221. *The Separation of Mixtures of Solutes by Distribution between Solvents. Part I. The Separation of the Components of a Fixed Quantity of a Mixture by a Continuous Counter-current Extraction Process.*

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The manner in which a fixed quantity of a solute, introduced all at once, distributes itself throughout a series of mixing vessels through which two solvents pass continuously in counter-current, is investigated theoretically. It is shown that mixtures of solutes having different distribution coefficients should be separable into their components to extents dependent on the number of vessels in the system, and on the relative flow rates of the solvents. The best condition for the separation of two solutes, present in equal proportions in the central vessel of an extended system, is when the ratio of the feed rates is the inverse of the geometric mean of the distribution coefficients. The theoretical conclusions have been investigated by reference to the behaviour of oxalic and succinic acids towards butanol saturated with water and water saturated with butanol, in 2- and 19-vessel systems; the experimental results are in substantial agreement with the theoretical requirements.

THE principles underlying the separation of solutes by systematic partition between two solvents have been treated by Jantzen (" Das fractionierte Destillieren und das fractionierte Verteilen als Methoden zur Trennung von Stoffgemischen," Dechema-Monographie No. 48, Vol. 5, p. 100, Berlin, Verlag Chemie, 1932), by Nash (*Ind. Eng. Chem.*, 1935, **27**, 836) and by Bush and Densen (*Analyt. Chem.*, 1948, **20**, 121). These authors have concerned themselves with the purification of an impure solute by a series of batch extractions which bears many analogies to systematic fractional crystallisation. The practical difficulties in operating in this fashion are, however, considerable. Each further stage of extraction involves still another additional operation of distribution, and the chances of confounding the various solutions are increased accordingly. Bush and Densen showed that the maximum separation of two solutes having distribution coefficients k_1 and k_2 results from the use of solvents in the volume ratio of $\sqrt{1/k_1k_2}$ at each step in the distribution process. This is strictly analogous to the condition deduced for the most satisfactory operation of the process described below.

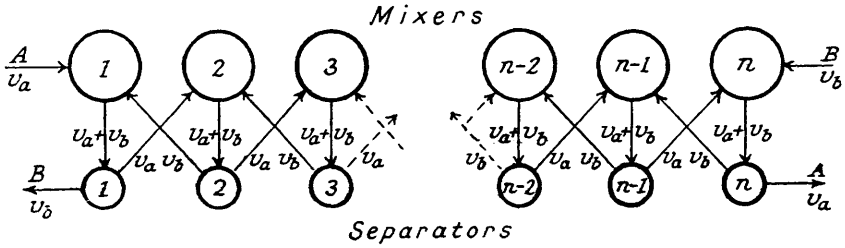
An ingenious method of conducting such batch-wise separations was devised by Craig (*J. Biol. Chem.*, 1943, **150**, 33; 1944, **155**, 519) who described an apparatus which prevented the confounding of the various solutions and permitted a large number of the steps to be performed by one manual operation. Craig (*loc. cit.*), Williamson and Craig (*ibid.*, 1947, **168**, 687), and Lieberman (*ibid.*, 1948, **173**, 63) have shown mathematically that the manner in which a solute should distribute itself in the several tubes which comprise the apparatus can be predicted, and that differences between the theoretical and the observed values point to inhomogeneity of the solute.

The behaviour of a continuously run counter-current system bearing analogies with the Craig apparatus has now been investigated and it has been shown to possess all the advantages of the latter, while at the same time no upper limit to the scale of operation is imposed.

Our counter-current separation process is concerned with a fixed quantity of material which is introduced *all at once* into the system. The separation or extraction of a solute, continuously, from a continuous feed of a raw material by counter-current methods is well-known practice, tubular extractors, or series of vessels which bear a formal resemblance to the apparatus now described, being the usual form of extraction equipment. As far as can be ascertained only two attempts have been made to deal with the separation of the components of a fixed quantity of a crude material fed into a system all at one time (Cornish, Archibald, Murphy, and Evans, *Ind. Eng. Chem.*, 1934, **26**, 397; Martin and Synge, *Biochem. J.*, 1941, **35**, 91). Our system consists of a series of mixing vessels each provided with a separator; the nature of the separator depends on the behaviour of the materials being studied but can be disregarded for the time being, except for the condition that the volumes of the separators must be small compared with those of the mixing vessels, the volumes of the latter being all supposed equal. The general arrangement of the mixers and separators, and the direction of flow of liquids, together with their rates, is shown in Fig. 1. Solvent A enters the system at mixer 1 and there mixes with solvent B; the mixture passes to the separator at a rate which is the sum of the separate rates of the two phases. (Volume changes on mixing the two solvents can be neglected since each is saturated with the other before introduction into the system.) The two phases separate in the separator 1, whence solvent B leaves the system and solvent A passes to mixer 2 at the same rate at which it enters mixer 1. In mixer 2 solvent A mixes with solvent B from separator 3, and the mixture

passes to separator 2, whence B proceeds to mixer 1 and A goes to mixer 3, and so on throughout the system. Solvent A moves, therefore, from left to right and solvent B contrariwise. The problems to be solved first are the behaviour of a solute placed in any one vessel, and the possibility of separating solutes, the distribution coefficients of which between the two solvents differ to greater or lesser degrees.

FIG. 1.



The following symbols are used below :

x = weight of solute in g.
V = volume of mixers in ml.
v_a = feed rate of solvent A in ml./hour.
v_b = feed rate of solvent B in ml./hour.
k, k_1, k_2 etc. = distribution coefficients of solutes between A and B.
n = number of mixers (or separators) in the system.
K = number of the vessel in which the solute is placed.
t = time in hours.
$p_1, p_2, \ldots p_n$ = concentration of solute in solvent A in vessel 1, 2, ... n.
$q_1, q_2, \ldots q_n$ = concentration of solute in solvent B in vessel 1, 2, ... n.

The distribution coefficient k is defined as p/q; p usually refers to aqueous solutions and q to solutions in the other solvent. Furthermore, the aqueous phase moves from left to right.

1-*Vessel System.*—At the moment when the solute is introduced into the 1-vessel system ($t = 0$) the following relationship holds :

$$\left(\frac{v_a}{v_a + v_b}\right) Vp + \left(\frac{v_b}{v_a + v_b}\right) V\frac{p}{k} = x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

since $\left(\frac{v_a}{v_a + v_b}\right) V$ and $\left(\frac{v_b}{v_a + v_b}\right) V$ are the volumes of solvents A and B present in the mixer.

Now at time t, the rate of loss of solute from the vessel (it will sometimes be a gain with multi-vessel systems) is given by

$$\left(\frac{v_a}{v_a + v_b}\right) V\frac{dp}{dt} + \left(\frac{v_b}{v_a + v_b}\right)\frac{V}{k} \cdot \frac{dp}{dt} = -v_b p/k - v_a p$$

If the conditions at zero time (equation 1) are taken into account, this equation leads to :

$$p = \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \cdot e^{-(v_a + v_b)t/V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of q is obtained by dividing this expression by k. Since this applies throughout the whole of the mathematical treatment, q is disregarded except in special cases.

If the vessel originally contained equal quantities of two solutes having different values of the distribution coefficient, the proportions in which they would be found in the two effluents, when all the solute had left the system, are given by

$$\int_0^\infty v_a p\,dt = v_a kx/(kv_a + v_b) \quad \text{and} \quad \int_0^\infty v_b q\,dt = \int_0^\infty \frac{v_b p}{k}\,dt = v_b x/(kv_a + v_b)$$

The ratio of the amounts of two solutes which should be present in solvent A effluent would be

$$\frac{v_a k_1 x}{k_1 v_a + v_b} \div \frac{v_a k_2 x}{k_2 v_a + v_b} = \left(\frac{k_2 v_a + v_b}{k_1 v_a + v_b}\right)\frac{k_1}{k_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the corresponding ratio in solvent B effluent would be similarly

$$(k_1 v_a + v_b)/(k_2 v_a + v_b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is clear that the extent of separation depends, not only on the distribution coefficients of the solutes, but also on the flow rates of the two solvents. If the best separation be defined as that which leads to the two products in an equal state of purity, we have

$$\left(\frac{k_2 v_a + v_b}{k_1 v_a + v_b}\right)\frac{k_1}{k_2} = \left(\frac{k_1 v_a + v_b}{k_2 v_a + v_b}\right)$$

whence it can be shown that

$$v_a/v_b = \sqrt{1/k_1 k_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the vessel is operated under conditions denoted by (5), the composition of the mixtures issuing from the vessel can be determined by substituting (5) in (3) or (4) which reduce to $\sqrt{k_1/k_2}$.

A 1-vessel system is not very effective as can be shown by substituting suitable values for k_1 and k_2 in this expression.

2-*Vessel System.*—The behaviour of this system may be used to illustrate two points of importance for the successful and most efficient operation of more complex systems.

(*a*) In the mathematical treatment of all systems it is assumed that the rate at which a solvent leaves the terminal separators is the same as that at which it enters the system and that at all intermediate points there is no hold-up or more speedy movement of the solvents. If the behaviour of the apparatus is such that a solvent tends to accumulate in, or to vanish from, one or more vessels of an extended system, the equilibrium is disturbed and a uniform movement of solutes cannot be expected. It is important, therefore, that the material leaving a mixer be a representative sample of its contents at the moment of departure; the agitation of the contents of the mixer must be sufficiently vigorous to prevent any settling and, in its journey to the separator, the mixture must have no opportunity of separating into layers, of which one or other might not pass completely into the separator.

(*b*) The solution entering a mixer from a separator is assumed to be a representative sample of the solution which exists at that precise moment in the mixer whence it comes. Because the concentrations in the various solutions throughout both simple and complex systems are continuously changing, a concentration gradient must exist within a separator of finite dimensions, but, if the separators, whatever their nature, have a content which is negligibly small compared with that of the mixers, the concentration gradient will also be negligibly small. The volumes of the separators must, therefore, be as small as possible compared with those of the mixers.

In a 2-vessel system the concentration in solvent A in the vessel in which the solute is placed, at the moment of introduction, is given by $p = \dfrac{kx(v_a + v_b)}{V(kv_a + v_b)}$, which can be derived by transposition of equation (1). At any moment the following relation holds for vessel 1 :

$$\left(\frac{v_a}{v_a + v_b}\right)V \cdot \frac{dp_1}{dt} + \left(\frac{v_b}{v_a + v_b}\right)\frac{V}{k} \cdot \frac{dp_1}{dt} = \frac{v_b}{k}p_2 - v_a p_1 - \frac{v_b}{k}p_1$$

Writing $V/(v_a + v_b) = R$ and $(kv_a + v_b)/k = S$, we have

$$RS\,dp_1/dt = p_2 v_b/k - Sp_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly, for vessel 2

$$RS\,dp_2/dt = v_a p_1 - Sp_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the solute is placed in vessel 1 (Case I), the solution is :

$$p_1 = \frac{kx(v_a + v_b)}{2V(kv_a + v_b)}\left\{e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 + \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} + e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 - \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t}\right\} \quad . \quad . \quad . \quad . \quad (8a)$$

and

$$p_2 = \sqrt{\frac{kv_a}{v_b}} \cdot \frac{kx(v_a + v_b)}{2V(kv_a + v_b)}\left\{e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 - \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} - e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 + \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t}\right\} \quad . \quad . \quad (8b)$$

Similarly, if the solute is placed in vessel 2 (case II),

$$p_1 = \sqrt{\frac{v_b}{kv_a}} \cdot \frac{kx(v_a + v_b)}{2V(kv_a + v_b)} \left\{ e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 - \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} - e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 + \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} \right\} \quad . \quad (9a)$$

$$p_2 = \frac{kx(v_a + v_b)}{2V(kv_a + v_b)} \left\{ e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 - \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} + e^{-\left(\frac{v_a + v_b}{V}\right)\left[1 + \frac{\sqrt{kv_a v_b}}{kv_a + v_b}\right]t} \right\} \quad . \quad . \quad . \quad . \quad (9b)$$

Although similar, these two sets of equations are not identical. If arbitrary values of the parameters are chosen the manner in which the concentration in solvent *A* varies with time in each vessel can be determined. Thus, if $v_a = v_b = x = 1$, $k = 4$, and $V = 10$, the values of p_1 and p_2 for the two cases, indicated by Fig. 2, are obtained.

