# Entropy Titrations: A Reassessment of Data for the Reaction of the Sulphate Ion with Protons and with Bivalent Metal Ions 

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The method of simultaneous determination of $\log K^{\circ}$ and $\Delta H^{\circ}$ for anion-cation interactions is critically discussed by reference to the reactions $\mathrm{H}^{+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{HSO}_{4}{ }^{-}$and $\mathrm{M}^{2+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{MSO}_{4}$. For these reactions the ' entropy titration ' approach is less reliable than that employing potentiometrically determined $K^{\circ}$ data to calculate $\Delta H^{\circ}$ from calorimetric measurements. The effects of the choice of activity coefficient expression are discussed. A general non-linear least-squares procedure has been used for the simultaneous determination of $\Delta H^{\circ}$ and $K^{\circ}$. The procedure is contrasted with the iterative method used by other workers. $\Delta H^{\circ}$ for $\mathrm{M}^{2+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{MSO}_{4}$ have been recalculated by use of potentiometric $K^{\circ}$ data and published calorimetric data.

Recently the method of simultaneous determination of $\Delta H^{\circ}$ and $K^{\circ}$ from enthalpimetric titrations has been developed. ${ }^{1}$ Results have been published for the re-
${ }^{1}$ J. J. Christensen, R. M. Izatt, L. D. Hansen, and J. A. Partridge, J. Phys. Chem., 1966, 70, 2003.
${ }_{2}$ R. M. Izatt, D. Eatough, J. J. Christensen, and C. H. Bartholomew, J. Chem. Soc. (A), 1969, 45.
${ }^{3}$ R. M. Izatt, D. Eatough, J. J. Christensen, and C. H. Bartholomew, J. Chem. Soc. (A), 1969, 47.
${ }_{4}$ J. J. Christensen, R. M. Izatt, and D. Eatough, Inorg. Chem., 1965, 4, 1278.
actions $\mathrm{H}^{+}+\mathrm{SO}_{4}{ }^{2-}{ }^{-} \mathrm{HSO}_{4}^{-}{ }^{-1,2} \mathrm{M}^{n+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons$ $\mathrm{MSO}_{4}{ }^{(n-2)+}, \mathbf{3}$ and many others. ${ }^{4-8}$ It has been shown ${ }^{1}$ that for a reaction $\mathrm{A}+\mathrm{B} \rightleftharpoons \mathrm{AB}$ an expression
${ }^{5}$ J. J. Christensen, D. P. Wrathall, R. M. Izatt, and D. O. Tolman, J. Phys. Chem., 1967, 71, 3001.

G R. M. Izatt, D. Eatough, and J. J. Christensen, J. Phys. Chem., 1968, 72, 2720.

7 P. Paoletti, A. Vacca, and D. Arenare, J. Phys. Chem., 1966, 70, 193.
${ }^{8}$ R. M. Izatt, J. H. Rytting, D. P. Nelson, B. L. Haymore, and J. J. Christensen, Science, 1969, 164, 443.
of the form (1) can be written, where $D, E$, and $F$ are expressions relating total concentrations of A and B ,

$$
\begin{equation*}
\frac{\Delta H}{K}=D(\Delta H)^{2}+E \Delta H+F \tag{1}
\end{equation*}
$$

volume, the measured heat change, and the product of activity coefficients for $\mathrm{A}, \mathrm{B}$, and AB . From a set of equations (1) for a given reaction $\Delta H$ and $K$ can be determined by solution of simultaneous equations ${ }^{1}$ such as (2) or by iterative ${ }^{7,9-11}$ or general non-linear least-squares procedures.
$\left(D_{2}-D_{1}\right) \Delta H^{2}+\left(E_{2}-E_{1}\right) \Delta H+\left(F_{2}-F_{1}\right)=0$
Different workers have pointed to the limitations ${ }^{\mathbf{9 , 1 0 , 1 2}}$ and to the advantages ${ }^{1,10}$ of this method which was first reported by Becker et al. ${ }^{13}$ and advanced by Christensen, Izatt, and their co-workers. This paper (i) discusses certain limitations and disadvantages of this method, (ii) discusses the dependence of the results on the relationships used for the minimisation of residuals, and (iii) refers to the profound effect of the chosen activity coefficient expression on the computed $\Delta H^{\circ}$ and $K^{\circ}$ for ion-pairing reactions.

