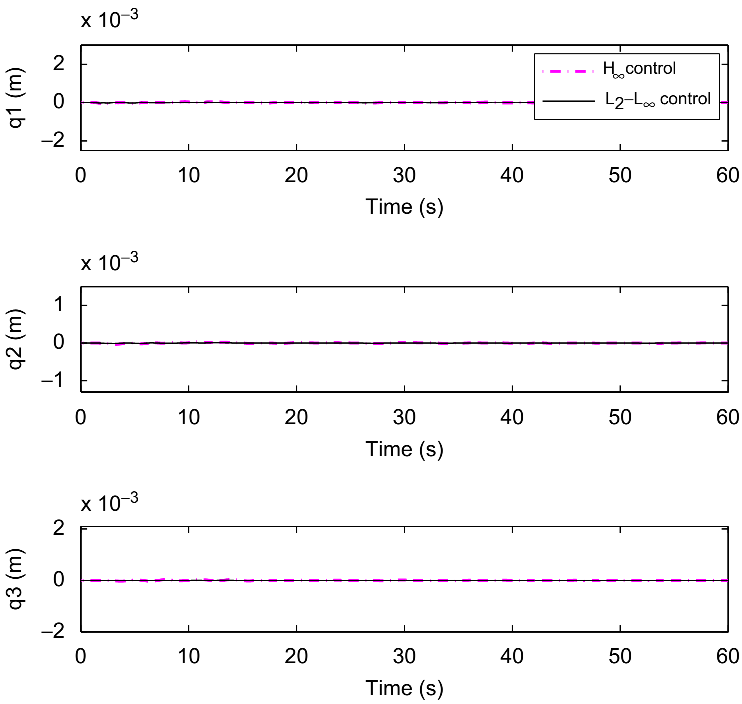

Fig. 19. Interstorey drifts of the three floors in Case 1.

The gain matrix of Controller H is obtained

$$\mathbf{K_{ah}} = 10^4 \times \begin{bmatrix} -0.3043 & -0.0779 & -0.1372 & -0.9237 & -0.3794 & -0.2098 \\ 0.0158 & -0.4971 & -0.0892 & -1.3034 & -1.1338 & -0.3797 \\ 0.1670 & -0.3170 & -0.5153 & -1.5133 & -1.3035 & -0.9239 \end{bmatrix},$$

and the corresponding minimum $\|\mathbf{G}(s)\|_\infty$ is $\eta_{\min} = 0.0624$.

Two fault conditions will be studied below:

$$\text{Case 1}: f_t = 1\ \text{s}, \quad \delta = 20\ \text{percent (20 percent loss of the actuator thrust);}$$

$$\text{Case 2}: f_t = 3\ \text{s}, \quad \delta = 80\ \text{percent (80 percent loss of the actuator thrust).}$$

Time responses of the interstorey drifts and accelerations of the three floors under the two cases are described in Figs. 19–24.

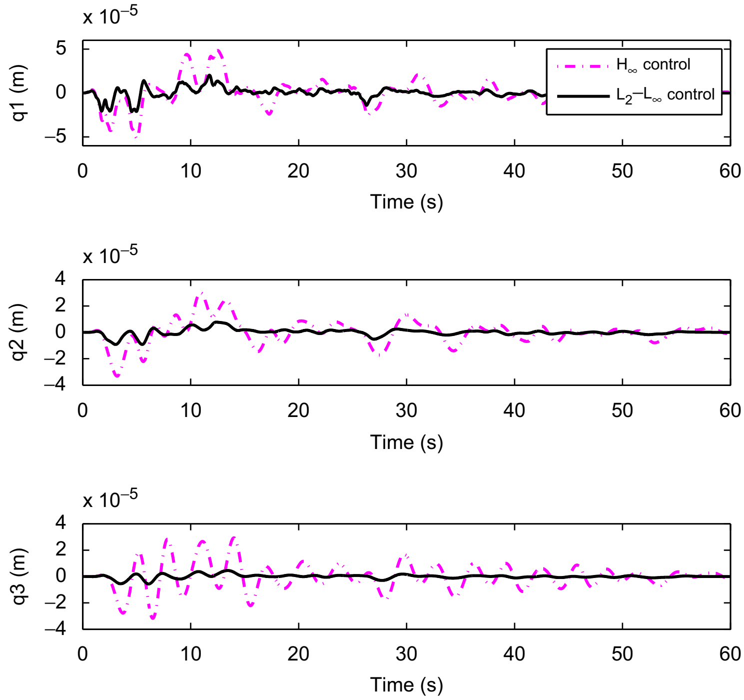

Fig. 20. Amplified interstorey drifts of the three floors in Case 1.

