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H_2 optimization of a non-traditional dynamic vibration absorber for vibration control of structures under random force excitation

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ABSTRACT

The H_2 optimum parameters of a dynamic vibration absorber of non-traditional form are derived to minimize the total vibration energy or the mean square motion of a single degree-of-freedom (sdof) system under random force excitations. The reduction of the mean square motion of the primary structure using the traditional vibration absorber is compared with the proposed dynamic absorber. Under optimum tuning condition, it is shown that the proposed absorber when compared with the traditional absorber, provides a larger suppression of the mean square vibrational motion of the primary system.

1. Introduction

The traditional damped dynamic vibration absorber (DVA) or tuned mass damper (TMD) is an auxiliary mass–spring system with a damper added between the absorber mass m and the primary mass M as shown in Fig. 1a. Its basic function is to limit the vibration amplitude of the primary mass when the lower resonance is experienced during system start-up and stopping. It was invented by Frahm [1] in 1911. In 1928, Ormondroyd and Den Hartog [2] pointed out that the damping of the DVA had an optimum value for the minimization of the amplitude response of sdof system at resonance. Such optimization criterion is known as H_∞ optimization. The fixed points theory of Den Hartog [3] was commonly used for determining the optimum tuning frequency and damping ratios of the DVA which was attached to a sdof vibrating system.

To extend the application of the DVA, Crandall and Mark [4] proposed another optimization principle of the damped DVA with the objective function of minimizing the total vibration energy or the mean square motion of the primary structure under white noise excitation, which is called H_2 optimization of dynamic vibration absorber. The H_2 optimization would be more desirable than the H_∞ optimization if the vibrating system is subjected to random excitation such as wind loading instead of harmonic excitation. The exact solution of the H_2 optimization for the traditional DVA attached to an undamped primary system was derived by Warburton [5] and Asami et al. [6]. Asami also derived with other researchers [7] the analytical solutions to H_∞ and H_2 optimization problems of the traditional DVA attached to damped sdof systems. Cheung and Wong [8] derived the H_∞ and H_2 optimum parameters of the traditional DVA for suppressing vibrations in plates.

There are several non-traditional configurations of dynamic vibration absorbers reported to have better vibration suppression performance than the traditional design of DVA. Some examples of non-traditional DVA are series TMD [9], parallel multiple TMDs [10], multi-degree-of-freedom TMDs [11] and three- or four-element TMDs [12]. A variant design of the damped dynamic vibration absorber as shown in Fig. 1b was proposed by Ren [13], and Liu and Liu [14] recently. Based on the fixed points theory, the H_∞ optimum tuning parameters of such a vibration absorber has been derived analytically for suppressing the resonant vibration of a sdof system subjected to force excitation [15] or caused by ground motions [16]. It has

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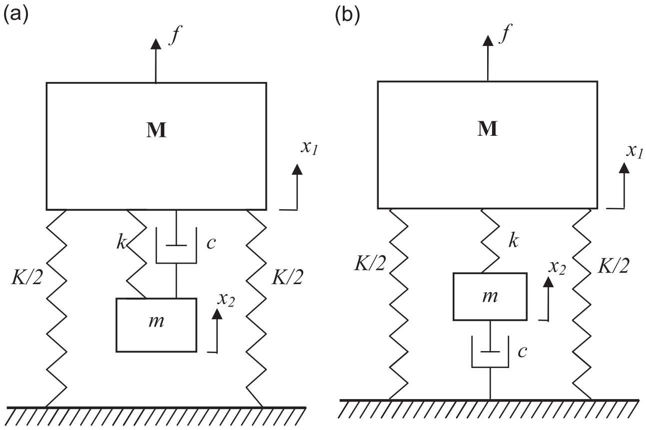

Fig. 1. A damped dynamic vibration absorber as an auxiliary mass–spring–damper (m–k–c) system attached to a primary system (M–K): (a) traditional design of the absorber [1], (b) the proposed design of the absorber for suppressing the vibration of the mass M due to a random force.

been proved that the H_∞ optimized non-traditional absorber can result in a larger reduction of the resonant vibration amplitude of the primary mass than the traditional damped dynamic absorber.

