# Magnetic Properties of Polynuclear Complexes. Part II. Superexchange in Some Binuclear Cobalt(II) Complexes

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The average magnetic susceptibilities of six binuclear complexes of cobalt(II) between 80 and 400 K are reported. The compounds studied are  $Co_2(dhph)_2X_4.nH_2O$  (dhph = 1.4-dihydrazinophthalazine, X = Cl or Br),  $Co_2-(dppn)_2X_4.nH_2O$  [dppn = 3.6-di-(2-pyridyl)pyridazine, X = NO<sub>3</sub> or ClO<sub>4</sub>],  $Co_2(dppn)(SO_4)_2.5H_2O$ , and  $Co_2(Me_2dppn)(NO_3)_4.2CH_3OH$  [Me\_2dppn = 3.6-di-(6-methyl-2-pyridyl)pyridazine]. These compounds exhibit weak antiferromagnetic exchange, and the values of the effective isotropic exchange parameter  $\mathscr{J}$  are estimated, making allowance for spin-orbit coupling and the effects of orbital reduction and axial distortion parameters. The values of  $\mathscr{J}$  are compared with those of the analogous nickel complexes reported earlier, and it is concluded that the  $t_{2g}$  spin of  $Co^{2+}$  probably makes a ferromagnetic contribution. The exchange interaction is analysed in a one-electron orbital basis, leading to an anisotropic form for the exchange parameter. The average susceptibility data do not distinguish between the isotropic and anisotropic models, however, and the precise identity of the orbitals responsible for the ferromagnetic contribution remains uncertain.

IN an earlier paper we described some binuclear nickel complexes of ligands containing a pair of adjacent nitrogen atoms, and interpreted their magnetic behaviour in terms of a superexchange interaction via the N-N bridge.<sup>1</sup> Three of the ligands, 1,4-dihydrazinophthalazine (dhph), 3,6-di-(2-pyridyl)pyridazine (dppn), and 3,6-di-(6-methyl-2-pyridyl)pyridazine (Me<sub>2</sub>dppn), gave compounds with cobalt(II) that appeared to have the same structures as their nickel analogues, and thus offered an opportunity to compare the superexchange behaviour of nickel and cobalt in similar environments. We now report the magnetic properties of these cobalt complexes, and use them to pursue the question of the mechanism of superexchange in systems of this type, particularly the effect of the unpaired spin in the  $t_{2g}$  orbitals of  $\text{Co}^{2+}$ . In doing this we shall have to take account of the orbital momenta of the ions, since they are in approximately octahedral co-ordination in all these complexes. A

<sup>1</sup> Part I, P. W. Ball and A. B. Blake, J. Chem. Soc. (A), 1969, 1415.

preliminary account of these results has appeared elsewhere.<sup>2</sup>



#### RESULTS

As before, we begin by presenting the evidence for binuclear structures, analogous to those previously assigned to the nickel complexes.

Spectra and Structures of the Complexes.—The compounds, which are listed in Table 1, are orange, paramagnetic,

### TABLE 1

The compounds studied and some magnetic properties

		μeff	$\mu_{\rm eff}$	
Number	Compound	(300  K)	(100 K)	$-\theta/K$
(1)	Co <sub>2</sub> (dhph) <sub>2</sub> ,Cl <sub>4</sub> ,5H <sub>2</sub> O	4.44	3.99	<b>67</b>
$(\mathbf{II})$	$Co_2(dhph)_2Br_4, 6H_2O$	4.45	3.93	<b>58</b>
(III)	$Co_2(dppn)_2(NO_3)_4, 2H_2O$	4.43	3.97	50
(LV)	$\operatorname{Co}_2(\operatorname{dppn})_2(\operatorname{ClO}_4)_4, 6\operatorname{H}_2O$	4.24	3.89	40
(V)	$Co_2(dppn)(SO_4)_2, 5H_2O$	4.53	4.21	35
(V1)	Co <sub>2</sub> (Me <sub>2</sub> dppn)(NO <sub>3</sub> ), 2CH <sub>3</sub> OH	[4.24]	3.94	45

crystalline solids which darken slowly in air (rapidly when moist). The diffuse reflectance spectra of compounds (III)—(V1) show a broad, slightly asymmetric band centred between 9000 and 11 000 cm<sup>-1</sup>, assigned to the  ${}^{4}T_{1g} \longrightarrow {}^{4}T_{2g}$  transition of Co<sup>2+</sup> in pseudo-octahedral symmetry, and up to three weaker bands between 18 000 and 24 000 cm<sup>-1</sup>, appearing as shoulders on an intense u.v. band, which probably correspond to the transitions to  ${}^{4}A_{2g}$  and  ${}^{4}T_{1g}(P)$ . [The sensitivity of compounds (I) and (II) to oxidation prevented satisfactory spectra from being obtained in these cases.]

The magnetic properties of all six compounds suggest the presence of a weak exchange interaction, in that the values of the Weiss constant  $\theta$  (Table 1) are considerably larger than those commonly observed in mononuclear cobalt(II) complexes.\* These compounds were prepared in a similar manner to the corresponding nickel complexes, and have similar formulae (differing by a molecule of solvent in some cases). The evidence for formulating both series of compounds as binuclear can be summarised as follows.

(i) The stoicheiometries and electronic absorption spectra of the nickel complexes are most simply accounted for by binuclear structures, and their magnetic behaviour between 80 and 300 K agrees closely with that predicted for pairs of  ${}^{3}A_{2g}$  ions with a weak antiferromagnetic exchange interaction (*J ca.* -40 cm<sup>-1</sup>).<sup>1</sup>

(ii) The compound Ni(dhph)Cl<sub>2</sub>,  $3H_2O$  has been shown by X-ray crystal-structure analysis to contain the centrosymmetric binuclear cation [Ni<sub>2</sub>(dhph)<sub>2</sub>(H<sub>2</sub>O)<sub>4</sub>]<sup>4+</sup>, Figure 1.<sup>4</sup> Comparison of single-crystal X-ray diffraction photographs of the hk0 and 0kl levels of this compound and its cobalt analogue, compound (I), shows conclusively that the two are isomorphous.

(iii) X-Ray powder photographs show that compound (III) is isomorphous with its nickel analogue.

(iv) The i.r. spectra of compounds (I)—(V) resemble those of their nearest nickel analogues to a degree that we believe reflects close similarities between the respective structures.<sup>†</sup> Thus, the spectra of compounds (III) and (V) between 4000 and 250 cm<sup>-1</sup> are virtually identical with those of the corresponding nickel compounds, and the resemblance is very close in the cases of (I), (II), and (IV). The spectrum of (VI) is sufficiently different from that of its nickel analogue to prevent a definite conclusion, though the differences can be accounted for partly by the different solvation.



FIGURE 1 Structure of  $[Ni_2(dhph)_2(H_2O)_4]^{4+}$  (ref. 4). The chloride of this ion is isomorphous with compound (I)

Taking the evidence as a whole, we feel reasonably confident in assigning binuclear structures with N-N bridges to all the complexes of cobalt and nickel in this study.