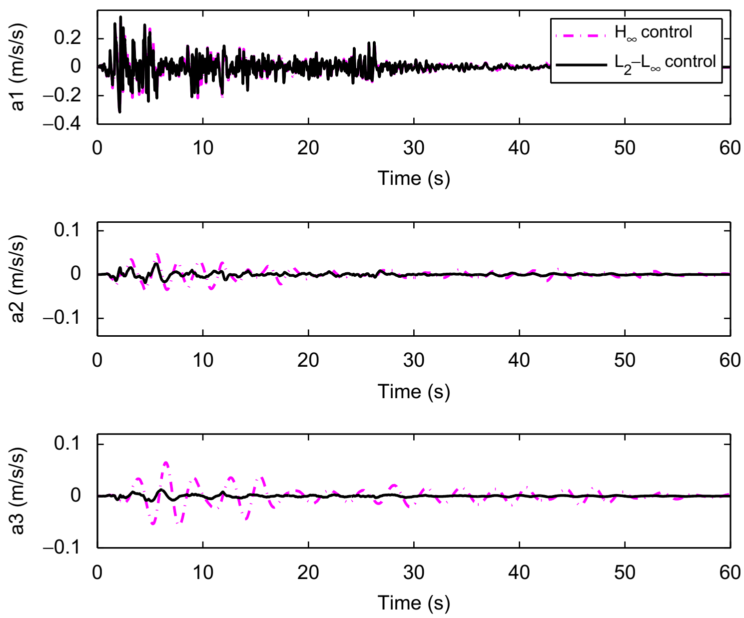

Fig. 21. Accelerations of the three floors in Case 1.

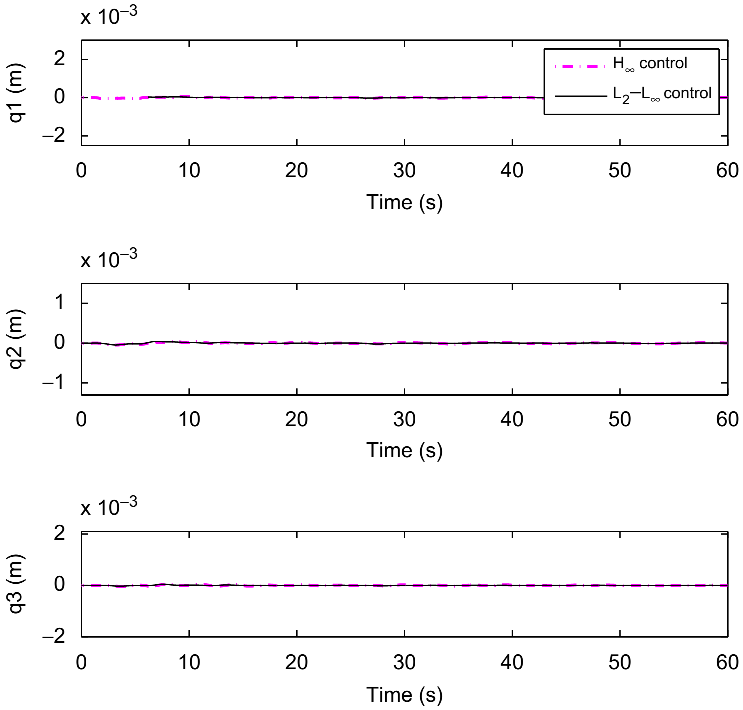

Fig. 22. Interstorey drifts of the three floors in Case 2.

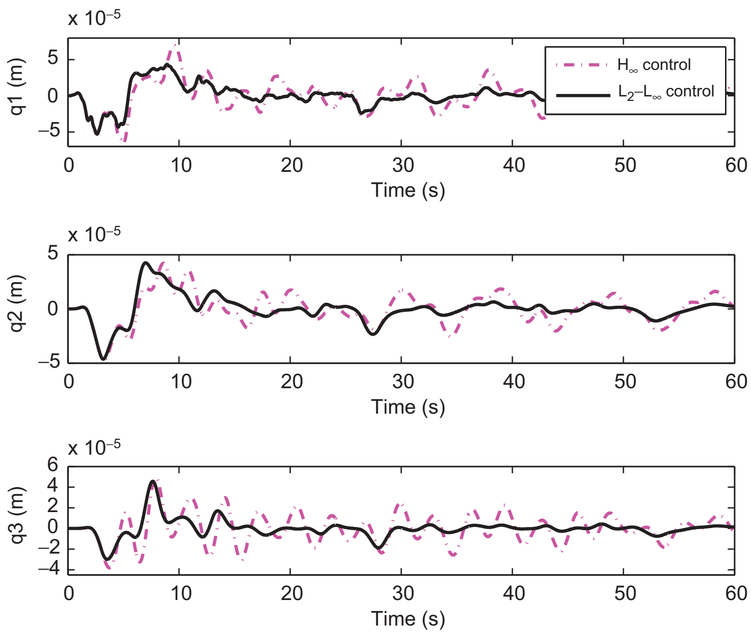

Fig. 23. Amplified interstorey drifts of the three floors in Case 2.

