Journal of Sound and Vibration 330 (2011) 1065–1073

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Controlled passage through resonance in mechanical systems

Alexander Fradkov [b,*], Olga Tomchina [a], Dmitry Tomchin [b]

[a] St. Petersburg Machine Building Institute, 14, Poliustrovskii Ave., Saint Petersburg, Russian Federation
[b] Institute for Problems of Mechanical Engineering, 61, Bolshoy Ave. V.O., Saint Petersburg 199178, Russian Federation

ARTICLE INFO

Article history:
Received 14 January 2008
Received in revised form
25 September 2010
Accepted 27 September 2010
Handling Editor: M.P. Cartmell
Available online 27 October 2010

ABSTRACT

Methods of passing through resonance zones in mechanical systems are discussed and a new method based on the speed-gradient energy control of two subsystems (rotor and support) is presented. Two typical problems of passing through resonance for one- and two-dimensional motion of the support are posed and analyzed by computer simulation. The control algorithms based on the speed-gradient method and averaging allow one to significantly reduce the required level of the controlling torque. The proposed algorithms have a small number of design parameters. Compared with the known algorithms the proposed ones are more simple for design and exhibit stronger robustness properties.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Vibrational units with unbalanced (eccentric) rotors are widely used in the industry. An important problem for their design is to reduce the maximum power of the driving motor achieved during the spin-up mode [1]. The decrease of the spin-up power leads to decrease of nominal power and, therefore to decrease of the weight and the size of the motor. Another problem is that in order to obtain the desired mode of vibration it is necessary to control the rotor speed in a broad range including both pre-resonance and post-resonance regions. The problem of passage through resonance becomes more difficult when the power of the motor decreases. Therefore solution of both problems is important for developing vibrational equipment with improved technological characteristics.

First approaches to the problem were based on open loop (nonfeedback) methods [2–6]. In Refs. [2,3] the so-called "double start-up method" was proposed suggesting to switch off and then to switch on again the electric motor in the near-resonance zone. Switching instants should be calculated in advance and therefore are sensitive to variations of the unit parameters. In Ref. [4] variations of the acceleration rate in order to minimize the motion during passage through resonance were proposed. In Ref. [5] the variations of the acceleration rate were optimized using a gradient-based optimization method. In Ref. [6] saturation control is proposed; special attention is focused on passage through resonance when the non-ideal excitation frequency is near the portal frame natural frequency and when the non-ideal system frequency is approximately twice the controller frequency (two-to-one internal resonance).

More recent solutions are based on interdisciplinary approaches encompassing mechanics, control theory and computer simulations. The key idea to reduce the spin-up power of the unbalanced rotor is to swing the rotor during the spin-up period by feedback control. Different approaches to the problem were proposed [7–11]. In Refs. [7,8] the control law for oscillating eccentric rotor was proposed based on optimal design using Pontryagin maximum principle. However the practical implementation of this law is difficult because of complex calculations required for solving nonlinear optimal

* Corresponding author.
E-mail address: fradkov@mail.ru (A. Fradkov).

0022-460X/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2010.09.031

control problem. In Refs. [9,10] the phenomenon of resonant capture arising in the dynamics of dual-spin spacecraft was considered. However the control design proposed in the above mentioned papers is based on the averaged equations depending on coordinates which represent composite functions of initial variables. The design procedure requires calculation of the averaged angle coordinate of transformed model which is a very labor-consuming process. In Ref. [11] an optimal control law is proposed providing passage through the first resonant peak with minimum amplitude of the main mass.

A new approach to feedback control of passage through resonance was proposed in Refs. [12,13] for one-dimensional motion of support based on the speed-gradient method with energy-based goal functions [14]. Speed-gradient method was also extended to synchronization and other problems of oscillations control [15,16]. It was shown [14,16] that the speed-gradient algorithms for energy control of conservative systems allow to achieve an arbitrary energy level by means of arbitrarily small level of control power. Using this approach for systems with losses allows to spend energy only to compensate the losses, and to reduce the power of driving motor significantly. However, reduction of the motor power for systems with several degrees of freedom may increase the influence of resonance and lead to emergence of Sommerfeld effect and capture. Note that Sommerfeld effect concerns nonlinear jump and capture phenomena induced due to the influence of the unbalance response on a non-ideal drive around resonance speed [1]. The case of plane motion was studied in Ref. [7], where the controller was designed using optimal control technique, see also Ref. [8].