In this article, the H_2 optimum tuning frequency and damping of a damped dynamic vibration absorber of non-traditional form as shown in Fig. 1b have been derived for minimizing the mean square motion of a single-degree-of-freedom system under random force excitation. To the author's knowledge, there is no research report found in literature on this topic. It is shown that the proposed absorber can always provide a larger suppression of the mean square motion of the primary structure under random force excitation than the traditional absorber if its parameters are optimized according to the procedure as described in Section 3 of this article.

2. The traditional damped dynamic vibration absorber

A schematic diagram of a traditional damped dynamic vibration absorber attached to an undamped mass–spring system is shown in Fig. 1a. This vibration model is called model A in the following discussion. The amplification ratio may be written as [3]

$$\left|\frac{X_1}{F/K}\right|_{\mathrm{A}} = |G_{\mathrm{A}}| = \left|\frac{\gamma^2-\lambda^2+2j\zeta\gamma\lambda}{[(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\lambda^2\gamma^2]+2j\zeta\gamma\lambda(1-\lambda^2-\mu\lambda^2)}\right| \tag{1}$$

where

$$\mu=\frac{m}{M},\quad \omega_a=\sqrt{\frac{k}{m}},\quad \omega_n=\sqrt{\frac{K}{M}},\quad \gamma=\frac{\omega_a}{\omega_n},\quad \zeta=\frac{c}{2\sqrt{mk}},\quad \lambda=\frac{\omega}{\omega_n},\quad \text{and}\quad j=\sqrt{-1}$$

The objective function of H_2 optimization of the vibration absorber is the minimization of the integral of the square of amplitude of its frequency response of the primary structure under white noise excitation expressed as [4,6] $\min_{\gamma,\zeta}(E[|G_{\mathrm{A}}|^2])$, where

$$E\left[|G_{\mathrm{A}}|^2\right]=\omega_n\int_{-\infty}^{\infty}|G_{\mathrm{A}}|^2 S_0\,\mathrm{d}\lambda=\frac{\pi\omega_n S_0}{2\mu\zeta\gamma}\left[(1+\mu)^2\gamma^4+(4\zeta^2(1+\mu)-2-\mu)\gamma^2+1\right] \tag{2}$$

and S_0 is the uniform power spectrum density function.

The exact solution of the H_2 optimization for the DVA attached to an undamped primary system can be derived as [6]

$$\gamma_{\mathrm{opt\_}A}=\sqrt{\frac{\mu+2}{2(\mu+1)^2}} \tag{3a}$$

and

$$\zeta_{\mathrm{opt\_}A}=\frac{1}{2}\sqrt{\frac{\mu(3\mu+4)}{2(\mu+2)(\mu+1)}} \tag{3b}$$

The dimensionless mean square motion $E[|G_{\mathrm{A}}|^2]/2\pi\omega_n S_0$ contours of the primary mass M of Model A is calculated according to Eq. (2) and plotted in Fig. 2 to show the H_2 optimum frequency ratio $\gamma_{\mathrm{opt\_A}}$ and damping ratio $\zeta_{\mathrm{opt\_A}}$ of the DVA leading to the global minimum mean square motion of the mass M.