Magnetic Results and Interpretation.—The experimental susceptibility data for the finely powdered solids between 80 and 400 K are collected in Table 2.<sup>‡</sup> The values listed have been corrected for diamagnetism as before,<sup>1</sup> and for temperature-independent paramagnetism (t.i.p.) as described in Appendix I. The t.i.p. correction is small (less than 2%), but we include it so that the cobalt and nickel results will be strictly comparable. The values of the effective magnetic moment  $\mu_{\text{eff}} = 2.828 (\chi_a T)^{\frac{1}{2}}$  at 300 and 100 K (where  $\chi_a$  is the susceptibility per mole of Co atoms)

<sup>‡</sup> Magnetic susceptibilities are given in this paper in the c.g.s. electromagnetic system, in the interest of uniformity with Part I. To convert to m<sup>3</sup> mol<sup>-1</sup> in SI, the values given should be multiplied by  $4\pi \times 10^{-6}$ .

<sup>2</sup> J. E. Andrew, P. W. Ball, and A. B. Blake, *Chem. Comm.*, 1969, 143.

<sup>3</sup> Landolt-Bornstein, New Series, Group II, Volume 2, 'Magnetic Properties of Co-ordination and Organometallic Transition Metal Compounds,' by E. König, Springer, Berlin, 1966.

<sup>4</sup> J. E. Andrew and A. B. Blake, J. Chem. Soc. (A), 1969, 1408. <sup>5</sup> A. B. Blake and L. R. Fraser, unpublished work.

<sup>\*</sup> Of 23 magnetically dilute, octahedral, cobalt(II) compounds whose  $\theta$  values in the range 80—300 K are listed by König,<sup>3</sup> 22 have  $\theta$  between -11 and -36 K, with a mean of -23 K. (The curious exception is CoSeO<sub>4</sub>,6H<sub>2</sub>O, -64 K.) The presumably mononuclear complex Co(Me<sub>2</sub>dppn)<sub>2</sub>(ClO<sub>4</sub>)<sub>2</sub>,3H<sub>2</sub>O has  $\theta = -26$  K.

<sup>&</sup>lt;sup>†</sup> We are aware that similarity of the i.r. spectra of such complex compounds is not an infallible indication of similarity of their structures, but we feel that it is very unlikely that such close resemblances as are observed here could arise from grossly different co-ordination of the ligands in cobalt and nickel compounds with analogous formulae. It is worth mentioning that the i.r. spectrum of  $\text{Co}_2(\text{dppn})_2(\text{NO}_3)_4$ ,  $2\text{H}_2\text{O}$  is quite different from (and indeed much simpler than) that of  $\text{Mn}(\text{dppn})_2(\text{NO}_3)_2$ , which is known from a crystal-structure analysis to be mononuclear, with bidentate  $\text{NO}_3^-$  and only one half of each dppn molecule co-ordinated.<sup>§</sup>

are given in Table 1, together with values of the Weiss constant  $\theta$  estimated graphically as before.<sup>1</sup>

In interpreting the magnetic data to obtain estimates of the exchange parameters, we are confronted with a much more difficult problem than in the nickel case. In both cases the metal ions are in approximately octahedral coordination with a tetragonal distortion; but the  ${}^{3}A_{2n}$ ground term of Ni<sup>2+</sup> is split (to first order) only by exchange, whereas the  ${}^{4}T_{1g}$  ground term of Co<sup>2+</sup> is split by both spinorbit coupling and the low-symmetry field even in the absence of exchange, and the contribution of these effects to the temperature dependence of the magnetic moment must be allowed for before a comparison of exchange in the  $\mathbf{k}T$ , with the orbital singlet levels lowest; the spin states are assumed to be coupled by an isotropic Heisenberg exchange interaction. The second model starts with the cubic-field  ${}^{4}T_{1q}$  terms, and assumes them to be perturbed by spin-orbit coupling and an isotropic exchange interaction; a small axial distortion can also be included if necessary. Finally, in the third model the orbital dependence of the individual exchange integrals is recognised, leading to an anisotropic (but, in a centrosymmetric complex, still symmetric) exchange operator.

(1) Ions in orbital singlet states. As in the nickel case,<sup>1</sup> the observed decrease in the magnetic moment with decreasing temperature is ascribed solely to an exchange term

Magnetic susceptib	oilities (cm³ mol <sup>-1</sup> ,	after correction i	for t.i.p. and dia	magnetism as desc	ribed in the text)
$T/{ m K}$	10 <sup>5</sup> x	$T/{ m K}$	- 10 <sup>5</sup> y	T/K	10 <sup>5</sup> Y
Co <sub>2</sub> (dhph)	2Cl1,5H2O	Co <sub>s</sub> (dhph).	Br, 6H,O	Co <sub>s</sub> (dppn) <sub>s</sub> (	NO.).2H.O
296.5	1701	381.6	1327	408.2	1235
273.0	1818	364.0	1383	395.1	1281
241.9	2006	$345 \cdot 3$	1456	376.0	1336
213.8	2193	322.7	1549	$362 \cdot 1$	1370
187.2	2434	310.2	1598	345.4	1437
160.0	2755	$295 \cdot 5$	1668	327.8	1505
137.8	3088	294.5	1679	313.1	1582
$102 \cdot 8$	3912	271.3	1795	297.4	1651
92.4	4163	238.8	1987	296.0	1663
80.5	4518	215.0	2146	270.8	1768
		190.6	2369	244.4	1941
		156.5	2767	216.5	2124
		136.2	3065	189.2	2348
		116.2	3471	167.4	2619
		98.0	3903	143.6	2963
Co <sub>2</sub> (dppn) <sub>2</sub>	$(ClO_4)_4, 6H_2O$	77.2	<b>4406</b>	122.0	3367
369.6	1239			98.4	3986
360.2	1280	Co <sub>2</sub> (dppn)(	$SO_4$ ), 5H,O	79.2	4510
345.5	1322	361.8	1455		
328.0	1388	347.1	1519	Co <sub>2</sub> (Me <sub>2</sub> dppn)	(NO <sub>3</sub> ) <sub>4</sub> ,2CH <sub>3</sub> OH
313.8	1446	330.0	1570	296.8	1666
$299 \cdot 2$	1498	320.6	1617	275.6	1778
292.8	1534	300.0	1719	251.8	1929
$266 \cdot 1$	1654	$275 \cdot 3$	1884	236.4	2050
238.2	1825	$242 \cdot 2$	2090	188.2	2413
$212 \cdot 2$	2018	210.0	2355	157.0	2806
183.8	2286	$178 \cdot 8$	2679	138.2	3010
149.3	2729	144.0	3255	116.1	3495
120.4	3302	112.8	4002	97.4	3933
95.3	3910	95.2	4607	86.8	4211
77.9	4554	86.4	4952		

TABLE 2

nickel and cobalt cases is possible. A further complication which arises in the cobalt case is that the three ' $t_{2g}$ ' orbitals contain only one unpaired spin, and since in the symmetry of a binuclear complex these three orbitals cannot be equivalent, the strength of the exchange will depend on which one is occupied on each ion. This not only contributes substantially to the magnetic anisotropy of the complex, but also means that more than one exchange parameter is required to describe its magnetic properties.