In this paper an approach to the problem of controlled passage through resonance zone for multidimensional systems is presented. The control algorithms based on speed-gradient method and averaging are proposed. Applications to the system with one-dimensional motion of support (TORA example) and to a two-dimensional system (plane motion of one-rotor vibrational unit) demonstrate efficiency of the approach. The proposed algorithms allow to significantly reduce the required level of the controlling torque. The efficiency of the algorithms is studied by means of computer simulation.

2. Problem statement

We use the standard Euler–Lagrange form of mechanical systems description

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{G}(\mathbf{q})=\mathbf{u}, \tag{1}$$

where $\mathbf{u}$ is the $n\times 1$ vector of control torques; $\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}$ are the $n\times 1$ vectors of generalized coordinates, velocities and accelerations correspondingly; $\mathbf{M}(\mathbf{q})$ is the $n\times n$ inertia matrix; $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the $n\times 1$ vector of Coriolis and centrifugal forces; $\mathbf{G}(\mathbf{q})$ is the $n\times 1$ vector of gravity forces; n is the number of the plant degrees of freedom. The losses (Rayleigh dissipation function) are not taken into account.

We will also use the Hamiltonian form which is convenient for the purpose of controller design. The controlled plant equations in Hamiltonian form are as follows:

$$\dot{\mathbf{p}}=-\left(\frac{\partial H}{\partial \mathbf{q}}\right)^{\mathrm{T}}+\mathbf{B}\mathbf{u},\quad \dot{\mathbf{q}}=\left(\frac{\partial H}{\partial \mathbf{p}}\right)^{\mathrm{T}}, \tag{2}$$

where $\mathbf{p},\mathbf{q}\in\mathbb{R}^n$ are the generalized coordinates and momenta; $H=H(\mathbf{p},\mathbf{q})$ denotes the Hamiltonian function (total energy of the system); $\mathbf{u}=\mathbf{u}(t)$ is the m-dimensional input vector (generalized forces), $\mathbf{B}$ is the $m\times n$ matrix, $m\le n$.

Formalize the main control objective as approaching the given energy level of the unforced system

$$H(\mathbf{p}(t),\mathbf{q}(t))\to H^* \quad \text{when} \quad t\to\infty. \tag{3}$$

Introduce the objective function as follows:

$$Q(\mathbf{p},\mathbf{q})=1/2[H(\mathbf{p},\mathbf{q})-H^*]^2. \tag{4}$$

Then the objective (3) can be reformulated as

$$Q(\mathbf{q}(t),\mathbf{p}(t))\to 0 \quad \text{when} \quad t\to\infty. \tag{5}$$

Let the additional inequality constraint be given

$$H_1(\mathbf{p}(t),\mathbf{q}(t))\le \Delta, \tag{6}$$

where H_1 is kinetic energy of first subsystem

$$H_1(\mathbf{p}(t),\mathbf{q}(t))=\frac{1}{2}\dot{\mathbf{q}}_1^{\mathrm{T}}\mathbf{M}_{11}(\mathbf{q}_1)\dot{\mathbf{q}}_1, \tag{7}$$

where $\mathbf{q}_1$ is the n_1-vector of generalized coordinates of the first subsystem; $\mathbf{M}_{11}(\mathbf{q}_1)$ is the corresponding $n_1\times n_1$ inertia submatrix. The problem is to design the control algorithm for the system (1) or (2), ensuring the objective (3) under constraint (6).

3. Design of control algorithms

The idea of solution is based on separation of motions near the resonance. It was noticed by several authors that relatively slow oscillations of angular velocity may appear near the resonance frequency [7,9,17,23]. In Ref. [17] it was shown by means of vibrational mechanics approach that the oscillations of the slow component may be described by the 1-dof pendulum-like equation. A more simple and general analysis including explicit conditions for appearance of slow motions was proposed in Ref. [23]. For passing through the resonance it was suggested in Ref. [12] to swing the slow motions up until the total energy H achieves the prescribed level and the energy of the subsystem H_1 becomes sufficiently small. Also it was suggested to apply the speed-gradient (SG) algorithm for swinging the system

$$\mathbf{u} = -\gamma \mathbf{\Psi}(\nabla_{\mathbf{u}} \dot{Q}), \tag{8}$$

where $\dot{Q}$ is derivative of Q with respect to the Eqs. (2), $\nabla_{\mathbf{u}}$ stands for the gradient in $\mathbf{u}$, $\gamma > 0$ is gain coefficient. The value of the vector-function $\mathbf{\Psi}(\mathbf{z})$ forms an acute angle with the vector $\mathbf{z}$, i.e. $\mathbf{\Psi}(\mathbf{z})^{\mathbf{T}}\mathbf{z} > 0$ for $\mathbf{z} \neq 0$.