As might have been expected, the solute is transported more readily in the direction of movement of the better solvent, although it still moves in both directions.

FIG. 2. FIG. 3.

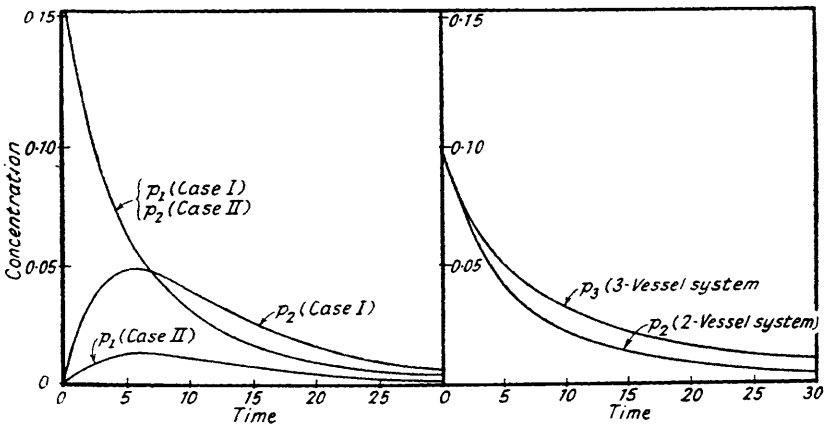


FIG. 2.—2-*Vessel system.* $v_a = v_b = x = 1$; $V = 10$; $k = 4$.

It is clear that a predominant movement in one direction could be checked, compensated, or even reversed by alteration in the relative feed rates of the two solvents. By proper choice of these rates, the solute could be caused to emerge at equal rates from the separators. The amounts of a solute which emerge from the two ends of the system can be ascertained from the expressions :

$$P = \int_0^\infty v_a p_2 \mathrm{d}t = \frac{k^2 v_a^2 x}{(k^2 v_a^2 + k v_a v_b + v_b^2)}; \quad Q = \int_0^\infty v_b \frac{p_1}{k} \mathrm{d}t = \frac{(kv_a + v_b)v_b x}{(k^2 v_a^2 + k v_a v_b + v_b^2)}$$

By choosing the arbitrary values $v_a = v_b = 1$, and various values for k, the following figures can be calculated (the solute being placed in vessel 1).

k	0·1	1·0	10·0
P	0·009x	0·333x	0·901x
Q	0·991x	0·667x	0·099x

These values of P and Q show that the components of a mixture of two solutes having $k = 1{\cdot}0$ and 10·0 would emerge from the system operated in this way 73% and 87% " pure." If the two solutes had $k = 0{\cdot}1$ and 1·0, they would emerge 97·4% and 60% " pure."

3-Vessel System.—The 3-vessel system (Fig. 1 reduced to three vessels) is the prototype of more complex systems, in that the central vessel is fed from each side. For the three vessels the differential equations which represent the movement of solute are :

Vessel 1 :

$$\left(\frac{v_a}{v_a + v_b}\right) V \frac{\mathrm{d}p_1}{\mathrm{d}t} + \left(\frac{v_b}{v_a + v_b}\right) \frac{V}{k} \frac{\mathrm{d}p_1}{\mathrm{d}t} = \frac{v_b}{k} p_2 - v_a p_1 - \frac{v_b}{k} p_1$$

Vessel 2 :

$$\left(\frac{v_a}{v_a + v_b}\right) V \frac{\mathrm{d}p_2}{\mathrm{d}t} + \left(\frac{v_b}{v_a + v_b}\right) \frac{V}{k} \frac{\mathrm{d}p_2}{\mathrm{d}t} = \frac{v_b}{k} p_3 + v_a p_1 - v_a p_2 - \frac{v_b}{k} p_2$$

Vessel 3 :

$$\left(\frac{v_a}{v_a + v_b}\right) V \frac{dp_3}{dt} + \left(\frac{v_b}{v_a + v_b}\right) \frac{V}{k} \frac{dp_3}{dt} = v_a p_2 - v_a p_3 - \frac{v_b}{k} p_3$$

These equations can be written in the simpler forms :

$$dp_1/dt = p_2 v_b/kRS - p_1/R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$dp_2/dt = p_3 v_b/kRS - p_2/R + p_1 v_a/RS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$dp_3/dt = -p_3/R + p_2 v_a/RS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Calling Case I that in which the solute is placed in vessel 1, and so on, and remembering that at time $t = 0$ the concentration p in solvent A in the vessel in which the solute is placed is $[kx(v_a + v_b)]/[V(kv_a + v_b)]$ and that in all other vessels it is zero, we can show the complete set of equations applicable to the 3-vessel system to be as follows :

Case I :

$$p_1 = \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{e^{\alpha t}}{2} + \frac{e^{\beta t}}{4} + \frac{e^{\gamma t}}{4}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

$$p_2 = \sqrt{\frac{kv_a}{v_b}} \cdot \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{\sqrt{2}}{4} e^{\gamma t} - \frac{\sqrt{2}}{4} e^{\beta t}\right] \quad . \quad . \quad . \quad . \quad (13b)$$

$$p_3 = \frac{kv_a}{v_b} \cdot \frac{kx(v_a + v_b)}{V(kv_a + b_b)} \left[-\frac{e^{\alpha t}}{2} + \frac{e^{\beta t}}{4} + \frac{e^{\gamma t}}{4}\right] \quad . \quad . \quad . \quad . \quad . \quad (13c)$$

Case II :

$$p_1 = \sqrt{\frac{v_b}{kv_a}} \cdot \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{\sqrt{2}}{4} e^{\gamma t} - \frac{\sqrt{2}}{4} e^{\beta t}\right] \quad . \quad . \quad . \quad . \quad (14a)$$

$$p_2 = \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{e^{\gamma t}}{2} + \frac{e^{\beta t}}{2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14b)$$

$$p_3 = \sqrt{\frac{kv_a}{v_b}} \cdot \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{\sqrt{2}}{4} e^{\gamma t} - \frac{\sqrt{2}}{4} e^{\beta t}\right] \quad . \quad . \quad . \quad . \quad (14c)$$

Case III :

$$p_1 = \frac{v_b}{kv_a} \cdot \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{e^{\beta t}}{4} + \frac{e^{\gamma t}}{4} - \frac{e^{\alpha t}}{2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (15a)$$

$$p_2 = \sqrt{\frac{v_b}{kv_a}} \cdot \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{\sqrt{2}}{4} e^{\gamma t} - \frac{\sqrt{2}}{4} e^{\beta t}\right] \quad . \quad . \quad . \quad . \quad (15b)$$

$$p_3 = \frac{kx(v_a + v_b)}{V(kv_a + v_b)} \left[\frac{e^{\alpha t}}{2} + \frac{e^{\beta t}}{4} + \frac{e^{\gamma t}}{4}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15c)$$

In these equations

$$\alpha = -\left(\frac{v_a + v_b}{V}\right)$$

$$\beta = -\left(\frac{v_a + v_b}{V}\right)\left(1 + \frac{\sqrt{2kv_a v_b}}{kv_a + v_b}\right)$$

$$\gamma = -\left(\frac{v_a + v_b}{V}\right)\left(1 - \frac{\sqrt{2kv_a v_b}}{kv_a + v_b}\right)$$

It is interesting to note the effect of adding an extra vessel to the 2-vessel system, especially case II (the 3-vessel system). In this case, the solute is placed in the central vessel, and those portions of it which pass to the adjacent mixers have a chance of being returned. Hence, it is to be expected that the concentration in the central vessel will be maintained at a higher level than in a 2-vessel system. Fig. 3 compares the concentrations in solvent A in 2- and 3-vessel systems when solute is placed in the central vessel of the latter, $v_a = v_b = 1$, $V = 10$, and $k = 1$ being chosen for the various parameters. In order to make the conditions in the two examples as nearly equal as possible, it is supposed that in the 2-vessel system the solute is placed in vessel 1. It might be expected that with systems of many vessels the concentration in the central vessel (when the solute is placed therein) would be maintained still higher, although there would undoubtedly be a peak value for such a concentration at a given time, reached only when the number of vessels becomes infinite.

Systems of Many Mixers and Separators.—The equations pertinent to a system of n vessels can be written as follows:

$$\left.\begin{aligned} \mathrm{d}p_1/\mathrm{d}t + p_1/\mathrm{R} - p_2v_\mathrm{b}/kRS &= 0 \\ \mathrm{d}p_2/\mathrm{d}t + p_2/R - p_3v_\mathrm{b}/kRS - p_1v_\mathrm{a}/RS &= 0 \\ \mathrm{d}p_3/\mathrm{d}t + p_3/R - p_4v_\mathrm{b}/kRS - p_2v_\mathrm{a}/RS &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ \mathrm{d}p_{n-2}/\mathrm{d}t + p_{n-2}/R - p_{n-1}v_\mathrm{b}/kRS - p_{n-3}v_\mathrm{a}/RS &= 0 \\ \mathrm{d}p_{n-1}/\mathrm{d}t + p_{n-1}/R - p_nv_\mathrm{b}/kRS - p_{n-2}v_\mathrm{a}/RS &= 0 \\ \mathrm{d}p_n/\mathrm{d}t + p_n/\mathrm{R} - p_{n-1}v_\mathrm{a}/RS &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

R and S have the significance given on p. 1070. The solution of these equations will depend on the number of the vessel in which the solute is originally placed. The general solution, although not difficult, is rather intricate and will be dealt with elsewhere. It has been shown, however, that if the solute is placed in the Kth vessel, the concentration p_r in the rth vessel at time t is given by

$$p_r = \left(\frac{2}{n+1}\right)\frac{kx(v_\mathrm{a}+v_\mathrm{b})}{V(kv_\mathrm{a}+v_\mathrm{b})}\left(-\sqrt{\frac{v_\mathrm{b}}{kv_\mathrm{a}}}\right)^{(K-r)}\sum_{j=1}^{j=n}\sin\frac{Kj\pi}{n+1}\cdot\sin\frac{rj\pi}{n+1}\cdot \mathrm{e}^{-\left(\frac{v_\mathrm{a}+v_\mathrm{b}}{V}\right)\left[1+\frac{\sqrt{kv_\mathrm{a}v_\mathrm{b}}}{kv_\mathrm{a}+v_\mathrm{b}}\left(2\cos\frac{j\pi}{n+1}\right)\right]t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In this expression j is an integer taking the successive values $(1 - n)$, whence p_r is the sum of n terms of the form $Ae^{-\phi t}$ which correspond to the n terms expected in the solution of a linear differential equation of the nth order. If r or $K = (n + 1)/2$, $\sin[Kj\pi/(n + 1)] = \sin(j\pi/2)$. For *even* values of j, $\sin(j\pi/2) = 0$, whence it follows that only the odd terms are required in order to find p_r. When $K = (n + 1)/2$, the solute has been placed in the central vessel and, since this seems the natural point at which to introduce a mixture of solutes which it is desired to separate into its components, the behaviour of a 19-vessel system will be considered in which $K = (n + 1)/2$; a 19-vessel system is chosen because it facilitates calculation and, while it is relatively small, yet it is sufficiently extensive to give some idea of the behaviour to be expected of more extended systems.