Dissociation of the Hydrogensulphate (Bisulphate) Ion.Cabani and Gianni ${ }^{9}$ have critically examined the simultaneous calculation of $\Delta H$ and $\log K^{\circ}$ for the reaction (3).

$$
\begin{equation*}
\mathrm{HSO}_{4}^{-}+\mathrm{H}_{2} \mathrm{O} \Longrightarrow \mathrm{SO}_{4}^{2-}+\mathrm{H}_{3} \mathrm{O}^{+} \tag{3}
\end{equation*}
$$

They drew attention to the dependence of $K^{\circ}$ and $\Delta H$ on the choice of activity coefficient expression and on the corrections made to $Q_{\text {obs }}$ for the side reactions occurring in the calorimeter, e.g., dilution of titrant $\left(\mathrm{HClO}_{4}\right)$ or formation of ion pairs $\left(\mathrm{NaSO}_{4}^{-}\right)$. However, in their calculations Cabani and Gianni ${ }^{9}$ and Izatt et al. ${ }^{1,2}$ have not allowed for the dependence of $\Delta H$ on the ionic strength; they have therefore derived an average value of the concentration-dependent molar enthalpy change, $\overline{\Delta H}$. Further, these workers ${ }^{1,2,9-11}$ and others ${ }^{7}$ have used only iterative procedures to calculate the 'best' values of $\Delta H$ and $K^{\circ}$. This procedure is less reliable and less rigorous than a general non-linear least-squares process which, by independently varying all parameters, seeks a minimum in the error square sum $U=\Sigma\left(F_{\text {obs }}-F_{\text {calc }}\right)^{2}$ where $F$ is an analytical or experimentally observed quantity; the process terminates when for all parameters $\partial(U) / \partial($ parameter $)=$ 0 . The iterative procedure ${ }^{7,10,11}$ varies only one parameter (usually $K$ ) and for each value of $K$

[^0]calculates the solution composition for each data point, an average value of the enthalpy change (4), and an error square sum (5) [or an equivalent expression (6) where $\left.\Delta H_{i}=\left(q_{r} / M_{\mathrm{AB}}\right)_{i}\right]$. The 'best pair,' $K^{\circ}, \overline{\Delta H}$,
\[

$$
\begin{align*}
\overline{\Delta H}_{\mathrm{AB}} & =\frac{1}{n} \sum_{i=1}^{n}\left(\frac{q_{r}}{M_{\mathrm{AB}}}\right)_{i}  \tag{4}\\
U & =\sum_{i=1}^{n}\left\{w_{i}\left(\overline{\Delta H}_{\mathrm{AB}}-\Delta H_{i}\right)^{2}\right\}  \tag{5}\\
U & =\sum_{i=1}^{n}\left\{w_{i}\left(q_{i}-\overline{\Delta H}_{\mathrm{AB}} M_{\mathrm{AB}}\right)^{2}\right\} \tag{6}
\end{align*}
$$
\]

corresponds to the minimum in the function $U$. However the quantity $\overline{\Delta H}$ is a function of $K^{\circ}$ and therefore $U=f\left(K,[\mathrm{~A}]_{t},[\mathrm{~B}]_{t}\right)$, whereas, as pointed out by Sillen, ${ }^{14}$ the expression used for $F$ must be complete and contain all terms which should be independently varied, i.e., it is required that $U=f\left(K^{\circ}, \Delta H^{\circ},[\mathrm{A}],[\mathrm{B}]_{t}\right)$ and that $U$ be minimised for both $K^{\circ}$ and $\Delta H^{\circ}$. Further, the minimum value of $U$ results for the value of $K^{\circ}$ which gives the smallest spread of $\Delta H_{i}$ values such that $\overline{\Delta H}-$ $\Delta H_{i}$ is a minimum through the data set. If there are any systematic uncertainties associated with $q_{r}$, $[\mathrm{A}]_{t}$, or $[\mathrm{B}]_{t}$ then the correct (rather than ' best') value of $K^{\circ}$ will give a set of $\Delta H_{i}$ which show a drift; the iterative process will choose an incorrect $K^{\circ}$ to minimise this drift. Also if (as assumed by the above workers ${ }^{\mathbf{1 , 2}, 9}$ ) $\Delta H_{i}$ is the concentration-dependent enthalpy change, then for the correct value of $K^{\circ}$ the $\Delta H_{i}$ values must span a significant range (ca. $0.7 \mathrm{~kJ} \mathrm{~mol}^{-1}$ for $\mathrm{H}^{+}+$ $\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{HSO}_{4}{ }^{-}$; Table 1). Thus minimisation of the sum of squared residuals $\left(\overline{\Delta H}_{\mathrm{AB}}-\Delta H_{i}\right)^{2}$ must yield an incorrect value for $K^{\circ}$.

In reassessing the data for $\mathrm{HSO}_{4}^{-}$proton dissociation Cabani and Gianni have used only those activity coefficient expressions employed by Izatt et al. ${ }^{1,2}$, viz., (7) and (8), although the latter expression leads to

$$
\begin{align*}
\log \gamma_{ \pm} & =\frac{-A z^{2} \sqrt{ } I}{1+1 \cdot 65 \sqrt{ } I}  \tag{7}\\
\log \gamma_{ \pm} & =-A z^{2}\left\{\frac{\sqrt{ } I}{1+1 \cdot 65 \sqrt{ } I}-0.5 I\right\} \tag{8}
\end{align*}
$$