Clearly, the magnetic behaviour of a binuclear cobalt(II) complex can be interpreted at several levels of approximation, and it would be useful to know how the results of these compare. We shall examine three models. The simplest ignores the effects of orbital degeneracy, and is equivalent to assuming that there is a distortion from octahedral symmetry large enough to split the  ${}^{4}T_{1g}$  terms by an energy  $\gg$ 

\* Energies involved in thermal equilibria are given here in units of  $\mathbf{k}$ K, where  $\mathbf{k}$  is Boltzmann's contant, equal to 8.310 J K<sup>-1</sup>.

<sup>6</sup> J. Kanamori, Progr. Theoret. Phys. (Japan), 1957, 17, 177.

 $-2 \mathscr{J} S_A$ .  $S_B$  in the Hamiltonian. The susceptibility of the pair of ions is given by equation (1), where  $x = \mathscr{J}/\mathbf{k}T$ . This

$$\chi_{\rm M} = \frac{2N\mu_{\rm B}^2 g^2}{\mathbf{k}T} \frac{{\rm e}^{-10x} + 5{\rm e}^{-6x} + 14}{{\rm e}^{-12x} + 3{\rm e}^{-10x} + 5{\rm e}^{-6x} + 7} \qquad (1)$$

expression was fitted to the experimental data for compounds (I)—(VI) by the method of least squares, giving the values for g and  $\mathcal{J}$  shown in Table 3, column (1).\* Values of the function  $R = [\Sigma(\chi_{obs} - \chi_{calc})^2 / \Sigma \chi_{obs}^2]^{\frac{1}{2}}$  are also listed, as a measure of the discrepancy between the experimental and calculated curves.

(2) Ions in  ${}^{4}T_{1q}$  states with isotropic exchange. Several authors have recently given attention to this type of problem.<sup>6-9</sup> Lines has discussed the effect of spin-orbit

<sup>7</sup> T. Nakamura and N. Uryû, J. Phys. Soc. Japan, 1956, **11**, 760; A. Abragam and B. Bleaney, 'Electron Paramagnetic Resonance of Transition Metal Ions,' Oxford, Clarendon, 1970.

 <sup>8</sup> M. E. Lines, J. Chem. Phys., 1971, 55, 2977.
 <sup>9</sup> (a) O. Kahn, J. Chim. Phys., 1973, 70, 392; (b) J. C. Bernier and O. Kahn, Chem. Phys. Letters, 1973, 19, 414.

coupling on the magnetic properties of two to four exchangecoupled Co<sup>2+</sup> ions, using an ingenious hybrid approximation which combines an exact spin Hamiltonian for the lowest Kramers doublets with a molecular-field representation of exchange in the upper levels of the ground multiplet.8 For a cluster of only two  $Co^{2+}$  ions, however, a perturbation treatment of the complete  ${}^{4}T_{1g} \times \, {}^{4}T_{1g}$  manifold is practicable, and should be more accurate. We use a computer program which does this for the general  ${}^{2S+1}T$  case  $(S = \frac{1}{2}, \frac{1}{2})$ 1,  $\frac{3}{2}$ , or 2).

The effect of spin-orbit coupling can be represented by the operator  $\mathscr{H}_{LS} = -\lambda (\mathbf{L}_A \cdot \mathbf{S}_A + \mathbf{L}_B \cdot \mathbf{S}_B)$  acting on product functions derived from pseudo- $^{4}P$  states of the ions A and  $B^{10}$  This operator must, however, be modified to take account of mixing of metal and ligand wavefunctions as a result of covalency (which causes the true matrix elements of  $\mathbf{L}$  to be smaller than those obtained using pure d wavefunctions), and mixing of the strong-field determinantal wavefunctions as a result of interelectronic repulsion. The first can be allowed for by introducing an orbital reduction factor k as an empirical parameter,<sup>11</sup> and the second by multiplying  $\mathbf{L}$  by another numerical factor A, whose value

number of parameters down, we have endeavoured to estimate D for each compound independently in the following way. In a spectroscopic study of a series of isomorphous tetragonal complexes of the type  $ML_4X_2$  (L = 3methylpyrazole, X = Cl, Br, I, or NO<sub>3</sub>), Reedijk found that the ratio  $D_{\rm Co}/D_{\rm Ni}$  varied between 0.31 and 0.37, with a



FIGURE 2 Co-ordinate system to which the orbitals of the metal ions are referred

mean of  $0.35.^{14}$  We therefore take D positive and equal to 0.35 times the splitting of the  ${}^{3}T_{1g}(F)$  term of the analogous nickel complex, determined from its reflectance spectrum<sup>1</sup>

TABLE 3

Some d	lerived	magnetic	parameters
		· · · · · · · ·	

Compound number	(1)			(2)				
	g	- <i>J</i> / <b>k</b> K	$10^2 R$	k	D/ <b>k</b> K	- <b>J/k</b> K	$10^2 R$	⁴∮Ni/⁰∮Co∥
(I)	2.39	10.6	1.12	1.03	2100	7.7	1.11	1.9
$(\hat{1}\hat{1})$	2.41	11.9	0.94	1.04	2100	9.0	0.86	1.7
(Ì11)	2.38	10.6	1.29	0.79	1100	7.6	1.21	2.0
(1 <b>V</b> )	2.26	$8 \cdot 2$	0.85	0.50	500	6.0	1.51	1.6
(V)	2.41	7.5	0.82	0.65	250	$3 \cdot 0$	1.79	2.6
(¥1)	$2 \cdot 41$	11.6	1.16	0.65	250	8.5	1.66	1.6

"  $\mathscr{J}_{Co}$  from Section (2) of this Table,  $\mathscr{J}_{Ni}$  from ref. 1.

lies between 1.0 and 1.5 and can be estimated independently from spectroscopic data.<sup>12</sup> From the reflectance spectra of compounds (3)–(5) we calculate  ${}^{13}$   $A = 1.39 \pm 0.02$  (see Appendix I). The free-ion spin-orbit coupling constant  $\lambda$ is thus replaced by  $kA\lambda$ , and the magnetic moment operator becomes  $-kA\mathbf{L} + 2\mathbf{S}$ . Note that the values of k are not necessarily the same in these two expressions, but we assume them to be equal; we also ignore the possibility of anisotropy in k, A, or  $\lambda$ . For <sup>4</sup>F of the free Co<sup>2+</sup> ion,  $\lambda/\mathbf{k} = -256$  K.<sup>10</sup>

According to the structures assigned to the nickel complexes,<sup>1</sup> the donor-atom set in each compound is either trans- $N_4O_2$  or cis- $N_2O_4$ , and the ligand field may therefore deviate considerably from cubic, though retaining roughly tetragonal symmetry about the z-axis in the co-ordinate system of Figure 2. An axial distortion in the z-direction can be represented approximately (provided we remain within the  ${}^{4}T_{1g} \times {}^{4}T_{1g}$  subspace) by the operator-equivalent  $\mathscr{H}_{LL} = D(L_{zA}^{2} + L_{zB}^{2} - 4/3)$ , where D is the splitting of the individual ion T-term by the axial field, and is positive if the orbital singlet lies lowest. Since we need to keep the

\* It must be emphasised that equally good fits can often be obtained with considerably different assumed values of D. We find, however, that with k, D, and  $\mathcal{J}$  as parameters, any constraint on D is accommodated mainly in the value of k, and vice versa, the best-fit value of  $\mathcal J$  being comparatively insensitive to other parameters. It is clear that too much significance should not be attached to the values of k listed in Table 3.