Let us demonstrate the SG design procedure for Q chosen as above and for single degree-of-freedom oscillator $\ddot{\psi} + \Pi'(\psi) = u$ with oscillating variable ψ, potential $\Pi(\psi)$, momentum $p = \dot{\psi}$ and the Hamiltonian $H(\psi, p) = p^2/2 + \Pi(\psi)$. In this case we obtain

$$\dot{Q} = (H - H^*)\dot{H} = (H - H^*)\left(\frac{\partial H}{\partial \psi}\dot{\psi} + \frac{\partial H}{\partial p}\dot{p}\right) = (H - H^*)(\Pi'(\psi)p - p\Pi'(\psi) + pu) = (H - H^*)pu.$$

Taking derivative in u we arrive at the SG-algorithm (8) which for $\Psi(z) = z$ looks as follows:

$$u = -\gamma (H - H^*)\dot{\psi}. \tag{9}$$

For relay-like functions Ψ the SG-algorithms are as follows:

$$u = -\gamma \, \mathrm{sign}[(H - H^*)\dot{\psi}], \tag{10}$$

$$u = -\gamma \, \mathrm{sgn}[(H - H^*)\dot{\psi}], \tag{11}$$

where sign $x = 1$, if $x \geq 0$, sign $x = -1$, if $x < 0$; sgn $x = 1$ if $x \geq 0$, sgn $x = 0$ if $x < 0$. For our purposes the above algorithms or their modifications can be used where ψ is chosen as the slow variable near the resonance. To implement the algorithms (9)–(11) it is necessary to measure the velocity $\dot{\psi}$ of the slow variable ψ.

In this paper it is suggested to obtain ψ as the averaged value of the fast variable velocity $\omega = \dot{\theta}$, i.e. $\dot{\psi}$ as the averaged value of $\ddot{\theta}$. The averaging is performed by the first-order low-pass filter ("dirty differentiator")

$$T\dot{\psi}(t) = -\psi(t) + \dot{\theta}(t). \tag{12}$$

Time constant T should allow to suppress fast motion, i.e. its value should lie in the interval between the periods of fast and slow oscillation.

It is also proposed to switch on the speed-gradient algorithms (9)–(11) with filter (12) when the first subsystem energy is large (inequality (6) violates) and to switch to the conventional control $u(t) \equiv \gamma > 0$ when Eq. (6) holds. Additionally, to achieve better performance, the current value of the energy H_1 is subject to filtering.

4. Example 1: translational oscillator–rotational actuator

Consider the problem of controlling translational oscillator–rotational actuator (TORA), consisting of a support (cart) attached to a wall by a spring (Fig. 1). A rotating eccentric mass (unbalanced rotor) connected to the cart is actuated by a DC motor. Initially this system has been used as a simplified model to study the resonance capture (Sommerfeld effect) [9,18]. Recall that the capture phenomenon represents the failure of a rotating mechanical system to be spun up by a torque-limited rotor to a desired rotational velocity due to its resonant interaction with another part of the system [18],

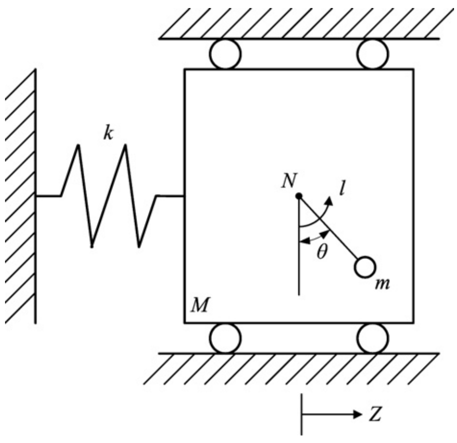

Fig. 1. TORA schematics (horizontal plane).

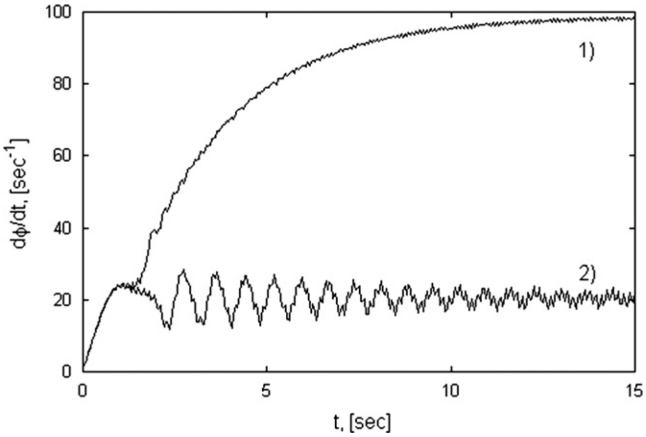

Fig. 2. Conventional control $u(t)\equiv\gamma_0$: (1) γ_0=0.49—capture and (2) γ_0=0.50—passage.

see also Fig. 2. Similar system (RTAC—Rotational/Translational ACtuator) was proposed by D. Bernstein with coworkers [19,20] as a benchmark example for nonlinear control and later was called TORA, see Ref. [21]. However, in the existing control related results only stabilization problem was addressed.