abnormally high activity coefficients. ${ }^{9}$ Two other empirical equations which have been proven applicable to hydrogensulphate or sulphate ion systems are the two forms of the Davies equation (9) where $\beta=0 \cdot 2^{15-20}$

$$
\begin{equation*}
\log \gamma_{ \pm}=-A z^{2}\left\{\frac{\sqrt{ } I}{1+\sqrt{ } I}-\beta I\right\} \tag{9}
\end{equation*}
$$

or $0 \cdot 3 .{ }^{21,22}$ Using each Davies equation and equation
${ }_{17}$ V. S. K. Nair and G. H. Nancollas, J. Chem. Soc., 1959, 3936.
${ }^{18}$ R. P. Bell and J. H. B. George, Trans. Faraday Soc., 1953, 49, 619.
${ }^{16}$ C. W. Davies, J. Chem. Soc., 1938, 2093.
${ }^{20}$ C. W. Davies, H. W. Jones, and C. B. Monk, Trans. Faraday Soc., 1952, 48, 921.
${ }^{21}$ C. W. Davies, ' Ion Association,' Butterworths, London, 1962, p. 44.
${ }^{22}$ W. D. Bale, E. W. Davies, and C. B. Monk, Trans. Faraday Soc., 1956, 52, 816.
(8) to calculate activity coefficients we have recalculated $\Delta H^{\circ}$ and $K^{\circ}$ for the dissociation of the hydrogensulphate ion from the data of Izatt et al. ${ }^{23}$ (Table 1). The data used were for the titrations of $0.7006 \mathrm{M}-\left(\mathrm{Me}_{4} \mathrm{~N}\right)_{2} \mathrm{SO}_{4}$ into $0.01665 \mathrm{M}-\mathrm{HClO}_{4}$ (ref. 23, run 12, 35 data points); $Q_{\text {ols }}$ was corrected for the heat of dilution of the titrant ${ }^{3}$ for the formation of the ion pair $\mathrm{Me}_{4} \mathrm{~N}^{+} \mathrm{ClO}_{4}{ }^{-}(\log K=$ $\left.0.27, \overline{\Delta H}=0.84 \mathrm{~kJ} \mathrm{~mol}^{-1}\right) .^{2}$ Molar concentrations were used because of the small temperature changes involved in the calorimetric measurements (max. 0.05 ${ }^{\circ} \mathrm{C}$ ). The calculations used the previously described ${ }^{24}$ general non-linear least-squares programme to determine the best value for the constants $\Delta H^{\circ}$ and $K^{\circ}$.

$$
\begin{array}{r}
-\log \gamma_{ \pm}=A z^{2}\left\{\frac{\sqrt{ } I}{1+B \sqrt{ } I}-C I\right\} \\
-\frac{\partial \log \gamma}{\partial T}=A z^{2} \frac{\sqrt{ } I}{1+B \sqrt{ } I}\left\{\frac{1}{A} \frac{\mathrm{~d} A}{\mathrm{~d} T}+\right. \\
\left.\frac{1}{1+B \sqrt{ } I} \frac{1}{2 I} \frac{\mathrm{~d} I}{\mathrm{~d} T}-\frac{B \sqrt{ } I}{1+B \sqrt{ } I} \frac{1}{B} \frac{\mathrm{~d} B}{\mathrm{~d} T}\right\} \\
-z^{2} A C I\left\{\frac{1}{A} \frac{\mathrm{~d} A}{\mathrm{~d} T}+\frac{1}{\bar{I}} \frac{\mathrm{~d} I}{\mathrm{~d} T}\right\} \tag{16}
\end{array}
$$

From the expressions for $A\left[=k(\varepsilon T)^{-3 / 2}\right]^{26}$ and $B\left[=k^{\prime}(\varepsilon T)^{-1 / 2}\right]^{26}$ and from the relationship $\mathrm{d} \varepsilon / \mathrm{d} T=$

Table 1
Simultaneous solutions of $\Delta H^{\circ}$ and $K^{\circ}$ from enthalpimetric titrations for the reaction $\mathrm{SO}_{4}{ }^{2-}+\mathrm{H}^{+} \rightleftharpoons \mathrm{HSO}_{4}^{-}(\mathrm{aq})$ at $25{ }^{\circ} \mathrm{C}{ }^{a}$