Figs. 19, 20, 22, 23 depict the drift curves of each floor, which demonstrate clearly that Controller L leads to less peak value and shorter duration of vibration, compared with Controller H. The relative accelerations of the three floors are depicted in Figs. 21 and 24, which also present better performances of Controller L in comparison with Controller H. Figs. 19–24 demonstrate that the energy-to-peak controllers can reduce the peak values of attenuation effectively and shorten the duration of vibration compared with the energy-to-energy controllers. The effectiveness of energy-to-peak control is completely proved.

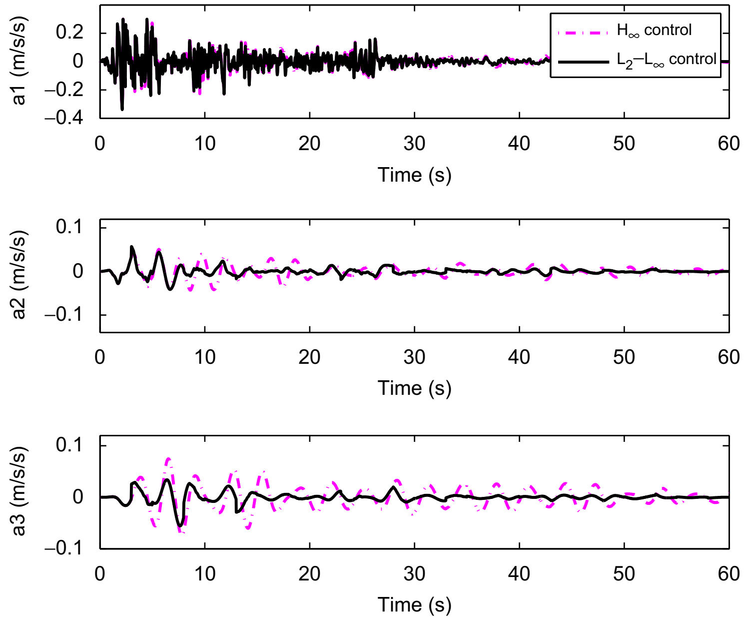

Fig. 24. Accelerations of the three floors in Case 2.

5. Conclusions

This paper has presented a method to design robust reliable energy-to-peak state-feedback controllers with actuator faults and parameter uncertainties for seismic-excited building vibration attenuation. Energy-to-peak performances of closed-loop systems are carefully analyzed. The objective to guarantee the asymptotic stability of closed-loop systems and attenuate building vibration caused by earthquakes has been achieved by solving the LMIs in the theorems.

Simulations of a 3-dof linear building structure have been employed to illustrate the effectiveness of the energy-to-peak control approach. Two different controlled outputs have been presented in the simulations, and different performances of closed-loop systems with energy-to-peak controllers and open-loop systems have been shown through the peak values of interstorey drifts and accelerations, as well as the duration of building vibration. Different controlled outputs can be chosen according to special requirements and constraints of real systems. The results of the simulations have demonstrated that energy-to-peak control for seismic-excited buildings has great effectiveness in comparison with open-loop systems. Comparisons have also been made between robust reliable energy-to-peak control and robust reliable H_∞ control through simulations which has further proved the effectiveness of the method proposed in this paper.

Since the algorithm developed in this paper is not very complex, we can expect it to be used in real building vibration control systems with the rapid development of sensors, processors, and actuators, and we want to do some experimental study to further check the effectiveness of the proposed method. After the similarities of the influence between wind and earthquakes on building vibration are taken into consideration, the approach of this paper can be promoted to the attenuation of wind-excited building vibration. Some future directions include consideration of time delay in measurements and actions, saturation of actuators, and synthesis of different building vibration control methods.

Acknowledgements

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Appendix

In the appendix, we will give proofs of the two theorems in this paper.