(assuming in each case that the ligand field is weakest along the tetragonal axis). The estimated values of D are shown in Table 3. [For compounds (V) and (VI) the splitting was not resolved in the spectrum of the nickel analogue, so that in these two cases the small value assigned to D is simply a reasonable guess.]

The complete perturbation is thus represented by the Hamiltonian (2), and the quantities to be determined by curve fitting to the experimental data are k and  $\mathcal{J}$ . The

$$\mathcal{H}_{\mathbf{I}} = D(L_{z\mathbf{A}^{2}} + L_{z\mathbf{B}^{2}} - 4/3) - hA\lambda(\mathbf{L}_{\mathbf{A}} \cdot \mathbf{S}_{\mathbf{A}} + \mathbf{L}_{\mathbf{B}} \cdot \mathbf{S}_{\mathbf{B}}) - 2\mathscr{J}\mathbf{S}_{\mathbf{A}} \cdot \mathbf{S}_{\mathbf{B}}$$
$$= \mathscr{H}_{\mathrm{LL}} + \mathscr{H}_{\mathrm{LS}} + \mathscr{H}_{\mathrm{SS}}$$
(2)

method of calculating the susceptibility is described in Appendix III. Since the dependence of  $\chi$  on k and  $\mathcal{J}$  cannot be expressed analytically, the usual method of least squares is not applicable, and a trial-and-error fit is necessary. k and  $\mathcal{J}$  were therefore varied by small amounts, and contours of the mean squared deviation were plotted until a minimum was located for each compound. The results are shown in Table 3, column (2).\*

10 J. S. Griffith, 'The Theory of Transition-Metal Ions, Cambridge, 1961.

 M. Gerloch and J. R. Miller, Progr. Inorg. Chem., 1968, 10, 1.
 B. N. Figgis, 'Introduction to Ligand Fields,' Interscience, 1966, p. 270. <sup>13</sup> B. N. Figgis, J. Chem. Soc. (A), 1968, 2086.

14 J. Reedijk, Rec. Trav. chim., 1970, 89, 993.

(3) Ions in  ${}^{4}T_{1g}$  states with anisotropic exchange.\* The spin-dependent part of the energy of interaction between two many-electron ions A and B was shown by Heisenberg and Dirac to have the form (3), where the summation is over the electrons i of ion A and j of ion B. When A and B

$$\mathscr{H}_{\rm SS} = -2\sum_{i,j} J(i,j) \mathbf{s}_{\rm A}(i) \cdot \mathbf{s}_{\rm B}(j)$$
(3)

are in orbitally non-degenerate states, (3) reduces to the usual exchange operator  $-2 \mathscr{J} S_A$ .  $S_B$ , where **S** is the total spin of the ion and  $\mathcal J$  is a constant. But, as Van Vleck pointed out some years ago,<sup>15</sup> when the ions have orbital degeneracy, the J(i,j) will be matrices connecting the orbital wavefunctions of the electrons, and the transformation of (3) to the form  $-2 \mathscr{J} \mathbf{S}_{A}$ .  $\mathbf{S}_{B}$  will then in general result in *J* being anisotropic and dependent on the orbital states of the ions. A detailed and general discussion of this problem has been given by Levy,<sup>16</sup> but because of the large number of parameters in the full theory, the possibility of anisotropic exchange is often ignored.8,9 Nevertheless, it is useful to return to (3) if we wish to understand in a qualitative way how the exchange interaction differs in the nickel and cobalt complexes. We shall take up later the question of whether the orbital dependence of  $\mathcal{J}$  can in fact be estimated from powder data.

The transformation of (3) to a basis of atomic states is carried out by elementary methods in Appendix II for the  $d^{8}$  and  $d^{7}$  cases. For two Ni<sup>2+</sup> ions we find that (3) is equivalent to the spin operator (4) acting on the states  $|\hat{M}_{\rm SA}\rangle|M_{\rm SB}\rangle$  of the direct product  ${}^{3}A_{2g} \times {}^{3}A_{2g}$ , where  $\theta$  and  $\varepsilon$  are real orbitals derived from the  $d_{z^3}$  and  $d_{x^2-y^3}$  orbitals,

$$\begin{aligned} \mathscr{H}_{\mathrm{SS}} &= -2[J(\theta\theta) + 2J(\theta\varepsilon) + J(\varepsilon\varepsilon)]\mathbf{S}_{\mathrm{A}} \cdot \mathbf{S}_{\mathrm{B}}/4S^2 \\ &= -2\mathscr{J}\mathbf{S}_{\mathrm{A}} \cdot \mathbf{S}_{\mathrm{B}} \end{aligned}$$
(4)

respectively, of a single metal ion in the co-ordinate system of Figure 2, and the exchange integrals  $J(\phi_A \phi_B)$  are the diagonal matrix elements  $\langle \phi_{\rm A} \phi_{\rm B} | J | \phi_{\rm A} \phi_{\rm B} \rangle$  of the orbital part of the exchange interaction (3). If is thus identified with the 'average 'exchange integral  $\tilde{J}(ee) = [J(\theta\theta) + 2J(\theta\varepsilon) +$  $J(\varepsilon\varepsilon)]/4$ ; as far as we are aware, there is no empirical way in which the orbital integrals  $J(\phi_A\phi_B)$  can be evaluated separately.

In the cobalt case, the ground term  ${}^4T_{1g}$  is a mixture of  $t_{2g}{}^4e_g{}^3$  and  $t_{2g}{}^5e_g{}^2$  functions. Neglecting the  $t_{2g}{}^4e_g{}^3$  contribution (which, according to spectroscopic evidence, is only ca. 6%), we find in Appendix II that (3) is equivalent to the

\* The authors are indebted to a referee of the original version of this paper for drawing their attention to the work of Levy <sup>16</sup> on anisotropic exchange.

† Actually, both diagonal and off-diagonal integrals  $\langle \phi'_{\mathbf{A}} \phi'_{\mathbf{B}} | J |$ - $\phi_A \phi_B$  appear in the expression for the exchange energy, and there is no theoretical justification for assuming that the off-diagonal terms are negligible. However, of the 625 integrals (for l = 2), Levy <sup>16</sup> has shown that 145 are distinct even when the two ions are related by a centre of inversion, and although the number will be reduced further in  $D_{2h}$  symmetry, it is clear that some very drastic assumptions are necessary before we have a model that can be related to experiment. Copland and Levy,<sup>17</sup> in a discussion of direct exchange between Co<sup>2+</sup> ions, chose for practical reasons to ignore the off-diagonal integrals, and Barraclough and Gregson  $^{18}$  have done the same in their treatment of  $\rm Ti_2Cl_9^{3-}$ . We here assume that the matrix of J is diagonal in a basis of real d-orbitals (the results are essentially the same if complex orbitals are used), with the aim of interpreting our results in terms of those diagonal elements that seem likely to be most important, but we must emphasise that this is to be regarded at present more as a simplifying convention than a justifiable approximation. Note, however, that  $\bar{J}(ee)$  is necessarily diagonal.