Distribution of the Components of a Complex Mixture in a 19-Vessel System.—The problem of how the components of a complex mixture distribute themselves in a 19-vessel system may be regarded as a particular example of the more general case. It is necessary, first, to ascertain the manner in which a simple solute passes from the central vessel, in which it is placed, to the ends of the system. If the distribution coefficient is unity and the feed rates of the two solvents are equal, then clearly the solute would leave the system at each end at equal speeds. If for this system the distribution coefficient is greater than unity, the solute would move faster in the direction of movement of the aqueous phase, and conversely. To illustrate the manner in which solutes move along the system, arbitrary values may be assigned to the various parameters in (17). Thus if $v_\mathrm{a} = v_\mathrm{b} = 1$, $V = 10$, and $k = 1$, equation (17) becomes

$$p_r = 0{\cdot}01x(-1)^{10-r}\sum_{j=1}^{j=19}\sin 90j^\circ\,.\,\sin 9rj^\circ\,.\,\mathrm{e}^{-1/5[1+\cos 9j^\circ]t} \quad . \quad . \quad . \quad (18)$$

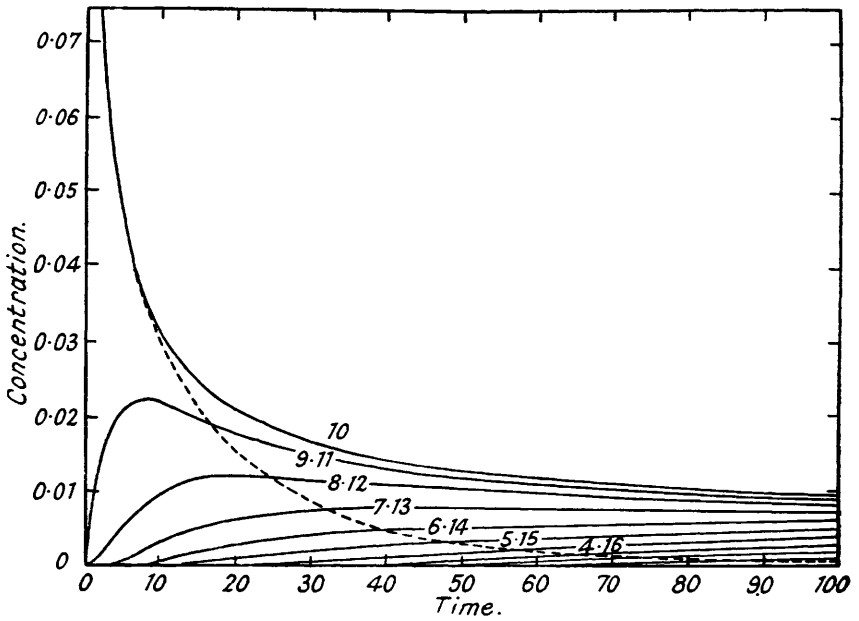
If $v_\mathrm{a} = v_\mathrm{b} = 1$ and $V = 10$, it is supposed that the residence time is 5, that is, material spends an average of 5 units of time in each vessel and, if this time is sufficiently long for equilibration, v_a and v_b can be made as large as desired. The feeds can even be stopped and restarted without detriment.

Calculation of individual values of p_r, that is, of p_1, p_2, p_3, etc., from (18) is no light task; seven-figure tables must be used throughout because the terms to be summed in (17) are partly positive and partly negative, whence, in the sequel, the value of p_r is frequently the difference of two large numbers. This is especially noticeable when the vessels remote from the point of introduction of the solute are considered.

The results of such calculations from equation (18) are shown in Fig. 4. The concentration in vessel 10, that in which the solute is placed originally, falls exponentially, but it is of interest to compare the rate of fall with that applying to the middle vessel of a 3-vessel system. The data applicable to the latter system are indicated by the broken line in the figure. It is seen

that the solute is retained in the central vessel of the 19-vessel system for a much longer period, since the concentration falls so much less rapidly.

FIG. 4.



Concentrations in the several vessels of a 19*-vessel system, when* $k = 1$; $v_a = v_b = 1$; $V = 10$; $K = 10$.
Broken curve: concentration in the central vessel of a 3*-vessel system, when* $k = 1$; $v_a = v_b = 1$; $V = 10$; $K = 2$.

If the distribution coefficient of the solute is 10, and $v_a = v_b = 1$ and $V = 10$ as before, the equation applicable to the system becomes

$$p_r = 0{\cdot}0181818(-0{\cdot}316228)^{10-r} \sum_{j=1}^{j=19} \sin 90j^\circ \,.\, \sin 9rj^\circ \,.\, e^{-\frac{1}{2}t[1 + 0{\cdot}5749596 \cos 9j^\circ]t} \quad . \quad . \quad (19)$$

and the distribution of the solute throughout the system takes on an entirely different character. Calculation is facilitated by noting that $\sin 9j^\circ = \sin 171j^\circ$ and so on, from which it can be shown that

$$p_1 = p_{19} \times 10^{-9}$$
$$p_2 = p_{18} \times 10^{-8}$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$p_9 = p_{11} \times 10^{-1}$$

Indeed, the concentration in the rth vessel is related to that in the $(n + 1 - r)$th in a very simple way generally, because the identity of

$$\sin \frac{rj\pi}{n+1} \quad \text{and} \quad \sin \frac{(n+1-r)j\pi}{n+1} = \sin\left(\pi j - \frac{rj\pi}{n+1}\right)$$

ensures the identity of the Σ terms in equations of the type of (17) for p_r and p_{n+1-r}; whence

$$p_r/p_{n+1-r} = (v_b/kv_a)^{\left(\frac{n+1}{2} - r\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

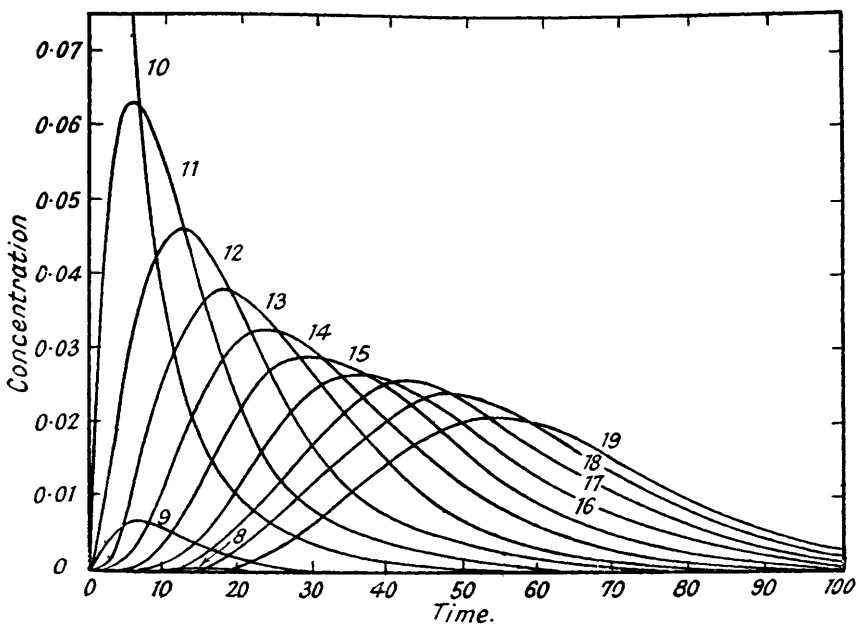
Moreover, it follows that the maximum concentration in vessels related in this way must be reached at the same moment.

The necessity for the use of seven-figure mathematical tables becomes more apparent here,

especially with vessels 19, 18, and 17. The results of such a calculation are illustrated in Fig. 5 which is drawn to the same scale as Fig. 4. First it is to be noted that the concentration in solvent *A* in vessel 10 is higher initially than it is if $k = 1{\cdot}0$, because the solute must naturally give a more concentrated solution in the solvent in which it is the more soluble. However, despite this high initial value the concentration falls much more rapidly than it does when $k = 1{\cdot}0$. The reason is as follows. The concentration in the same solvent in vessel 11 rises very rapidly to a high figure and then falls for a similar reason, *i.e.*, the solute is being passed rapidly from vessel to vessel towards mixer 19. On the other side of vessel 10 the reverse is occurring. The concentration in vessel 9 rises to a moderate figure and soon falls, but in all the remaining vessels it never reaches an important level.

With a solute having $k = 0{\cdot}1$ the situation is the reverse, solute leaving the system essentially in solvent *B* from separator 1.

FIG. 5.



Concentration in the several vessels of a 19*-vessel system, when* $k = 10$; $v_a = v_b = 1$; $V = 10$; $K = 10$.

A better picture of the manner in which the solutes leave a system is obtained if the concentrations of solute in *A* in each of the vessels, at fixed times, are considered. These concentrations can be taken from Figs. 4 and 5; and Figs. 6 and 7 indicate the distribution of solutes having $k = 1{\cdot}0$ and $10{\cdot}0$ respectively at the times indicated. For a solute having $k = 0{\cdot}1$ the situation is much the same as that for the solute having $k = 10$, except that the solute moves in the other solvent in the reverse direction.

Similar curves, plotted for different values of t, show that the peak concentration of a solute passes along the system like a wave of gradually diminishing amplitude, the rate of travel depending on the value of k for given values of v_a and v_b.

Craig, in dealing with the behaviour of his apparatus, has indicated how the peak concentrations are found in different tubes according to the number of transfers and the differences in the distribution coefficients of the solutes. An experimental approach to the problem of detecting inhomogeneity, used by him, has been to determine some property of the contents of the several tubes, whence a plot of this property (such as the biological activity, etc.) against the number of the tube, reveals inhomogeneity by a divergence of the experimental from the calculated curve, or, more obviously, by abrupt changes of slope of the experimental curves. A similar phenomenon is to be expected in the continuous counter-current process now described. If equal quantities of three solutes for which data have just been considered, are introduced into the central vessel of the 19-vessel system, the dispositions at fixed time intervals can be regarded as analogous to the distribution of the solutes after a given number of transfers in the

Craig apparatus. Fig. 8 shows how the three solutes would be distributed in solvent *A* throughout the system at $t = 50$, the broken line indicating the sum of the several concen-

FIG. 6.