| Method | $\Delta H^{\circ}$ |  | $\log K^{\circ}$ | $\Sigma\left(F_{\text {obs }}-F_{\text {calc }}\right)^{2}$ | $R^{\text {b }}$ (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overline{\mathrm{kJ} \mathrm{mol}}{ }^{-1}$ | $K^{\circ}$ |  |  |  |
| $\log \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ } I}{1+\sqrt{ } I}-0.2 I\right\}^{\circ}$ | $26.4 \pm 0.5$ | $99 \cdot 5 \pm 4 \cdot 2$ | $1.998 \pm 0.018$ | $1.20 \times 10^{-6}$ | $0 \cdot 29$ |
| $\log \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ }}{1+\sqrt{ } I}-0.3 I\right\}^{d}$ | $23.3 \pm 0 \cdot 4$ | $109 \cdot 4 \pm 4 \cdot 2$ | $2.039 \pm 0.018$ | $1.18 \times 10^{-6}$ | $0 \cdot 29$ |
| $\log \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ } \text { I }}{1+1.65 \sqrt{ } I}-0.5 I\right\}^{e}$ | $19 \cdot 6 \pm 0 \cdot 2$ | 142.4土 $\mathbf{3} \cdot 6$ | $2 \cdot 153 \pm 0.011$ | $0.81 \times 10^{-6}$ | $0 \cdot 24$ |
| $\begin{aligned} & \log \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ }}{1+1.65 \sqrt{ } I}-0.5 I\right\}^{f} \\ & \quad \text { assuming } \Delta H=\Delta H^{0} \end{aligned}$ | $17 \cdot 8 \pm 0 \cdot 6$ | $161 \cdot 6 \pm 14 \cdot 7$ | $2 \cdot 208 \pm 0.038$ | $1.65 \times 10^{-5}$ | 1.09 |
| Reported by Izatt et al. | $20.5 \pm 0.8$ | $93 \cdot 3 \pm 6 \cdot 7$ | $1.97 \pm 0.03$ |  |  |
| Calc. $\overline{\Delta H}$ by imposing Izatt's $\log K^{\circ}$ and and $\log \gamma_{ \pm}$values on least-squares refinement | $21.74 \pm 0.08$ | $93 \cdot 3$ | 1.97 | $2.28 \times 10^{-5}$ | 1.26 |
| a Data from ref. $23 ; 35$ data points for th and $\mathrm{R}_{4} \mathrm{~N}^{+} \mathrm{ClO}_{4}^{-}$ion pairing (ref. 2). b $R$ defin | ation of ( $\left.\mathrm{Me}_{4} \mathrm{~N}\right)$ | 9) ${ }^{\text {a }}$ (ref. into $\mathrm{HClO}^{\text {a }}$. | Corrections app | d for titrant dil | - r (ref. |
| $\Delta H$ range $24.2-23.6 \mathrm{~kJ} \mathrm{~mol}^{-1}$; $q_{\text {diln }}$ in range | as in-0.874 J a | $q$ (ion pair) in | $\text { omputed } 10 n 1 \mathrm{c}$ | J for individual | ata poin |
| hich range from 1.021 to 21.43 J . ${ }^{2}$ Compu nic strength range $0.0172-0.0915 \mathrm{~m} ; \Delta H$ | onic strength $18 \cdot 9-18 \cdot 4 \mathrm{k}$ | $\text { ge } 0.0175-0.0$ | ; $\Delta H$ range 22 dionic strength | $22 \cdot 1 \mathrm{~kJ} \mathrm{~mol}^{-1}$. <br> ange $0.0171-0.0$ | Compu |

For the reaction $\mathrm{H}^{+}+\mathrm{SO}_{4}{ }^{2-} \Longrightarrow \mathrm{HSO}_{4}^{-}$we obtain equation (11) where $\left[\mathrm{HSO}_{4}\right]$ is given by (12) and $\Delta H$ by (13) ${ }^{25}$ and (14) for the protonation of sulphate ion.

$$
\begin{align*}
K_{\mathrm{v}} & =K^{\circ} \frac{\gamma_{\mathrm{H}+}+\gamma_{\mathrm{SO}_{4}{ }^{2-}}}{\gamma_{\mathrm{HSO}}^{4}}=  \tag{10}\\
& =\frac{\left[\mathrm{HSO}_{4}\right]}{\left([\mathrm{H}]_{t}-\left[\mathrm{HSO}_{4}{ }^{2-}\right.\right.}=K^{\circ} \gamma_{ \pm}  \tag{11}\\
{\left[\mathrm{HSO}_{4}\right] } & =\frac{Q_{\text {corr }}(\mathrm{J})}{V\left(\left[\mathrm{SO}_{4}\right]_{t}-\left[\mathrm{HSO}_{4}\right]\right)}  \tag{12}\\
\Delta H & =\Delta H^{\circ}-R T^{2} \Sigma\left[n_{\mathrm{s}} \frac{\partial \ln f_{\mathrm{s}}}{\partial T}\right]_{p}  \tag{13}\\
& =\Delta H^{\circ}+2 \cdot 303 R T^{2}\left[\frac{\partial \log \gamma_{\mathrm{SO}_{4} 2^{--}}}{\partial T}\right]_{p} \tag{14}
\end{align*}
$$

For the activity coefficient expressions employed, (15), we obtain equation (16), if $C$ is taken as constant.