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spin operator (5) acting on the states  $|M_{\rm LA}M_{\rm SA}\rangle|M_{\rm LB}M_{\rm SB}\rangle$ of the direct product  ${}^{4}T_{1g} \times {}^{4}T_{1g}$ , where L (= 1) is the

$$\mathscr{H}_{\rm SS} = -2 \mathscr{J}(M_{\rm LA}, M_{\rm LB}) \mathbf{S}_{\rm A} \cdot \mathbf{S}_{\rm B}$$
(5)

'effective' orbital angular momentum quantum number. The three distinct exchange parameters  $\mathscr{J}(M_{\mathrm{LA}},M_{\mathrm{LB}})$  are given by (6), in which  $\xi$ ,  $\eta$ , and  $\zeta$  are the  $t_{2g}$  orbitals,  $\tilde{J}(ee)$ 

$$\begin{aligned} \mathcal{J}(0,0) &= [4\bar{f}(ee) + 4J(e\zeta) + J(\zeta\zeta)]/9 \\ \mathcal{J}(1,0) &= [4\bar{f}(ee) + 2\bar{f}(e\zeta) + 2\bar{f}(e\zeta) + J(\zeta\zeta)]/9 \\ \mathcal{J}(1,1) &= [4\bar{f}(ee) + 4\bar{f}(e\zeta) + \frac{1}{2}J(\zeta\zeta) + \frac{1}{2}J(\zeta\zeta)]/9 \end{aligned}$$
(6)

is defined as before, and  $\overline{J}(e\phi) \equiv \frac{1}{2}[J(\theta\phi) + J(\epsilon\phi)](\phi = \zeta \text{ or }$  $\xi$ ). † Note that (6) contains eleven distinct orbital exchange integrals [cf. equation (15), Appendix II], and again there is no way in which they can be evaluated individually, even if the three  $\mathscr{J}$ 's can be experimentally determined.

If we assume that  $\overline{J}(ee)$  will not be greatly different in the nickel and cobalt cases, the main differences between  $\mathscr{J}$  for nickel and the  $\mathcal{J}(M_{\text{LA}}, M_{\text{LB}})$  for cobalt, apart from the anisotropy of the latter, are the factor 4/9 arising from the different total spins, and the addition of terms of the type  $J(\phi_{\rm A}, \phi_{\rm B}'), \phi' \neq \phi$ , in the cobalt case. Such terms are likely to be positive, *i.e.* ferromagnetic, in nature,<sup>19</sup> and their net effect might be expected to outweigh that of the additional terms  $J(\zeta\zeta)$  and  $J(\xi\xi)$  (whose signs are probably negative but which are statistically less important), making all three  $\mathscr{J}$ 's less negative than in the absence of  $t_{2g}$  contributions. If the  $t_{2g}$  orbitals made no contribution, we should, of course, have a single isotropic exchange parameter  $\mathscr{J} =$  $(4/9)\tilde{J}(ee).$ 

To determine whether average susceptibility measurements over the range 80-400 K can reveal anything about the orbital dependence of exchange in complexes of the type we are concerned with, we selected the data for compound (III). With  $D/\mathbf{k}$  equal to 1100 K and k equal to the value estimated by using the isotropic model, the values of  $\mathcal{J}(0,0)$ ,  $\mathcal{J}(1,0)$ , and  $\mathcal{J}(1,1)$  were systematically varied over the range -14 to +2 K in an effort to find a minimum in the value of the discrepancy index R. Not unexpectedly, the fit, though quite sensitive to the average, was found to be rather insensitive to the individual values of the  $\mathcal{J}$ 's; low values of R were confined not to a point in the threedimensional  $\mathcal{J}$ -space, but to a plane defined approximately by  $[\mathcal{J}(0,0) + \mathcal{J}(1,0) + 1.5 \mathcal{J}(1,1)]/\mathbf{k} = -26.4 \text{ K}$ . The best fit had  $\mathcal{J}(0,0)/\mathbf{k}$  ca. -12,  $\mathcal{J}(1,0)/\mathbf{k}$  ca. -10, and  $\mathcal{J}(1,1)/\mathbf{k}$  ca. -3 K, with R = 0.0117, but almost equally good fits could be obtained with considerably different values, the only consistent observation being an apparent preference for  $\mathcal{J}(1,1)$  to be less negative than  $\mathcal{J}(0,0)$  or  $\mathcal{J}(1,0)$ . The individual estimation of the three exchange parameters in compounds of this type must evidently await single-crystal magnetic measurements.

#### DISCUSSION

We have used three models to interpret the magnetic properties of the binuclear cobalt(II) complexes. The first, in which the temperature dependence of the mag-

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- 14, 189.
  16 P. M. Levy, *Phys. Rev.*, 1969, 177, 509.
  17 G. M. Copland and P. M. Levy, *Phys. Rev. B.*, 1970, 1, 3043.
  18 C. G. Barraclough and A. K. Gregson, *J.C.S. Faraday II*,
- 1972, 177. <sup>19</sup> P. W. Anderson, in 'Magnetism,' eds. G. T. Rado and H. Suhl, vol. I, Academic Press, 1963, ch. 2.

netic moment is assumed to be due entirely to exchange, is obviously inappropriate: the effective magnetic moment of an octahedral cobalt(II) complex normally decreases over the temperature range 400-80 K, as a result of the splitting of the  ${}^{4}T_{1g}$  ground term by spinorbit coupling and distortion,3 and in neglecting these effects we must inevitably overestimate the importance of any antiferromagnetic exchange interaction. We were interested to know the extent of this overestimation, which can be seen by comparing the results obtained by using models (1) and (2) in Table 3. (Note that the spin-only model gives quite a good fit to the data, probably because the orbital effects are partly accommodated in the false parameter g.)

The second model takes the spin-orbit coupling and distortion into account, and leads to values of the effective isotropic exchange parameters that should be much more realistic. Unfortunately, in doing so it introduces two new parameters, both of rather uncertain value, so that the accuracy of the results will not be high. We assume a possible error of  $\pm 10\%$  in  $\mathcal{J}$ , arising from these uncertainties.

In the third model, we have attempted to include the orbital dependence of the exchange resulting from the inherently low symmetry of a binuclear complex, but we find that our powder data are barely capable of distinguishing this from an isotropic interaction. We therefore use the results of model (2) in comparing the strengths of exchange in the binuclear cobalt and nickel complexes discussed in this paper and Part I.<sup>1</sup>

From Table 3 it may be seen that the ratio  $(4/9) \mathscr{J}_{Ni'} \mathscr{J}_{Co}$ for the six pairs of compounds lies between 1.6 and 2.6, with five of the values being between 1.6 and 2.0. Because of the experimental uncertainties in the  $\mathscr{J}$ 's it is doubtful that the deviations from the mean of 1.9 are of much significance, and we conclude that the trends in  $\mathcal J$ from one complex to another for a given metal are largely the result of structural variations that affect the interaction rather similarly for cobalt and nickel. As was noted previously, we are not able to explain these variations in  $\mathcal{J}$ .

