$$r_0 = 1 + 1.0000\,2223x + .5000\,0271x^2 + .1664\,8913x^3 + .04164497x^4 + .00868659x^5 + .0014\,3229x^6;$$

$$|p(x) - r_0| = |q_0| \cdot |T_7(x)| \leqq 3.61 \times 10^{-6} \quad (-1 \leqq x \leqq 1).$$

Therefore $|f(x) - r_0| \leqq 3.91 \times 10^{-6}$ $(-1 \leqq x \leqq 1)$. Dividing $T_7(x)$ by r_0,

$$q_1 = -270{,}998.81 + 44{,}683.688x.$$

$$r_1 = 270{,}998.81 + 226{,}314.15x + 90{,}815{,}458x^2 + 22{,}832.391x^3 + 3{,}846.3890x^4 + 381.2048x^5.$$

$a_0 = 1$; $b_0 = -q_0$

$a_1 = -q_1$; $b_1 = 1 + q_1 q_0$

Therefore

$$p(x) = \frac{r_1}{a_1} - \frac{b_1}{a_1} T_7 = -\frac{r_1}{q_1} + \frac{1 + q_1 q_0}{q_1} T_7(x).$$

The second term on the right is

$$\frac{.1394\,0940 + .0368\,86598x + .0056\,281054x^2 + .0019\,269840x^3}{-270{,}998.81 + 44{,}683.688x} T_7(x)$$

whose absolute value is bounded by 8.121×10^{-7} for $-1 \leqq x \leqq 1$. Thus e^x may be approximated on this interval by

$$-\frac{r_1}{q_1} = \frac{1 + .8351\,1123x + .3351\,1386x^2 + .0842\,5274x^3 + .0141\,9338x^4 + .0014\,0667x^5}{1 - .1648\,8518x},$$

where the error is bounded by $\pm(3 \times 10^{-7} + 8.1 \times 10^{-7}) = \pm 1.1 \times 10^{-6}$.

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The Complete Factorization of $2^{132} + 1$

By K. R. Isemanger

The integer $2^{132} + 1$ is divisible by $2^{44} + 1 = 17 \cdot 353 \cdot 2931542417$ and the quotient, $2^{88} - 2^{44} + 1$, is divisible by $241 \cdot 7393$. There remains the formidable problem of factoring the resultant quotient N, where N is the integer

1 73700 82040 22350 83057.

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By a method of exclusion, using small prime moduli, the following binary representations of N were obtained:

$$N = 88073\ 81316^2 + 98046\ 34351^2$$

$$N = 82086\ 25547^2 + 2^3 \cdot 3^3 \cdot 11^2 \cdot 37^2 \cdot 4243^2 \cdot 7 \cdot 17 \cdot 19 \cdot 73$$

$$N = 1\ 28399\ 72408^2 + 7^2 \cdot 11^2 \cdot 13^2 \cdot 3137^2 \cdot 3 \cdot 19 \cdot 79 \cdot 199$$

$$N = 23334\ 17999^2 + 2^4 \cdot 13^2 \cdot 17^2 \cdot 2707^2 \cdot 3 \cdot 7 \cdot 79 \cdot 89 \cdot 199$$

$$N = 1\ 06383\ 10009^2 + 2^{10} \cdot 17^2 \cdot 3853^2 \cdot 3 \cdot 107 \cdot 167 \cdot 257$$

$$N = 1\ 05791\ 40907^2 + 2^3 \cdot 3^3 \cdot 13^2 \cdot 3169^2 \cdot 17 \cdot 23 \cdot 29 \cdot 89 \cdot 167$$

$$N = 1\ 58709\ 07595^2 - 2^3 \cdot 3^3 \cdot 17^2 \cdot 37^3 \cdot 3061^2 \cdot 7 \cdot 13 \cdot 29$$

$$N = 1\ 47403\ 24637^2 - 2^3 \cdot 7^2 \cdot 23^2 \cdot 73^2 \cdot 2803^2 \cdot 3 \cdot 7 \cdot 239$$

$$N = 1\ 51299\ 86183^2 - 2^9 \cdot 7^2 \cdot 3259^2 \cdot 3 \cdot 37 \cdot 73 \cdot 107 \cdot 239$$

$$N = 1\ 49963\ 46199^2 - 2^5 \cdot 3^4 \cdot 11^2 \cdot 17^2 \cdot 2711^2 \cdot 13 \cdot 23 \cdot 257$$

If these ten relations are written as congruences in the form

$$x^2 \equiv Dy^2 (\text{mod } N)$$

and then are multiplied together, there results the congruence

$$A^2 \equiv B^2 (\text{mod } N)$$

where

$$A = 3030\ 76720\ 24193\ 56872$$

$$B = 85638\ 82032\ 43137\ 98848.$$

The greatest common divisor of $A + B$ and N was found to be 9 86182 73953, which yields the factorization

$$N = 17613\ 45169 \cdot 9\ 86182\ 73953.$$

The primality of both factors was verified on the SILLIAC computer at the University of Sydney.

This result supplements information previously published by R. M. Robinson [1].

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1. Raphael M. Robinson, "Some factorizations of numbers of the form $2^n \pm 1$," *MTAC*, v. 11, 1957, p. 265–268.