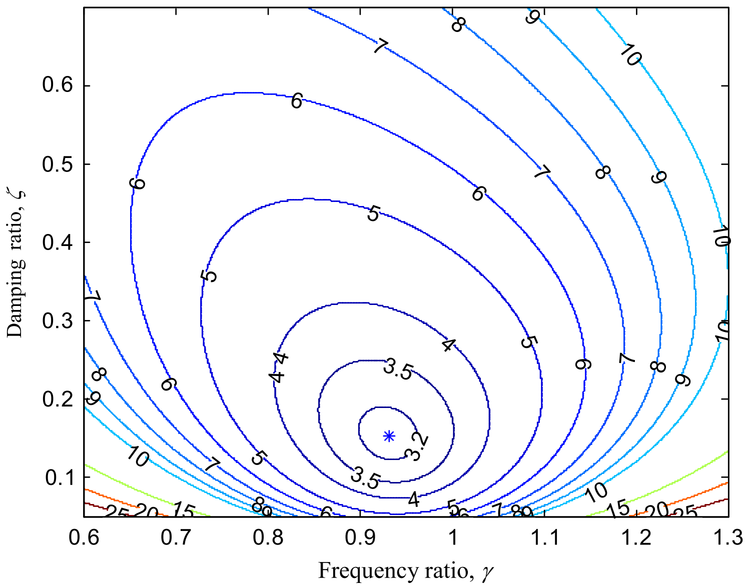

Fig. 2. Dimensionless mean square motion $E[|G_A|^2]/2\pi\omega_n S_0$ contours of the mass M of Model A. μ=0.1.

3. A variant form of the damped dynamic vibration absorber

A variant form of the damped dynamic vibration absorber as shown in Fig. 1b is called model B in the following discussion. The amplification ratio may be written as [13]

$$\left|\frac{X_1}{F/K}\right|_B = |G_B| = \left|\frac{\gamma^2-\lambda^2+2j\zeta\gamma\lambda}{[(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\lambda^2\gamma^2]+2j\zeta\gamma\lambda(1-\lambda^2+\mu\gamma^2)}\right| \tag{4}$$

The objective function of H_2 optimization of the vibration absorber is the minimization of the integral of the square of amplitude of its frequency response of the primary structure under white noise excitation expressed as $\min_{\gamma,\zeta}\left(E[|G_B|^2]\right)$, where

$$E\left[|G_B|^2\right] = \omega_n \int_{-\infty}^{\infty} |G_B|^2 S_0 \, d\lambda$$
$$= \omega_n S_0 \int_{-\infty}^{\infty} \left|\frac{\gamma^2-\lambda^2+2j\zeta\gamma\lambda}{[(1-\lambda^2)(\gamma^2-\lambda^2)-\mu\lambda^2\gamma^2]+2j\zeta\gamma\lambda(1-\lambda^2+\mu\gamma^2)}\right|^2 d\lambda \tag{5}$$

The following formula from Gradshteyn and Ryzhik [17] is used to solve the integration problem in Eq. (5)

$$\text{If } H(\lambda) = \frac{-j\lambda^3 B_3-\lambda^2 B_2+j\lambda B_1+B_0}{\lambda^4 A_4-j\lambda^3 A_3-\lambda^2 A_2+j\lambda A_1+A_0}$$
$$\text{then } \int_{-\infty}^{\infty} |H(\lambda)|^2 \, d\lambda = \pi\frac{[(B_0^2/A_0)(A_2A_3-A_1A_4)+A_3(B_1^2-2B_0B_2)+A_1(B_2^2-2B_1B_3)+(B_3^2/A_4)(A_1A_2-A_0A_3)]}{A_1(A_2A_3-A_1A_4)-A_0A_3^2} \tag{6}$$

Comparing Eqs. (5) and (6), we may write

$$A_0=\gamma^2, \quad A_1=2\zeta\gamma(1+\mu\gamma^2), \quad A_2=1+\gamma^2+\mu\gamma^2, \quad A_3=2\zeta\gamma, \quad A_4=1,$$
$$B_0=\gamma^2, \quad B_1=2\zeta\gamma, \quad B_2=1, \quad B_3=0 \tag{7}$$

Using Eqs. (6) and (7), Eq. (5) can be simplified as

$$E\left[|G_B|^2\right] = \frac{\pi\omega_n S_0}{2\mu\zeta\gamma^5}\left[\gamma^4+(\mu+4\zeta^2-2)\gamma^2+1\right] = \frac{\pi\omega_n S_0}{2\mu\zeta\gamma}\left[1+\frac{(\mu+4\zeta^2-2)}{\gamma^2}+\frac{1}{\gamma^4}\right] \tag{8}$$