[^1]$-4.5 \times 10^{-3} \varepsilon_{\mathrm{H}_{2} \mathrm{O}} \quad\left(25{ }^{\circ} \mathrm{C}\right)^{27}$ the terms $(1 / A)(\mathrm{d} A / \mathrm{d} T)$ and $(1 / B)(\mathrm{d} B / \mathrm{d} T)$ are calculated as $1.85 \times 10^{-3}$ and $0.62 \times 10^{-3}$ respectively at $25{ }^{\circ} \mathrm{C}$. If the solution is assumed to have the same thermal coefficient of expansion as the solvent then we obtain equation (17) at $25^{\circ} \mathrm{C}$, where $V_{\mathrm{m}}$ is the molar volume of water. ${ }^{28}$

If equation (16) is written as a function of $I, C^{\prime}(I)$, rearrangement of equation (11) gives equation (18).

$$
\begin{equation*}
\frac{1}{I} \frac{\mathrm{~d} I}{\mathrm{~d} T}=-\frac{1}{V_{\mathrm{m}}} \frac{\mathrm{~d} V_{\mathrm{m}}}{\mathrm{~d} T}=-2.58 \times 10^{-4} \tag{17}
\end{equation*}
$$

$$
\begin{align*}
& {\left[\mathrm{SO}_{4}\right]_{t}=} \\
& \frac{Q_{\text {corr }}}{K^{\circ} \gamma_{ \pm}}+Q_{\text {corr }}\left[\mathrm{H}^{+}\right]_{t}-\frac{\left(Q_{\text {corr }}\right)^{2}}{\left[\Delta H^{\circ}+2 \cdot 303 R T^{2} C^{\prime}(I)\right] V}  \tag{18}\\
& {[\mathrm{H}]_{t} V\left[\Delta H^{\circ}+2 \cdot 303 R T^{2} C^{\prime}(I)\right]-Q_{\text {corr }}}
\end{align*}
$$

The least-squares process varied both the parameters $\Delta H^{\circ}$ and $K^{\circ}$ to minimise the error square sum $\Sigma w_{i^{-}}$ $\left\{\left[\mathrm{SO}_{4}\right]_{t}(\mathrm{obs})-\left[\mathrm{SO}_{4}\right]_{t}(\mathrm{calc})\right\}_{i}{ }^{2}$. Following the first least-
${ }^{25}$ H. K. J. Powell and N. F. Curtis, J. Chem. Soc. (B), 1966, 1205.
${ }_{26}$ R. A. Robinson and R. H. Stokes, ' Electrolyte Solutions,'
Butterworths, London, 1959, 2nd edn., p. 230.
${ }^{27}$ R. L. Kay, G. A. Vidulich, and K. S. Pribadi, J. Phys. Chem., 1969, 73, 445.
${ }^{28}$ Ref. 26, p. 457.
squares cycle, which used trial values of $K^{\circ}, \Delta H^{\circ}$, and $I$, each cycle commenced by using the improved $K^{\circ}$ to calculate the solution composition, and hence to calculate an improved value of $I$ and of $\gamma_{ \pm}$for use in equation (18). The least-squares process terminated when the error square sums for successive cycles of refinement differed by less than $0 \cdot 1 \%$. A weighting factor $w_{i}=$ 1.0 was used.

The results in Table $\mathbf{l}$ indicate the strong dependence of both $\Delta H^{\circ}$ and $K^{\circ}$ on the equation used for $\log \gamma_{ \pm}$. The titration data span the ionic strength range $c a$. $0.017-0 \cdot 10$, and although equations (8) and (9) give similar values for $\gamma_{ \pm}$at $I=0.02$ \{viz., 0.569 [equation (9), $\beta=0.2$ ]; 0.574 [equation (9), $\beta=0.3$ ]; 0.612 [equation (8)]\}, the calculated coefficients differ markedly at $I=0.09 \quad(0.369,0.384$, and 0.481 respectively). For comparison, Table 1 records results obtained by use of Izatt's equation and the assumption ${ }^{1}$ that $\Delta H^{\circ}=$ $\overline{\Delta H}$. Footnotes record the computed ionic strength range and the range of values for $\Delta H$ for each calculation. The programme gave convergence in 4 or 5 cycles ( 3 min on an IBM 360/44 computer) except for the calculation which assumed that $\Delta H^{\circ}=\overline{\Delta H}$; in the latter case, after 10 cycles, the parameters were ranging in the area $K^{\circ}=160-1701 \mathrm{~mol}^{-1}$ and $\Delta H^{\circ}=17 \cdot 4-17.9$

$$
\begin{equation*}
R=\sqrt{\left(\frac{\Sigma\left(F_{\text {obs }}-F_{\text {calc }}\right)^{2}}{\Sigma\left(F_{\text {obs }}\right)^{2}}\right) \cdot \frac{100}{1} \%} \tag{19}
\end{equation*}
$$

$\mathrm{kJ} \mathrm{mol}^{-1}$. Each calculation achieved an $R$ factor ${ }^{26}$ of ca. $0.3 \%$.