A.1. Proof of Theorem 1

Proof. According to Lemma 1 and Lemma 2, in order to design an energy-to-peak controller, we need to guarantee that there exists a symmetric matrix $\mathbf{X}>0$ satisfying $\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}+\mathbf{B_w}\mathbf{B_w^T}<0$ and $\mathbf{CXC^T}<\gamma^2\mathbf{I}$ for closed-loop system (7). Since uncertainties exist in matrix $\overline{\mathbf{A}}$, we need to find the upper bound of $\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}+\mathbf{B_w}\mathbf{B_w^T}$ and then we can use this upper bound to design controllers. According to this idea, together with Lemma 1 and (7), in this paper, we get

$$\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}=(\mathbf{A}+\mathbf{\Delta_A}+\mathbf{B_u}\mathbf{M_a}\mathbf{K_a})\mathbf{X}+\mathbf{X}(\mathbf{A^T}+\mathbf{\Delta_A^T}+\mathbf{K_a^T}\mathbf{M_a^T}\mathbf{B_u^T}). \tag{26}$$

According to (16), (26) can be further described as

$$\begin{aligned}\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}&=[\mathbf{A}+\mathbf{\Delta_A}+\mathbf{B_u}\mathbf{M_{a0}}(\mathbf{I}+\mathbf{L_a})\mathbf{K_a}]\mathbf{X}+\mathbf{X}[\mathbf{A^T}+\mathbf{\Delta_A^T}+\mathbf{K_a^T}(\mathbf{I}+\mathbf{L_a^T})\mathbf{M_{a0}^T}\mathbf{B_u^T}]\\&=(\mathbf{AX}+\mathbf{XA^T})+(\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T})+(\mathbf{B_u}\mathbf{M_{a0}}\mathbf{K_a}\mathbf{X}+\mathbf{X}\mathbf{K_a^T}\mathbf{M_{a0}^T}\mathbf{B_u^T})+(\mathbf{B_u}\mathbf{M_{a0}}\mathbf{L_a}\mathbf{K_a}\mathbf{X}+\mathbf{X}\mathbf{K_a^T}\mathbf{L_a^T}\mathbf{M_{a0}^T}\mathbf{B_u^T}).\end{aligned} \tag{27}$$

Taking the structure of $\mathbf{M_{a0}}$ and $\mathbf{L_a}$ into consideration, we can easily prove that $\mathbf{M_{a0}L_a}=\mathbf{L_aM_{a0}}$. Therefore, by defining

$$\mathbf{S}\triangleq\mathbf{M_{a0}K_aX},$$

we can get

$$\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}=(\mathbf{AX}+\mathbf{XA^T})+(\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T})+\mathbf{B_u}\mathbf{S}+\mathbf{S^T}\mathbf{B_u^T}+\mathbf{B_u}\mathbf{L_a}\mathbf{S}+\mathbf{S^T}\mathbf{L_a^T}\mathbf{B_u^T}. \tag{28}$$

According to Lemma 2, for any scalar $\varepsilon_1>0$ and symmetric matrix $\mathbf{X}$, we have

$$\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T}\le\varepsilon_1\mathbf{\Delta_A}\mathbf{\Delta_A^T}+\varepsilon_1^{-1}\mathbf{XX}\le\varepsilon_1\alpha^2\mathbf{I}+\varepsilon_1^{-1}\mathbf{XX}. \tag{29}$$

By using Lemmas 2 and 3, for any scalar $\varepsilon_2>0$ and matrices $\mathbf{S}$, we have

$$\mathbf{B_u}\mathbf{L_a}\mathbf{S}+\mathbf{S^T}\mathbf{L_a^T}\mathbf{B_u^T}\le\varepsilon_2\mathbf{B_u}\mathbf{J_a^T}\mathbf{B_u^T}+\varepsilon_2^{-1}\mathbf{S^T}\mathbf{J_a}\mathbf{S}. \tag{30}$$

Till now, we have

$$\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}+\mathbf{B_w}\mathbf{B_w^T}\le(\mathbf{AX}+\mathbf{XA^T})+\mathbf{B_u}\mathbf{S}+\mathbf{S^T}\mathbf{B_u^T}+\varepsilon_1\alpha^2\mathbf{I}+\varepsilon_1^{-1}\mathbf{XX}+\varepsilon_2\mathbf{B_u}\mathbf{J_a^T}\mathbf{B_u^T}+\varepsilon_2^{-1}\mathbf{S^T}\mathbf{J_a}\mathbf{S}+\mathbf{B_w}\mathbf{B_w^T}. \tag{31}$$

Finally, we can see that if (31) holds, (12) will be guaranteed. Then, with the help of Schur complement, we come to the conclusion that although parameter uncertainties exist, Lemma 1 will be satisfied if (18) and (19) hold, which indicates that closed-loop system (7) is asymptotically stable and the energy-to-peak performance in (10) is satisfied. □