$\min_{\gamma,\zeta}(E[|G_B|^2])$ requires zero derivatives of $E[|G_B|^2]$, i.e.

$$\frac{\partial}{\partial\zeta}E\left[|G_B|^2\right] = -\frac{\pi\omega_n S_0}{2\mu\zeta^2\gamma^5}\left[\gamma^4+(\mu-4\zeta^2-2)\gamma^2+1\right]=0 \tag{9a}$$

$$\frac{\partial}{\partial \gamma}E\left[|G_B|^2\right] = -\frac{\pi\omega_n S_0}{2\mu\zeta\gamma^6}\left[\gamma^4+(3\mu+12\zeta^2-6)\gamma^2+5\right]=0 \tag{9b}$$

Solving Eqs. (9a) and (9b) lead to the optimum damping ratio written as

$$\zeta_{\text{opt\_}B} = \sqrt{\frac{\gamma^4+(\mu-2)\gamma^2+1}{4\gamma^2}} \tag{10}$$

and two local minimum and maximum of tuning ratios written as

$$\gamma_{\text{H2\_min}} = \frac{1}{2}\sqrt{6-3\mu-\sqrt{(6-3\mu)^2-32}} \tag{11a}$$

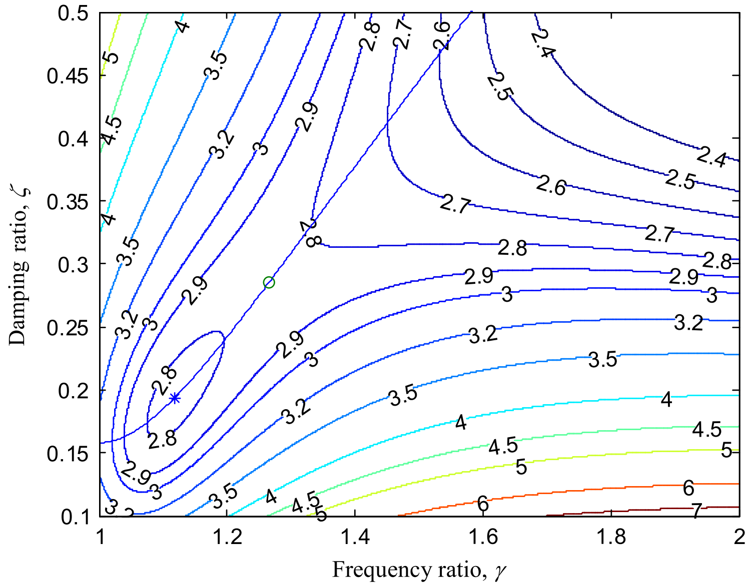

Fig. 3. Dimensionless mean square motion $E[|G_B|^2]/2\pi\omega_n S_0$ contours of the mass M of Model B. μ=0.1.

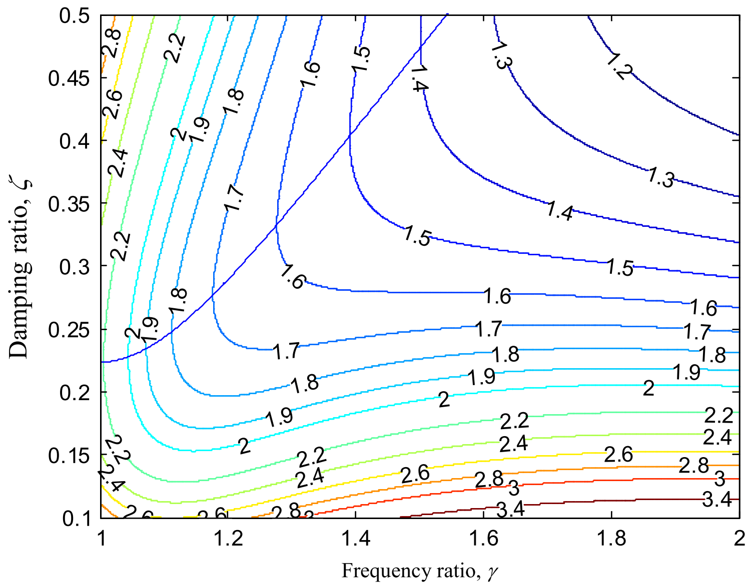

Fig. 4. Dimensionless mean square motion $E[|G_B|^2]/2\pi\omega_n S_0$ contours of the mass M of Model B. μ=0.2.