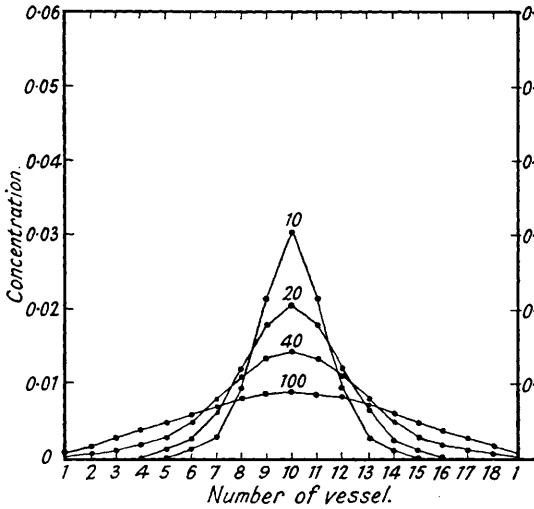


FIG. 7.

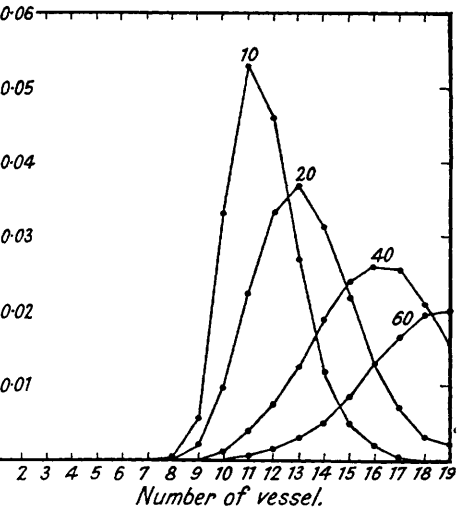
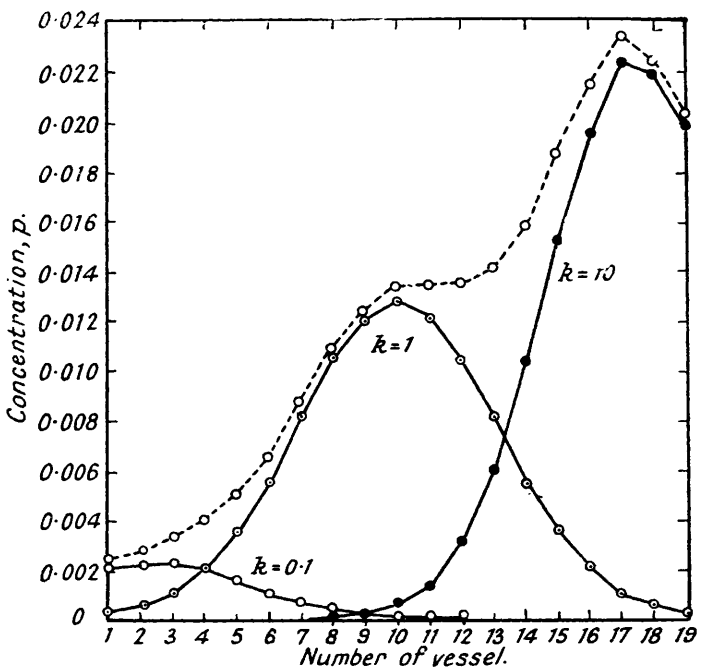


FIG. 6.—*Distribution of concentration in a 19-vessel system at times* 10, 20, 40, *and* 100. $k = 1$; $v_a = v_b = 1$; $V = 10$; $K = 10$.

FIG. 7.—*Distribution of concentration in a 19-vessel system at times* 10, 20, 40, *and* 60. $k = 10$; $v_a = v_b = 1$; $V = 10$; $K = 10$.

FIG. 8.



19-*Vessel system.* *Distribution at* $t = 50$ ($v_a = v_b = 1$; $V = 10$).

trations. The presence of the third solute could not be deduced from the composite curve, but this is because the third solute is essentially in solvent *B*. The composite curve for concentrations in solvent *B* would be the mirror image, about the ordinate through 10, of that shown

in Fig. 8, so that the presence of the third solute could readily be detected. It is, therefore, extremely desirable to investigate each phase separately. While the examination of an aliquot of the mixed contents of each of the vessels in the example just considered would easily reveal the presence of the three solutes, in other cases when the proportions of the solutes are not equal, or when the distribution coefficients are not so far apart, or when the feed rates of the solvents are not equal, the presence of one or more components of the mixture might be overlooked, if this method of examination only were adopted.

Optimum Conditions for the Operation of a Multi-vessel System.—In dealing with a 2-vessel system the amounts of material leaving the end vessels by the expiration of a given time were calculated for $t = \infty$. It is possible to calculate the amount of a solute which will leave the ends of a multi-vessel system in much the same way although the expression to be integrated appears somewhat formidable. It is just as useful, and much simpler, to determine the *ratio* of the quantities of a given solute which leave the two ends of the system, which is readily done for systems of an odd number of vessels. The integrals

$$\int_0^\infty p_n v_a dt \quad \text{and} \quad \int_0^\infty q_1 v_b dt = \int_0^\infty \frac{p_1}{k} v_b dt$$

are required, where p_n and p_1 are given by equation (17).

$$\int_0^\infty p_n v_a dt = \int_0^\infty \frac{2}{n+1} \cdot \frac{v_a k x (v_a + v_b)}{V(k v_a + v_b)} \left(-\sqrt{\frac{v_b}{k v_a}} \right)^{K-n} \sum_{j=1}^{j=n} \sin \frac{Kj\pi}{n+1} \cdot \sin \frac{nj\pi}{n+1} \cdot e^{-\left(\frac{v_a+v_b}{V}\right)\left[1 + \frac{\sqrt{k v_a v_b}}{k v_a + v_b}\left(2\cos\frac{j\pi}{n+1}\right)\right] t}$$

$$= \frac{2 v_a k x}{n+1} \left(-\sqrt{\frac{v_b}{k v_a}} \right)^{K-n} \sum_{j=1}^{j=n} \frac{\sin \frac{Kj\pi}{n+1} \cdot \sin \frac{nj\pi}{n+1}}{k v_a + v_b + \sqrt{k v_a v_b}\left(2\cos\frac{j\pi}{n+1}\right)} \quad . \quad . \quad . \quad . \quad (21)$$

Similarly,

$$\int_0^\infty \frac{p_1}{k} v_b dt = \frac{2 v_b x}{n+1} \left(-\sqrt{\frac{v_b}{k v_a}} \right)^{K-n} \sum_{j=1}^{j=n} \frac{\sin \frac{Kj\pi}{n+1} \cdot \sin \frac{j\pi}{n+1}}{k v_a + v_b + \sqrt{k v_a v_b}\left(2\cos\frac{j\pi}{n+1}\right)} \quad . \quad . \quad . \quad . \quad (22)$$

Now, when j is odd $\sin \frac{nj\pi}{n+1} = \sin \frac{j\pi}{n+1}$ and, since, with a central feed of solute, even values of j are irrelevant, it follows that the ratio,

$$\frac{\int_0^\infty p_n v_a dt}{\int_0^\infty \frac{p_1}{k} v_b dt} = \frac{v_a k}{v_b} \left(\sqrt{\frac{v_b}{k v_a}} \right)^{1-n} = \left(\frac{k v_a}{v_b} \right)^{\frac{n+1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The ratio of the amounts of solute which have left the two ends of the system at the end of a given time is shown by the same expression, since the ratio of the concentrations in a given solvent in the first and the last vessel is a constant at all times, as has been shown above (equation 20).

If specific values are given to v_a, v_b, k, and n, the efficiency with which a solute is transported to one end of the system becomes clear. Thus, with $v_a = v_b = 1$ and $k = 10$ in a 19-vessel system, the ratio of the amounts which leave the nineteenth and the first vessel is 10^{10}; with $k = 2$ it is 1024. Thus, even with such a low value of k as 2, 99·9% of the solute travels in one direction.

Of most importance is the degree of separation of two solutes for which the distribution coefficients are close. If, as before, the aim is to get each solute equally pure, then the best conditions can be deduced thus. Calling the ratio of the amounts of the one solute leaving the nineteenth and the first vessel, Q_1, and that of the amounts of the other solute leaving the first and the nineteenth vessel, Q_2, we have $Q_1 = (k_1 v_a / v_b)^{(n+1)/2}$ and $Q_2 = (v_b / k_2 v_a)^{(n+1)/2}$. If the two solutes are present in equal amounts in the mixture, Q_1 must equal Q_2, whence

$$v_a / v_b = \sqrt{1/k_1 k_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

This is an important relation since it defines the ratio of solvent flow rates most favourable for separation of the two solutes.

From equation (24) we can deduce the value of Q for the system operated under its most favourable conditions, *viz.*:

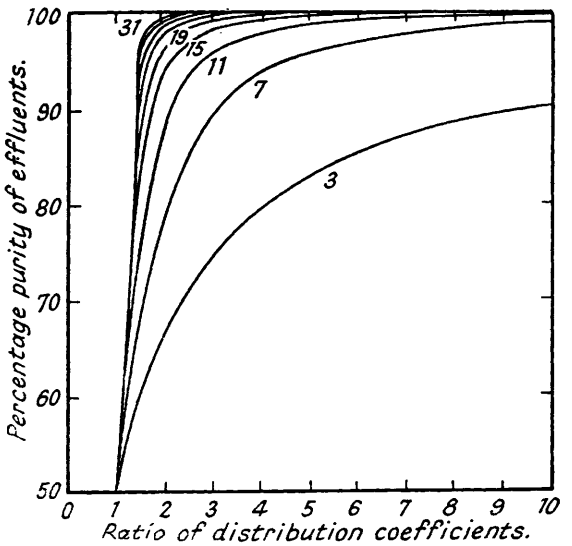
$$Q = (k_1 v_a/v_b)^{(n+1)/2} = (k_1 \sqrt{1/k_1 k_2})^{(n+1)/2} = (k_1/k_2)^{(n+1)/4} \quad . \quad . \quad . \quad (25)$$

This expression makes possible a choice of the number of vessels to be used where the distribution coefficients of the two solutes are known.

It is also important that Q is highly dependent on the constancy and correctness of the flow rate ratio, and on k, when the system is an extended one, because a small error in the choice of the flow rate of one or other solvent is raised to the $[(n + 1)/2]$th power.

The expression $100[Q/(Q + 1)]$ defines the percentage purity of each solute emerging from the system operating under optimum conditions. Fig. 9 shows the relation of the percentage purity to the ratio of the distribution coefficients and the number of vessels. With a 19-vessel system, solute at least 99·9% pure results if the ratio of the distribution coefficients is 4 or more. Even with ratios as low as 2, the products should each be 96·8% pure.

FIG. 9.