We pose the problem as spinning the system up to the desired energy level under restriction on the energy of its specified subsystem. The goal is to achieve the desired average angular velocity of the motor under constraint imposed on the translational oscillations of the cart. The motor torque is assumed to be a control variable. The model of the system is as follows:

$$(M+m)\ddot{z}+k_1\dot{z}+ml(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta)+kz=0, \tag{13}$$

$$J\ddot{\theta}+k_\theta\dot{\theta}+ml\ddot{z}\cos\theta=u, \tag{14}$$

where z is the displacement of the cart from its equilibrium position, θ is rotational angle of the rotor, u=$N(t)$ is motor torque (control variable). The system has the state vector $x=(z,\dot{z},\theta,\dot{\theta})^{\mathrm{T}}\in\mathbb{R}^4$. The total energy of the system is as follows:

$$H=\frac{1}{2}(M+m)\dot{z}^2+ml\dot{z}\dot{\theta}\cos\theta+\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}kz^2. \tag{15}$$

The model (13) and the energy (15) can be expressed also in the Hamiltonian form. We want to achieve the control goal

$$Q(x(t))\rightarrow 0, \tag{16}$$

where $Q=(H-H^*)^2/2$ and H_* is the desired energy level under the constraint $H_1(x(t))\leq\Delta$, where $\Delta>0$ and H_1 is the kinetic energy of the cart $H_1(x)=M\dot{z}^2/2$. For control the modification of SG-algorithm (11), (12) was chosen

$$u=\begin{cases}\gamma & \text{if } (H-H^*)\dot{\psi}<0,\\ 0 & \text{else,}\end{cases} \tag{17}$$

where ψ is the slow variable. Note that the filter (12) allows to obtain the sign of $\dot{\psi}(t)$ just comparing the values of $\dot{\theta}(t)$ and $\psi(t)$.

The numerical investigation of the system performance for different values of controller parameters γ and T was carried out by means of MATLAB software. The system parameters were taken as in Ref. [22]: J=0.014 kg m^2, M=10.5 kg, m=1.5 kg, l=0.04 m, k_θ=0.005 J s, k=5300 N/m, k_1=5 kg/s. For comparison the time history of angular velocity of the rotor driven by constant torque $u(t)$=γ_0 is presented in Fig. 2. The capture (for γ_0=0.49) and pass-through (for γ_0=0.50) phenomena can be observed.

In the first series of simulations with the algorithm (17), (12) the value of the controlled torque was chosen as γ_0=0.2, which is significantly less than the capture threshold γ_0=0.5. The values of time constant T were tested in the interval from T=0.1 to 2 s. The values of T corresponding to the fastest passage through resonance frequency were close to the period of slow motions: $T_*\approx 0.7$ s. In the second series of simulations we chose T=0.7 s and decreased γ. The results are shown in Figs. 3 and 4 (for γ_0=0.26 and H_*=11).

The simulation results confirm that the algorithm (17), (12) achieves significant reduction of the torque required for passage through resonance.

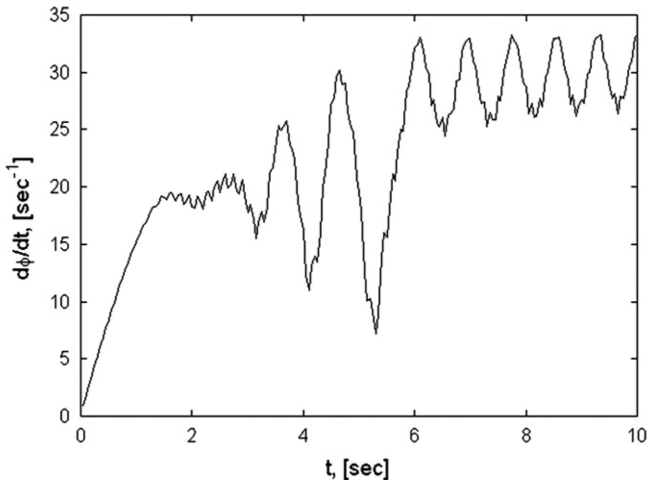

Fig. 3. Controlled passage through resonance, γ_0=0.26.