It is appreciated ${ }^{29,30}$ that a good fit between calculated and experimental data is often insufficient support for hypotheses on an equilibrium system. For protonation of the sulphate ion at least three approaches yield a good fit (Table 1). One has to decide which combination of $\Delta H^{\circ}$ and $K^{\circ}$ (which result from different activity coefficient expressions) most closely approaches the 'correct' result; this is a problem which does not arise when an independently determined $K^{\circ}$ value is used in the calculation of $\Delta H^{\circ}$.

The results in Table $\mathbf{1}$ can be compared with the ' best' values of $\log K^{\circ}$ (potentiometric) and $\Delta H^{\circ}$, viz., $\log$ $K^{\circ}=1.988^{9,31}$ and $\Delta H^{\circ}=24.0 \mathrm{~kJ} \mathrm{~mol}^{-1}$ (Austin and Mair; ${ }^{32} \quad \mathrm{HCO}_{4}$ dilution into $\mathrm{H}_{2} \mathrm{SO}_{4}, I=0.02$, corrected to $I=0.0$ ) and $22.6 \mathrm{~kJ} \mathrm{~mol}^{-1}$ (Readnour and Cobble, ${ }^{33}$ data recalculated by Cabani and Gianni; ${ }^{9}$ $\mathrm{Na}_{2} \mathrm{SO}_{4}$ solution in $\mathrm{HCl}, I=0.01-0.03$, extrapolated to $I=0.0$ ). This comparison suggests that results obtained by use of the Davies equation ( $\beta=0.2$ or 0.3 ) for the activity coefficient are the most acceptable. The lack of agreement between the results in Table 1 and Izatt's results relates to the different methods used in the minimisation process. Imposition of the value $\log K^{\circ}=1.97$ on the least-squares programme

[^2]does yield $\overline{\Delta H}=21.74 \pm 0.09$ in concurrence with Izatt's result but the error square sum is significantly higher for this ' solution ' (Table 1).

The agreement between, on the one hand, calorimetric $\Delta H^{\circ}$ and potentiometric $K^{\circ}$ values, and on the other $\Delta H^{\circ}$ and $K^{\circ}$ values derived simultaneously from enthalpimetric titrations on this reaction, has been used to establish the reliability of the entropy titration method. ${ }^{1}$ However, on the basis of the above arguments and of extensive studies on bivalent cationsulphate ion interactions (next section) we believe that the agreement achieved in the case of the hydrogensulphate proton dissociation reaction is fortuitous and does not justify confidence in the method.
Formation of Metal(II)-Sulphate Ion Pairs.-The dependence of the computed $\Delta H^{\circ}$ and $K^{\circ}$ on the activity coefficient expression will now be more marked because (i) the relationships $K_{\mathrm{c}}=K^{\circ} \gamma_{ \pm}$and $\Delta H=\Delta H^{\circ}+$ $2 \cdot 303 R T^{2} C^{\prime}(I)$, valid for sulphate protonation, are replaced by $K_{\mathrm{c}}=K^{\circ}\left(\gamma_{ \pm}\right)^{2}$ and $\Delta H=\Delta H^{\circ}+2[2 \cdot 303-$ $\left.R T^{2} C^{\prime}(I)\right]$ respectively, and (ii) for the reactions $\mathrm{M}^{2+}+$ $\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{MSO}_{4}$ the enthalpy titrations reported ${ }^{3,34}$ involve a higher and wider range of ionic strengths (ca. $0.06-0.14 \mathrm{~m}$ ) than for the reaction $\mathrm{H}^{+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons$ $\mathrm{HSO}_{4}{ }^{-2,23}$
The $\Delta H^{\circ}$ and $K^{\circ}$ data in Table 2 have been computed from published enthalpimetric titration data ${ }^{34}$ by use of the Davies equation and equation (8) to calculate activity coefficients. The titration data refer to the addition of $\left(\mathrm{Me}_{4} \mathrm{~N}\right)_{2} \mathrm{SO}_{4}(0.7006 \mathrm{M})$ into a metal perchlorate solution ( $c a .0 .02 \mathrm{~m}, \mathrm{pH} c a .3-5$ ). The nonlinear least-squares programme minimised the error square sum $\Sigma\left\{\left[\mathrm{SO}_{4}\right]_{t}(\mathrm{obs})-\left[\mathrm{SO}_{4}\right]_{t}(\mathrm{calc})\right\}_{i}{ }^{2}$ where $\left[\mathrm{SO}_{4}\right] t^{-}$ (calc.) is given by equation (20). Following the first

$$
\begin{align*}
& {\left[\mathrm{SO}_{4}\right]_{t}(\text { calc })=} \\
&  \tag{20}\\
& \quad \frac{Q_{\text {corr }}}{\left[K^{\circ}\left(\gamma_{ \pm}\right)^{2}\right.}+Q_{\text {corr }[\mathrm{M}]_{t}}\left(Q_{\text {cort }}\right)^{2} \\
& \left.\frac{\left\{\Delta H^{\circ}+2\left[2 \cdot 303 R T^{2} C^{\prime}(I)\right]\right] V}{}\right] \\
& \\
& [\mathrm{M}] t\rangle\left\{\Delta H^{\circ}+2\left[2 \cdot 303 R T^{2} C^{\prime}(I)\right]\right\}-Q_{\text {corr }}
\end{align*}
$$