N.B.—*The numbers refer to the number of vessels in the system.*

Introduction of Solute Mixture in Vessels Other than the Central One.—It is, of course, not essential that the solute be placed in the central mixer. It has, however, not been possible to deduce a simple expression for the optimum conditions for operating a system when the solute is placed in any other vessel. If a solute is placed in a vessel to the left of the central one, the extra vessels to the right will tend to prevent the loss of solute from that end, and to favour its elimination on the left. To counteract this, an augmented feed of solvent A would be required. Hence, we may generalise by stating that the nearer the solute is introduced to the vessel into which a given solvent enters the system, the faster must be the feed of this solvent for the system to behave in a fashion comparable to that observed when the point of introduction is the central vessel.

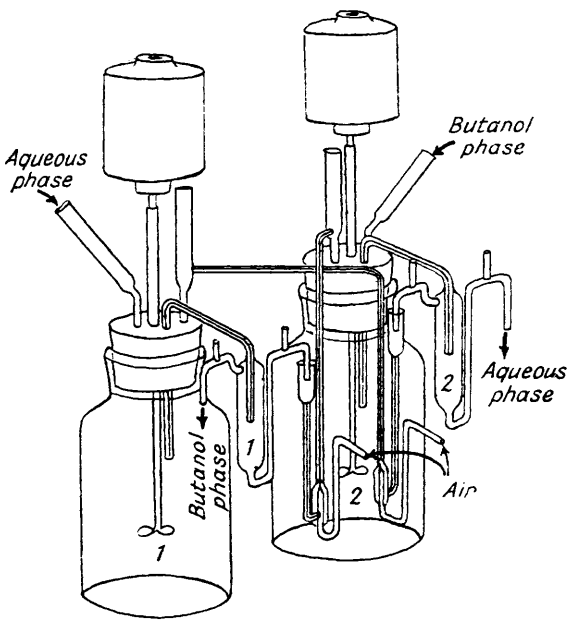
Separation of Oxalic and Succinic Acids.—The principles described above have been tested with mixtures of oxalic and succinic acids, which have been distributed between water and butanol saturated with each other, in 2- and 19-vessel systems. These acids were chosen because their distribution coefficients are close to unity, the distribution coefficient of one (succinic acid) is nearly independent of concentration, whereas that of oxalic acid is widely affected by concentration, and because both are easily determined in admixture.

Distribution coefficients are usually recorded as the ratio of the concentration in the aqueous phase to that in the other phase. The coefficients are by no means constant as a rule, rising with

decrease in concentration in almost all the examples recorded in International Critical Tables. Therefore, as the concentration decreases, there is an added tendency for a solute to pass into the aqueous phase if k is variable. The effect of this variable value of k on the behaviour of a solute subjected to the operation of our counter-current system can be ascertained as follows.

If the distribution coefficient applicable at the moment the solute is introduced into the system is chosen as the basis of calculations using equation (17), then, as time proceeds, k will rise for the vessel of introduction and the solute will move faster in the direction of the flow of the aqueous phase. In the vessel to the right of the central one, the value of k applicable when $t = 0$ will initially be higher than that chosen for the basis of calculations, whence solute will leave this vessel at a rate faster than that calculated. Consequently, the peak concentration in solvent A in this vessel may not be as high as expected (although the actual figure will depend on the relative values of augmented rate of entry into, and exit from, the vessel); in general, therefore, a solute with variable value of k will move faster to the right in the system, and form a more diffuse wave, than would be expected on the basis of calculations in which k is assumed to be constant with the value appertaining to stronger solutions.

FIG. 10.



2-*Vessel system.* The behaviour of succinic and oxalic acids separately in a 2-vessel system was first studied, the acids being placed in mixer 2 (Fig. 10). The separators in the apparatus used had a capacity about 5% of that of the mixers, and were, therefore, rather larger than desirable. The effect of this can be seen clearly in Fig. 11 which illustrates the concentration of succinic acid in the aqueous phase (p_2) from vessel 2 and in the butanol phase (q_1) from vessel 1. Obviously, at the start of the experiment p_2 should have had its maximum value, whereas, the maximum was reached only after the expiration of a short lag. The explanation is, of course, that the first sample taken for analysis from the separator was largely composed of the neat solvent which the separator contained at the moment of introduction of the solute into the mixer. Shortly afterwards, the neat solvent was displaced by concentrated solution, but after this the latter was displaced by the weaker solution then present in the mixer. Hence, at later times the concentration in this mixer, as determined by samples taken from the separator, was somewhat higher than that calculated. The deviation produced by the finite relative size of the separators was more noticeable with the aqueous than with the butanol phase because, arbitrarily, the feed rate of the former was only about 20% of that of the latter. Consequently, the deviations of q_1 from the theoretical are within the limits of experimental error.

Fig. 12 illustrates the results of experiments with oxalic acid with the same solvents which were, however, fed at rates more nearly equal and somewhat faster on the average than in the

experiment with succinic acid. As the experiment progressed, the difference between p_2 observed and calculated became greater proportionally, although the general agreement left little to be desired. The discrepancy is partly due to the factors discussed for succinic acid and partly to the effect of the variable value of k for oxalic acid. The value of k applicable at the beginning of the experiment (1·708) was deduced as described in the Experimental portion, but this was no longer valid towards the end of the run. The later results are higher than those calculated on the basis of $k = 1·708$. The discrepancy between the observed and calculated values is much more pronounced with the butanol solution issuing from vessel 1. The observed maximum concentration is only slightly more than one-half of the calculated. However, the experimental results show that the concentration never reached the region in which the distribution coefficient is 1·708 and, if a figure of 2·20 is taken as more applicable (based on an average concentration equal to one-half of the observed maximum value), the calculated values for q_2 are in much closer agreement. (It is not suggested that such a figure for k should be chosen

FIG. 11. FIG. 12.

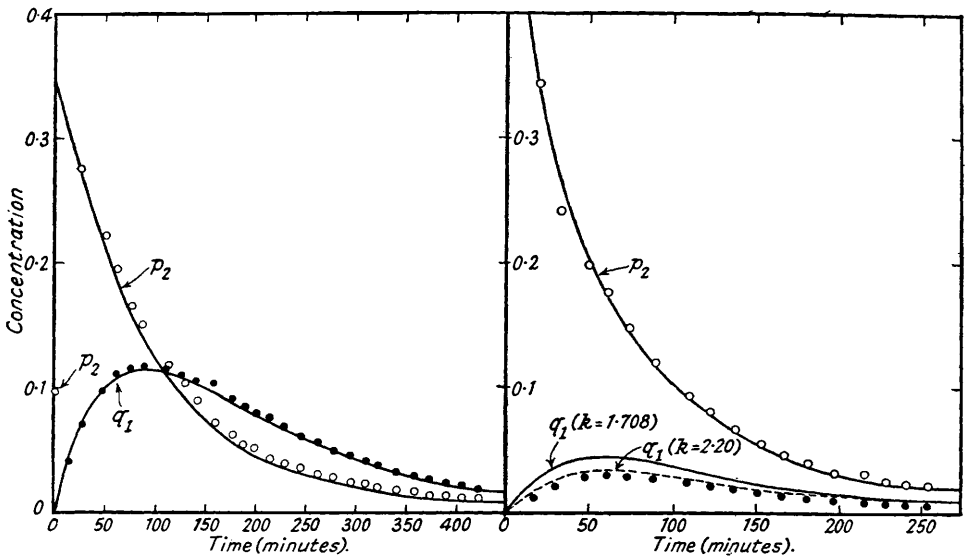


FIG. 11.—*Distribution of succinic acid in a 2-vessel system.* p_2 = *concentration in aqueous phase;* q_1 = *concentration in butanol phase.*

FIG. 12.—*Distribution of oxalic acid in a 2-vessel system.* p_2 = *concentration in aqueous phase;* q_1 = *concentration in butanol phase.*

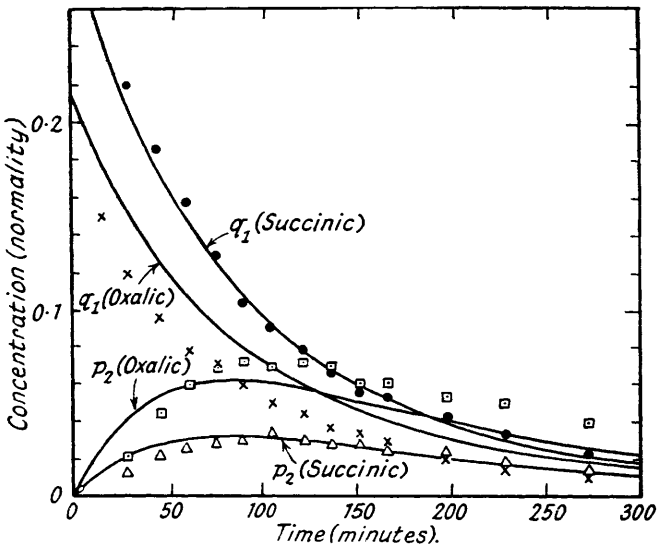
so as to bring the theoretical and observed values into line; the higher value for k has been used only to illustrate that the reasoning given above regarding the expected behaviour of solutes with variable values of the distribution coefficient is sound.)

It was desirable to supplement the two experiments described, by an experiment on the behaviour of a mixture of oxalic and succinic acids in a 2-vessel system. Taking the distribution coefficients of the two acids as 1·8 and 0·935 respectively, the ideal ratio of the feed rates of the two solvents is 0·755. As k for oxalic acid is probably greater than 1·8 for most of the time the best ratio is probably somewhat smaller than this; the flow rates actually used gave a ratio of 0·412, which was undoubtedly lower than necessary. On this occasion the solutes were placed in vessel 1. Fig. 13 shows the agreement between the theoretical and observed values and, as with the experiments conducted on the separate acids, the behaviour of the succinic acid was extremely similar to that calculable, whereas that of the oxalic acid was less so.

Although the effluent acids were not collected and analysed, the analyses of the samples taken can be used to show that the succinic–oxalic acid mixture emerging from vessel 1 comprised 63·5% of the former acid, whereas that issuing from vessel 2 contained 27% of this acid. The mixture used comprised equal weights of the two acids.

19-*Vessel system.* The experience gained with the 2-vessel system suggested the adoption of certain features in the 19-vessel system which are described in the Experimental section. The apparatus used is illustrated in Fig. 14 where the last three vessels are shown. A mixture

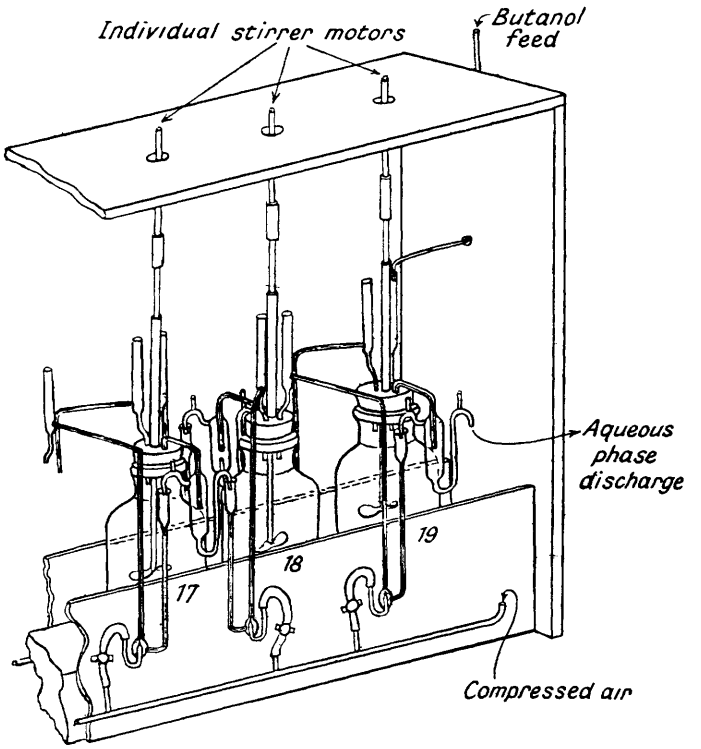
FIG. 13.