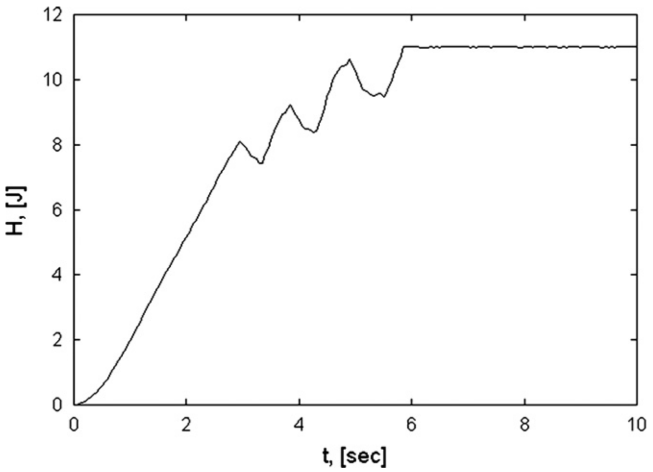

Fig. 4. Energy plot for proposed control: γ_0=0.26, H^*=11.

5. Example 2: vibrational unit with plane motion of support

5.1. Problem statement

Consider the following system of differential equations describing the plane motion of one-rotor vibrational unit (see Fig. 5) [13]:

$$\begin{aligned} &J\ddot{\varphi}+k_\varphi\dot{\varphi}+m\varepsilon(\ddot{x}\cos\varphi+\ddot{y}\sin\varphi+g\sin\varphi)=u(t),\\ &(M+m)\ddot{x}+k_x\dot{x}+c_x x+m\varepsilon(\ddot{\varphi}\cos\varphi-\dot{\varphi}^2\sin\varphi)=0,\\ &(M+m)\ddot{y}+k_y\dot{y}+cy+m\varepsilon(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi)=Mg, \end{aligned} \tag{18}$$

where φ is the rotor angle, x, y are the coordinates of the platform, $u(t)$ is the control action (rotating torque of a motor), J is the moment of inertia of an unbalanced rotor (disk), m is the mass of a rotor, M is the mass of a platform, ε is the eccentricity of the rotor center of mass, c, c_x are the shaft torsional stiffness, k_φ, k_x, k_y are the redamping factors.

It is well-known [23], that the "capture" of angular velocity of a rotor (Sommerfeld effect) sometimes takes place in the near-resonance zone. The capture phenomenon happens when the level of constant control action $u(t)\equiv M_0$ is small. If the level of constant control action $u(t)\equiv M_0$ is higher, the system passes the resonance zone (see Fig. 6) for the nominal values of system parameters: J=0.014 kg m^2, M=10.5 kg, m=1.5 kg, ε=0.04 m, k_φ=0.01 J s, c=5300 N/m, c_x=530 N/m, k_x=k_y=5 kg/s.

The problem is to design the control algorithm $u=\mathbf{U}(x,\dot{x},y,\dot{y},\varphi,\dot{\varphi})$, providing the spin-up of unbalanced rotor until the speed exceeds critical resonant value. The level of control signal is restricted and does not allow the passage through resonance when the control signal is constant. The existing optimal control algorithms [7] are hard for implementation and are not sufficiently robust.

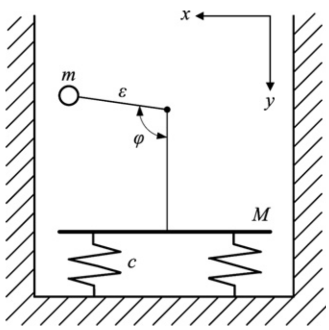

Fig. 5. One-rotor vibrational unit.

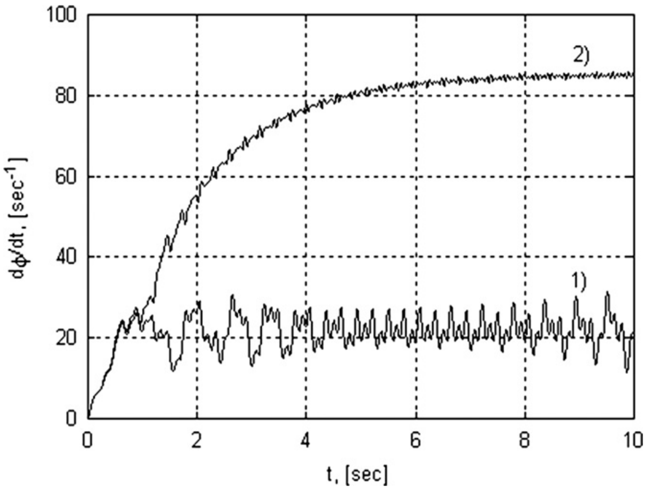

Fig. 6. Angular velocity for conventional control $u(t) \equiv M_0$: (1) M_0=0.80—capture and (2) M_0=0.81—passage.