A.2. Proof of Theorem 2

Proof. Firstly, according to Lemma 4, we perform a congruence transformation to (22) by matrix diag[$\mathbf{W}^{-1}$,**I**,**I**]. By denoting $\mathbf{X}=\mathbf{W}^{-1}$, we get that (22) is equivalent to

$$\begin{bmatrix}\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}} & \mathbf{B_w} & \mathbf{XC^T}\\ * & -\mathbf{I} & \mathbf{0}\\ * & * & -\eta^2\mathbf{I}\end{bmatrix}<0.$$

Similar to the proof of Theorem 1, $\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}$ can be written as

$$\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}=(\mathbf{A}+\mathbf{\Delta_A}+\mathbf{B_u}\mathbf{M_a}\mathbf{K_a})\mathbf{X}+\mathbf{X}(\mathbf{A^T}+\mathbf{\Delta_A^T}+\mathbf{K_a^T}\mathbf{M_a^T}\mathbf{B_u^T}). \tag{32}$$

By (16) we know that (32) can be further described as

$$\begin{aligned}\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}&=[\mathbf{A}+\mathbf{\Delta_A}+\mathbf{B_u}\mathbf{M_{a0}}(\mathbf{I}+\mathbf{L_a})\mathbf{K_a}]\mathbf{X}+\mathbf{X}[\mathbf{A^T}+\mathbf{\Delta_A^T}+\mathbf{K_a^T}(\mathbf{I}+\mathbf{L_a^T})\mathbf{M_{a0}^T}\mathbf{B_u^T}]\\&=(\mathbf{AX}+\mathbf{XA^T})+(\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T})+(\mathbf{B_u}\mathbf{M_{a0}}\mathbf{K_a}\mathbf{X}+\mathbf{X}\mathbf{K_a^T}\mathbf{M_{a0}^T}\mathbf{B_u^T})+(\mathbf{B_u}\mathbf{M_{a0}}\mathbf{L_a}\mathbf{K_a}\mathbf{X}+\mathbf{X}\mathbf{K_a^T}\mathbf{L_a^T}\mathbf{M_{a0}^T}\mathbf{B_u^T}).\end{aligned}$$

Notice that $\mathbf{M_{a0}L_a}=\mathbf{L_aM_{a0}}$. Therefore, by defining

$$\mathbf{Y}\triangleq\mathbf{M_{a0}K_aX},$$

we can get

$$\overline{\mathbf{A}}\mathbf{X}+\mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}}=\mathbf{AX}+\mathbf{XA^T}+\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T}+\mathbf{B_u}\mathbf{Y}+\mathbf{Y^T}\mathbf{B_u^T}+\mathbf{B_u}\mathbf{L_a}\mathbf{Y}+\mathbf{Y^T}\mathbf{L_a^T}\mathbf{B_u^T}.$$

According to Lemma 2, for any scalar $\varepsilon_3>0$ and symmetric matrix $\mathbf{X}$, we have

$$\mathbf{\Delta_A}\mathbf{X}+\mathbf{X}\mathbf{\Delta_A^T}\le\varepsilon_3\mathbf{\Delta_A}\mathbf{\Delta_A^T}+\varepsilon_3^{-1}\mathbf{XX}\le\varepsilon_3\alpha^2\mathbf{I}+\varepsilon_3^{-1}\mathbf{XX}.$$

By using Lemmas 2 and 3, for any scalar $\varepsilon_4 > 0$ and matrix $\mathbf{Y}$, we have

$$\mathbf{B_u L_a Y} + \mathbf{Y^T L_a^T B_u^T} \le \varepsilon_4 \mathbf{B_u J_a^T B_u^T} + \varepsilon_4^{-1} \mathbf{Y^T J_a Y}.$$

Till now, we have

$$\overline{\mathbf{A}}\mathbf{X} + \mathbf{X}\overline{\mathbf{A}}^{\mathbf{T}} \le \mathbf{AX} + \mathbf{XA^T} + \mathbf{B_u Y} + \mathbf{Y^T B_u^T} + \varepsilon_3 \alpha^2 \mathbf{I} + \varepsilon_3^{-1} \mathbf{XX} + \varepsilon_4 \mathbf{B_u J_a^T B_u^T} + \varepsilon_4^{-1} \mathbf{Y^T J_a Y}. \tag{33}$$

Obviously Lemma 4 is guaranteed if (33) holds. Furthermore, by Schur complement we can know that Lemma 4 is satisfied if (23) holds, which shows that the closed-loop system (7) is asymptotically stable and the H_∞ performance in (21) is satisfied. □

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