The value of  $1.9 \pm 0.3$  for  $(4/9) \mathscr{J}_{\rm Ni}/\mathscr{J}_{\rm Co}$  may be compared with the value of ca. 1.3 found in the oxides.\* If the  $t_{2g}$  electrons of Co<sup>2+</sup> made no net contribution to the exchange, this ratio would be equal to  $\bar{J}(ee)_{\rm Ni}/\bar{J}(ee)_{\rm Co}$ . Nesbet has calculated a theoretical value of ca. 1.6 for the latter ratio in the oxides,<sup>22</sup> and although the approximations involved in this calculation have been questioned by Anderson,<sup>23</sup> both authors' theories seem to indicate that this ratio should not be strongly dependent on the

\* According to the simple molecular-field theory, the Néel \* According to the simple molecular-held theory, the Neel temperature for the NaCl structure with spin ordering of the second kind is given by  $T_{\rm N} = -4 \int S(S+1)/\mathbf{k}$ , where  $\int$  is the exchange parameter for next-nearest neighbours.<sup>20</sup> For CoO and NiO,  $T_{\rm N} = 292$  and 523 K respectively.<sup>21</sup> and hence (4/9)  $\int_{\rm NI}/\int_{\rm Co} = 1.49$ . However, allowance for the orbital magnetism of CoO reduces this to  $1.34.^6$  Note that above the Néel tempera-ture of CoO reduces the coupling makes in decrease with in ture of CoO, spin-orbit coupling makes  $\mu_{eff}$  decrease with increasing temperature, and hence the correction results in a higher value of  $|\mathcal{J}|$ , the reverse of the situation we have been considering.

nature of the ligands. (The  $e_g$  contribution to  $\mathcal{J}$  by Anderson's mechanism is  $2b^2/U$ , where b is roughly equal to one third of the ligand-field splitting and U is the energy required to transfer an electron from one cation to the other.<sup>24</sup>) It thus seems that the  $t_{2g}$  contribution to the exchange in our compounds is considerably more positive than in CoO, and although we cannot be certain about the actual sign of the contribution in either case, it seems likely that it is in fact ferromagnetic in the binuclear complexes.

The major ferromagnetic terms in equation (6) are probably  $4\bar{J}(e\zeta)$  and  $4\bar{J}(e\xi)$ . There is a slight indication that  $\mathcal{J}(1,1)$  is less negative than  $\mathcal{J}(0,0)$  or  $\mathcal{J}(1,0)$ , which would suggest that the  $d\pi$  orbitals ( $\xi$  and  $\eta$ ) provide the strongest ferromagnetic contributions, perhaps through the azine  $\pi$ -system. In the absence of magnetic anisotropy information, however, further speculation is unfruitful.

## EXPERIMENTAL

Magnetic susceptibilities were measured on finely powdered samples by the Gouy method as described previously.<sup>1</sup> Diffuse reflectance spectra were recorded between 5000 and 30 000 cm<sup>-1</sup> on a Beckman DK2 spectrophotometer.

Preparation of Complexes .- The methods were similar to those used for the nickel complexes,<sup>1</sup> except for the compounds with dihydrazinophthalazine, which are easily oxidised and were prepared using air-free solutions in an atmosphere of oxygen-free nitrogen and subsequently manipulated in a nitrogen-filled glove box. Microanalysis for C, H, N, Cl, and Br was performed commercially, with the results: (I), Found: C, 26.3; H, 4.1; N, 23.2; Cl, 19.2.  $C_{16}H_{10}Cl_4Co_2N_{12}O_{15} \ requires \ C, \ 26\cdot4\,; \ \ H, \ 4\cdot1\,; \ \ N, \ 23\cdot0\,; \ \ Cl,$ 19.4. (II), Found: C, 20.9; H, 3.5; N, 18.5; Br, 34.5.  $C_{16}H_{32}Br_4Co_2N_{12}O_{16}$  requires C, 20.8; H, 3.5; N, 18.2; Br, 34.6. (III), Found: C, 38.7; H, 2.7; N, 19.3. C<sub>28</sub>H<sub>24</sub>- $Co_2N_{12}O_{14}$  requires C, 38.7; H, 2.7; N, 19.3. (IV), Found: C, 30.8; H, 3.0; N, 10.2; Cl, 13.2. C<sub>28</sub>H<sub>52</sub>Cl<sub>4</sub>Co<sub>2</sub>-N<sub>8</sub>O<sub>22</sub> requires C, 30.8; H, 2.9; N, 10.2; Cl, 13.0. (V), Found: C, 26.6; H, 3.2; N, 8.6. C<sub>14</sub>H<sub>20</sub>Co<sub>2</sub>N<sub>4</sub>O<sub>13</sub>S<sub>2</sub> requires C, 26.6; H, 3.1; N, 8.8. (VI), Found: C, 31.3; H, 3.1; N, 16.1. C<sub>18</sub>H<sub>22</sub>Co<sub>2</sub>N<sub>8</sub>O<sub>14</sub> requires C, 31.2; H, 3.2; N, 16.2%.

## APPENDIX I

Configuration Mixing and T.I.P. for Co<sup>2+</sup> in an Octahedral Field.—The  ${}^{4}T_{1}$  wavefunctions of a  $d^{3}$  or  $d^{7}$  ion in cubic symmetry have the form (7), where  $a^2 + b^2 = 1$ , a/b =(10Dq + 9B + R)/12B, and  $R^2 = (10Dq)^2 + 180 BDq +$  $(15B)^2$ , the separation of the  ${}^4F$  and  ${}^4P$  terms in the limit

$$\begin{aligned} |T_1^+\rangle &= a |t_2^2 e T_1\rangle + b |t_2 e^2 T_1\rangle \\ |T_1^-\rangle &= b |t_2^2 e T_1\rangle - a |t_2 e^2 T_1\rangle \end{aligned}$$
(7)

Dq = 0 being 15B. The values of Dq and B for an octahedral cobalt(11) complex may be calculated from spectroscopic data by equations (8), where  $E_1$ ,  $E_2$ , and  $E_3$  are the

- <sup>23</sup> Ref. 19, p. 48.
- <sup>24</sup> P. W. Anderson, Phys. Rev., 1959, 115, 2.

<sup>&</sup>lt;sup>20</sup> J. S. Smart, 'Effective Field Theories of Magnetism,' <sup>21</sup> M. Foëx, Compt. rend., 1948, 227, 193.
 <sup>22</sup> R. K. Nesbet, Phys. Rev., 1960, 119, 658.

