5[I].—A. N. LOWAN, *The Operator Approach to Problems of Stability and Convergence of Solutions of Difference Equations and the Convergence of Various Iteration Procedures*, Scripta Mathematica, New York, 1957, x + 104 p., Office of Technical Services, Department of Commerce, Washington 25, D. C. 26 cm. Price $3.00.

In the numerical solution of linear second-order differential equations by difference methods, one has to solve

$$A_k u_{k+1} + B_k u_k + C_k u_{k-1} = D_k \tag{1}$$

where A_k, B_k, C_k, D_k are sparse matrices with regular structure, the u's are vectors, and the integer subscripts refer to time-steps, iteration cycles, etc. In many important cases A_k, B_k, C_k are independent of k and closely related. Stability and mesh-convergence of "stepping-ahead" solutions (parabolic, hyperbolic equations) and convergence of iterative solutions (elliptic equations) can be discussed in terms of the operators $A_k^{-1}B_k$ and $A_k^{-1}C_k$. In the von Neumann technique, as formalized and extended by the reviewer, one essentially takes the Fourier transform (in the extended sense) of (1), thus introducing immediately the eigenvectors and eigenvalues of these operators. This is equivalent to a change of coordinates in vector space under which A_k, B_k, C_k take very simple forms. Various authors, however, have retained the original coordinates and worked directly with (1); prominent among these are S. Frankel, D. Rutherford, A. Mitchell, P. Lax, R. Richtmyer, J. Douglas, J. Todd, and (unpublished) C. Leith. Professor Lowan here gives a detailed, connected account of this second method which he calls the "operator approach". He covers the usual parabolic, elliptic, and hyperbolic partial differential equations, homogeneous and non-homogeneous, with some attention to equations with variable coefficients. He discusses stability and mesh-convergence for parabolic and hyperbolic equations, and convergence of the various iterative schemes for elliptic equations. Original contributions, besides the organization of the material, include a discussion of iteration schemes for solving "implicit" difference approximants to parabolic and hyperbolic equations and a novel "second-order" Richardson scheme for elliptic difference equations. In addition, Professor Lowan has written down a number of "folk theorems", rather widely known but unpublished. There are eight "Sections" and six "Appendices". Most of the typographical errors have been detected by the author and listed on the "Errata" sheet. However, the figures on page 5 and at the top of page 33 should be corrected; in the second line from the bottom of page 44, read $(-1)^{h+1}\sqrt{2}\sin rh\pi/M$; page 26, line 5 and page 47, line 18 should show the respective qualifications "$r > 1$" and "$r \leqq \frac{1}{2}$". Moreover, the reader should be aware that the estimates of truncation error in difference solutions given during the discussion of mesh-convergence require that various functions be "sufficiently continuous." Those of us interested in the numerical solution of partial differential equations are indebted to Professor Lowan for a very worthwhile addition to the literature of this field.

MORTON A. HYMAN

IBM Research Laboratory
Yorktown Heights, New York