5.2. Design of control algorithms

To describe the proposed control algorithm first describe the way to define the time of passing through resonance zone. It is proposed to measure the depth of the resonance by the average energy of the rotor center of mass motion, which is proportional to $(\dot{x}^2+\dot{y}^2)$. A related measure for oscillatory motion is the average sum of coordinate squares x^2+y^2 which is decreasing when system is passing through the resonance zone. This fact is confirmed by simulation: the sum x^2+y^2 increases when the level of constant control action is small and does not allow system to pass through the resonance zone. In case of higher control torque, the average sum of coordinate squares x^2+y^2 increase in the pre-resonance zone and decrease in the post-resonance zone.

In order to smooth the variable x^2+y^2 we introduce the additional low-pass filter:

$$T_\theta \dot{\theta}(x,y,t) = -\theta + x^2 + y^2, \quad \theta(0) = \dot{\theta}(0) = 0, \tag{19}$$

where $T_\theta > 0, T_\theta$=const is the algorithm parameter. The filtered variable $\theta(x, y, t)$ increases when the system is in the pre-resonance or resonance zone. In the post-resonance zone the value of $\theta(x, y, t)$ decreases significantly in comparison with the maximum value. Thus measuring the variable $\theta(x, y, t)$ of the filter (19) allows estimate the time of the passage through resonance. This fact is confirmed by computer simulation, see Fig. 7.

To synthesize the control algorithm we use the speed-gradient method. At this stage we suppose that the control plant is conservative, i.e. the friction equals to zero. The control goal is to find controlling function $u(t)$ providing the goal equality $H(x,\dot{x},y,\dot{y},\varphi,\dot{\varphi}) = H^*$, where $H(\cdot)$ is a current energy, H_* is the given energy level corresponding to the desired average rotation speed. Choose the goal functional $Q(z) = 1/2(H(z)-H^*)^2$, where $z = [x,\dot{x},y,\dot{y},\varphi,\dot{\varphi}]^{\mathrm{T}}$. The "relay" form of speed-gradient algorithm designed according to the proposed approach is as follows:

$$\begin{cases} u = \begin{cases} M_0 & \text{if } (H-H^*)(\dot{\varphi}-\psi) > 0, \\ 0 & \text{else,} \end{cases} \\ T_\psi \dot{\psi} = -\psi + \dot{\varphi}, \end{cases}$$

where $\psi(t)$ is the filtered variable, $T_\psi > 0$, T_ψ=const.

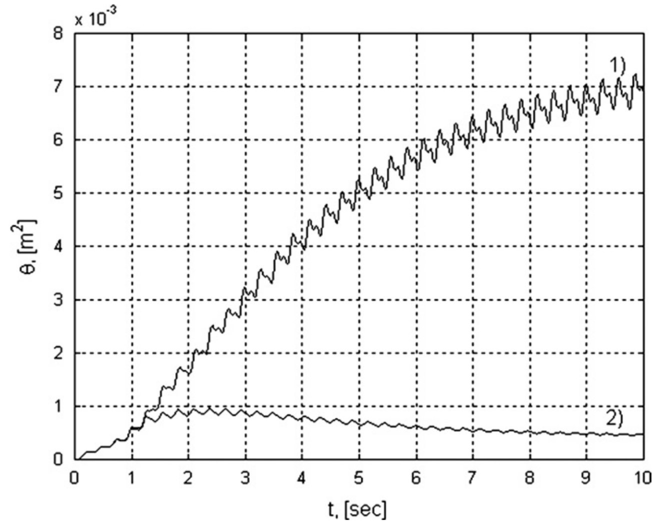

Fig. 7. Plot of $\theta(t)$ for $u(t)\equiv M_0$: (1) M_0=0.80—capture and (2) M_0=0.81—passage.

However the efficiency of this algorithm is rather low because of high amplitude of rotor oscillations. So the value H_* may be achieved in the resonance zone. Also this algorithm requires choosing the value H_* for every set of plant parameters, and this task has no evident solution.

Thus we propose to exclude the factor $H-H_*$ having the negative sign in the post-resonance zone, from the algorithm. We also propose to switch off the feedback term in the post-resonance zone, keeping only constant control torque. The algorithm is modified as follows.