2)

energies of the transitions from  ${}^{4}T_{1}(F)$  to  ${}^{4}T_{2}$ ,  ${}^{4}A_{2}$ , and  ${}^{4}T_{1}(P)$ , respectively. For compounds (III)—(V) discussed

$$15B = \frac{1}{2}[E_3 - 2E_1 + (E_3^2 + E_1E_3 - E_1^2)^{\frac{1}{2}}]$$
  
$$10Dq = 2E_1 - E_3 + 15B = E_2 - E_1$$
(8)

in this paper,  $E_1 = 9000-11\ 000\ \mathrm{cm^{-1}}$  and  $E_2$  and  $E_3$  are both in the range 18 000-24 000 cm<sup>-1</sup>, and we can obtain reasonable fits to the observed spectra with  $10Dq = 10\ 000$ -12 000 and  $15B = 12\ 000-14\ 000\ \mathrm{cm^{-1}}$ . By analogy with the nickel complexes <sup>1</sup> we expect similar values in compounds (I), (I1), and (VI). Hence  $a = 0.97 \pm 0.01$  and  $b = 0.25 \pm 0.01$ , *i.e.* the ground configuration is about  $94\%\ t_2^{1}e^2$ .

The factor A which appears in equation (2) is defined so that the matrix of **L** within the ground  $T_1$  term is -A times the matrix within a P term, and it follows that A is equal to  $\frac{1}{2}(2a^2 + 4ab - b^2)$ . Hence in our compounds  $A = 1.39 \pm 0.02$ .

The second-order perturbation of the ground term by the magnetic field contributes a temperature-independent paramagnetic susceptibility given by (9), where the sum is over all m states  $\psi_{oi}$  of the ground term and all excited states  $\psi_{nj}$ 

$$\chi_{\rm tip} = \frac{2N\mu_{\rm B}^2}{3m} \frac{|\langle \psi_{nj} | \mathbf{L} | \psi_{oi} \rangle|^2}{E_n - E_o} \tag{9}$$

such that  $E_n - E_o \gg \mathbf{k}T$ . Since  $\mathbf{L}$  transforms as  $T_1$  in cubic symmetry, both the  ${}^{4}T_2(F)$  and  ${}^{4}T_1(P)$  terms will contribute to the t.i.p. of a  ${}^{4}T_1(F)$  ground term. The contributions in the  $d^3$  or  $d^7$  case are found to be  $N\mu_{\rm B}^2(2k'a + kb)^2/E_1$  from  ${}^{4}T_2$  and  $N\mu_{\rm B}^2(2k'b^2 + 3kab - 2k'a^2)/3E_3$  from  ${}^{4}T_1(P)$ , where k and k' are orbital reduction factors for  $\langle t_2|\mathbf{l}|t_2 \rangle$  and  $\langle t_2|\mathbf{l}|e \rangle$ , respectively. If we assume k' = k, we have:

$$\chi_{\rm tip} = N\mu_{\rm B}^2 k^2 (2+2A) [1/E_1 + (3-2A)/3E_3]$$

The result for  $d^2$  or  $d^8$  is the same. For our compounds only the  ${}^4T_2$  contribution is significant, and  $\chi_{\rm tip} = 0.000125k^2$ . The average value  $k^2 = 0.8$  was used in making the t.i.p. corrections.

## APPENDIX II

Orbital Exchange Parameters for  $d^8-d^8$  and  $d^7-d^7$  Interactions.—The  ${}^3A_2$  ground term of  $d^8$  belongs to the strongfield 'hole' configuration  $e_g^2$ , and we may therefore write the exchange matrix in the nickel case as (10), where  $\mathscr{H}_{SS}$ is defined by (3) and the abbreviation  $|\phi\rangle_{A,B}$  is used for the

$$\langle e^{2} {}^{3}A_{2}a_{2}M_{S}' |_{\mathbf{A},\mathbf{B}} \mathscr{H}_{SS} | e^{2} {}^{3}A_{2}a_{2}M_{S} \rangle_{\mathbf{A},\mathbf{B}}$$
(10)

product  $|\phi\rangle_{\rm A}|\phi\rangle_{\rm B}$ . The orbital and spin functions are separable, and the spin part of (3) can be shown to be equivalent to the operator  $-2\mathbf{S}_{\rm A} \cdot \mathbf{S}_{\rm B}/4S^2$  acting on the components of  ${}^{3}A_{2} \times {}^{3}A_{2}$ . In terms of the real  $e_{g}$ -orbitals, the orbital function is  $|e^{2}A_{2}a_{2}\rangle = |\theta\varepsilon\rangle$ , and hence (3) reduces to (4).

The  ${}^{4}T_{1}(F)$  term of  $d^{7}$  is a mixture of  $t_{2}{}^{2}e^{1}$  and  $t_{2}{}^{1}e^{2}$  functions, with the latter contributing *ca.* 94% of the electron density in the present case, as noted above. In view of the many other sources of uncertainty in the estimation of the exchange parameters from the data, we make the approximation of neglecting the  $t_{2}{}^{2}e^{1}$  component, and take the ground-state wavefunctions as  $|e^{2}t_{2}{}^{4}T_{1}M_{\rm L}M_{\rm S}\rangle$ . By partially expanding these determinantal functions, bearing in mind that the only term of  $e^{2}$  that is a parent of  ${}^{4}T_{1}$  is  ${}^{3}A_{2}$ , we

obtain (11), where  $\mu$  represents one of the real  $t_2$  orbitals  $\xi$ ,  $\eta$ , and  $\zeta$ , and the two-electron functions are still antisymmetrised. It can again be shown that the spin part of (3)

$$\begin{aligned} |e^{2}t_{2} \ ^{4}T_{1}M_{L}M_{8}\rangle &= \sqrt{\frac{1}{3}}\sum_{\bar{M}S}\sum_{mS}\sum_{\mu} \\ \{|e^{2}(12)^{3}A_{2}a_{2}\bar{M}_{8}\rangle|t_{2}(3)\mu m_{8}\rangle \\ &+ |e^{2}(23)^{3}A_{2}a_{2}\bar{M}_{8}\rangle|t_{2}(1)\mu m_{8}\rangle \\ &+ |e^{2}(31)^{3}A_{2}a_{2}\bar{M}_{8}\rangle|t_{2}(2)\mu m_{8}\rangle\} \\ &\langle a_{2}\mu|T_{1}M_{L}\rangle\langle \bar{M}_{8}m_{8}|\frac{3}{2}M_{8}\rangle \end{aligned}$$
(11)

is equivalent to the operator  $-2\mathbf{S}_{A}$ .  $\mathbf{S}_{B}/4S^{2}$  acting on the components of  ${}^{4}T_{1} \times {}^{4}T_{1}$ , and the exchange matrix is therefore given by (12), where the operators are defined by (13) and the subscript A, B indicates, as before, the product of the corresponding terms for the A and B ions.