and

$$\gamma_{H2\_max} = \frac{1}{2}\sqrt{6-3\mu+\sqrt{(6-3\mu)^2-32}} \tag{11b}$$

These local minimum and maximum of tuning ratios would not exist if $\left(\sqrt{(6-3\mu)^2-32}\right) < 0$ or $\mu < 2-4\sqrt{2}/3 \approx 0.11$. The dimensionless mean square motion $E[|G_B|^2]/2\pi\omega_n S_0$ contours of the primary mass M of Model B is calculated according to Eq. (8) with mass ratio μ=0.1 and the results are plotted in Fig. 3. No global minimum of mean square motion exists and the local minimum and maximum points of the mean square motion of mass M are marked in Fig. 3 for illustration. Eq. (10) is also plotted in Fig. 3 to show the optimum damping ratio ζ_{opt_B} for any chosen frequency ratio γ of the variant DVA leading to the minimum mean square motion of the mass M of Model B. It can be seen in Fig. 3 that the variation of the mean square motion is relatively small at any point on the dotted line in Fig. 3 and it means small variation of frequency ratio or damping ratio of the DVA would not cause a big change of the mean square motion of the primary mass M in Model B. On the other hand, the variation of the mean square motion is relatively higher at the minimum point in Fig. 2 for the case of Model A. The mean square motion contours of the primary mass M of Model B is calculated according to Eq. (8) with mass ratio μ=0.2 and the results are plotted in Fig. 4 for illustration.

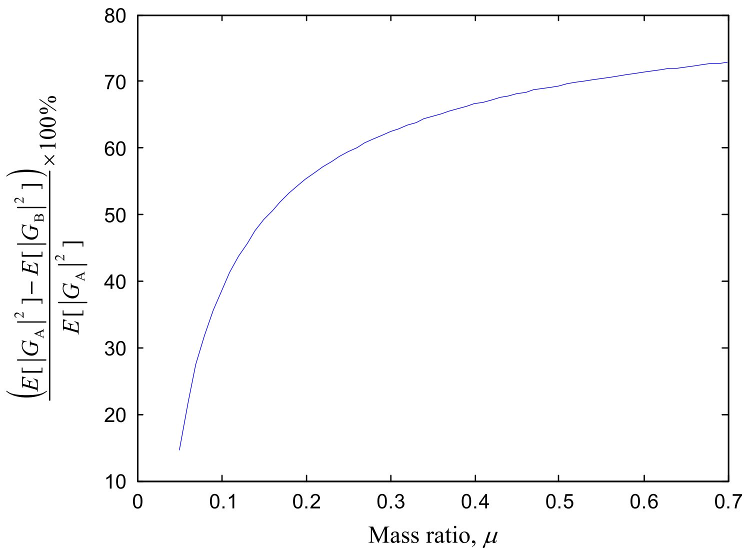

Fig. 5. Percentage reduction of the mean square motion of the proposed absorber relative to the traditional absorber at different mass ratios. γ=2.