The variable $\gamma_1(t)$:

$$\gamma_1(t)=\max_{[0,t]}\operatorname{sgn}[K\sup_{[0,t]}\theta(t)-\theta(t)],$$

is introduced, where $K>0$ is the algorithm parameter. The properties of the variable $\theta(t)$ allows to say that $\gamma_1(t)$=0 means that the system is in the pre-resonance or resonance zone (there was no significant decrease of $\theta(t)$). Also $\gamma_1(t)$=1 means that the system is in the post-resonance zone. Thus, $\gamma_1(t)$ characterizes the current behavior of the system if K is properly chosen. The value of K should be sufficiently small to guarantee that the system is already in the post-resonance zone. At the same time the unjustified decrease of K may reduce the efficiency and transient time of the proposed algorithm.

Finally, the algorithm takes the form

$$\begin{cases} u(t)=\begin{cases} M_0 & \text{if} \quad \gamma_1(t)=1, \\ M_0 & \text{if} \quad \gamma_1(t)=0 \quad \text{and} \quad (\dot{\varphi}-\psi)<0, \\ 0 & \text{else}, \end{cases} \\ T_\psi\dot{\psi}=-\psi+\dot{\varphi}, \\ \gamma_1(t)=\max\limits_{[0,t]}\operatorname{sgn}\left[K\sup\limits_{[0,t]}\theta(t)-\theta(t)\right], \\ T_\theta\dot{\theta}(t)=-\theta(t)+x^2+y^2, \quad \theta(0)=\dot{\theta}(0)=0. \end{cases} \tag{20}$$

The value of T_ψ (time constant of the angular velocity filter) should be more then the period of the resonant oscillations. At the same time, if the value of T_ψ is too high, the algorithm works slowly.

5.3. Simulation results

The proposed control algorithm was numerically examined to analyze its efficiency. Numerical integration was made in MATLAB environment by means of Runge–Kutta method of second order. The value of the fixed step equal to 0.00025 s was chosen so as the relative simulation error does not exceed 5%.

The nominal values of system parameters were chosen as in Ref. [22]: J=0.014 kg m^2, M=10.5 kg, m=1.5 kg, ε=0.04 m, k_φ=0.01 J s, c=5300 N/m, c_x=530 N/m, k_x=k_y=5 kg/s. It is seen that the value of the rotating torque of a motor, allowing system to pass the resonance zone for $u(t)\equiv M_0$, is reduced in almost two times by means of the proposed algorithm (Fig. 8).

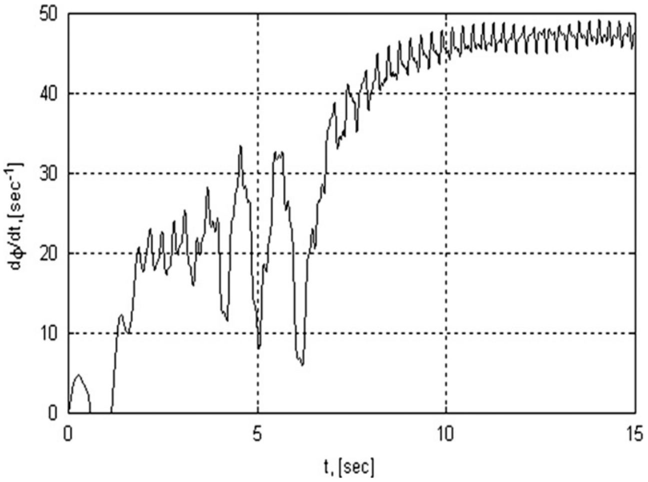

Fig. 8. Angular velocity of controlled system, M_0=0.46.

6. Conclusions

New algorithms of passing through resonance zone for mechanical systems are proposed and analyzed by computer simulation. The algorithms are based on speed-gradient method and allow to significantly reduce the required level of the controlling torque. The algorithms have small number of design parameters and, compared with the known algorithms are more simple for design. It is planned to test the proposed method experimentally on the two-rotor vibrational set-up (promising numerical results are presented in Ref. [22]).

Acknowledgements

The work was supported by RFBR (grants 05-01-00775, 07-01-92166-CNRS) and Russian Federal Program “Cadres” (contract 16.740.11.0042).