cycle, which used trial values of $K^{\circ}, \Delta H^{\imath}$, and $I$, and calculated an approximate correction for titrant dilution, successive cycles commenced by using the improved $K^{\circ}$ to calculate the solution composition and thus calculate improved values of $I$ and $\gamma_{ \pm} ; Q_{\text {obs }}$ was corrected for the formation of the ion pair $\mathrm{Me}_{4} \mathrm{~N}^{+} \mathrm{ClO}_{4}^{-}$and for the protonation of the sulphate ion.

The results are critically dependent on the activity coefficient expression used. Convergence was not achieved for each combination of reaction and activity coefficient expression. For each combination one of the two following results was obtained: (i) the least-squares values of $K^{\circ}$ and $\Delta H^{\circ}$ were quite unrelated to those determined by direct methods, e.g., for zinc, with

[^3]$\log \gamma_{ \pm}=-2.034\{[\sqrt{ } I /(1+\sqrt{ } I)]-0.3 I\} ;$ in these cases it was further established that the solution obtained did correspond to a minimum value of the error square sum (see below); (ii) the programme could not determine a minimum value for the error square sum: either
values reported by Nair and Nancollas ${ }^{\mathbf{1 6 , 1 7}}$ from potentiometric measurements gave $\Delta H^{\circ}$ values in fair agreement with those obtained by direct calorimetric measurements. ${ }^{35}$ However, in each case the use of these fixed values for $K^{\circ}$ gave a least-squares solution with a

Table 2
Least-squares $K^{\circ}$ and $\Delta H^{\circ}$ values from enthalpimetric titrations ${ }^{a}$ for reactions $\mathrm{M}^{2+}+\mathrm{SO}_{4}{ }^{2-} \Longrightarrow \mathrm{MSO}_{4}(\mathrm{aq})$

| Method ${ }^{\text {b }}$ | $\frac{\Delta H^{\circ}}{\mathrm{kJ} \mathrm{~mol}}{ }^{-1}$ | $K^{\circ}$ | $\log K^{\circ}$ | $\Sigma\left(F_{\text {obs }}-F_{\text {calc }}\right)^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Zinc sulphate |  |  |  |
| Davies equation, $\beta=0.2$ | NC 0 | NC |  |  |
| Davies equation, $\beta=0.3$ | $9 \cdot 5 \pm 0 \cdot 4$ | $95.0 \pm 8 \cdot 1$ | 1.98 | $2.1 \times 10^{-6}$ |
| Equation (8) | NC | NC |  |  |
| Davies equation, $\beta=0 \cdot 2, K^{\circ}=240{ }^{\text {d }}$ imposed | $6.72 \pm 0.03$ |  | 2.38 | $4.5 \times 10^{-5}$ |
| Davies equation, $\beta=0 \cdot 3, K^{\circ}=240{ }^{\text {d imposed }}$ | $6.44 \pm 0.02$ |  | $2 \cdot 38$ | $3.2 \times 10^{-5}$ |
| Equation (8), $K^{\circ}=309^{\circ}$ imposed, $\Delta H^{\circ}=\Delta H$ | $2.68 \pm 0.01$ |  | $2 \cdot 49$ | $2 \cdot 3 \times 10^{-4}$ |
| Equation (8), reported values * | $2.64 \pm 0.08$ |  | $2.49 \pm 0.04$ |  |
|  | Magnesium sulphate |  |  |  |
| Davies equation, $\beta=0.2$ | 4.4-5.2 | 230-310 |  |  |
| Davies equation, $\beta=0.3$ | $4 \cdot 6-4 \cdot 8 \pm 0.5$ | 265-324 |  |  |
| Equation (8) | NC | NC |  |  |
| Equation (8), $K=169{ }^{\circ}$ imposed, $\Delta H^{\circ}=\Delta H$ | $2 \cdot 11 \pm 0.05$ |  | ${ }_{2}^{2 \cdot 23}$ | $4.1 \times 10^{-4}$ |
| Equation (8), reported values ${ }^{\text {e }}$ | $2 \cdot 24 \pm 0 \cdot 12$ |  | $2 \cdot 23$ |  |
|  | Cobalt sulphate |  |  |  |
| Davies equation, $\beta=0.2$ | $11.4 \pm 0.8$ | $71.0 \pm 9.0$ | 1.85 | $2.2 \times 10^{-6}$ |
| Davies equation, $\beta=0.3$ | $8 \cdot 15 \pm 0.32$ | $116 \pm 10$ | 2.066 | $2.8 \times 10^{-6}$ |
| Equation (8) $\Delta H^{\circ}=\Delta H$ | NC | NC |  |  |
| Davies equation, $\beta=0 \cdot 2, K=224{ }^{\text {f imposed }}$ | $6.53 \pm 0.02$ |  | $2 \cdot 35$ | $2.8 \times 10^{-8}$ |
| Equation (8), $K=490{ }^{\text {e imposed, }} \Delta H^{\circ}=\Delta H$ | $2.19 \pm 0.01$ |  | ${ }_{2.69 .69}^{2.03}$ | $1.4 \times 10^{-9}$ |
| Equation (8), reported value ${ }^{\text {e }}$ | $2.09 \pm 0.08$ |  | $2.69 \pm 0.03$ |  |