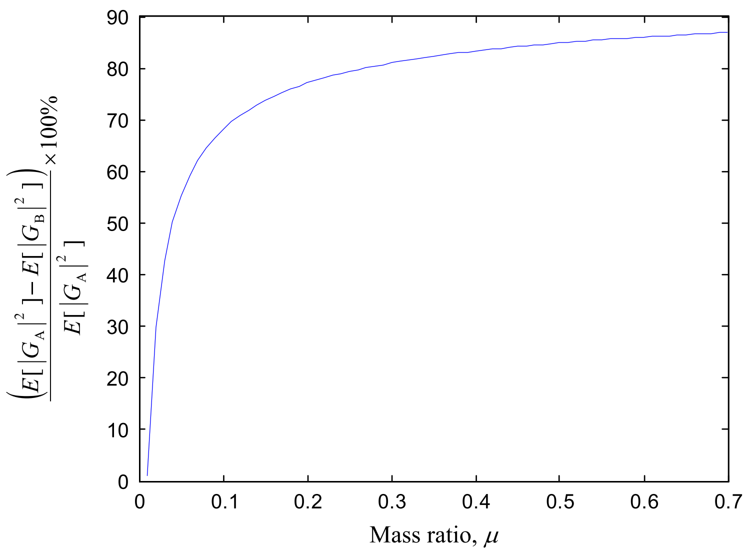

Fig. 6. Percentage reduction of the mean square motion of the proposed absorber relative to the traditional absorber at different mass ratios. γ=3.

Eq. (10) is also plotted in Fig. 4 to show the optimum damping ratio ζ_{opt_B} for any chosen frequency ratio γ of the variant DVA leading to the minimum mean square motion of the mass M of Model B. As predicted in the theoretical analysis before, no local minimum or maximum of the mean square motion exists and the mean square motion of the primary mass M decreases as frequency ratio or damping ratio increases. In practice, an as-high-as-possible frequency ratio of the DVA should be chosen and the corresponding damping ratio of the DVA is shown by the dotted line in Figs. 3 and 4 for μ=0.1 and 0.2, respectively, leading to the minimum mean square motion of the primary mass M.

To compare the variant DVA with ordinary DVA with the same mass ratio, the reduction of mean square motion by using a variant DVA with frequency ratio γ=2 and 3 are shown as Figs. 5 and 6, respectively. The mass ratio of the DVA in practice is about 10–30%. As shown in Fig. 5 the proposed DVA can provide 35% or more reduction of mean square motion of the primary mass M if the frequency ratio γ=2 is chosen for the proposed DVA. In Fig. 6, it shows that 60% or more reduction of mean square motion of the primary mass M if the frequency ratio γ=3 is chosen. In general, if γ is chosen to be larger than one and the damping ratio ζ and mass ratio μ are kept constant, Eq. (8) shows that the mean square motion $E[|G_B|^2]$ of the mass M of model B decreases as γ increases. It is therefore an as-high-as-possible value of γ is recommended to be chosen such that a practical minimum of mean square motion $E[|G_B|^2]$ of the mass M of model B can be achieved.

4. Conclusion

The H_2 optimum parameters of a non-traditional DVA attached to a sdof primary system is derived for minimizing the mean square motion of a sdof system under random force excitation. It has been shown that the performance of the variant DVA can be better than the ordinary one if the frequency and damping ratios of the DVA are chosen properly. To the author's knowledge, there is no research report found in literature on this topic. No global optimum tuning condition exists in the proposed absorber and it is recommended that a high tuning frequency ratio be used if possible. The best value of damping ratio after selecting the tuning frequency ratio is derived and stated in Eq. (10).

The optimally tuned variant DVA adds both damping and stiffness to the primary structure in comparison with the traditional DVA and therefore the mitigation of structural vibration subjected to external force is more significant for this variant dynamic vibration absorber. The comparison revealed that model B provides a 60 more percent of reduction of mean square motion of the primary mass M than the optimized model A if the frequency ratio of the model B is chosen to be three or above. It provides an alternative design for the traditional damped dynamic vibration absorber.

The H_2 optimization of the variant DVA to control the structural vibration under random ground motion will be reported elsewhere because the derivation of the H_2 optimal parameters of the variant DVA to control the structural vibration under random ground motion is more complicated than the present case and the comparison result of the effectiveness of the traditional and the variant dynamic vibration absorbers is found to be quite different from the result of the present case.

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