***Calculated** and observed concentrations of succinic and oxalic acids in the aqueous and butanol phases issuing from a 2-vessel system.*

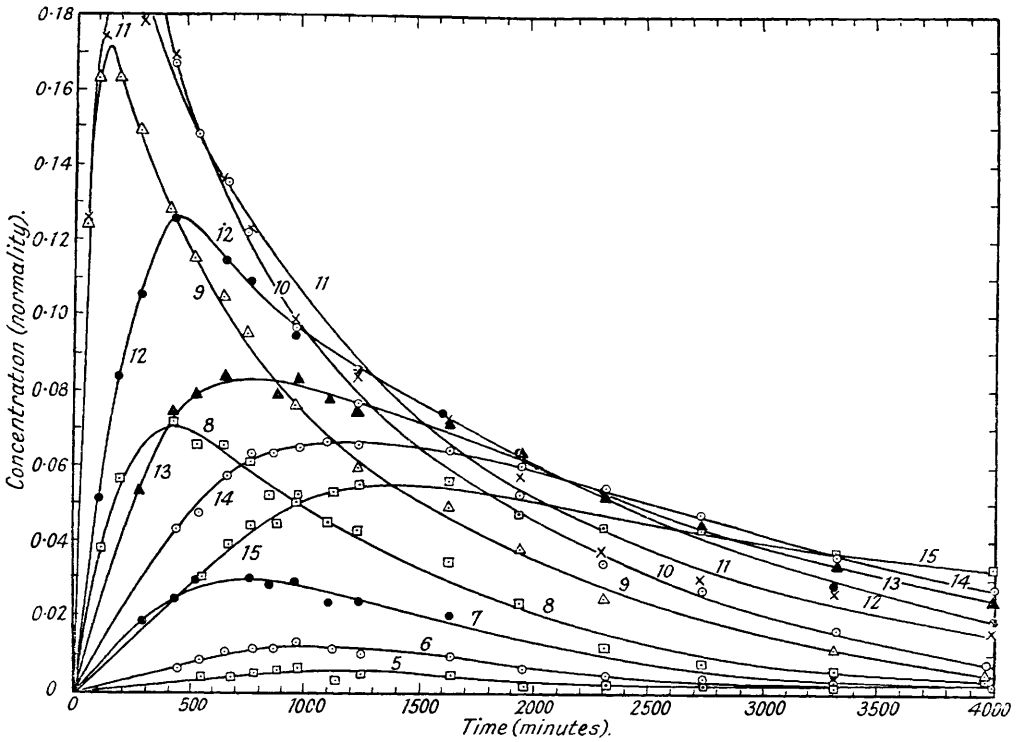
Vessel 1 { ● *Succinic concentration in butanol.*
× *Oxalic concentration in butanol.*
Vessel 2 { □ *Oxalic concentration in water.*
△ *Succinic concentration in water.*
Continuous lines represent calculated concentrations.

FIG. 14.



of 25 g. each of succinic and oxalic acid (on a water-free basis) was introduced into the central vessel; aqueous butanol, and water saturated with butanol, only approximating in ratio to the ideal, were passed through the system. The separation of the two acids proceeded as expected. Both solutions in each vessel were examined for succinic and oxalic acids at various times and Figs. 15, 16, 17, and 18 give the results obtained. (Great accuracy for individual results is not claimed since titrations were carried out on 1-ml. samples, of which over 500 were taken.) From Fig. 15 it is clear that the concentration of oxalic acid fell rapidly in vessel 10 in the aqueous phase; in vessel 11 it rose rapidly and then fell in such a way that it soon exceeded the concentration in vessel 10. With vessels further along the system, the concentration of oxalic acid rose less and less rapidly, but in each case it ultimately rose, or would have risen, above that in

FIG. 15.



Concentration of oxalic acid in the aqueous phase in vessels 5—15 *of a* 19*-vessel system. Details for vessels* 1—4 *and* 16—19 *omitted for clarity. Concentrations below* 0·004N. *of no significance.*

the preceding vessel. This indicated that the oxalic acid was passing towards vessel 19, that is, in the direction of the water-flow. With vessels 1—10, the concentration of oxalic acid in the aqueous phase rose at first and then declined in such a way that it was always higher in the vessel furthest along the system from vessel 1. This accords with expectation. Furthermore, Fig. 15 shows that the peak concentrations in vessels 9 and 11, 8 and 12, and so on (which pairs of vessels may be called conjugate), were reached at approximately the same time as had been predicted (see p. 1074).

For a 19-vessel system (see p. 1074) the concentrations in conjugate vessels should be related thus:

$$p_1 = p_{19}(v_b/kv_a)^9$$
$$p_2 = p_{18}(v_b/kv_a)^8$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$p_9 = p_{11}(v_b/kv_a)$$

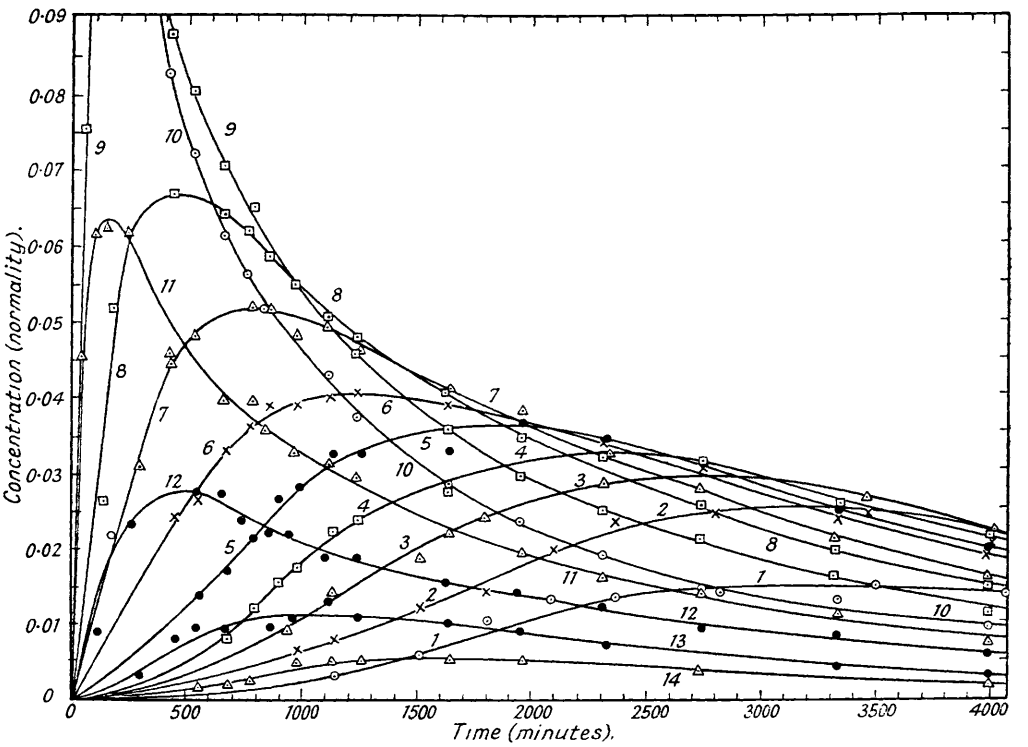
For the concentrations of oxalic acid actually found in all the vessels except the tenth after a relatively short time, the value of k applicable must have been over 2 and probably approached 3 in some cases. If it is assumed that $k = 2{\cdot}0$, p_9/p_{11} should have been approximately 0·83 and

it should have been applicable at all times, including that at which the peak concentrations were observed. Fig. 15 shows that this prediction was substantially realised.

Concentrations of succinic acid in the aqueous phase in each of the vessels are depicted in Fig. 16 and it is clear that the solute is gradually moving towards vessel 1. Similarly, oxalic acid in the butanol phase (Fig. 17) is moving towards vessel 19 and this must necessarily be so since, at all times, the distribution of the acid between the two phases in a given vessel must be in accordance with the value of k applicable at the concentration concerned.

Interest, of course, is mainly centred in the manner in which the oxalic acid moves in the aqueous phase and that in which the succinic acid moves in the butanol phase. The data for succinic acid are not likely to be so accurate as those obtained for oxalic acid, since they are the differences of two sets of experimental results, but here again (Fig. 18) it is clear that the acids

FIG. 16.



Concentration of succinic acid in the aqueous phase in vessels 1—14 *of a* 19*-vessel system. Details for vessels* 15—19 *omitted for clarity. Concentrations below* 0·004N. *of no significance.*

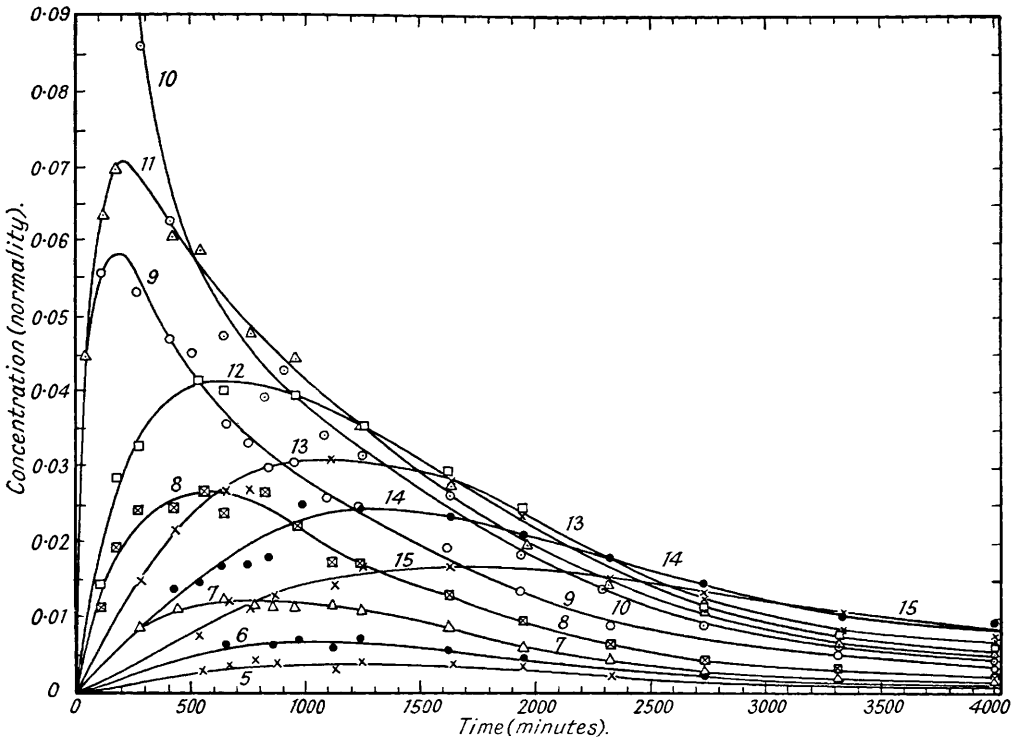
behaved as expected. The oxalic acid moved essentially with the aqueous phase accompanied by a little succinic acid, and the latter, mixed with oxalic acid, moved with the butanol phase.