$$\langle {}^{4}I_{1}M_{L}'M_{B}'|_{A,B}\mathcal{H}_{SS}|{}^{4}I_{1}M_{L}M_{S}\rangle_{A,B} = \\ \langle M_{S}'|_{A,B} - 2\mathbf{S}_{A} \cdot \mathbf{S}_{B}/4S^{2}|M_{S}\rangle_{A,B} = \\ \times \sum_{\boldsymbol{\mu}_{A}} \sum_{\boldsymbol{\mu}_{B}} \{\langle T_{1}M_{L}'|a_{2}\boldsymbol{\mu}\rangle_{A,B}\langle e^{2}A_{2}a_{2}|_{A,B}J^{(4)}|e^{2}A_{2}a_{2}\rangle_{A,B} + \\ + \sum_{\boldsymbol{\mu}_{B}'} \langle T_{1}M_{L}'|a_{2}\boldsymbol{\mu}\rangle_{A}\langle T_{1}M_{L}'|a_{2}\boldsymbol{\mu}'\rangle_{B} \\ \times \langle e^{2}A_{2}a_{2}|_{A}\langle t_{2}\boldsymbol{\mu}'|_{B}J^{(2)}|e^{2}A_{2}a_{2}\rangle_{A}|t_{2}\boldsymbol{\mu}\rangle_{B} \\ + (a \text{ similar term with } A \text{ and } B \text{ interchanged}) \\ + \\ \sum_{\boldsymbol{\mu}_{A}'} \sum_{\boldsymbol{\mu}_{B}'} \langle T_{1}M'|a_{2}\boldsymbol{\mu}'\rangle_{A,B}\langle t_{2}\boldsymbol{\mu}'|_{A,B}J^{(1)}|t_{2}\boldsymbol{\mu}\rangle_{A,B} \} \\ \times \langle a_{2}\boldsymbol{\mu}|T_{1}M\rangle_{A,B} \qquad (1$$

$$J^{(1)} = J(3,3)$$

$$J^{(2)} = \sum_{i=1}^{2} J(i,3)$$

$$J^{(4)} = \sum_{i=1}^{2} \sum_{j=1}^{2} J(i,j)$$
(13)

If the matrix of  $J^{(1)}$  is diagonal in  $\mu_A$  and  $\mu_B$ , (12) becomes diagonal in  $M_{LA}$  and  $M_{LB}$  and  $\mathscr{H}_{88}$  takes the simple form (5), where  $\mathscr{J}(M_{LA}, M_{LB})$  is defined by (14) and its non-zero components are given by (15) [equivalent to (6)].

$$\begin{aligned} \mathscr{J}(M_{\mathrm{LA}}, M_{\mathrm{LB}}) &= \frac{1}{9} \{ \langle e^2 A_2 a_2 |_{\mathrm{A},\mathrm{B}} J^{(4)} | e^2 A_2 a_2 \rangle_{\mathrm{A},\mathrm{B}} \\ &+ \sum_{\mu_{\mathrm{A}}} \langle e^2 A_2 a_2 |_{\mathrm{A}} \langle t_2 \mu |_{\mathrm{B}} J^{(2)} | e^2 A_2 a_2 \rangle_{\mathrm{A}} | t_2 \mu \rangle_{\mathrm{B}} | \langle T_1 M_{\mathrm{L}} | a_2 \mu \rangle |^2_{\mathrm{B}} \\ &+ (\mathrm{a \ similar \ term \ with \ A \ and \ B \ interchanged}) \\ &+ \sum_{\mu_{\mathrm{A}}} \sum_{\mu_{\mathrm{B}}} \langle t_2 \mu |_{\mathrm{A},\mathrm{B}} J^{(1)} | t_2 \mu \rangle_{\mathrm{A},\mathrm{B}} | \langle T_1 M_{\mathrm{L}} | a_2 \mu \rangle |^2_{\mathrm{A},\mathrm{B}} ) \tag{14}$$

$$\begin{split} \mathscr{J}(\pm 1, \pm 1) &= \mathscr{J}(\pm 1, \mp 1) = [J(\theta\theta) + 2J(\theta\varepsilon) + \\ J(\varepsilon\varepsilon) + 2J(\theta\xi) + 2J(\varepsilon\xi) + \frac{1}{2}J(\xi\xi) \\ &+ \frac{1}{2}J(\xi\eta)]/9 \\ \mathscr{J}(\pm 1, 0) &= \mathscr{J}(0, \pm 1) = [J(\theta\theta) + 2J(\theta\varepsilon) + J(\varepsilon\varepsilon) + \\ J(\theta\zeta) + J(\varepsilon\zeta) + J(\theta\xi) + J(\varepsilon\xi) + \\ J(\xi\zeta)/9 \\ \mathscr{J}(0, 0) &= [J(\theta\theta) + 2J(\theta\varepsilon) + J(\varepsilon\varepsilon) + 2J(\theta\zeta) + \\ &+ \\ 2J(\varepsilon\zeta) + J(\zeta\zeta)]/9 \end{split}$$
(15)

## APPENDIX III

Susceptibility of a Pair of Ions in  $T_1$  or  $T_2$  Orbital States.— The magnetic behaviour of a pair of  ${}^4T_{1g}$  ions perturbed by the Hamiltonian (2) with the exchange operator defined by (5) was calculated by means of two computer programs. Program TTMAG1 prepares a magnetic tape containing the necessary vector-coupling coefficients and the matrices of  $\mathscr{H}_{LL}$ ,  $\mathscr{H}_{LS}$ , and  $\mathscr{H}_{SS}$  with parameters omitted. Program TTMAG2 is supplied with this tape and one or more sets of the parameters  $\lambda$ , *D*, etc., and calculates the susceptibilitytemperature curves. These programs can also deal with  ${}^{2}T_{2}$  and  ${}^{3}T_{1}$  ions. They are written in Fortran and require (for  ${}^{4}T_{1}$ ) core storage for at least 40 000 real numbers, or 16 000 if disc backing store is used. A program is also available for the case of two  ${}^{2}T_{2}$ ,  ${}^{3}T_{1}$ ,  ${}^{4}T_{1}$ , or  ${}^{5}T_{2}$  ions in effectively cubic environments with isotropic exchange.

Testing of the Computer Programs.—The correctness of the programs was checked as follows (assuming isotropic exchange).

(i) With  $D = \mathscr{J} = 0$  and  $S = \frac{1}{2}$ , 1,  $\frac{3}{2}$ , and 2, the programs gave results identical with those calculated for isolated  ${}^{2}T_{2}$ ,  ${}^{3}T_{1}$ ,  ${}^{4}T_{1}$ , and  ${}^{5}T_{2}$  ions respectively.<sup>10</sup> (ii) With D = k = 0 and  $S = \frac{1}{2}$ , 1,  $\frac{3}{2}$ , and 2, they gave results identical with those calculated for a pair of exchange-coupled spinonly ions with  $g = 2.2^{6}$  (iii) With  $\mathscr{J} = 0$  and  $S = \frac{1}{2}$  they gave results that agreed with those calculated for isolated

<sup>2</sup> $T_2$  ions with an axial distortion.<sup>26</sup> (iv) With  $\mathscr{J} \neq 0$ , as  $D \longrightarrow \pm \infty$  the results approached those calculated for a pair of exchange-coupled spin-only ions. (v) With D = 0 and  $S = \frac{1}{2}$ , the calculated curves of  $\mu_{\text{eff}}$  against  $\mathbf{k}T/\lambda$  for various  $\mathscr{J}/\lambda$  agreed with those of Kahn.<sup>9a</sup>

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