a Data from ref. 34 for reactions described in ref. 3. ${ }^{b}$ The expression used to calculate activity coefficients; where a $K^{\circ}$ value was imposed, only $\Delta H^{\circ}$ varied in the least-squares process. ${ }^{\circ}$ No convergence achieved. d Ref. 16. e Ref. 3. f Ref. 17.
$K^{\circ}$ and $\Delta H^{\circ}$ fluctuated within a broad range (e.g. for magnesium, with $\log \gamma_{ \pm}=-2 \cdot 034\{[\sqrt{ } I /(1+\sqrt{ } I)]-$ $0.3 I\}$ or one of the parameters was divergent.

The effect of ignoring the ion pairing reaction between $\mathrm{Me}_{4} \mathrm{~N}^{+}$and $\mathrm{ClO}_{4}^{-}$was to increase $\Delta H^{\circ}$ by $40-65 \%$; the profound effect of this side reaction between countercation and anion can be avoided by use of metal
larger error square sum than that for the solution achieved by allowing both parameters to vary.

Because of the uncertainties associated with the $K^{\circ}$ and $\Delta H^{\circ}$ values determined by the 'entropy titration' method, $\Delta H^{\circ}$ has been recalculated for the reactions $\mathrm{M}^{2+}+\mathrm{SO}_{4}{ }^{2+} \rightleftharpoons \mathrm{MSO}_{4}$ by using the best $K^{\circ}$ values (from potentiometric or conductivity measurements) and

Table 3
Enthalpy data ${ }^{a}$ for reactions $\mathrm{M}^{2+}+\mathrm{SO}_{4}{ }^{2-} \rightleftharpoons \mathrm{MSO}_{4}(\mathrm{aq})$ at $25^{\circ} \mathrm{C}$

$$
\begin{aligned}
& \begin{array}{l}
\log K^{\circ} / \mathrm{K}^{\circ} / \mathrm{k} \mathrm{~mol}^{-1}
\end{array} \\
& \text { (assuming log } \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ } I}{1+\sqrt{ } I}-0.2 I\right\} \text { ) } \\
& \Delta H^{\circ} / \mathrm{kJ} \mathrm{~mol}^{-1} \\
& \text { (assuming } \left.\log \gamma \pm=-2.034\left\{\frac{\sqrt{ } I}{1+\sqrt{ } I}-0.3 I\right\}\right) \\
& \begin{array}{l}
\Delta H^{\circ} / \mathrm{kJ} \mathrm{~mol}^{-1} \\
\quad\left(\text { assuming } \log \gamma_{ \pm}=-2.034\left\{\frac{\sqrt{ } I}{1+\sqrt{ } I}-0.2 I\right\}\right) \\
\text { (ref. 35) }
\end{array}
\end{aligned}
$$

Using $\log K$ (Izatt) and equation (8) $\overline{\Delta H}$ (calc.), this work Reported values (ref. 3):
$2.13 \pm 0.13 \quad 3.39 \pm 0.34 \quad 2.55 \pm 0.04 \quad 2.09 \pm 0.09 \quad 1.72 \pm \begin{array}{llllll}0.08 & 5.10 \pm 0.13 & 2.64 \pm 0.08 & 4.10 \pm 0.17\end{array}$
Ref. 16 .
chloride solutions. ${ }^{35}$ Further insight into the uncertainties associated with previously described calculation methods ${ }^{9-11}$ was achieved by imposing a fixed value of $K^{\circ}$ on the programme and then determining the leastsquares value of $\Delta H^{\circ}$ (Table 2). Use of $K^{\circ}$ values published by Izatt et al. ${ }^{3}$ gave $\overline{\Delta H}$ values in agreement with those reported by these workers. Use of $K^{\circ}$
${ }^{35}$ G. R. Hedwig and H. K. J. Powell, J.C.S. Dalton, 1973, 798.
the reported enthalpimetric titration data. ${ }^{34}$ The programme was similar to that described previously ${ }^{\mathbf{3 5}}$ but replaced the approximate expression $\Delta H=\Delta H^{\circ}+$ $6.80 \log \gamma_{ \pm}$with the expression derived in this paper, $\Delta H=\Delta H^{\circ}+2\left[2 \cdot 303 R T^{2} C^{\prime}(I)\right]$; the results agree with those reported from direct calorimetric measurement (Table 3).
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