The actual ratio of solvent feed rates was not the ideal, but somewhat lower. The ratio used (0·615) favoured the flow of acids towards vessel 1. Consequently the oxalic acid leaving vessel 19 was rather purer than corresponds to the " ideal " ratio while the succinic acid was somewhat less pure.

It has been indicated above (p. 1075) that Craig (*J. Biol. Chem.*, 1944, **155**, 519) was able to infer the presence of more than one solute by the " kinks " or abrupt changes of slope when change of a physical property was plotted against the number of the tubes. The analogous operation with our system is to determine the total acidity of samples from all vessels at fixed intervals of time, the two phases being examined separately for the reason given on p. 1077. The samples were too numerous to be removed simultaneously and the time differences were sometimes considerable, so that it has been deemed desirable to take points at 500, 1000, 1500, 2000, 2500, 3000, and 4000 minutes from the smoothed curves drawn through the experimental data. These points are plotted in Fig. 19, the continuous curves referring to solutions in the aqueous phase

and the broken lines to solutions in butanol. At 1000 minutes, corresponding to eight passes (approximately) in the Craig apparatus, there are no signs of separation of the two solutes, insofar as discontinuities in the curves are concerned, but the fact that the two sets of concentrations appropriate to that time are not, vessel by vessel, proportional to one another, is an indication that more than one solute is present. If one solute only were present, the ratio of the concentration of this solute in the one phase to that in the other, in each vessel, would be equal to the distribution coefficient. While this can be variable, there must be some symmetry about the curves representing the behaviour of the solute in the two solvents. This symmetry is lacking in Fig. 19, even at $t = 1000$. As the run proceeds, the asymmetry of the curves becomes more apparent and by $t = 4000$, there is a pronounced inflection in the curve relating to the aqueous phase. The presence of two solutes is no longer in doubt, and it is clear that the one solute is at one end of the system with the second solute at the other.

FIG. 17.



Concentration of oxalic acid in the butanol phase in vessels 5—15 *of a* 19*-vessel system. Details for vessels* 1—4 *and* 16—19 *omitted for clarity. Concentrations below* 0·004N. *of no significance.*

Finally, in Fig. 20 curves are given depicting the movement of the two acids from the system; this should be compared with Figs. 6 and 7.

The purities of the acids from the ends of the system were 82% for succinic and 99% for oxalic whereas, if the flow rates of the two solvents had been ideal, each would have emerged about 95% pure. It is concluded, therefore, that the 19-vessel system behaved substantially in accordance with expectation, because the effect of using solvent flow rates different from the ideal should be to lead to the isolation of the one solute less pure and the other more pure than in the " ideal " operation of the apparatus.

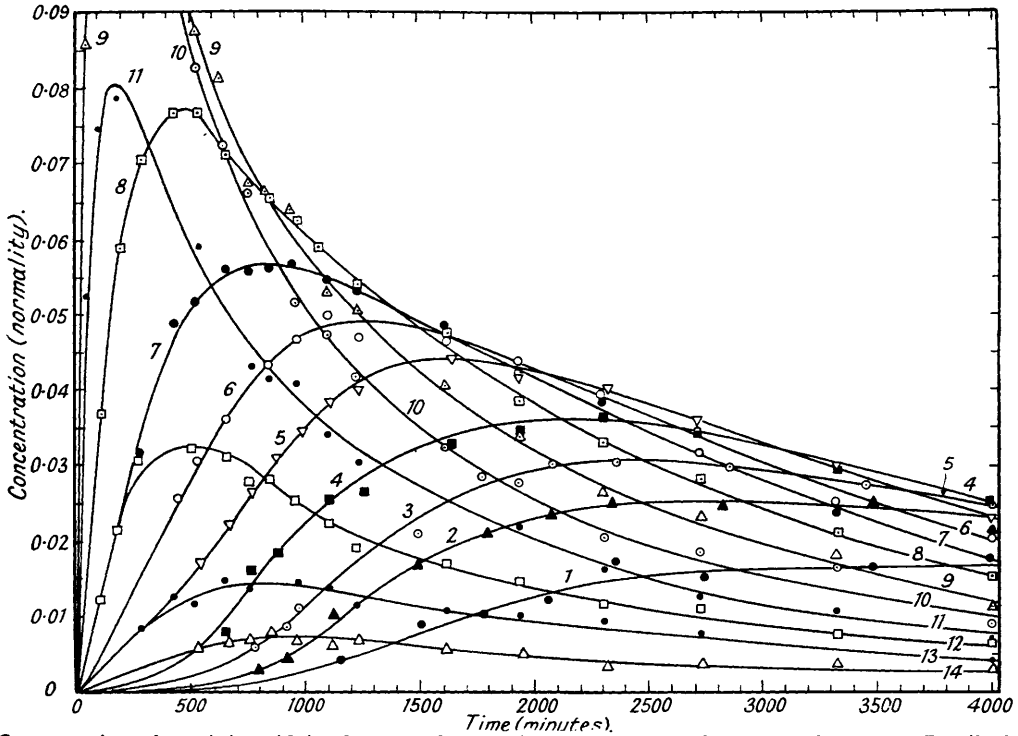
Our experiments show that the theoretical reasoning is sound and there is reason to believe that its application to the separation of the components of complex mixtures of solutes is likely to be successful.

EXPERIMENTAL.

Distribution Coefficients of Oxalic and Succinic Acids between Water and Butanol mutually saturated with One Another.—The distribution coefficients were determined in the conventional way at 25°, the concen-

Succinic acid (25·0°).			*Oxalic acid* (25·1°).		
Concn. of acid in aqueous phase, N.	Concn. of acid in butanol phase, N.	*k*.	Concn. of acid in aqueous phase, N.	Concn. of acid in butanol phase, N.	*k*.
0·458	0·473	0·968	1·439	0·813	1·770
0·260	0·275	0·947	0·650	0·384	1·691
0·0826	0·0883	0·935	0·434	0·254	1·711
0·0740	0·0792	0·934	0·225	0·122	1·852
0·0464	0·0497	0·933	0·105	0·0455	2·314
0·0251	0·0270	0·930	0·0644	0·0238	2·706

FIG. 18.



Concentration of succinic acid in the butanol phase in vessels 1—14 *of a* 19*-vessel system. Details for vessels* 15—19 *omitted for clarity. Concentrations below* 0·004N. *of no significance.*

trations of the acids being varied over a moderate range. These experiments were " static," *i.e.*, after vigorous shaking to mix the two phases, ample time was given to permit settling. In continuous counter-current studies, settling in the separators is relatively rapid and, if there were a pronounced tendency for a solute to be adsorbed positively or negatively at an interface, then with dilute solutions the apparent distribution coefficient would differ from that observed in static experiments. The degree of dispersion of the dispersed phase and the dilution of the solutions necessary for this phenomenon to become apparent are both such as to make the effect lie within the limits of the experimental error likely to be encountered with the apparatus used in the present studies.

Continuous Counter-Current Distribution of Succinic and Oxalic Acids between Water and Butanol mutually saturated with One Another.

2-*Vessel System.—Apparatus.* The apparatus is illustrated in Fig. 10. The two mixing vessels, of equal volume (±1%), consisted of wide-mouthed bottles which, together with the ancilliary separators, air-lifts and connecting tubes, were estimated to have a volume of 900 ml. each. The tube connecting the mixer with the separator was of *ca.* 2-mm. bore in order that the passage of the mixture from the bottle to the separator should proceed without delay. This narrow tube took the mixture of liquid phases from near the middle of the bottle to reduce centrifugal effects and it passed to the middle of the separator, which consisted of a bulbous tube of about 50-ml. capacity. The phases separated at this point into 2 layers which proceeded by the exit tubes either from the system, or to an air-lift. The relative heights of the exit tubes are of great importance; that discharging the lighter liquid must be a little higher than the other. It is essential that correct positioning be secured, otherwise the line of separation of the phases in the separator is too high or too low. In early experiments the liquid was delivered straight from the separator to the air-lift without a break, but it was found that the small air

current required to raise the liquid was sufficient to cause uncontrollable suction throughout the system, which soon went out of balance. Even if the tube carrying the liquid were provided with a " breather " at the top, the small fluctuations provided by the air current caused erratic operation.

FIG. 19.

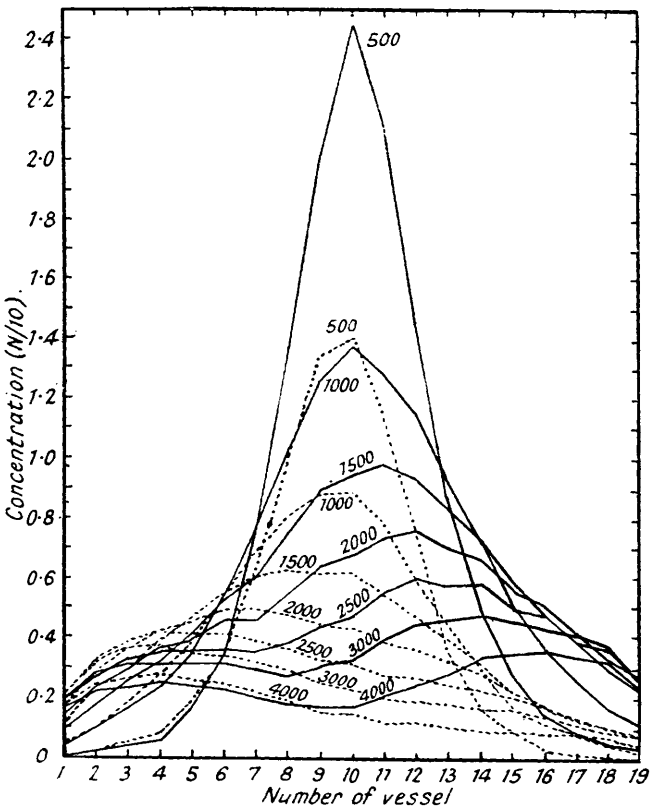
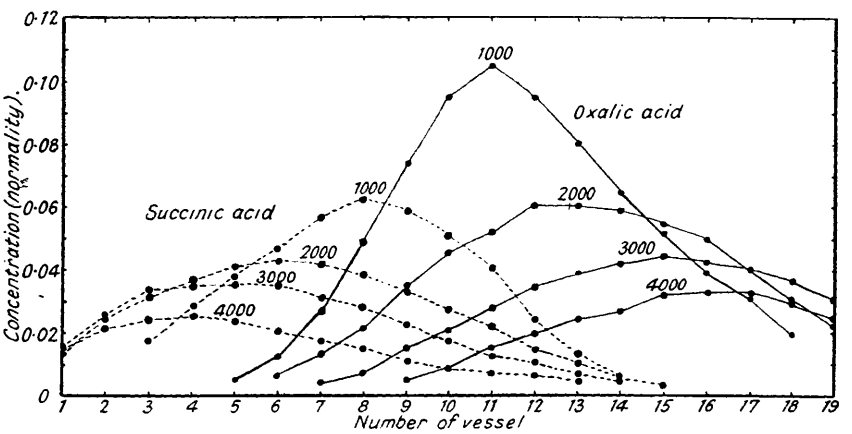


FIG. 20.