References

[1] I.I. Blekhman, *Vibrational Mechanics*, World Scientific, Singapore, 2000, (in Russian: 1994).
[2] V.V. Gortinskii, A.D. Savin, A.B. Demskii, M.A. Boriskin, E.A. Alabin, USSR Inventor’s Certificate no. 255760, Byull. Izobret., no. 33, 1969.
[3] V.V. Gortinskii, B.G. Khvalov, A method for control of the start-up of a vibrational system with an inertial actuator, in: *Mekhanika Mashin (Mechanics of Machines)*, Nauka, Moscow, 1981, issue 58, pp. 42–46.
[4] T. Iwatsubo, H. Kanki, R. Kawai, Vibration of asymmetric rotor through critical speed with limited power supply, *Journal of Mechanical Engineering Science* 14 (3) (1972) 184–194.
[5] H. Yamakawa, S. Murakami, Optimum designs of operating curves for rotating shaft systems with limited power supplies, in: H. Chung (Ed.), *Current Topics in Structural Mechanics, PVP*, Vol. 179, ASME, New York, NY1989, pp. 181–185.
[6] J.L.P. Felix, J.M. Balthazar, R.M.L.R.F. Brasil., On Saturation control of a non-ideal vibrating portal frame foundation type shear-building, *Journal of Vibration and Control* (1) (2005) 121–136*11* (1) (2005) 121–136.
[7] L.M. Malinin, A.A. Pervozvansky, Optimization of passage of an unbalanced rotor through critical speed, *Mashinovedenie* 4 (1983) 36–41 (in Russian).
[8] A.S. Kel’zon, L.M. Malinin, *Control of Rotor Oscillations*, Politekhnika, Saint Petersburg, 1992, (in Russian).
[9] R.H. Rand, R.J. Kinsey, D.L. Mingori, Dynamics of spin up through resonance, *International Journal of Nonlinear Mechanics* 27 (3) (1992) 489–502.
[10] R.H. Rand, R.J. Kinsey, D.L. Mingori, Nonlinear control of dual spin spacecraft during despin through precession phase lock, *AIAA Journal of Guidance, Control and Dynamics* 19 (1) (1996) 60–67.
[11] J.M. Balthazar, B.I. Cheshankov, D.T. Ruschev, L. Barbanti, H.I. Weber, Remarks on passage through resonance of a vibrating system with two degrees of freedom, excited by a non-ideal energy source, *Journal of Sound and Vibration* 239 (5) (2001) 075–1085.
[12] O.P. Tomchina, Passing through resonances in vibratory actuators by speed-gradient control and averaging, in *Proceedings of the International Conference on Control of Oscillations and Chaos*, Vol. 1, IEEE, St. Petersburg, 1997, pp. 138–141.
[13] O.P. Tomchina, K.V. Nechaev, Controlling passage through resonances in vibratory actuators, in *Proceedings of the Fifth European Control Conference*, Karlsruhe, September 1999.
[14] A.L. Fradkov, Swinging control of nonlinear oscillations, *International Journal of Control* 64 (6) (1996) 1189–1202.
[15] I.I. Blekhman, A.L. Fradkov, O.P. Tomchina, D.E. Bogdanov, Self-synchronization and controlled synchronization: general definition and example design, *Mathematics and Computers in Simulation* 58 (4–6) (2002) 367–384.
[16] A.S. Shiriaev, A.L. Fradkov, Stabilization of invariant sets for nonlinear systems with applications to control of oscillations, *International Journal of Robust and Nonlinear Control* 11 (2001) 215–240.
[17] A.V. Pechenev, About motion of oscillating system with bounded excitation near resonance, *Physics-Doklady* 32 (1986) (Doklady Academii Nauk SSSR, vol. 290, No. 1).
[18] R.M. Evan-Iwanowski, *Resonance Oscillations in Mechanical Systems*, Elsevier, Amsterdam, 1976.
[19] C.-J. Wan, D.S. Bernstein, V.T. Coppola, Global stabilization of the oscillating eccentric rotor, in *Proceedings of the 34th IEEE Conference on Decision and Control*, 1994, pp. 4024–4029.

[20] R.T. Bupp, D.S. Bernstein, V.T. Coppola, A benchmark problem for nonlinear control design, *International Journal of Robust and Nonlinear Control* 8 (1998) 307–310.
[21] M. Jankovic, D. Fontaine, P.V. Kokotovic, TORA example: cascade and passivity based designs, *IEEE Transactions on Control Systems Technology* 4 (1996) 292–297.
[22] D.A. Tomchin, A.L. Fradkov, Control of passage through a resonance area during the start of a two-rotor vibration machine, *Journal of Machinery Manufacture and Reliability* 36 (4) (2007) 380–385.
[23] I.I. Blekhman, D.A. Indeitsev, A.L. Fradkov, Slow motions in systems with inertially excited vibrations, *Journal of Machinery Manufacture and Reliability* (1) (2008) 21–27.