Disposition of oxalic acid in the aqueous phase and of succinic acid in the butanol phase in a 19*-vessel system at the times shown in minutes. The data shown have been taken from smooth curves drawn through all the experimental results obtained. Concentrations below* 0·004N. *are without significance.*

The air-lifts used in this series of experiments had quite an appreciable hold-up of liquid (about 10 c.c.) but those used in the 19-vessel experiments were reduced so that the total capacity was about 3 c.c. The air for the operation of the air-lifts was saturated with the liquid to be raised by bubbling the supply

through a vessel containing the appropriate liquid immersed in the thermostat, into which the mixers and separators were also immersed as much as possible.

The feeds of solvents were maintained steady by using a fixed head and passing the liquids through capillary tubes surrounded by constant-temperature jackets fed from the thermostat. Before reaching the capillary tubes the liquids passed through sintered glass discs of ample dimensions, to remove particles which might cause blockages, and were mutually saturated before use.

Stirring was vigorous for the reasons given earlier. Since this tended to produce emulsions, the speed of entry of the two liquids to the system was kept sufficiently low to allow time for the emulsion to break continuously in the separator. There was little tendency for the formation of stable emulsions when the acids were present in solution; hence, in the preliminary stages of bringing the system to equilibrium, about 0·01 g. or less of the appropriate acid was added into each mixer right from the start; this quantity could not affect the results obtained.

Method of operation. The approximate feed rates of solvents to be used having been decided, each of the vessels was filled with a mixture of the solvents in the ratio of the feed rates. The feeds, stirrers, and air-lifts were then started, and after about an hour's running, readings of the solvent input and output, were taken each 15 minutes. When the rates of input and output for a given solvent were equal, the system was in equilibrium. The feeds only were stopped and about 100—150 ml. of mixed solvent were withdrawn from the vessel into which it was proposed to introduce the solute. The weighed solute was dissolved in some of the solvent mixture, warm if necessary, and the solution was re-introduced into the partly empty bottle. The residual solvent mixture was used for washing in the solution drainings, any solvent mixture over and above that required just to fill the mixer being discarded. The feeds were re-started and samples of the effluent butanolic and aqueous phases from the two separators were collected at intervals. Measurements were on a time basis but, since the experiments were interrupted, the timing of the run was based on the volumes of solvents used.

Succinic acid was determined alkalimetrically, as was oxalic acid when alone. When mixtures of the two were used, the total acid was determined in this way, and the oxalic acid by permanganate titration. It was necessary to remove the butanol from all solutions before determining the oxalic acid. Steam was passed through 1-ml. portions of the samples (which amounted to about 5 c.c.) for about 3 minutes, and the hot liquid titrated immediately in the usual way.

Distribution of succinic acid. Acid (19·18 g.) was introduced into vessel 2. The aqueous phase A was fed into vessel 1, and the butanol phase B into vessel 2, at the rates $v_a = 104$ and $v_b = 510$ ml./hour; V was 900 ml. On the assumption that $k = 0{\cdot}935$,

$$p_2 = 1{\cdot}710(e^{-0{\cdot}932t} + e^{-0{\cdot}432t})\text{N}/10$$

and

$$q_1 = 4{\cdot}18(e^{-0{\cdot}432t} - e^{-0{\cdot}932t})\text{N}/10$$

The results are recorded in Fig. 11, and the continuous curves are plots of the above two expressions.

Distribution of oxalic acid. Acid (20 g.) was introduced into vessel 2. The aqueous phase A was fed into vessel 1 and the butanol phase B into vessel 2, at the rates $v_a = 471$ and $v_b = 561$ ml./hour. V was 900 ml. The value of k assumed to apply at the beginning of the run was deduced from equation (1) [which becomes $p = 363{\cdot}8k(471k + 561)$N. if the appropriate figures are introduced], and the results recorded above for the distribution coefficient of oxalic acid. When the expression just given and the results mentioned are plotted, the intercept gives 1·708 for the value of k and 0·455N. for the initial concentration in the aqueous phase in vessel 2.

The data for this acid are recorded in Fig. 12, the continuous and broken lines being the theoretical values based on considerations given earlier.

Distribution of a mixture of oxalic and succinic acids. Each acid (15 g.) was introduced into vessel 1. The aqueous phase was fed into vessel 1, and the butanol phase into vessel 2, at the rates $v_a = 208$ and $v_b = 504$ ml./hour, V being 900. It was assumed that k (succinic acid) = 0·935, and k (oxalic acid) initially = 1·73. The experimental results are given in Fig. 13. The theoretical curves are the continuous lines.

19-*Vessel System.—Description of apparatus.* The more elaborate apparatus (Fig. 14) required for the study of a 19-vessel system was based on the experience gained with the 2-vessel system described above. The total volume of a separator and its two relevant air-lifts was reduced to 20—25 ml. at most. The air-lifts were fashioned mainly from wide-bore capillary tubing and, by trial, the best diameter of bore for the feeds desired appeared to be about 2—3 mm. The bulb of the air-lift was made very small and the air-jet, which entered the capillary tube to the extent of 7—10 mm., was drawn out sufficiently fine to permit the free flow of liquid by the side of the jet.

The flow of the two liquid phases was maintained from large reservoirs, and the fine control of the rates was ensured by stopcocks having elongated arms. Between the stopcocks and the flowmeters, screw clips were provided so that the feeds could be arrested without affecting the setting of the former. The flowmeters were of the familiar pattern, a fine capillary inverted U-tube providing the resistance path for liquid flow; the difference in pressures between the two ends was measured on a mercury manometer.

Each mixing vessel was provided with two air-lifts, a stirrer, and a separator, except that the terminal vessels had only one air-lift, each terminal vessel having, however, a tube through which the liquid phases made their entry into the system.

Alternative arrangements for separating and raising the liquid phases were rejected for various reasons. Of two designed to reduce the number of air-lifts, one required too rigid an assembly, and the other led to considerable emulsification.

In the 2-vessel system, the air supplies were passed through vessels containing the two solvents so as to saturate the air before it reached the lifts, and so reduce evaporation. In the larger apparatus this was dispensed with, because the air-lifts were so efficient that a mere trickle of air was capable of ensuring efficient elevation of the liquids. The use of a thermostat for the 19-vessel system was considered

unnecessary because it was not expected that the theoretical and observed concentrations of solutes would be in very close agreement, since the removal of the large number of samples envisaged would necessarily disturb the system to some extent.

Method of operation. The following method for the removal of samples affords least disturbance to the system. The two air-lifts raising liquid issuing from one mixer were placed out of action and about 3—4 ml. each of butanol and water (mutually saturated) were introduced into the mixing vessel being sampled. This caused displacement of suitable quantities, and samples were taken immediately from the side-arms of the air-lifts. The air-lifts were then brought into operation again. The rationale of adding the small quantities of the liquid phases to the system is that the removal of a sample leads to alteration in feed rates to a number of vessels on each side of that sampled. Rather than disturb the system in an undetermined manner, it was decided that samples removed should be replaced by an approximately equal volume of solvent and, since introduction of such solvents causes speedy discharge of an equal volume of the contents of a mixing vessel, the introduction was used as a means of assistance in sampling with the minimum of disturbance. The removal of samples, of course, meant the removal of solute from the system; this could not be avoided but the samples were made as small as possible, it being understood that the cumulative effect of many samples would show itself most clearly towards the end of a run.

In starting a run with a mixture of oxalic and succinic acids, it was assumed that the distribution coefficient of the former between butanol and water was 2 The aim was to use as strong solutions as possible in the middle vessel so that the titre of the solutions withdrawn from the end vessels would be reasonably accurate. Although this would imply a value of k less than 2 in the central vessel at the start, it was expected that k would not be less than 2 at any time for the remainder of the vessels, because the concentration would never be sufficiently high in these vessels (except possibly vessels 9 and 11). Now, in accordance with equation (24) the most desirable ratio of feed rates of the two solvents would be $v_a/v_b = \sqrt{1/(1 \times 2)} = 0{\cdot}707$, a figure lower than this being more likely to correspond to most efficient separation of the solutes. The actual average ratio of feed rates used was 0·615 but there were slight variations during the course of the experiment. Initially each vessel carried volumes of solvents in this same ratio.

The apparatus was run for 5—6 hours before introduction of the mixture of solutes, in order to ensure equilibration of the solvents.

The mixture of acids was introduced into the system by withdrawing a suitable volume of mixed solvents from vessel 10 and dissolving the appropriate weights of succinic acid and crystallised oxalic acid in some of the mixture, from which the correct volume of water was withdrawn (to allow for the water present in the oxalic acid). The mixture of solvents was warmed to effect dissolution, and the solution was then introduced into vessel 10, sufficient of the residual mixture of solvents being used to wash in the drainings and just to fill the mixing vessel. When the stirrers, air-lifts, and feeds were started, the whole system began to operate smoothly within a few moments and this was taken to be zero time.

It was expected that there would be a fairly rapid fall in concentration in the central vessel and that acids would pass into the two adjacent vessels quite soon, but that the passage of the acids along the system would then be rather slow because of the closeness of the values of k. At the beginning of the run, therefore, samples were taken at relatively close intervals from vessels near the middle of the apparatus and then, as the run proceeded, the intervals were extended and more and more vessels were sampled. At the start, minutes were of importance, but later, *e.g.*, after 3000 minutes, 10-hourly intervals were not excessive and the exact moment of sampling was of no consequence.

Slight trouble with emulsions was encountered. It occurred first in vessel 6 and was attributed to something removed from the rubber bungs by the butanol, since it passed along the system towards vessel 1. In vessels carrying high concentrations of acids, no emulsions were encountered; judicious use of small quantities of N-HCl removed all emulsions, in a few seconds. The emulsions were confined to the terminal vessels after a few hours, and with vessel 1 it was found that 0·2 ml. was ample to remove the emulsion for a considerable time. At the other end of the system 1 ml. of acid was used, and samples from this and the adjacent vessels were tested for Cl′; if Cl′ was present, the hydrochloric acid was determined and taken into account. The titre of hydrochloric acid never exceeded 0·0003N. and, in general, throughout the system Cl′ was not detectable.

The solutions leaving the two ends of the system were suitably treated for recovery of butanol and concentration of the solutes. The butanolic phase was steam-distilled and the aqueous residue concentrated at reduced pressure. The aqueous phase leaving the 19-vessel system was concentrated at reduced pressure. The recovered butanol was re-circulated through the apparatus. Reservoirs of 5-l. capacity were sufficiently large to feed the counter-current extraction system, solvents which had passed through being worked up when approximating to 2 l.

25 G. of each (anhydrous) acid were used in the experiment described. The volume of a mixer and its ancilliary equipment was nearly 900 ml. and the feed rates of the two solvents were 157·7 ml./hour (aqueous phase) and 256·6 ml./hour (butanol phase).

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